



PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Approved by **AICTE** and Affiliated to **Anna University**)

sky is the limit

Department of Mechanical Engineering

Regulation 2021

III Year – VI Semester

ME3691 HEAT AND MASS TRANSFER

UNIT - I

Heat Transfer

It's can be defined as the transmission of energy from one region to another due to temp difference.

Modes of heat transfer

- * Conduction
- * Convection
- * Radiation

Conduction:

Heat conduction is a mechanism of heat transfer from a region of high temp to region of low temp. within medium (solid, liquid, gas)

OR

Different medium in direct ^{physical} contact

Convection:

It's a process of heat transfer that will occur between a solid surface & a fluid medium, when they are diff temp.

Convection is possible only in the presence of fluid medium.

Radiation:-

The heat transfer from 1 body to another body without any transmitting medium.

It's a electro-magnetic wave phenomenon.

Fourier's law of conduction.

$$Q = -KA \frac{dT}{dx}$$

A = Area in m^2

$\frac{dT}{dx}$ - Temp gradient $K, K/m$

k - Thermal conductivity W/mK

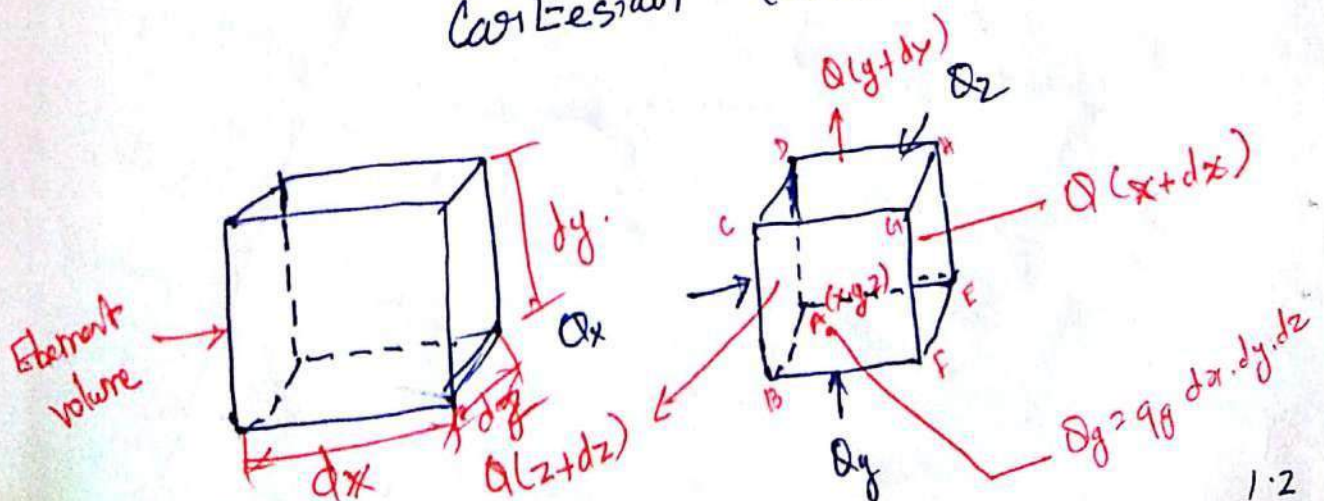
Q - Rate of heat conduction

The rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temp gradient in that direction.

Thermal conductivity:-

'Ability of a substance to conduct heat.'

General heat conduction equation in Cartesian coordinates:-



* Consider small rectangular element of sides dx, dy, dz

* The energy balance of the rectangular element is obtained from 1st law of Thermodynamics.

$$\Delta u = Q - W$$

$$\Delta u + W = Q \quad \Delta u + W = Q$$

$$\left[\begin{array}{l} \text{Net heat} \\ \text{conducted in to} \\ \text{element from} \\ \text{all coordinate} \\ \text{directions} \end{array} \right] + \left[\begin{array}{l} \text{Heat} \\ \text{generated} \\ \text{within the} \\ \text{element} \end{array} \right] = \left[\begin{array}{l} \text{Heat} \\ \text{stored} \\ \text{in the element} \end{array} \right] \quad \text{--- -- -- -- --> (1.1)$$

Net heat conducted into element from all the coordinate directions:

Let Q_x be the heat flux in a direction of face ABCD and

$Q_x + dQ_x$ be the heat flux in " " " EFGH

the rate of heat flow into the element in x direction through the

face ABCD is

$$Q_x = q_x dy dz$$

$$\boxed{q_x = -k_x \frac{\partial T}{\partial x} dy dz} \quad \text{--- -- -- -- --> (1.2)}$$

where $k = T/K$

$$\frac{\partial T}{\partial x} = T/m$$

The rate of heat flow out of the element in x direction through the

face EFGH is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x} (Q_x) dx$$

$$-k_x \frac{\partial T}{\partial x} dy dz + \frac{\partial}{\partial x} \left[-k_x \frac{\partial T}{\partial x} dy dz \right] dx$$

$$\boxed{Q_{x+dx} = -k_x \frac{\partial T}{\partial x} dy dz - \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx dy dz} \quad (1.3)$$

Subtracting (1.2) - (1.3)

$$\cancel{Q_x} - \cancel{Q_{x+dx}}$$

$$Q_x - Q_{x+dx} = -k_x \frac{\partial T}{\partial x} dy dz - \left[-k_x \frac{\partial T}{\partial x} dy dz - \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx dy dz \right]$$

$$= -k_x \frac{\partial T}{\partial x} dy dz + k_x \frac{\partial T}{\partial x} dy dz + \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx dy dz$$

$$\boxed{Q_x - Q_{x+dx} = \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx dy dz} \rightarrow (1.4)$$

similarly:

$$Q_y - Q_{y+dy} = \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] dx dy dz$$

$$Q_z - Q_{z+dz} = \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] dx dy dz$$

Adding (1.4) + (1.5) + (1.6)

$$\boxed{= \left[\frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] \right] dx dy dz}$$

Heat stored in the element

WKT,

Heat stored in the element = Mass of the element \times Sp. heat of the element \times Rise in temp of element

$$= m \times C_p \times \frac{\partial T}{\partial t}$$

$$= \rho \times dx dy dz \times C_p \times \frac{\partial T}{\partial t}$$

|| Density, $\frac{\text{mass}}{\text{volume}}$
mass = $\rho \times \text{Vol}$

$$\boxed{\text{Heat stored in element} = \rho C_p \frac{\partial T}{\partial t} dx dy dz} \rightarrow (1.8)$$

Heat generated within the element:

" " is given by

$$\boxed{Q = \dot{q} dx dy dz} \rightarrow (1.9)$$

Sub equation 1.7 & 1.8 & 1.9 in eqn (1.1)

$$1.1 = \left[\frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] \right] dx dy dz + \dot{q} dx dy dz = \rho C_p \frac{\partial T}{\partial t} dx dy dz$$

net heat conducted into elem from all the coordinate direction

Heat generated in elem = Heat stored in elem

Considering the material is isotropic & isotropic

$$\boxed{k_x = k_y = k_z = k = \text{constant}}$$

$$\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] k + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

$$\div k$$

$$\therefore \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\alpha = \text{thermal diffusivity} = \frac{k}{\rho c_p} \text{ - m}^2/\text{s}.$$

Case (i) No heat source:

equ 1.10 $\cdot \quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \dots \dots \dots 1.11$

diffusion eqn or Fourier's eqn

case (ii) Steady state conditions:-

" the temp does not change with time.

So, $\frac{\partial T}{\partial t} = 0$ So, (1.10) reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = 0$$

(or)

$$\boxed{\nabla^2 T + \frac{q}{k} = 0} \quad \text{Poisson's eqn.}$$

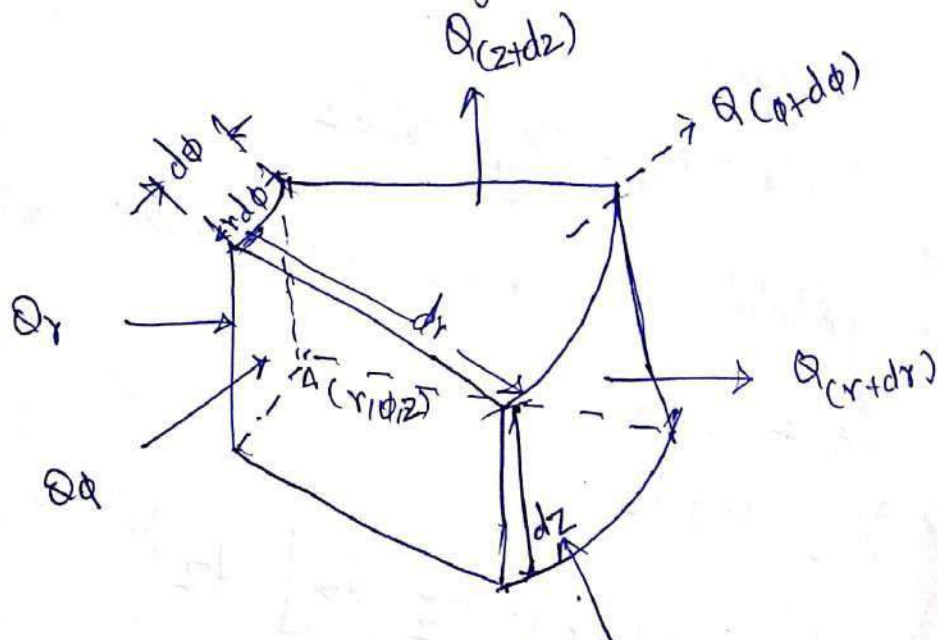
absence of internal heat generation

$$\nabla^2 T = 0 \quad \text{Laplace equation.}$$

General heat conduction Equ in cylindrical co-ordinates.

The Cartesian coordinate system is not applicable for the solids like cylinder, cones, sphere etc.

Consider a small cylinder element of sides dr , $d\phi$ & dz



The volume of the element ~~is~~ dV

$$dV = r dr d\phi dz$$

Let us assume that thermal conductivity k , specific heat c_p & ρ density are constant

The energy balance to 1st law of thermodynamics

Net heat
conduct into
element from
all 4 co-ord
directions

+

Heat
generated
within the
element

=

Heat
stored
in the
element

→ ①

Net heat conducted into element from all the co-ordinate directions.

I Heat entering the element through (r, ϕ) plane in time dt

$$Q_z = -k(r \cdot d\phi \cdot dr) \frac{\partial T}{\partial z} dt$$

" " " " " "

" leaving " " " " "

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z} (Q_z) dz$$

Net heat conducted into the element through (r, ϕ) plane in time

$$= Q_z - Q_{z+dz}$$

$$= - \frac{\partial}{\partial z} (Q_z) dz$$

$$= - \frac{\partial}{\partial z} [k(r \cdot d\phi \cdot dr) \cdot \left(\frac{\partial T}{\partial z}\right) dt] dz$$

$$= k \left[\frac{\partial^2 T}{\partial z^2} \right] [dr \cdot r \cdot d\phi \cdot dz] dt$$

$$\text{Net heat conducted through } (r, \phi) \text{ plane} = k \left[\frac{\partial^2 T}{\partial z^2} \right] [dr \cdot r \cdot d\phi \cdot dz] dt$$

II) Heat " " " " (ϕ, r) plane in time dt .

$$Q_r = -k(r \cdot d\phi \cdot dz) \frac{\partial T}{\partial r} dt$$

" " leaving " " " " " "

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r} (Q_r) dr$$

Net heat conducted into the element through (z, r) plane is Q_{in} due

$$Q_{in} - Q_{out} + dQ = k \left[\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] (dr \cdot r d\phi \cdot dz) d\theta \quad \text{--- (4)}$$

Add 2, 3 + 4

$$k (dr \cdot r d\phi \cdot dz) d\theta \left[\frac{1}{r^2} \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right]$$

All coordinates
direction

$$= k \cdot (dr \cdot r d\phi \cdot dz) d\theta \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

Heat generated within the element:

Total ... is given by

$$Q = \dot{q} (dr \cdot r d\phi \cdot dz) d\theta \quad \text{--- (5)}$$

Heat stored in the element

The increase in internal energy of the element is equal to the net heat stored in the element.

Increase in internal energy = Net heat stored in the element

1.12 (14)

Net heat conducted into the element through (ϕ, z) Plane is

$$= Q_r - Q_{r+dr}$$

$$= - \frac{\partial}{\partial r} (Q_r) dr$$

$$= - \frac{\partial}{\partial r} \left[-k (r d\phi dz) \cdot \left(\frac{\partial T}{\partial r} \right) d\theta \right] dr$$

$$= k (dr d\phi dz) \frac{\partial}{\partial r} \left[r \cdot \frac{\partial T}{\partial r} \right] d\theta$$

$$= k (dr \cdot r d\phi \cdot dz) \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] d\theta$$

Net heat
conducted through
 (ϕ, z) Plane

$$= k (dr \cdot r d\phi \cdot dz) \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] d\theta$$

III Heat " entering " (z, r) " in time dt

$$Q_\theta = -k (dr \cdot dz) \cdot \frac{\partial T}{r \partial \theta} d\theta$$

" leaving " (z, r) " in time dt

$$Q_{\theta+d\theta} = Q_\theta + \frac{\partial}{\partial \theta} (Q_\theta) r d\theta$$

$$= \rho (dr r d\phi \cdot dz) c_p \frac{\partial T}{\partial \theta} \times d\theta \rightarrow (5)$$

Substn eqn 5, 4, 3 in eqn (1).

\therefore by $(dr r d\phi \cdot dz) d\theta$

$$k \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] + \dot{q} = \rho \cdot c_p \frac{\partial T}{\partial \theta}$$

$$+ \frac{\dot{q}}{k} = \frac{\rho c_p}{k} \cdot \frac{\partial T}{\partial \theta}$$

"

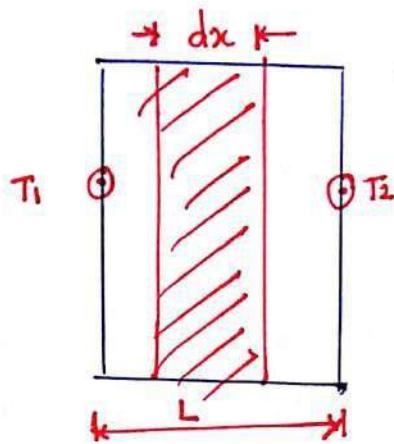
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$$\therefore \alpha = k / \rho c_p$$

$$= \frac{1}{\alpha} \frac{\partial T}{\partial \theta}$$

$$\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = 0$$

Conduction of heat through a slab or plane wall



Consider a slab of uniform \$k\$, \$L\$ thickness, with inner temp \$T_1\$ & outer temp \$T_2\$

Let us consider a small element of thickness \$dx\$

From Fourier law of conduction, wkt

$$Q = -kA \frac{dT}{dx}$$

$$Q \cdot dx = -kA dT$$

Integrate the above eqn betw the limits of 0 to \$L\$ & \$T_1\$ to \$T_2\$

$$\Rightarrow \int_0^L Q dx = - \int_{T_1}^{T_2} kA dT$$

$$\Rightarrow Q \int_0^L dx = -kA \int_{T_1}^{T_2} dT$$

$$\Rightarrow Q [x]_0^L = -kA [T]_{T_1}^{T_2}$$

$$\Rightarrow Q (L-0) = -kA (T_2 - T_1)$$

$$\Rightarrow Q \times L = kA (T_1 - T_2)$$

$$\Rightarrow Q = \frac{kA}{L} [T_1 - T_2]$$

$$Q \Rightarrow \frac{T_1 - T_2}{L/kA} = Q = \frac{\Delta T_{\text{Overall}}}{R} \rightarrow \text{①}$$

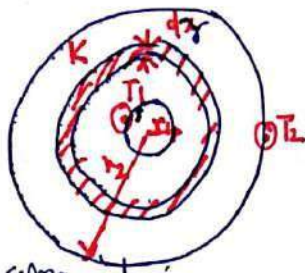
Where

$$\Delta T = T_1 - T_2$$

\$R = L/kA\$ - Thermal resistance of slab.

Conduction of Heat through Hollow cylinder

Consider a hollow cylinder of inner radius r_1 & outer radius r_2 , inner temp & outer temp T_1 & T_2



From Fourier law of conduction

$$Q = -k A \frac{dT}{dr}$$

let us consider small element area dr

Area of cylinder is $2\pi r L$

So,

$$Q = -k 2\pi r L \frac{dT}{dr}$$

$$Q \times \frac{dr}{r} = -k 2\pi L dT$$

Integration the above eqn from r_1 to r_2 & T_1 to T_2

$$\Rightarrow Q \int_{r_1}^{r_2} \frac{dr}{r} = -k 2\pi L \int_{T_1}^{T_2} dT$$

$$\Rightarrow Q [\ln r]_{r_1}^{r_2} = -k 2\pi L [T]_{T_1}^{T_2}$$

$$\Rightarrow Q [\ln r_2 - \ln r_1] = -k 2\pi L [T_2 - T_1]$$

$$\Rightarrow Q \ln \left[\frac{r_2}{r_1} \right] = 2\pi L k [T_1 - T_2]$$

$$\Rightarrow Q = \frac{2\pi L k [T_1 - T_2]}{\ln \left(\frac{r_2}{r_1} \right)}$$

$$\Rightarrow Q = \frac{T_1 - T_2}{\frac{1}{2\pi L k} \ln \left(\frac{r_2}{r_1} \right)}$$

$$\Rightarrow Q = \frac{\Delta T_{\text{overall}}}{R}$$

Where,

$$T_1 - T_2 = \Delta T$$

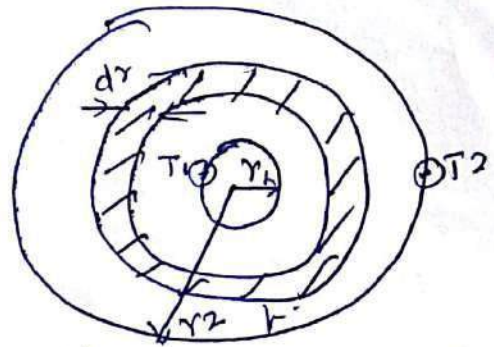
$$R = \frac{1}{2\pi L k} \ln \left(\frac{r_2}{r_1} \right)$$

Thermal Resistance of the hollow cylinder.

Conduction of heat through hollow sphere.

Consider a hollow sphere of inner radius r_1 , outer radius r_2 ,

Inner & Outer temp : T_1 & T_2



Let us consider a small element area of thickness 'dr' from Fourier law of heat conduction

$$Q = -kA \frac{dT}{dr}$$

Area of sphere is $4\pi r^2 = A$

$$Q = -k \cdot 4\pi r^2 \frac{dT}{dr}$$

$$Q \times \frac{dr}{r^2} = -k \times 4\pi dr$$

Integrate on both sides

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} dT$$

$$Q \left[\frac{-1}{r} \right]_{r_1}^{r_2} = -4\pi k [T]_{T_1}^{T_2}$$

$$Q \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = -4\pi k [T_2 - T_1]$$

$$Q \left[\frac{r_2 - r_1}{r_1 r_2} \right] = 4\pi k [T_1 - T_2]$$

$$Q = \frac{4\pi k [T_1 - T_2]}{\frac{r_2 - r_1}{r_1 r_2}}$$

$$Q = \frac{T_1 - T_2}{\frac{r_2 - r_1}{4\pi k [r_1 r_2]}}$$

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$

$$\int \frac{dr}{r^2} = -\frac{1}{r}$$

$$\int \frac{dT}{T} = \ln T$$

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$

Wk. $\Delta T = T_1 - T_2$

$$R = \frac{r_2 - r_1}{4\pi k (r_1 r_2)} = \text{Thermal Resistance of hollow sphere.}$$

Newton's Law of cooling

Heat transfer by convection is given by Newton's law of cooling

$$Q = h A (T_s - T_\infty)$$

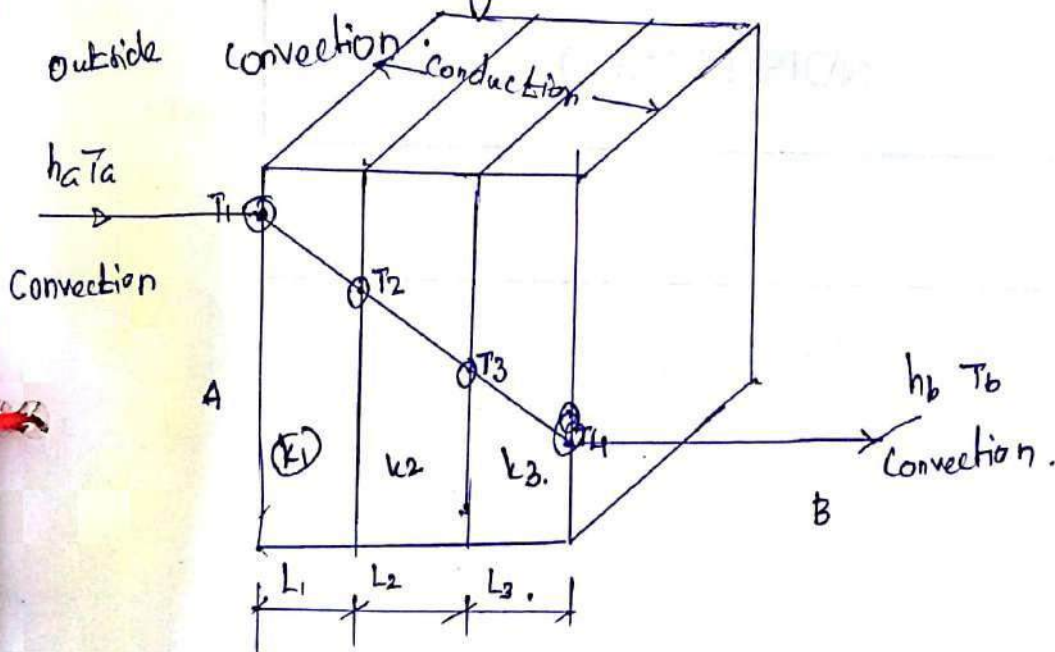
A - Area exposed to heat transfer in m^2

h - heat transfer coefficient in W/m^2K

T_s - Temp of the surface in K

T_∞ = Temp of the fluid in K .

Heat transfer Through a composite plane wall with Inside & outside convection



Composite wall thickness L_1, L_2, L_3

Thermal conductivity k_1, k_2, k_3

Internal & External ~~low~~ surface of the system are subjected to convection at mean temp T_a & T_b .

heat transfer coefficient h_a & h_b

From Newton law of cooling we get

Heat transfer by convection at side A is

$$Q = h_a A [T_a - T_1] \rightarrow (1)$$

Heat " " " at slab 1 is

$$Q = \frac{k_1 A (T_1 - T_2)}{L_1} \rightarrow (2)$$

Heat " " " at slab (2) is

$$Q = \frac{k_2 A (T_2 - T_3)}{L_2} \rightarrow (3)$$

Heat " " " at slab (3) is

$$Q = \frac{k_3 A (T_3 - T_4)}{L_3} \rightarrow (4)$$

Heat transfer by convection at side B is

$$Q = h_b A (T_4 - T_b) \rightarrow (5)$$

NKT:-

$$T_a - T_1 = Q \times \frac{1}{h_a A}$$

$$T_1 - T_2 = Q \times \frac{L_1}{k_1 A}$$

$$T_2 - T_3 = Q \times \frac{L_2}{k_2 A}$$

$$T_3 - T_4 = Q \times \frac{L_3}{k_3 A}$$

$$T_4 - T_b = Q \times \frac{1}{h_b A}$$

Adding both sides of the above equ

$$T_a - T_b = Q \left[\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A} \right]$$

$$Q = \frac{T_a - T_b}{\left[\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A} \right]}$$

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$

Where

$$\Delta T = T_a - T_b$$

$$\text{Thermal Resistance } R = R_a + R_1 + R_2 + R_3 + R_b$$

T_a & T_b = heat transfer co-efficients

$$\therefore R_a = \frac{1}{h_a A}$$

$$R_1 = \frac{L_1}{k_1 A}$$

WKT

$$R = \frac{1}{UA}$$

$$Q = \frac{T_a - T_b}{\frac{1}{UA}}$$

$$\dot{Q} = UA [T_a - T_b]$$

U = overall heat transfer co-efficient W/m²k.

Heat transfer through composite pipe or cylinder with
Inside & outside convection.

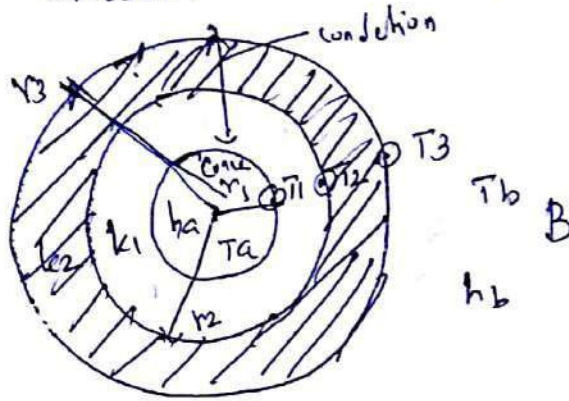
T_a - hot fluid temp

h_a - heat transfer coefficient A

k_1, k_2 - Thermal conductivity.

T_b - cold fluid temp

h_b - heat transfer coefficient B



Heat transfer by convection at side A is

$$Q = h_a A (T_a - T_1)$$

$$\text{Area} = 2\pi r_1 L$$

So, $Q = 2\pi r_1 L h_a (T_a - T_1)$

Heat transfer by conduction at section 1 is

$$Q = \frac{2\pi L k_1 (T_1 - T_2)}{\ln(r_2/r_1)}$$

Similar At section 2

$$Q = \frac{2\pi L k_2 (T_2 - T_3)}{\ln(r_3/r_2)}$$

Heat transfer by convection at side B is

At section B

$$Q = h_b A (T_3 - T_b)$$

$$Q = 2\pi r_3 L h_b (T_3 - T_b)$$

Wkt.

$$T_a - T_1 = \frac{Q}{2\pi L r_1 h_a}$$

$$T_1 - T_2 = \frac{Q}{2\pi L k_1} \ln\left(\frac{r_2}{r_1}\right)$$

$$T_2 - T_3 = \frac{Q}{2\pi L k_2} \ln\left(\frac{r_3}{r_2}\right)$$

$$T_3 - T_b = \frac{Q}{2\pi r_3 L h_b}$$

Add both side above eqn

$$T_a - T_b = \frac{Q}{2\pi L} \left[\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_b r_3} \right]$$

$$Q = \frac{2\pi L (T_a - T_b)}{\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_b r_3}}$$

$$T_a - T_b$$

$$Q =$$

$$\frac{1}{2\pi L} \left[\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_b r_3} \right]$$

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$

$$R = \frac{1}{2\pi L} \left[\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_b r_3} \right]$$

$$\Delta T_{\text{overall}} = T_a - T_b$$

WKT

$$R = \frac{1}{UA}$$

$$Q = \frac{T_a - T_b}{1/UA}$$

$$Q = UA [T_a - T_b]$$

where

U - Overall heat transfer coefficient $\text{W/m}^2\text{K}$

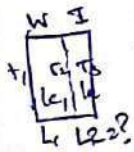
$$A = \text{Area} = 2\pi r_3 L$$

Problems for Slabs

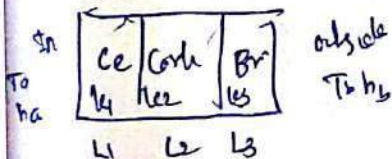
- ① Determine the heat transfer through the plane of length 6m, height 4m & thickness 0.30m. The temp of inner & outer surface are 100°C & 40°C . Thermal conductivity of wall is 0.55W/mK Ans: Watts.
(HMT Pg no: 44 Sec 4 edition)



- ② A wall of 0.5m thickness having (k) of 1.2W/mK . The wall is to be insulated with a material having an average ~~heat~~ k of 0.3W/mK . $\Delta T = T_a - T_b$ (or $T_1 - T_3$). Inner & outer temp are 100°C & 10°C respectively. If heat transfer rate is 1400W/m^2 calculate the thickness of insulation. HMT Pg no: 44/45
Ans: $L_2 = ?$ m



- ③ The wall of a cold room is composed of three layers. The outer layer is brick 30cm thick. The middle layer is cork 20cm thick. The inner layer is cement 15cm thick. The temp of outside air is 25°C & on the inner air is -20°C . The film co-efficient for outside air & brick is $55.4\text{W/m}^2\text{K}$. Film co-efficient for inner air & cement is $17\text{W/m}^2\text{K}$. Take k for brick = 2.5W/mK , cork = 0.05W/mK , cement = 0.28W/mK . Find heat flow rate Q/A . HMT Pg: 44/45



4) A wall of cold room is composed of three layers. The outer layer is brick Firewall 20cm thick, the middle layer is asbestos 10cm thick, the inside layer is cement 5cm thick. The temp of outside layer air is 25°C and that on the inside air is -20°C .

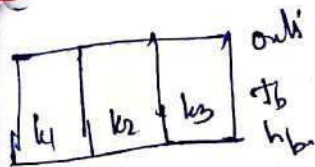
The film co-efficient h_1 For outside air is $45.4 \text{ W/m}^2\text{K}$

The film co-efficient h_2 Inside air Cement is $17 \text{ W/m}^2\text{K}$.

(i) Thermal resistance $= R = ?$

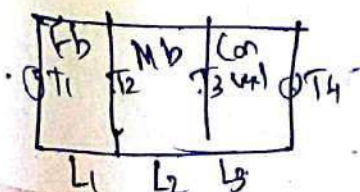
(ii) The heat flow rate $Q/A = ?$

k for Firewall $= 3.45 \text{ W/mK}$
 k for asbestos $= 0.043 \text{ W/mK}$
 k for cement $= 0.294 \text{ W/mK}$



For unit Area $A = 1$

5) A Furnace wall consists of 3 layer. The inner layer of 10cm thickness is made of Firebricks ($k = 1.04 \text{ W/mK}$). The intermediate layer of 25 cm thickness is made of masonry brick ($k = 0.69 \text{ W/mK}$) followed by 5cm thick concrete wall ($k = 1.37 \text{ W/mK}$). When the furnace is in continuous operation the inner surface of the furnace is at 800°C while the outer concrete surface is at 50°C . Calculate the rate of heat transfer loss per unit area wall, the temp at the interface of Firebricks & masonry brick & the temp at the interface of masonry brick & concrete.

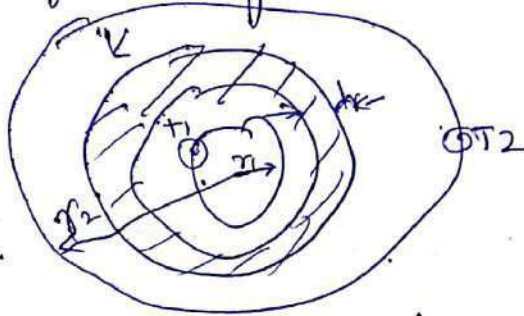


To find 1) Q/A
 2) T_2
 3) T_3

1.23

Conduction of heat through hollow cylinder

①



inner & outer radius r_1, r_2
Temp T_1, T_2
 k

Small element area 'dr'

$$Q = -kA \frac{dT}{dr}$$

Area of cylinder is $2\pi rL$

$$Q = -k 2\pi rL \frac{dT}{dr}$$

$$Q \times \frac{dr}{r} = -k 2\pi L dT$$

Integrate eq r_1 to r_2 T_1 to T_2 .

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -k 2\pi L \int_{T_1}^{T_2} dT$$

$$Q \ln \left[\frac{r_2}{r_1} \right] = -k 2\pi L [T]_{T_1}^{T_2}$$

$$Q \ln \left(\frac{r_2}{r_1} \right) = -k 2\pi L [T_2 - T_1]$$

$$Q \ln \left(\frac{r_2}{r_1} \right) = 2\pi L k [T_1 - T_2]$$

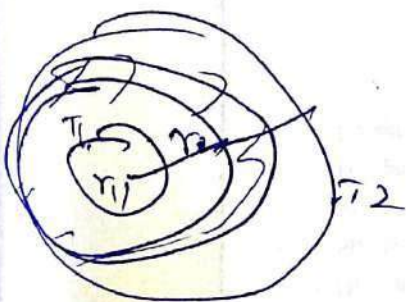
$$Q = \frac{2\pi L k (T_1 - T_2)}{\ln \frac{r_2}{r_1}}$$

$$\frac{T_1 - T_2}{\ln \frac{r_2}{r_1}} = \Delta T$$

$$\frac{1}{\ln \frac{r_2}{r_1}} = R$$

$$Q = \frac{\Delta T}{R}$$

T_b
 h_b



Area of sphere is $4\pi r^2$

$$Q = -k 4\pi r^2 \frac{dT}{dr}$$

$$Q \times \frac{dr}{r^2} = -k 4\pi dT$$

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} dT$$

$$Q \left[\frac{-1}{r} \right]_{r_1}^{r_2} = -4\pi k [T]_{T_1}^{T_2}$$

$$Q \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = 4\pi k [T_2 - T_1]$$

$$Q \frac{r_2 - r_1}{r_1 r_2} = 4\pi k [T_2 - T_1]$$

$$Q = \frac{4\pi k [T_1 - T_2]}{\left(\frac{r_2 - r_1}{r_1 r_2} \right)}$$

$$Q = \frac{4\pi k r_1 r_2 [T_1 - T_2]}{r_2 - r_1}$$

$S_{b \& b}$

Solved Problems on cylinders.

$$R = \frac{1}{2\pi L} \left[\frac{1}{h_{a1}} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_{b3}} \right] \quad \left| \begin{array}{l} \text{cylinder} \\ R = \frac{1}{2\pi L k} \ln\left(\frac{r_2}{r_1}\right) \end{array} \right.$$

$$Q = UA(T_a - T_b) \quad \left| \begin{array}{l} U = W/m^2K \\ A = 2\pi r_2 L \end{array} \right.$$

A hollow cylinder 5cm inner radius & 10cm outer radius has inner surface temp of $200^\circ C$ & outer surface temp of $100^\circ C$. If the thermal conductivity is $70 W/mK$. Find heat transfer per unit length.



$$Q/L = \frac{2\pi k (T_1 - T_2)}{\ln(r_2/r_1)}$$

Determine thermal conductivity of asbestos powder packed in b/w concentric copper pipes 85mm & 36mm diameters length. The inner pipe flowing has a heating oil to which 120W power is supplied. The average temp of inner & outer pipes are $42.4^\circ C$ & $27.9^\circ C$ respectively.

$$k = \frac{T_1 - T_2}{\frac{1}{2\pi L} \ln\left(\frac{r_2}{r_1}\right)} \quad 1.24$$



Problem for hollow sphere.

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$

$$\Delta T = T_a - T_b$$

$$R = \frac{1}{4\pi} \left[\frac{1}{h r_1^2} + \frac{1}{k_1} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] + \frac{1}{k_2} \left[\frac{1}{r_2} - \frac{1}{r_3} \right] + \frac{1}{h r_3^2} \right]$$

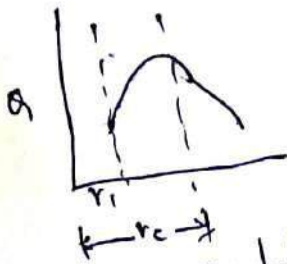
- 1) A hollow sphere ($k = 65 \text{ W/mK}$) of 120 mm inner dia & 350 mm outer dia is covered 10 mm layer of insulation ($k = 10 \text{ W/mK}$). The inside & outside temp are 500°C & 50°C respectively. Calculate the rate of heat flow through this sphere.

CRITICAL RADIUS OF INSULATION:

Addition of insulation material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is max is called CRITICAL RADIUS.

$$\text{Critical radius} = r_c = m$$

$$\text{Critical thickness} = r_c - r_i$$



Critical Radius of Insulation for a cylinder

$$Q = \frac{T_i - T_o}{\ln \left(\frac{r_o}{r_i} \right) / 2\pi k L}$$

$$r_c = k/h = r_c (m)$$

Problem for Critical thickness:

- 1) An electrical wire of 10m length & 1mm dia dissipates 200W in air at 25°C . The convection heat transfer coefficient b/w the wire surface & air is $15 \text{ W/m}^2\text{K}$. k for wire is 0.582 W/mK . Calculate the critical radius of insulation & also determine the temp of the wire if it is insulated to the critical thickness of insulation.

Additional Critical thickness $a = \frac{(T_a - T_b)}{\frac{1}{2\pi L} \left[\frac{\ln T/r_1}{k_1} + \frac{1}{h r_c} \right]}$

$$Q = \frac{2\pi T_{\text{wire}}}{\ln \left(\frac{r_o}{r_i} \right)}$$

Problem for Planewall with IHG (Internal heat generation)

① An electric current is passed through a plane wall of thickness 150mm which generates heat at the rate of $50,000 \text{ W/m}^3$. The convective heat transfer coefficient between wall & ambient air is $65 \text{ W/m}^2\text{K}$. ambient air temp is 28°C & the k of the wall material is 22 W/mK . Calculate (i) Surface temp (ii) Max temp.

② An electric current is passed through a plane wall of thickness 25mm & 120mm wide, which used to heat a fluid at 30°C . The heat generation rate is $65 \times 10^5 \text{ W/m}^3$. k of the plate is 25 W/mK . Calculate the heat transfer coefficient to maintain the temp of plate below 150°C .

③ An electric current is passed through a composite wall made up of 2 layers. 1st layer is steel of 10cm thick & 2nd layer is brass of 8cm thick. The outer surface temp of steel & brass are maintained at 120°C & 65°C respectively. Assume that the contact b/w 2 slab is perfect & the heat generation is 1.65 MW/m^3 . Determine:

① Heat flux through the outer surface of brass slab q_2 .

$$q_{\text{eq}} = q_1 + q_2$$

② Interface temp T_2 .

Problem for cylinder with IHG

A copper wire of 4mm dia carries 950 A & has a resistance of $0.25 \times 10^{-4} \Omega/\text{cm}$ length surface temp of copper wire is 250°C & the ambient air temp is 10°C . k of copper wire is 175 W/mK . Calculate

① Heat transfer coefficient b/w wire surface & ambient air (h)

② max temp of the wire (T_{max})

Problem for sphere with IHG

① A sphere of 100mm dia having k of 0.18 W/mK . The outer surface temp is 8°C & 250 W/m^2 of energy is released due to heat source. Calculate

① Heat generated $q = Q/V = 4/3 \pi r^3$

② Temp at the center of the sphere.

$$h/A = \frac{Q/A}{V}$$

$$\frac{h}{4\pi r^2} = \frac{Q/A}{4/3 \pi r^3}$$

Heat conduction with Heat Generation:

In many practical cases, there is a heat generation within the system.

Ex: 1) Electric coils,

2) Resistance heater.

3) Nuclear Reactors

4) Combustion of fuel in the fuel of boiler furnaces

Plane wall with internal heat generation:

$$(i) T_{max} = T_w + \frac{\dot{q} L^2}{8k} \quad (\text{Mid})$$

Where

T_{∞} = Fluid temp

\dot{q} = Heat generation rate

L = Thickness m

h = heat transfer co

k = Thermal conduct

$$(ii) T_w = T_{\infty} + \frac{\dot{q} L}{2h} \quad (\text{Surface temp})$$

Cylinder with internal heat generation:

$$(i) T_{max} = T_w + \frac{\dot{q} r_o^2}{4k} \quad \left| \begin{array}{l} \text{heat} \\ \text{generation} \end{array} \right. \dot{q} = \frac{Q}{V}$$

$$(ii) T_w = T_{\infty} + \frac{r_o \dot{q}}{2h}$$

Sphere with internal heat generation:

$$T_c = T_w + \frac{\dot{q} r_o^2}{6k}$$

also $\left[\begin{array}{l} V - \text{volume} \\ r - \text{radius} \end{array} \right]$

FINS

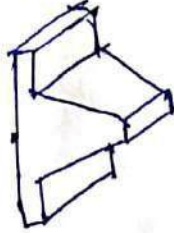
It's possible to increase the heat transfer rate by increasing the surface of heat transfer. The surface used for increasing heat transfer are called extended surfaces or fins.

TYPES:-

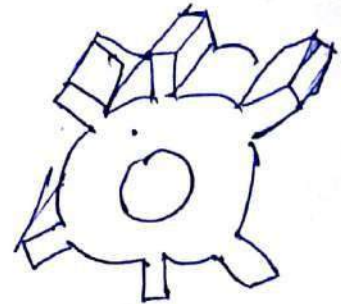
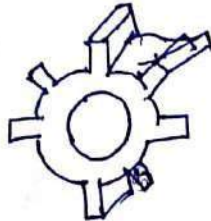
(1) Uniform straight fin



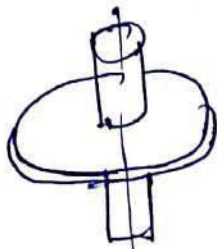
(2) Tapered straight fin



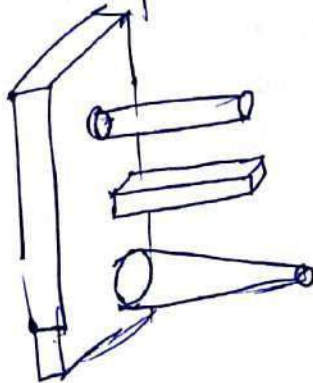
(3) Splines



(4) Annular fin



(5) Pin fins



: types

(1) Infinitely long fin

(2) Short fin (end is insulated)

(3) Short fin (end is not insulated).

Applications:-

* Cooling of electrical components, motor cycle engines, small capacity compressors, transformers, radiators, refrigerator etc.

Fin efficiency

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max}}$$

The ratio of actual heat transferred Q_{fin} to the maximum possible heat transferred by the fin.

For insulated end

$$\eta_{fin} = \frac{k_{an} b (mL)}{mL}$$

Fin effectiveness:

It is defined as the ratio of heat transfer with fin to heat transfer without fin.

$$\text{Fin Effectiveness } E = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

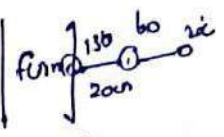
For insulated end

$$\text{Fin effectiveness } E = \frac{k_{an} b (mL)}{\sqrt{hA/RP}}$$

Problem for long fin or infinitely long fin

① Find the heat loss from a rod of 3 mm in diameter & infinitely long when its base is maintained at 140°C . The conductivity of the material is 150 W/mK and the heat transfer coefficient on the surface of the rod is $3000\text{ W/m}^2\text{K}$. The temp of the surrounding the rod is 15°C .

② A long rod 5 cm dia its base is connected to a furnace wall at 150°C , while the end is projecting into the room at 20°C . The temp of the rod at distance of 20 cm apart from its base is 60°C . The conductivity of the material is 200 W/mK . Determine convective heat transfer coefficient. $e = \frac{h}{k}$



③ One end of the long solid rod of 50 mm dia is inserted into a furnace with the other end is projected the atmosphere at 25°C . Once the steady state is reached, the temp of the rod is measured at 2 points 20 cm apart are found to be 150°C & 100°C . The convective heat transfer coefficient b/w the rod & the surrounding air is $20\text{ W/m}^2\text{K}$. Calculate the thermal conductivity of the rod material.

④ A carbon steel ($k = 55\text{ W/mK}$) 90 mm long rod with cross sectional area $5 \times 10^{-3}\text{ m}^2$ and perimeter 0.69 m is attached to a plane wall which is maintained at a temp of 400°C . The surrounding environment is at 50°C & $h = 90\text{ W/m}^2\text{K}$. Calculate the heat dissipated by the rod (a)

A turbine blades are made of stainless steel. Each blade carries 85 W heat. The cross sectional area of each blade is 4.5 cm^2 , Perimeter of each blade is 7 cm . The gas temp flowing over the blade is 800°C . The temp of the root of the blade is 125°C . $k = 22\text{ W/mK}$ & $h = 110\text{ W/m}^2\text{K}$. Determine the height of the blade neglecting the heat flow from the gas to the end of the blade.

INFINITELY LONG FIN (OR) LONG FIN

Important
Formula for
Fin

(a) Temp distribution:

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-m\alpha}$$

Where

T_b = Base Temp, K

T_{∞} = Surrounding Temp K

T = Intermediate temp K

α : Distance, m

$$m, \text{ m}^{-1} = \sqrt{\frac{hp}{KA}}$$

Where
 h = heat transfer co-efficient
 $\text{W/m}^2\text{K}$

P = Perimeter $\pi \times D$
 k = Thermal cond W/mK
 A = Area $\pi/4 d^2$

(b) Heat transfer

$$Q = (T_b - T_{\infty}) \sqrt{hpKA}$$

SHORT FIN

(a) Temperature distribution

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh m(L - \alpha)}{\cosh mL}$$

(b) Heat transferred:

$$Q = (hpLA)^{1/2} (T_b - T_{\infty}) \tanh mL$$

$$\text{Number of fin} = \frac{\text{Heat generated}}{\text{Heat transfer per fin}}$$

Perimeters = $2 \times L$ (Approximately)

Area = $E \times L$

L-1 heat Analysis [Negligible internal resistance]

⑥ An aluminium alloy fin of 7mm thick & 50mm long protrudes from a wall, which is maintained at 120°C . The ambient air temp is 22°C . The h & $k = 140\text{W/m}^2\text{K}$ & 55W/mK respectively. Determine.

① Temp at the end of the fin $x=L$ % calc (m)

② Temp at the middle of the fin $x=L/2$

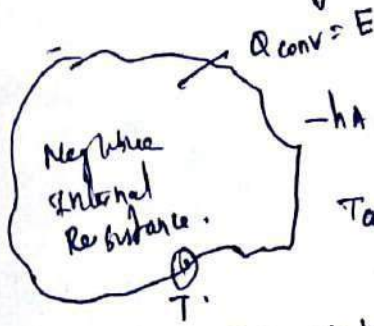
③ Total heat dissipated by the fin

$$\left| \begin{array}{l} P_c = 2 \times L \\ A = L \times T \end{array} \right.$$

⑦ An rectangular aluminium fins of 0.5mm square and 12mm long are attached on a plane plate which is maintained at 80°C . $T_{\infty} = 22^{\circ}\text{C}$. Calculate the number of fins required to generate $35 \times 10^3\text{W}$ of heat. $k = 165\text{W/mK}$, $h = 10\text{W/m}^2\text{K}$. Assume no heat loss from the tip of the fin.

$P = 2$ (b & d)
Number of fins
Heat transfer
fin

Lumped heat Analysis (Negligible internal resistance)



Convective heat loss from the body = Rate of change of internal energy.

$$T = T_0 \text{ at } t = 0$$

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = e^{\left[\frac{-hA}{\rho C_p V} t \right]}$$

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where

T_0 = Initial temp

T = Intermediate temp

T_{∞} = Surface or final temp.

A = Surface

The lumped parameter system (Biot number) value is less than 0.1.

$$\text{ie } Bi < 0.1 = \frac{hL_c}{k} < 0.1$$

$$L_c = \frac{V}{A}$$

Problem for Lumped heat Analysis

- ① A $50 \times 50 \text{ cm}^2$ aluminium slab of 6 mm thick is at 400°C initially & it is suddenly immersed in water, so its surface temp is lower to 50°C . Determine the time required for slab to reach 120°C . Take $h = 100 \text{ W/m}^2\text{K}$.
 Properties of Aluminium: ρ, c_p, k

Slab $L/2$, $Bi = \frac{hL_c}{k} = 1.46 \times 10^{-3} < 0.1$

- ② A copper rod of outer dia 20 mm initially at a temp of 380°C is suddenly immersed in water at 100°C . Determine the time req for reach 240°C . $h = 95 \text{ W/m}^2\text{K}$.
 Properties of Copper: ρ, c_p, k

- ③ A solid copper cylinder of 7 cm dia is at 200°C initially & it is suddenly immersed in water. $T_{\infty} = 90^\circ\text{C}$. It is kept for 6 minutes. Determine the time taken for the cylinder to reach 100°C .
 Properties of Copper: ρ, c_p, k

- ④ A spherical sphere made of copper of dia 6 mm, $T_0 = 200^\circ\text{C}$, is placed in a quenching furnace. The temp of the furnace is 450°C . Calculate the time required for the sphere to reach a temp of 82°C , $h = 30 \text{ W/m}^2\text{K}$.

- ⑤ A cylindrical stainless steel ingot 170 mm dia & 50 cm long passes through a heat treatment furnace. The temp of the furnace is 1300°C . The initial temp of ingot is 120°C . The combined radiation & convection $h = 105 \text{ W/m}^2\text{K}$. Calculate the maximum speed with which the ingot moves through the furnace in order to achieve 800°C . Take $k = 15 \text{ W/mK}$, $\alpha = 0.46 \times 10^{-5} \text{ m}^2/\text{s}$.

alternator.

Problem for Semi-Infinite Solids.

① A large concrete highway initially at a temp of $T_i = 70^\circ\text{C}$ & stream water is directed on the highway to that surface temp is suddenly lowered to $T_o = 40^\circ\text{C}$. Determine the time required to reach $T_x = 55^\circ\text{C}$ at a depth of 4cm from the surface (birologhen).

A large block of steel is initially at $T_i = 35^\circ\text{C}$. The surface temp is suddenly raised & maintained at $T_o = 250^\circ\text{C}$. Calculate the temp at a depth of 2.5cm after a time of 30s. The $\alpha = 1.4 \times 10^{-5} \text{ m}^2/\text{s}$ $k = 45 \text{ W/m}^\circ\text{C}$.

② A large wall 2cm thick has uniform temp $T_i = 30^\circ\text{C}$ initially and the wall temp is suddenly raised & maintained at $T_o = 400^\circ\text{C}$ find

① The temp at a depth of 0.8cm from the surface of the wall after 10s

② Instantaneous heat flow rate through that surface per m^2 per hour
Take $\alpha = 0.008 \text{ m}^2/\text{h}^2$ $k = 6 \text{ W/m}^\circ\text{C}$
 $q_{2x} = \frac{k(T_o - T_i)}{\sqrt{\alpha \pi t}}$ $\left[\frac{20^\circ}{400^\circ} \right]$

③ Total heat energy $Q_T = 2k[T_o - T_i] \sqrt{\frac{t}{\pi \alpha}}$

④ A semi infinite slab of aluminium is exposed to a constant heat flux at the surface of 0.25 MW/m^2 . Initial temp of slab is $T_i = 25^\circ\text{C}$. Calculate the surface temp after 10 minutes & also find the temp at a distance of 30cm from the surface after 10 minutes.
 $q_o = \frac{k(T_o - T_i)}{\sqrt{\alpha \pi t}}$

(1.4)

Q Alloy steel $\frac{1}{3}$ ball 9 mm dia heated to 808°C is quenched in a bath at 150°C .
 properties of the ball are: $k = 205 \text{ kJ/m hr}^\circ\text{C}$, $\rho = 7860 \text{ kg/m}^3$, $C_p = 0.45 \text{ kJ/kg}^\circ\text{C}$, $h = 1500 \text{ W/m}^2\text{K}$.
 Determine (i) Temp of ball after 10 sec & (ii) Time for ball to cool to 400°C .

$205 \text{ kJ/m hr}^\circ\text{C}$

$$\frac{205 \times 10^3}{3600 \times \text{mk}}$$

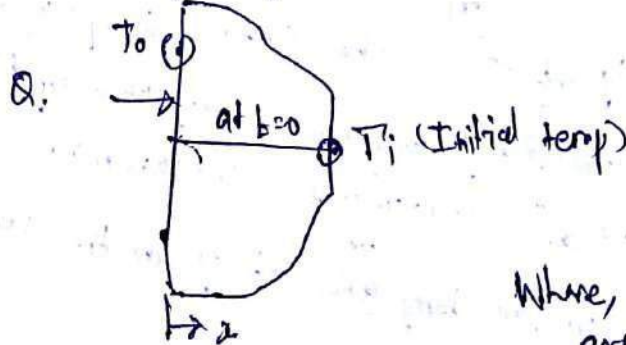
$$\boxed{J/S = W}$$

INFINITE SOLIDS:

A solid which extends itself infinitely in all directions of space is known as infinite solids.

SEMI INFINITE SOLIDS:

If an infinite solid is split in the middle by a plane, each half is known as semi infinite solids.
 (Surface temp)



$$\frac{T_x - T_o}{T_i - T_o} = \text{erf} \left[\frac{x}{2\sqrt{\alpha t}} \right]$$

Note:

(i) If Semi infinite Solid heat transfer Co-efficient (h) is infinite then number value is ∞ ie $h \rightarrow \infty$ or $Bi \rightarrow \infty$

Initial T_i , T_o , T_x
 Intermediate

Where,

erf = error function of
 α = Thermal diffusivity m^2/s

t = Time s

T_i = Init temp $^\circ\text{C}$

T_o = Surface temp $^\circ\text{C}$

T_x = Intermediate temp $^\circ\text{C}$

Transient Heat Flow in an Infinite plate

(1)

$$\frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = f \left[\frac{x}{L}, \frac{hL}{k}, \frac{\alpha t}{L^2} \right]$$

Note:

In infinite solids biot number. value is in between 0.1 & 100

$$\text{ie } 0.1 < Bi < 100.$$

- Q. A aluminium slab of 5cm thick initially at a temp of T_i is suddenly immersed in a liquid at T_{∞} . Calculate the mid plane temp after 1 minute & also calculate the temp at the plate at a distance of 10mm from the mid plane. $h = 1800 \text{ W/m}^2\text{K}$, $\alpha = \frac{hL}{k}$ 11 HMT: 66
- (i) L_c , (ii) $Bi = 0.1$ to 100 Case (ii) Formula: $\frac{\alpha t}{L^2}$ Case: $\frac{hL}{k}$

Case (ii) $X_{axis}: Bi = \frac{hL}{k}$
Curve: $\frac{k}{L_c}$

1) $\frac{T_0 - T_{\infty}}{T_i - T_{\infty}}$

2) $\frac{T_x - T_{\infty}}{T_0 - T_{\infty}}$

- Q. A slab of aluminium 10cm thick is originally at a temp of T_i is suddenly immersed in a liquid at T_{∞} resulting in a $h = 1200 \text{ W/m}^2\text{K}$. Determine the temp at the centre line & the surface 1 minute after immersion. Also calculate the total thermal energy removed per unit area of the slab during this period.
- Given: $k = 215 \text{ W/mK}$, $\rho = 2700 \text{ kg/m}^3$, $C_p = 0.9 \text{ kJ/kgK}$
 $Q = 8.14 \times 10^{-5} \text{ m}^2/\text{s}$ (1) T_0 (2) $X = L_c$ (Temp at the surface) (3) $X_{axis} = \frac{h^2 \alpha t}{k^2}$

$Q_{\infty} = 0.84$

$Q_0 = \rho C_p L [T_i - T_{\infty}]$

$Q_{\infty} = \text{J/m}^2$

mid plane

T_i - Initial Temp

T_{∞} = Final Temp

T_0 = Centre line Temp

T_x = Intermediate Temp

③ A 10 cm dia apple approximately spherical in shape is taken from a T_1 20°C environmental & placed in a refrigerator where temp is T_2 5°C & average heat $h = 6 \text{ W/m}^2\text{K}$. Calculate the temp at the center of the apple after a period of 1 hour. Properties $\rho = 998 \text{ kg/m}^3$, $C = 4180 \text{ J/kgK}$, $k = 0.6 \text{ W/mK}$.

$$Bi = \frac{hR}{k}$$

$$\alpha = \frac{k}{\rho C_p} \quad \text{num} \frac{\alpha t}{R^2}$$

$$\text{dim: } \frac{hR}{k}$$

Cylinder

At center

$$\text{Cyl no: } hR/k$$

$$\alpha = \alpha t / R^2$$

UNIT: II

CONVECTION

CONVECTIVE HEAT TRANSFER

Dimensional analysis
is a mathematical method which makes use of study
of dimensions for solving several engineering problems.

Dimensions:-

Mass - M Temp - θ
Length - L Time - T } are commonly used quantities in heat transfer

Buckingham π Theorem

If there are n variables in a dimensionally
homogeneous equation & if these contain m fundamental dimensions.
then the variables are arranged into $(n-m)$ dimensionless terms.

Advantages of Dimensional analysis)

- 1) It expresses the functional relationship b/w the variable in dimensional terms
- 2) It enables getting up a theoretical solution in a simplified dimensionless form.

Limitations

- 1) No information is given about the internal mechanism of physical phenomenon.
- 2) It does not give any clue regarding the selection of variables

Reynolds number (Re)

$$Re = \frac{UL}{\nu}$$

u - velocity m/s

L - length m

ν - kinematic viscosity m^2/s

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

Prandtl Number (Pr)

$$Pr = \frac{\mu C_p}{k} = \frac{\nu}{\alpha} = \frac{\nu - \text{kin viscosity } m^2/s}{\alpha - \text{Thermal diff } m^2/s}$$

Nusselt Number (Nu)

$$Nu = \frac{hL}{k}$$

h - W/m^2K

L - m

k - W/mK

$$\frac{q_{conv}}{q_{cond}}$$

Grashof Number (Gr)

$$Gr = \frac{g \times \beta \times L^3 \times \Delta T}{\nu^2}$$

β - coefficient of expansion K^{-1}

L - m

ν - m^2/s

ΔT - Temp difference K.

Stanton Number (St)

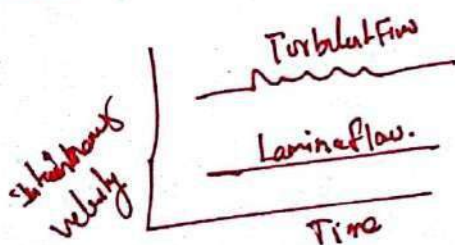
$$St = \frac{h}{\rho u C_p}$$

Newtonian & Non-Newtonian Fluids:

They Fluid which obey the
- not obey

Newton's law of viscosity - Newtonian
- Non-Newtonian

Laminar & Turbulent flow



Irregular flow. Zig Zag.

Smooth & continuous path.
(Stream line flow)

Types of Boundary layers

- 1) Hydrodynamic Velocity
- 2) Thermal

velocity of fluid less than 99% of stream velocity
Temp of the fluid less than 99% of stream velocity.

Convection - solid to liquid

Natural convection:

$$Q = hA(T_w - T_{\infty})$$

$$h = W/m^2K$$

$$A = m^2$$

T_w = Surface or wall temp $^{\circ}C$

T_{∞} = Temp of fluid $^{\circ}C$

Free ~~con~~ Natural convection:

If the fluid motion is produced due to change in density resulting from temp gradients, the mode of heat transfer is said to be free or natural convection.

Forced convection:

If the fluid motion is artificially created by means of an external force like a blower or fan, this type of heat transfer is known as FC.

Heat transfer from flat surface - Formulae

1) Velocity is given then forced convection problems.

$$2) \text{ Film Temp} = T_f = \frac{T_w + T_{\infty}}{2} \quad \left| \begin{array}{l} T_w = \text{Plate Temp or Surface} \\ T_{\infty} = \text{fluid Temp} \end{array} \right.$$

$$1) Re = \frac{UL}{\nu} < 5 \times 10^5 \Rightarrow \text{Laminar Flow}$$

$$Re = \frac{UL}{\nu} > 5 \times 10^5 \Rightarrow \text{Turbulent Flow}$$

U - velocity m/s
 L - m
 ν - m²/s kinematic viscosity

For Flat Plate: Laminar Flow HMT 115

1) Local Nusselt Number

$$Nu_x = 0.332 (Re)^{0.5} (Pr)^{0.333} \quad \left| \begin{array}{l} 0.0296 (Re)^{0.4} (Pr)^{0.333} \end{array} \right.$$

2) Local Nusselt Number

$$Nu_x = \frac{h_x L}{k}$$

h_x = Local heat transfer coefficient W/m^2K
 $k = W/mK$

3) Average heat transfer coeff $h = 2 \times h_x \quad \left| \quad 1.25 h_x \right.$

1) Air at 20°C at a pressure of 1 bar is flowing over a flat plate at a velocity of 3 m/s . If the plate is maintained at 60°C . Calculate the heat transfer per unit width of the plate. Assuming the length of the plate along the flow of air is 2 m .

1) T_f 2) Properties of air at $T_m = (T_f + T_s)/2$ 3) Re 4) Nu_x 5) h_x 6) Q

2) Air at 25°C flows over a flat plate at a speed of 5 m/s & heated to 135°C . The plate is 3 m long & 1.5 m wide. Calculate the local h_x at $x = 0.5$ & the Q from the first 0.5 m of the plate.

1) T_f 2) Properties of air at $T_m = (T_f + T_s)/2$ 3) Re 4) Nu_x 5) h_x 6) Q

3) Air at 20°C at atmospheric pressure flows over a flat plate at a velocity of 3 m/s . If the plate is 1 m wide & 80°C . The following at $x = 300\text{ mm}$.

1) T_f 2) Properties of air at $T_m = (T_f + T_s)/2$ 3) Re 4) Nu_x 5) h_x 6) Q

- 1) Hydrodynamic boundary layer thickness δ
- 2) Thermal boundary layer thickness δ_t
- 3) Local friction coefficient C_{fx}
- 4) Average friction coefficient C_{fL}
- 5) Local heat transfer coefficient h_x
- 6) Average heat transfer coefficient h
- 7) Q

4) Air at 20°C flows over a flat plate at a velocity of 3.5 m/s . The plate is 5 m long & 2 m wide. Calculate the following:

- 1) Length of the plate over which the boundary layer is laminar
 - 2) Thickness of boundary layer δ at laminar flow
 - 3) Shear stress at the location where boundary layer is laminar
 - 4) Total drag force on both sides of the plate where boundary layer is laminar
- Take $\rho = 1.205\text{ kg/m}^3$
 $\mu = 15.06 \times 10^{-6}\text{ m}^2/\text{s}$
- 5) T_s - Shear stress
 $C_{fx} = \frac{\tau_x}{\frac{1}{2}\rho U^2}$
 $F_D = \text{Area} \times \text{Average shear stress} (\tau)$
 $W \times L \times \tau$
 $C_{fL} = \frac{\tau}{\frac{1}{2}\rho U^2}$

5) Air at 10°C flows over a flat plate at a velocity of 2 m/s . The plate is maintained at 100°C . The length of the plate is 2.5 m . Calculate the heat transfer per unit width. Using 1) Exact method 2) Approximate method

In a crank flow h/c will find ...

Q Air at $T_w = 25^\circ\text{C}$ flows over $1\text{m} \times 3\text{m}$ (3m long) horizontal plate maintained at 20°C . at 10m/s. Calculate the average heat transfer coefficient for both laminar & turbulent regions. Take $Re = 3.5 \times 10^5$ (critical)

A particular engine the underside of the crankcase can be idealised as a flat plate measuring $80\text{cm} \times 20\text{cm}$. The engine runs at 80km/hr and the crankcase is cooled by air flowing past it at the same speed. Calculate the loss of heat from the crankcase surface by temp 75°C to the ambient temp 25°C .

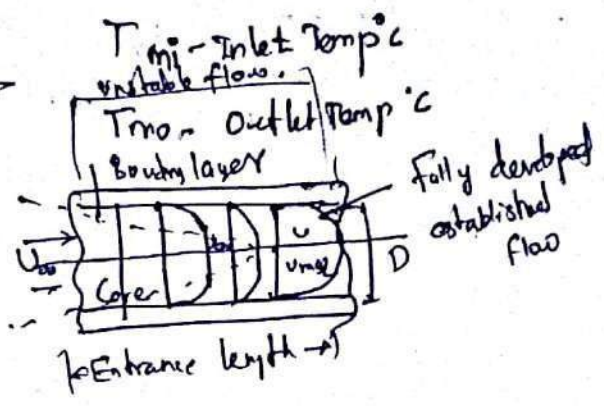
$A = 0.6\text{m}^2$

Q?

Formulae used for Flow through cylinders (Internal flow).

① Bulk mean temp $T_m = \frac{T_{mi} + T_{mo}}{2}$

② Reynolds Number $Re = \frac{UD}{\nu}$
 $Re < 2300$ Hemi laminar.
 $Re > 2300$ Turbulent



③ Laminar flow: $Nu = 366$

④ Turbulent (general eq: $Nu = 0.023 (Re)^{0.8} (Pr)^n$

$0.6 < Pr < 160$
 $Re > 10000$
 $L/D > 60$
 $n = 0.4$ Heating process.
 $n = 0.3$ Cooling process

$Nu = 0.036 (Re)^{0.8} Pr^{0.33} (D/L)^{0.55}$

This eqn valid for $10 < L/D < 400$

$Re < 10,000$

⑤ equivalent dia for rectangular section

$D_h \text{ or } D_e = \frac{4A}{P} = \frac{4(L \times W)}{2(L+W)}$

A - Area m^2
P - Perimeter m
L - Length m
W - width m

⑥ " " for hollow cylinder $D_h \text{ or } D_e = \frac{4A}{P}$

$= \frac{4 \times \pi/4 (D_o^2 - D_i^2)}{\pi (D_o + D_i)}$

D_o - Outer dia
 D_i - Inner dia

Problem for internal flow.

Pg: no: 22

① Water flows inside a tube of 20mm dia & 3m long at U of 0.03 m/s. The water gets heated from 40°C to 120°C while passing through the tube. The tube wall is maintained at $T_w = 160^\circ\text{C}$. Find Q & h .

$Re = \frac{UD\mu}{\mu} < 2300$ $Pr = \frac{c_p \mu}{k}$ $Nu = 3.66 + \frac{hD}{k} \left(\frac{D}{4L} \right) \left(\frac{T_w - T_m}{T_w - T_m} \right)$ $Q = \pi D L h (T_w - T_m)$

Water at 20°C is passed through a tube of 2cm dia, it is found to be heated from 20°C to 60°C . The heating is achieved by condensing steam on the surface of tube and subsequently the surface temp. of the tube is maintained at 90°C . Determine the length of the tube req for fully developed flow. ① $m = \rho A U$

$Q = m (c_p \Delta T)$ $Q = \pi D L h (T_w - T_m)$

Water at 50°C enters 50mm dia and 4m long tube with a velocity of 0.8 m/s. The tube wall is maintained at a const temp of 90°C . Determine h & Q if exit temp is 70°C .

$Re = 836 \times 10^4$ (turbulent) $Pr = 0.023 \times (Re)^{0.8} (Pr)^{0.4}$

Water flows through 0.8 cm dia, 3m long tube at an average temp of 40°C . The flow velocity is 0.15 m/s & tube wall temp is 140°C . Calculate h .

$Re = 375$ $Pr = 0.036 (Re)^{0.8} (Pr)^{0.33} (D/L)^{0.55}$

Air at 15°C , 35 m/s, flows through a hollow cylinder of 4cm inner dia & 6cm outer diameter & length at 45°C . Tube wall is maintained at 60°C . Calculate the convective co-efficient b/w the air & the inner tube.

$h = \frac{Q}{A (T_w - T_m)}$

Engine oil flows through a 50mm dia tube at an average temp. of 147°C . The flow velocity is 80 cm/s. Calculate the average h , if the tube wall is maintained at a temp of 200°C & it is 2m long.

Free Convection:

If the fluid motion is produced due to change in density resulting from temp gradients. the mode of heat transfer is said to be Free or natural convection.

Formulae used for free convection:

① Film temp $T_f = \frac{T_w + T_\infty}{2}$ T_w - Surface °C
 T_∞ - Fluid °C

② Coefficient of thermal expansion: ③ Nusselt Number $Nu = \frac{hL}{k}$

$$\beta = \frac{1}{T_f \text{ in } ^\circ\text{K}}$$

$$Gr = \frac{g \times \rho \times L^3 \times \Delta T}{\mu^2}$$

④ Grashof Number for vertical plate

$$GrPr = \frac{g \times \rho^2 \times L^3 \times \Delta T}{\mu^2 \times k}$$

$$10^4 < GrPr < 10^9 \quad \text{Eq. 13.6}$$

⑤ Nu for laminar flow (vertical plate)

$$Nu = 0.59 (GrPr)^{0.25}$$

⑥ Nu for Turbulent flow (vertical plate)

$$Nu = 0.10 (GrPr)^{0.333}$$

⑦ Q for vertical plate

$$Q = hA(T_w - T_\infty)$$

$A = \pi DL$ (cylinder)
 $A = \text{Area plate}$

$$L = \frac{h}{2}$$

⑧ Grashof Number for horizontal plate:

$$Gr = \frac{g \times \rho^2 \times L_c^3 \times \Delta T}{\mu^2}$$

⑨ For horizontal plate, upper surface heated,

$$Nu = 0.54 (GrPr)^{0.25}$$

This expression valid for

$$10^4 < GrPr < 10^9$$

$$Nu = 0.15 [GrPr]^{0.333}$$

$$10^5 < GrPr < 10^9$$

$$\frac{12.7}{13.6}$$

⑩ For horizontal plate lower surface heated

$$Nu = 0.27 (GrPr)^{0.25}$$

$$10^5 < GrPr < 10^9$$

⑪ Q for horizontal plate $2(h_u + h_l) \times A \times (T_w - T_\infty)$

(a) Explain generalized

for horizontal cylinder

$$Nu: C (Gr Pr)^m$$

$$\left(\frac{Pr}{138} \right)$$

$$Q = hA (T_w - T_\infty) \quad | A = \pi D L$$

for sphere

$$Nu: 2 + 0.43 [Gr Pr]^{0.25}$$

$$\left(\frac{Pr}{138} \right)$$

$$Q = h \times A \times (T_w - T_\infty)$$

$$h = 471 r^2$$

Boundary layer thickness

$$\delta_x = \left[3.93 \times (Pr)^{-0.5} (0.952 + Pr)^{0.25} \times (Gr)^{-0.25} \right] \times \frac{2}{114} \left(\frac{135}{138} \right)$$

Maximum velocity

$$u_{max} = 0.766 \times u \times (0.952 + Pr)^{-1/2} \left[\frac{g \beta (T_w - T_\infty)}{u^2} \right]^{1/2} \times x^{1/2}$$

mass flow rate

$$\dot{m} = 1.7 \times \rho \times u \left[\frac{g \beta}{(Pr)^2 (1 + 0.952)} \right]^{0.25}$$

PHASE CHANGE UNIT: II HEAT TRANSFER & HEAT EXCHANGERS

31 Boiling & condensation:

Boiling & condensation are such convective heat transfer processes that are associated with change in phase of liquid.

Boiling:

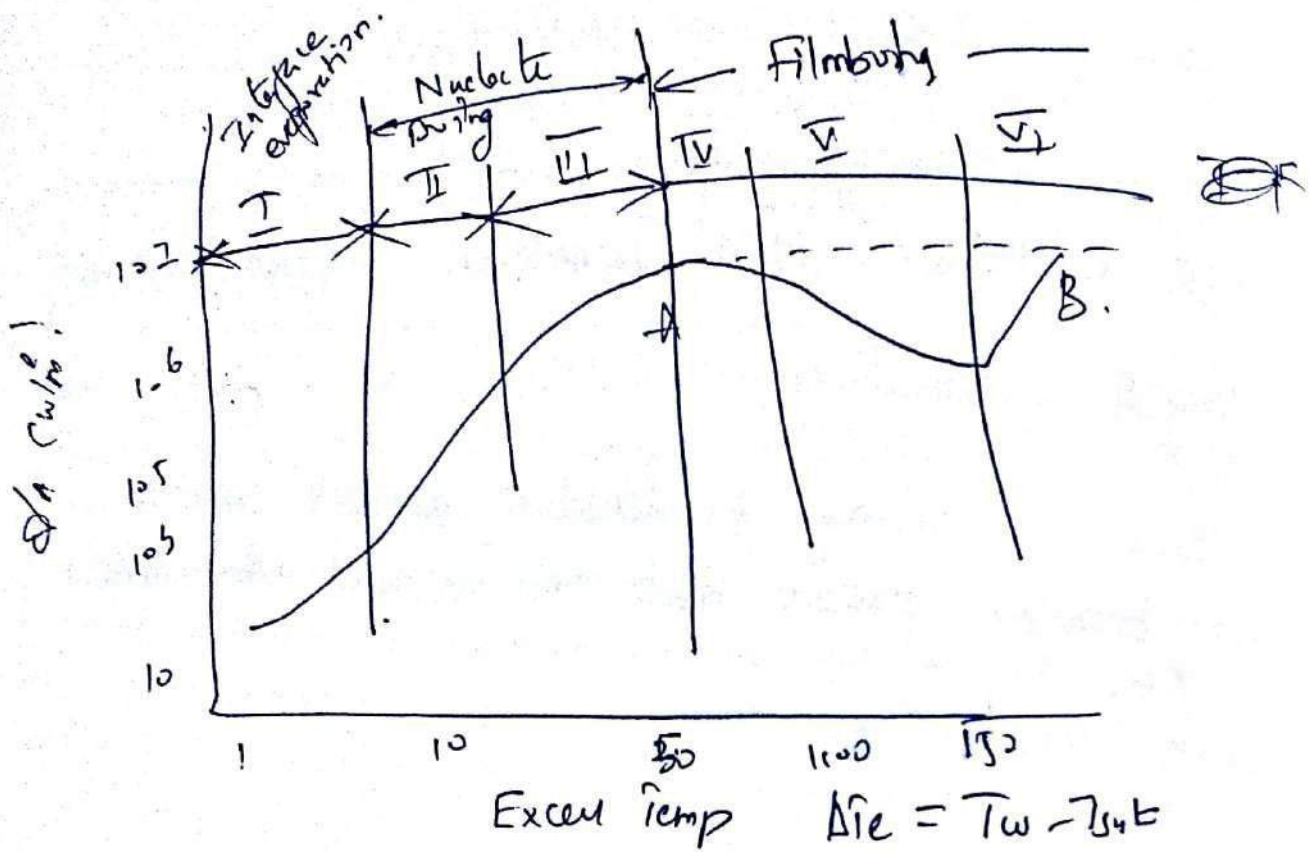
The change of phase from liquid to vapour state.

Condensation

The change of phase from vapour to liquid.

Application:-

- * Thermal & Nuclear power plants
- * Refrigerating system
- * Heating metal in furnaces
- * Air conditioning system



- I - Free convection
- II - Bubble condense in super heated liquid
- III - Bubble near to surface
- IV - Unstable film
- V - Stable film
- VI - Radiation coming into play.

① Nucleate Pool Boiling

HMT DB : 143

② Heat flux $\frac{Q}{A} = \mu_L h_{fg} \left[\frac{g \times (\rho_L - \rho_V)}{\sigma} \right]^{0.5} \times$

$$\left[\frac{c_{pL} \times \Delta T}{C_{sf} \times h_{fg} \cdot Pr^n} \right]^3$$

$\frac{Q}{A} = q = \text{heat flux } W/m^2$

μ_L - Dynamic viscosity of liquid Ns/m^2

h_{fg} - enthalpy of evaporation J/kg

g - Acceleration due to gravity m/s^2

ρ_L - Density of liquid kg/m^3 ρ_V - density vapour

c_{pL} - Sp. heat liquid J/kgK

C_{sf} - Surface fluid constant

Pr - Prandtl Num ΔT - Excess temp

T_w - Surface temp

T_{sat} - Sat temp.

$n = 1$ for water

$n = 1.7$ for other fluid

(b) Critical head flux

$$\frac{Q}{A} = 0.18 \, h_{fg} P_v \left[\frac{\sigma g (P_1 - P_v)}{P_v^2} \right]^{0.25}$$

(c) Excess temp:

$$\Delta T = T_w - T_{sat} < 50^\circ\text{C} \text{ for Nickel pool boiling}$$

(d) Heat transfer $Q = m \times h_{fg}$.

② Film Pool Boiling:

① Heat transfer coefficient

$$h = h_{\text{conv}} + 0.75 h_{\text{rad}}$$

$$h_{\text{conv}} = 0.62 \left[\frac{k_v^3 \rho_v (p_k - p_v) \times g \times C_{p_g} + 0.4 (C_{p_v} \Delta T)^{0.25}}{\mu_v \Delta T} \right]$$

② k_v - g vapor w/mk

C_{p_g} - specific heat of vapor at constant pressure.

μ_v - Dynamic viscosity of vapor.

$$h_{\text{rad}} = \sigma E \frac{T_w^4 - T_{\text{sat}}^4}{T_w - T_{\text{sat}}}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$$

E - Emissivity

σ - Stefan Boltzmann constant.

T_w - Surface Temp $^{\circ}\text{C}$

T_{sat} - Sat. Temp.

Excess Temp

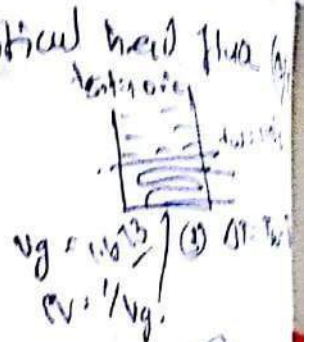
$$\Delta T = T_w - T_{\text{sat}} > 5^{\circ}\text{C} \text{ for film boiling}$$

Problem for boiling

Q.22

① Water is to be boiled at atm pressure in a polished copper pan by means of an ele heater. The dia of the pan is 0.5 m & is kept at 115°C . Calculate following

- FB
- ① Power req. to boil the water (P)
 - ② Rate of evaporation (m) or (m) kg water
 - ③ Critical heat flux (q_c)



④ Properties of water at 100°C (sub)

⑤ $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ at 100°C

CSF = 124

⑥ $q_c = \frac{1}{\sqrt{C}} \times \sqrt{h_{fg} \rho \Delta T}$

6) $q_c = 4$

② It is desired to boil water at atmospheric pressure on a copper surface which is electrically heated. Estimate the heat from the surface to the water, if the surface is maintained at 115°C and also the peak heat flux q_c .

$q_c = 4$

$T_w = 260.173^{\circ}\text{C}$

FB ③ A heating element clad with metal is 8 mm dia & 0.92 m long. The element is horizontally immersed in a water bath. Temp of the metal is 260°C under steady state boiling conditions. Calculate power dissipation per unit length of the heater

1) $\Delta T = T_s - T_w$ 2) $T_s = 115^{\circ}\text{C}$ 3) h_{fg} 4) $q_c = 5.67 \times 10^{-8}$

Boiling

- ① It's desired to boil water at atmospheric pressure on a Copper surface which is electrically heated. Estimate the heat flux from the surface to the water, if the surface maintained at 110°C and also the peak heat flux.

Condensation :-

The change of phase from vapour to liquid state.

Modes of condensation.

- Two types
- ① filmwise condensation
 - ② Dropwise condensation.

FWC :-

The liquid condensate wets the solid surface, spreads out & forms a continuous film over the entire surface.

DWC

The vapour condenses into small liquid droplets of various size which fall down the surface in a random fashion.

✓ Q for DWC is 10 times more than FWC

⊗ Note

Laminar flow $Re < 1800$

Turbulent flow $Re > 1800$

$$Re = \frac{4m}{P \mu}$$

(3.7)

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Vertical

① Film thickness for laminar flow vertical surface. (See) - Boundary layer thickness δ (m)

$$\delta_x = \left[\frac{4\mu k x (T_{sat} - T_w)}{g \times h \rho \times \rho^2} \right]^{0.25}$$

μ - dynamic viscosity
 $\mu = \rho \times \nu$

② Local heat transfer coefficient (h_x)

|| for vertical surface
Laminar flow

$$h_x = \frac{k}{\delta_x}$$

③ Average heat transfer coefficient (h)

for vertical surface
laminar flow.

$$h = 0.943 \left[\frac{k^3 \rho^2 g h \rho}{\mu L (T_{sat} - T_w)} \right]^{0.25}$$

The factor 0.943 may be replaced by 1.13 for more accurate result suggest by Mc Adams.

Horizontal

① Average heat transfer coefficient for horizontal surface, laminar flow

$$h = 0.728 \left[\frac{k^3 \rho^2 g h \rho}{\mu L (T_{sat} - T_w)} \right]^{0.25}$$

μ - Newtonian
Avg. h for bank of tubes

② Average heat transfer coefficient for vertical surface (turbulent flow).

$$h = 0.0077 (Re)^{0.4} \left[\frac{k^3 \rho^2 g}{\mu^2} \right]^{0.333}$$

$Re < 1800$ L.F
 $Re > 1800$ T.F

$$Re = \frac{Lm}{\rho \mu} = \frac{P \cdot \text{Perimeter}}{\rho \mu}$$

3-8

Laminar flow vertical surface.

① Dry saturated steam at a pressure of 3 bar, condenses on the surfaces of a vertical tube of height of 1m. The tube surface temp is kept 110°C . Calculate the following

① Thickness of the condensate film (δ_x)

② Local heat flux $x=0.25\text{m}$ (h_{xx}).

② A vertical tube of 65mm outside dia and 1.5m long is exposed to steam at atmospheric pressure. The outer surface of the tube is maintained at a temp of 60°C by circulating cold water through the tube. Calculate the following

① $T_{\text{sat}} = 100$

② Overall rate of heat transfer to the condenser (Q)

③ T_f ④ properties H_2O

⑤ Overall rate of condensation of steam (\dot{m})

⑥ h , ⑦ $Q = h A \Delta T$

③ A vertical Flat Plate in the form of Fin is 500mm in height and its exposed to steam at atmospheric pressure. Its surface of the plate is maintained at 60°C .

$A = \pi D \times L$

Calculate the following.

$x = L = 0.5\text{m}$

① Film thickness at the rear trailing edge (δ_x)

② Overall heat transfer coefficient (h)

③ Heat transfer rate (Q)

④ The condensation mass flow rate. (\dot{m})

④ A vertical plate 0.4m height and 0.2m wide at 40°C , is exposed to saturated steam at atmospheric pressure. Find the following

① Film thickness at the bottom of plate. δ_x

② Max velocity (u_{max}) = $\frac{\rho g (\delta_x)^2}{2\mu}$

Advantage: Easy construction, more economical, more surface area for heat transfer

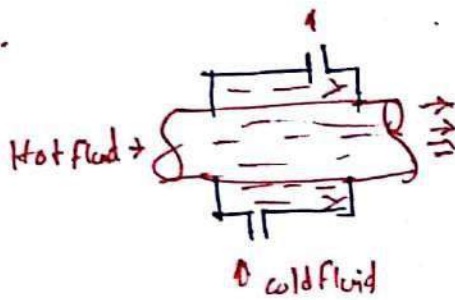
Disadvantage: Less h, less generative capacity.

II Relative direction of Fluid motion:

Types:

(a) Parallel flow H/E (b) counter flow H/E (c) Cross flow Heat Exchanger
→ same direction (Hot & cold fluid)

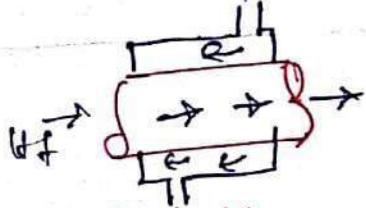
(a) Parallel Flow H/E:



(b) Counter flow H/E:

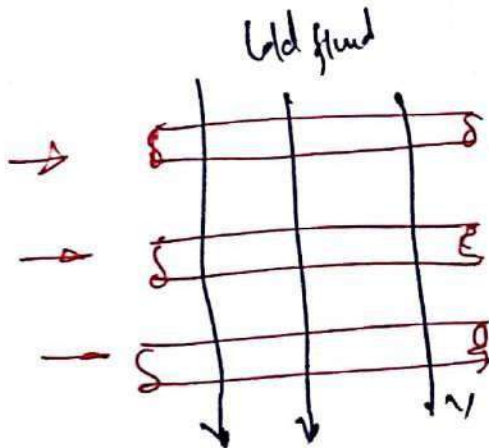
Cold fluid

In this type hot & cold fluids move in parallel but opposite directions



(c) Cross flow H/E:

h/f



[hot & cold fluid move opposite directions]

Parallel Flow & Counter Flow heat exchanger

① Heat transfer $Q = UA(\Delta T)_m$ - $(\Delta T)_m$ Logarithmic mean Temp Diff (LMTD)

For Parallel Flow

$$\Delta T_m = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln \left[\frac{T_1 - t_1}{T_2 - t_2} \right]}$$

$T_1 \text{ \& } T_2 \Rightarrow$ Entry & Exit of hot fluid
 $t_1 \text{ \& } t_2 \Rightarrow$ Entry & Exit cold fluid

For Counter Flow:

$$(\Delta T)_m = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln \left[\frac{T_1 - t_2}{T_2 - t_1} \right]}$$

② Heat lost by hot fluid = Heat gained by cold fluid

$$Q_h = Q_c$$

$$m_h C_{ph} (T_1 - T_2) = m_c C_{pc} (t_2 - t_1)$$

m_h - mass flow rate of h/f
 m_c - " " c/f
 C_{ph} - sp. heat h/f
 C_{pc} - sp. heat c/f

③ Surface area of tube

$$A = \pi D_i L$$

D_i - Inner dia

$$Q = m_h C_{ph} (T_1 - T_2)$$

$$Q = m_c C_{pc} (t_2 - t_1)$$

④ $Q = \dot{m} \times h_{fg}$

⑤ mass flow rate

$$\dot{m} = \rho AC$$

Problem for heat exchanger

- ① In a counter flow double pipe H/E, oil is cooled from 75°C to 55°C by water entering at 25°C . The mass flow rate of oil is 9800 kg/h & sp heat of oil is $2000 \text{ J/kg}^{\circ}\text{C}$. The mass flow rate of water is 8000 kg/h & sp heat of water is $4180 \text{ J/kg}^{\circ}\text{C}$. Determine the H/E area & heat transfer rate for an overall heat transfer coefficient $U = 9280 \text{ W/m}^2\text{K}$.
- Hot fluid T_1, T_2
Cold fluid t_1, t_2
- ② $Q_h = Q_c$ ③ $Q = U A \Delta T_m$

- ② Water flows at the rate of 65 kg/s through a double pipe parallel flow H/E. Water is heated from 50°C to 75°C by an oil flowing through the tube. The sp oil is $1780 \text{ J/kg}^{\circ}\text{C}$. The oil enters at 115°C & leaves 70°C . $U = 340 \text{ W/m}^2\text{K}$. Calculate the following:
① Heat transfer Area ② Rate of heat transfer

- ③ In a counter flow single pass H/E is used to cool the engine oil from 150°C to 55°C with water, available at 23°C as a cooling medium. Sp of oil is $2125 \text{ J/kg}^{\circ}\text{C}$. The flow rate of cooling water through the inner tube of 0.1 m dia is 2.4 kg/s . $U = 240 \text{ W/m}^2\text{K}$. How long must the H/E be to meet its cooling requirement?

- ④ Saturated steam at 126°C is condensing on the outer tube surface of a single pass H/E. The H/E heats 1050 kg/h of water from 20°C to 95°C . $U = 1800 \text{ W/m}^2\text{K}$. Calculate the following:
① Area of H/E ② Rate of condensation of steam. Take $h_{fg} = 2185 \text{ kJ/kg}$.

① $Q = ?$ ② $Q = \dot{m} c_p \Delta T$ ③ $Q = U A \Delta T_m$

T_1, T_2
 t_1, t_2

U
 \dot{m}
 c_p

2.4 kg/s

1) In a ~~counter~~ Counter flow double pipe heat Exchanger is used to cool the oil from 200°C to 120°C with water available at 25°C as the cooling medium. The exit temp of water is 70°C . Sp. heat of oil is $1.5 \text{ kJ/kg}^{\circ}\text{C}$ & $m_h = 0.5 \text{ kg/s}$ if $U = 1000 \text{ W/m}^2\text{K}$. Find the following

- ① Overall heat transfer (Q)
- ② Mass flow rate of water m_c
- ③ Area of heat exchanger (A)

① $Q = m_h c_p (T_1 - T_2)$
② $\Delta T = 25^{\circ}\text{C}$ ③ $\Delta T = 125^{\circ}\text{C}$

Problem on cross flow Heat Exchanger on Shell & tube Heat Exchanger.

Pg No: 151.

① $Q = FUA(\Delta T)_m$ (counter flow)

F - Correction factor HMT/DB

$$(\Delta T)_m = \frac{(T_1 - T_2) - (T_2 - T_1)}{\ln \left[\frac{T_1 - T_2}{T_2 - T_1} \right]}$$

② $Q_h = Q_c$

$$m_h c_p (T_1 - T_2) = m_c c_p (T_2 - T_1)$$

③ X-axis value $P = \frac{T_2 - T_1}{T_1 - T_1}$

Y-axis value $R = \frac{T_1 - T_2}{T_2 - T_1}$

3-16

formulas used for NTU method

1) Capacity rate of hot fluid: $C = \dot{m}_h \times c_{ph}$

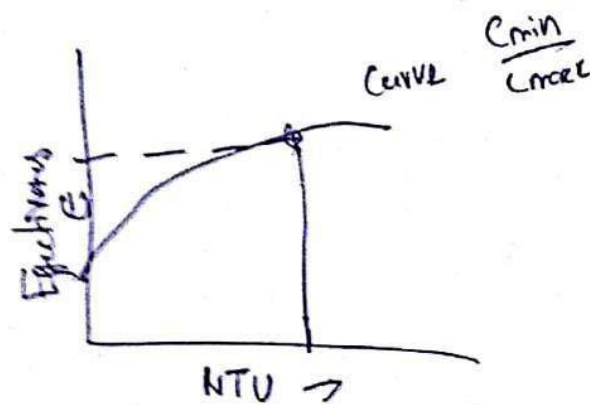
2) Capacity rate of cold fluid $= C = \dot{m}_c \times c_{pc}$

W.K.

3) $\frac{C_{min}}{C_{max}}$

4) $NTU = \frac{UA}{C_{min}}$

5) Effective to find



6) $Q_{max} = C_{min} (T_1 - t_1)$ (max possible heat transfer)

7) Actual heat transfer rate $Q = E \times Q_{max}$

8) Heat transfer $Q = \dot{m}_c c_{pc} (t_2 - t_1)$ (cold fluid)

Heat transfer $Q = \dot{m}_h (h_1 - T_2)$ (hot fluid)

16 : New method is used to determine the inlet and exit temp of heat exchanger.

A parallel flow heat exchanger is used to cool 4.2 kg/min of hot liquid of sp. heat $3.5 \text{ kJ/kg}^\circ\text{C}$ at 13°C . A cooling water of sp. heat $4.18 \text{ kJ/kg}^\circ\text{C}$ is used for cooling purpose at a temp of 15°C .

The mass flow rate of cooling water is 17 kg/min . Calculate the following
 ① Outlet temp of liquid (T_2) ② Outlet temp of water (t_2) ③ Efficiency (%)

Take $U = 1100 \text{ W/m}^2\text{K}$ Area = 0.3 m^2

17 In a counter flow H/E water at 20°C flowing at the rate of 1200 kg/h . It is heated by oil of sp. heat $2100 \text{ J/kg}^\circ\text{C}$ flowing at the rate of 520 kg/h at inlet temp of 95°C . Determine the following

① Total heat transfer (Q)

② Outlet temp of water

③ Outlet temp of oil

Take $U = 1000 \text{ W/m}^2\text{K}$
 $A = 1 \text{ m}^2$

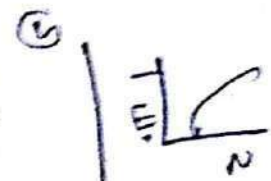
Cold fluid
 water
 t_1, t_2

Hot fluid - oil
 T_1, T_2

④ Capacity max (pr max) $\frac{m_2 c_p}{m_1 c_p}$

① $\frac{C_{min}}{C_{max}}$

② $NTU = \frac{UA}{C_{min}}$



⑤ $Q_{max} = C_{min}(T_1 - t_2)$

⑥ $Q = \epsilon Q_{max}$

$Q = m_c c_p (t_2 - t_1)$
 $Q = m_h c_p (T_1 - T_2)$

UNIT: 4 RADIATION.

Introduction:

The heat is transferred from one body to another without any transmitting medium.

This is an electromagnetic wave phenomenon.
 $3 \times 10^8 \text{ m/s}$

Emission Properties

- 1) The wavelength
- 2) The temp, the surface
- 3) The nature of the surface.

Emissive power $[E_b]$

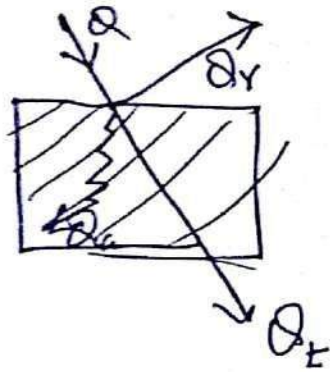
$E_b = \text{Total amount of radiation emitted by unit time \& body unit area}$

$$E_b = \text{W/m}^2$$

Monochromatic Emissive power ($E_{b\lambda}$)

The energy emitted by the surface at a given length per unit time per unit area in all directions.

Absorption, Reflection & Transmission



R - Reflection
T - Transmission
a - Absorption.

$$Q = Q_a + Q_r + Q_t$$

$$\div Q \quad = \quad \frac{Q}{Q} = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q}$$

$$1 = \alpha + \rho + \tau$$

$$\text{Absorptivity } (\alpha) = \frac{\text{Radiation absorbed}}{\text{Incident radiation}}$$

$$\text{Reflectivity } (\rho) = \frac{\text{Radiation reflected}}{\text{Incident radiation}}$$

$$\text{Transmissivity } (\tau) = \frac{\text{Radiation transmissivity}}{\text{Incident radiation.}}$$

Concept of Blackbody:

* A black body absorbs all incident radiation, regardless of wave length & direction.

* For a prescribed temp & wave length, no surface can emit more energy than black body



Enclosure at uniform temp

PLANCK'S DISTRIBUTION LAW

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$E_{b\lambda}$ = monochromatic emissive power W/m^2

λ = Wavelength - m

$$C_1 = 0.374 \times 10^{-15} \text{ W-m}^2$$

$$C_2 = 14.4 \times 10^{-3} \text{ mK}$$

The relationship b/w the MEP of a Black body & Wavelength of radiation at a particular temp.

WIEN'S DISPLACEMENT LAW :

The Wien's Law gives the relationship b/w temp & Wavelength corresponding to the max spectral emissive power of the black body at that temp.

$$\lambda_{\text{max}} \cdot T = 2898 \mu\text{mK}$$

Pg-81

$$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ mK}$$

STEFAN-BOLTZMANN LAW

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

Emissive power of a black body, is proportional to the 4th power of absolute temp.

Emissive power W/m^2
Stefan's constant $5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$
Temp - K

MAXIMUM EMISSIVE POWER $(E_b)_{max}$

A combination of T & λ yields the condition for the max monochromatic emissive power for a black body.

$$(E_b)_{max} = C_1 T^5$$

$$C_1 = 1.307 \times 10^{-5} \text{ (Radiation constant)}$$

$$(E_b)_{max} = 1.307 \times 10^{-5} T^5 \rightarrow$$

EMISSIVITY

$$\epsilon = \frac{E}{E_b}$$

$$= \frac{\text{Emissive power of any body}}{\text{Emissive power of a black body}}$$

GRAY BODY

* If a body absorbs a definite percentage of incident radiation irrespective of their wave length.

* The emissive power of a grey body is always less than black body

IRCHOFF'S LAW OF RADIATION

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3}$$

INTENSITY OF RADIATION (I_b)

$$I_b = \frac{E_b}{\pi}$$

LAMBERT'S COSINE LAW

$$E_b \propto \cos \theta$$

FORMULAE USED:

Hint D.8: 82

1. Emissive power (or) Total Emissive power:

$$E_b = \sigma T^4 \text{ W/m}^2$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$$

2. Wien's Law

$$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ mK}$$

3. Monochromatic Emissive power (or) Spectral Emissive power (E_b)

$$E_b = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$$C_1 = 0.374 \times 10^{-15} \text{ W-m}^2$$

$$C_2 = 14.4 \times 10^{-3} \text{ mK}$$

4. Max Emissive power (E_b)_{max}

$$(E_b)_{\text{max}} = C_4 T^5$$

$$C_4 = 1.307 \times 10^{-5}$$

5. Intensity of Radiation (I_b)

$$I_b = E_b / \pi$$

Black body High Emissivity } Grey body low Emissivity • compare black body

6. Absorptivity (α) = $\frac{\text{Radiation absorbed}}{\text{Incident radiation}}$

$$\text{Transmittivity } \tau = \frac{P_T}{P_R}$$

7. Reflectivity (ρ) = $\frac{P_R}{P_R}$

Problem for radiation:

Assuming sun to be black body emitting radiation with maximum intensity at $\lambda = 0.5 \mu$, calculate its surface temperature & emissive power $E_b = \sigma T^4$ $\frac{W}{m^2}$

$\lambda_{max} T = 2.9 \times 10^3 mK$

$1 \mu = 10^{-6} m$

A black body at 3000K emits radiation.

Calculate the following:

- Monochromatic Emissive power at $1 \mu m$ wave length. $1 \mu = 10^{-6} m$
- The length at which emission is maximum (λ_{max}) $\lambda_{max} T = 2.9 \times 10^3 mK$
- Maximum emissive power $(E_b)_{max}$ $E_b = 1.307 \times 10^{-5} T^5 \frac{W}{m^2}$ $E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1} \frac{W}{m^2}$
- Total emissive power $(E_b = \sigma T^4)$
- Calculate the total emissive of the furnace if it is assumed as a real surface having emissivity equal to $\frac{0.85}{\epsilon}$.

$$(E_b)_{real} = \epsilon \sigma T^4$$

A grey surface is maintained at a temp of $900^\circ C$ & max emissive power at that temp is $1.4 \times 10^{10} \frac{W}{m^2}$. Calculate the emissivity of the body & wave length corresponding to the maximum intensity of radiation.

$$① (E_b)_{max} = C_4 T^5$$

$$\epsilon = \frac{1.4 \times 10^{10}}{2.90 \times 10^{10}}$$

$C_4 = 1.307 \times 10^{-5} \frac{W}{m^2 K^5}$

800 W/m^2 of radiant energy is incident upon a surface, out of which 300 W/m^2 is absorbed, 100 W/m^2 is reflected & the remainder is transmitted through the surface. Calculate α , ρ , τ = $800 - (300 + 100)$

A black body of $\frac{1200 \text{ cm}^2}{1200 \times 10^{-4} \text{ m}^2}$ emits radiation at 1000 K .

Calculate the

- 1) Total rate of energy emission $E_b = \sigma T^4 \text{ W/m}^2$
- 2) Intensity of normal radiation $(I_n) = \frac{E_b}{T_1} \text{ W/m}^2$

Radiation Exchange b/w Surfaces:

- ① All surfaces are considered to be either black or gray.
- ② Radiation & reflection process are assumed to be diffuse.

Formulae used:

- ① Heat Exchange b/w two large parallel plates is given by

$$Q_{12} = \bar{\epsilon} \sigma A (T_1^4 - T_2^4)$$

$$\text{emissivity} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

ϵ_1 ϵ_2 Surface Emissivity ① ②

- ② Heat exchange b/w two large concentric cylinder or sphere is given by

$$Q_{12} = \bar{\epsilon} A_1 \sigma (T_1^4 - T_2^4)$$

for cylinder

$$A = 2\pi r L$$

$$\text{or } 2\pi L$$

$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$$

for sphere $4\pi r^2$

- ③ Heat transfer with n shields is given by

n -number shields

ϵ_s - Emissivity of shields

$$Q_{1n} = \frac{A \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2n}{\epsilon_s} - (n+1)}$$

Problems Radiation 2 Parallel Plates, Cylinders, Spheres

Calculate the net radiative interchange $\frac{W}{A}$ per sq. m. for two large plates at a temp of 900K & 500K respectively. Assume that the emissivity of hot plate is 0.9 & that of cold plate is 0.7. $\frac{W}{A}$ $\frac{W}{m^2}$

Estimate the net radiative heat exchange per square meter from a very large plate at a temp of 550°C & 32°C. Assume that emissivity of hot plate is 0.8 & cold plate is 0.6. $\frac{W}{A}$ $\frac{W}{m^2}$

Two large parallel plates are maintained at a temp of 900K & 500K respectively. Each plate has an area of $6 m^2$. Compare the net heat exchange b/w the plates for the following cases:

① Both plates are black $\epsilon = 1$

② Plates have an emissivity of 0.5 $\epsilon_1 = \epsilon_2 = 0.5$

Calc.
Calc.

③ Calculate the heat exchange by radiation b/w the surface of 2 long cylinders having radii 200mm & 600mm respectively. The axis of the cylinder are parallel to each other. The inner cylinder is maintained at a temp of 130°C & emissivity of 0.6. Outer cylinder is maintained at a temp of 30°C & emissivity of 0.5. $A = \pi D L$ $L = 4 = 4m$

④ A liquid oxygen is stored in double walled spherical vessel. Inner wall temp is -160°C & outer wall temp is 30°C. Inner dia of sphere is 20cm & outer dia is 32cm. Calculate the following:

① Heat transfer if emissivity of spherical surface is 0.05.

② Rate of evaporation of liquid oxygen if its rate of vaporization of latent heat is 200 kJ/kg, $\epsilon_1 = 0.05 = \epsilon_2$ Latent heat 200 kJ/kg $200 \times 10^3 J$

$$A = 4\pi r^2$$

[(-) sign indicates heat is transferred from outer surface to inner surface]. Rate of evaporation: $\frac{\text{Heat transfer}}{\text{Latent heat}}$

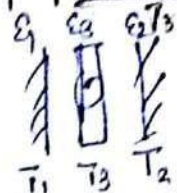
Calc

Problem for Radiation shield:

- ① Emittances of two large parallel plates maintained at 800°C & 300°C are 0.5 & 0.5 respectively. Find net radiant heat exchange per square meter for these plates.

Find the percentage reduction in heat transfer when a polished aluminium radiation shield of emissivity 0.05 is placed b/w them. Also find the temp of the shield.

$$\frac{Q_{\text{between plates}}}{Q_{\text{w/o sh}}} = \frac{Q_{12}}{Q_{12}}$$



- ② A pipe dia 30cm, carrying steam runs in a large room & is exposed to air at a temp of 25°C . The surface temp of the pipe is 300°C . Calculate the loss of heat to surrounding per meter length of pipe due to thermal radiation. The emissivity of pipe surface is 0.8.

What would be the loss of heat due to radiation of the pipe which is enclosed in a 55cm dia brick of emissivity 0.91? $A = \pi D L$

$$Q/L = \text{W/m}$$

- ③ Emittances of two large parallel plates maintained at T_1 & T_2 are 0.6 & 0.6 respectively. Heat transfer is reduced 75 times when a polished aluminium radiation shield of emissivity 0.04 are placed in b/w them. Calculate the number of shields required.

$$Q_{1n} = \frac{A \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2n}{\epsilon_3} - (n+1)}$$

$$\frac{Q_{\text{unshielded}}}{Q_{\text{with shield}}} = \frac{Q_2}{Q_{1n}} = 75$$

$$= \frac{\frac{A \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}}{\frac{A \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2n}{\epsilon_3} - (n+1)}}$$

By 11
③

① Introduction:

The heat is transferred from one body to another without any transmitting medium.
It is an electromagnetic phenomenon. ($3 \times 10^8 \text{ m/s}$)

② Emission Properties:

- ① The wavelength or frequency of radiation
- ② the temp of surface
- ③ the nature of the surface.

③ Emissive power [E_b]

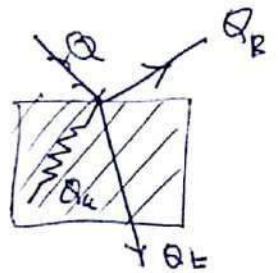
It is defined as the total amount of radiation emitted by a body per unit time & unit area. It is expressed in W/m^2

④ MONOCHROMATIC EMISSIVE POWER ($E_{b\lambda}$)

The energy emitted by the surface at a given length per unit time per unit area in all directions is known as MEP.

⑤ Absorption, Reflection & Transmission.

When the radiant energy falling on a body, 3 things happen.
A part is reflected back, a part is transmitted through the surface, and the remainder is absorbed.



$$Q = Q_a + Q_r + Q_t$$

$\therefore \alpha$ for Absorptivity:

$$\frac{Q}{Q} = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q}$$

where α = Absorptivity
 P = Reflectivity
 τ = Transmissivity

$$1 = \alpha + P + \tau$$

$$\alpha = \frac{\text{Radiation absorbed}}{\text{Incident radiation}}, \quad P = \frac{\text{Radiation Reflected}}{\text{Inc / Rad}}, \quad \tau = \frac{\text{Radiation Transmitted}}{\text{Inc / Rad}}$$

⑥ Black Body.

① A Black body absorbs all incident radiation, regardless of wave length & direction

② For a prescribed temp & wavelength, no surface can emit more energy than black body.



④ Planck's Distribution Law:

The relationship b/w the monochromatic emissive power of a black body and wave length of radiation at a particular temp is given by the following expression

$$E_{b\lambda} = \text{monochromatic emissive power } W/m^2$$

$$\lambda = \text{Wave length (m)}$$

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$$C_1 = 0.374 \times 10^{-15} W \cdot m^2$$

$$C_2 = 14.4 \times 10^{-3} mK$$

⑤ Wien's displacement Law:

Wien's law gives the relationship b/w temp & wave length corresponding to the maximum spectral emissive power of the black body at the temp.

$$\lambda_{max} T = 2898 \mu m K$$

$$\lambda_{max} T = 2.9 \times 10^{-3} mK$$

$$\therefore \mu = 10^{-6} m$$

⑥ Stefan-Boltzmann Law

The emissive power of a black body is proportional to the fourth power of absolute temp.

$$E_b \propto T^4 \quad / \quad \therefore E_b = \sigma T^4$$

E_b - Emissive power

σ - Stefan Boltzmann constant
 $5.67 \times 10^{-8} W/m^2 K^4$

⑦ Maximum Emissive power (E_b)_{max}

A combination of Planck's law and Wien's displacement law yields the condition for the max monochromatic emissive power for a black body.

$$(E_b)_{max} = C_4 T^5 \Rightarrow \frac{1.307 \times 10^5}{C_4} T^5 \quad (\text{Radiation constant})$$

⑧ Emissivity $\Rightarrow \epsilon = E/E_b$

The ratio of emissive power of any body to the emissive power of a black body at the same temp.

⑨ Grey Body: If a body absorbs a definite % of incident radiation irrespective of their wave length, the body is known as grey body. Emissive power of grey body < emissive power of black body.

⑩ Intensity of Radiation (I_b)

$$I_b = E_b / \pi$$

⑪ Lambert's cosine law

$$E_b \propto \cos \theta$$

⑤ A furnace wall emits radiation at 2000K . Treating it as black body radiation, calculate.

① Monochromatic radiant flux density at given wave length. (E_b)

② Wave length at which emission is maximum & the corresponding (λ_{max})
emissive power. $\lambda_{max} T = 2.9 \times 10^{-3}$

③ Total emissive power $E_b = \sigma T^4$

⑥ The temp of a black body, surface 0.25m^2 area is 650°C

Calculate,

① The total rate of energy emission. $E_b = \sigma T^4$

② The intensity of normal radiation = $\frac{E_b}{\pi}$

③ The wave length of max monochromatic emissive power.
 $\lambda_{max} T = 2.9 \times 10^{-3}$

Problem for Radiation

1) Assuming sun to be black body emitting radiation with maximum intensity at $\lambda = 0.5 \mu$, calculate its surface temp & emissive power. $\lambda_{max} = 2.9 \times 10^{-3}$ $E_b = \sigma T^4$

2) A black body at 3000K emits radiation. Calculate the following:

① Monochromatic emissive power at $1 \mu m$ wave length, $E_{b\lambda}$ at $1 \mu m = 1 \times 10^{-6} m$

② Wave length at which emission is maximum, (λ_{max}) Wien's law

③ maximum emissive power $(E_{b\lambda})_{max}$

④ Total emissive power (E_b) is T^4

⑤ Calculate the total emissive of the furnace if it is assumed as a real surface having emissivity equal to $\frac{0.85}{\epsilon}$

① E_b

$$(E_b)_{real} = \epsilon \sigma T^4$$

③ A gray surface is maintained at a temp of $900^\circ C$ & the max. emissive power at that temp is $1.4 \times 10^6 W/m^2$. $(E_{b\lambda})_{max}$

Calculate the emissivity of the body (ϵ)

$$\epsilon = \frac{E}{E_{max}}$$

$$(E_{b\lambda})_{max} = C_1 T^5$$

$$\lambda_{max} = 0.52 \times 10^{-6} m$$

④ The sun emits max radiation at $\lambda = 0.52 \mu$. Assuming the sun to be a black body. Calculate the surface temp of the sun.

Also calculate the monochromatic emissive power of the sun's surface

Temp T , $(E_{b\lambda})$

$$\lambda_{max} = 2.9 \times 10^{-3}$$

Formulae Used

① Emissive power (or)

Total emissive power

$$E_b = \sigma T^4 \text{ W/m}^2$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$$

Stefan Boltzmann constant

② Wien's Law

$$\lambda_{\max} T = 2898 \mu\text{mK} = 2.89 \times 10^{-3} \text{ mK}$$

$$\mu = 10^{-6} \text{ m}$$

③ Monochromatic Emissive power (or)

Spectral Emissive power

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$$C_1 = 0.374 \times 10^{-16} \text{ Wm}^2$$

$$C_2 = 14.4 \times 10^3 \text{ mK}$$

④ Maximum Emissive power ($E_{b\lambda}$)_{max}

$$[E_{b\lambda}]_{\max} = C_4 T^5$$

$$C_4 = 1.307 \times 10^{-5}$$

⑤ Intensity of Radiation (I_n)

$$I_n = \frac{E_b}{\pi}$$

$$\sigma T_1^4 - \sigma T_2^4$$

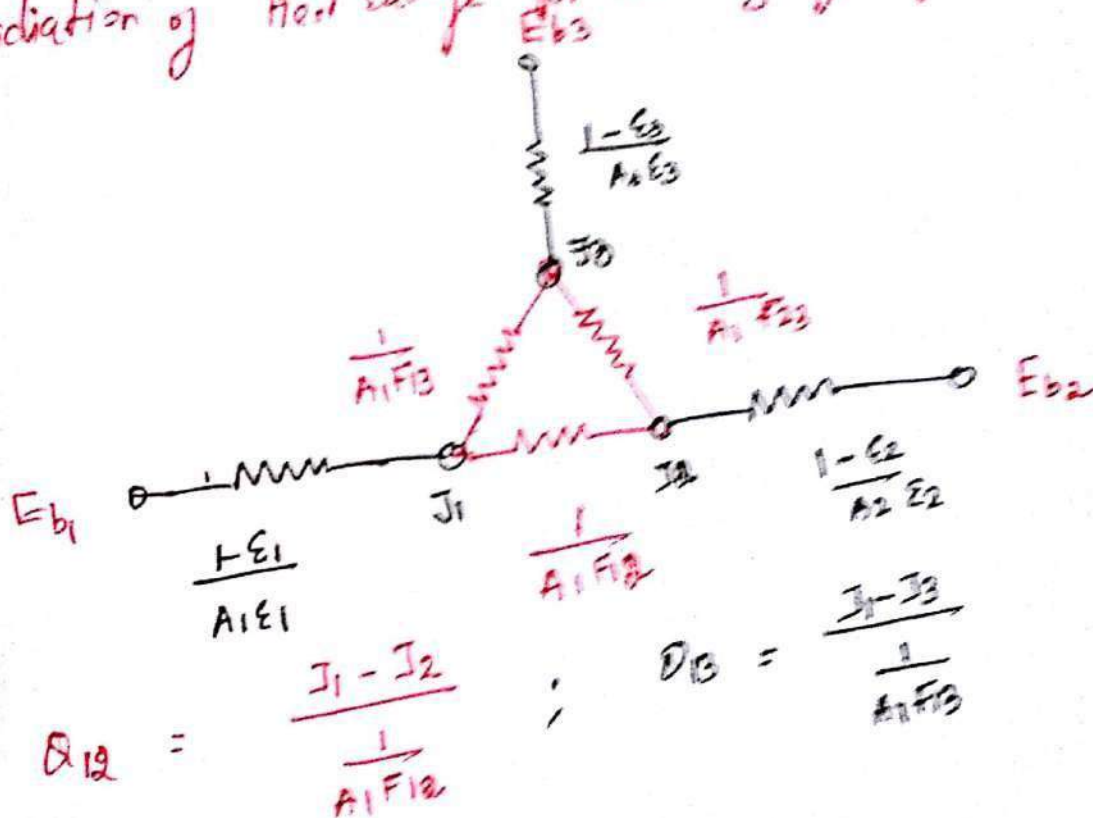
$$Q_{12} =$$

$$\frac{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}{\epsilon_1 = \epsilon_2 = 1}$$

Where F_{12} - shape factor

$$Q_{12} = \sigma (T_1^4 - T_2^4) A_1 F_{12}$$

Radiation of Heat exchange for three gray surfaces.



$$Q_{12} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}}; \quad D_{13} = \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}}$$

$$E_{b3} = \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}}$$

13

UNIT: 5

MASS TRANSFER

Introduction:

In a system consisting of 2 or more components whose concentrations vary from point to point, there is a natural tendency for species (particles) to be transported from a region of higher concentration side to a " " lower " " side.

Example of mass transfer

- * Humidification of air in cooling tower.
- * Evaporation of petrol in the carburettor of an IC engine
- * Transfer of water vapour into dry air
- * Dissolution of sugar added to a cup of coffee

Modes of Mass transfer

- ① Diffusion mass transfer
- ② Convective mass transfer

Diffusion mass transfer:

- * Molecular diffusion
- * Eddy diffusion

Molecular diffusion:

The transport of matter on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquid or gases

Concentrations:

(i) Mass concentration = $\frac{\text{Mass of component}}{\text{Unit volume of mixture}}$ kg/m^3

(ii) Molar concentration = $\frac{\text{Number of molecules of component}}{\text{Unit volume of mixture}}$

kg-mole/m^3

$C_A = \frac{\rho_A}{M_A} = \frac{\text{Density of component A}}{\text{Molecular weight of component A}}$

(iii) Mass fraction = $\frac{\text{Mass concentration of a species}}{\text{Total mass density}}$

$m_A = \frac{\rho_A}{\rho}$

(iv) mole fraction = $\frac{\text{Mole concentration of a species}}{\text{Total molar concentration}}$

$x_A = C_A / C$

Fick's law of diffusion:

$\boxed{-D_{ab} \frac{dC_a}{dy}} = N_a = \frac{m_a}{A}$ - molar flux units kg-mole/s-m^2
 or
 mass flux - kg/s-m^2

D_{ab} - Diffusion co-efficient of species a & b m^2/s

$\frac{dC_a}{dy}$ - Concentration gradient.

- Used in industries for water pumping

Steady state diffusion through a Plane membrane:

$$\text{Molar flux } \frac{m_a}{A} = \frac{D_{ab}}{L} [C_{a2} - C_{a1}]$$

C_{a1} - inner side } concentration kg. mol/m^3
 C_{a2} - outer side }

For cylinder,

$$L = r_2 - r_1$$

$$A = \frac{2\pi L(r_2 - r_1)}{\ln(r_2/r_1)}$$

For sphere

$$L = r_2 - r_1$$

$$A = 4\pi r_1 r_2$$

r_1 - inner } radius (m)
 r_2 - outer }

L - length (m)

using gears to reduce the power consumption level in order to take the water from the interior level make this project most successful in the future

• Gaseous hydrogen is stored in a rectangular container. The walls of the container are of steel having $\frac{2.5 \text{ mm}}{L}$ thick. At the inner surface of the container the molar concentration of hydrogen in the steel is $\frac{1.2 \text{ kg}}{\text{cm}^3} \text{ mole/m}^3$ while at the outer surface the molar concentration is zero. Take $D_{\text{H}_2} = 0.24 \times 10^{-12} \text{ m}^2/\text{s}$.

Hydrogen gases at $\frac{3 \text{ bar}}{P_1}$ & $\frac{1 \text{ bar}}{P_2}$ are separated by a plastic membrane having thickness 0.25 mm . The binary diffusion coefficient of hydrogen in the plastic is $\frac{9.1 \times 10^{-8} \text{ m}^2/\text{s}}{D_{\text{H}_2}}$. The

Solubility of H_2 in the membrane is $2.1 \times 10^3 \text{ kg-mole/m}^3\text{-bar}$.

As isotherm temp condition of 20°C is assumed. Calculate follow

- (i) Molar concentration of H_2 on both side - C_{H_1} & $C_{\text{H}_2} \rightarrow S_{\text{H}_2} \times \text{Outer pressure}$
- (ii) Molar flux of hydrogen $\frac{\text{mole}}{\text{m}^2 \cdot \text{s}}, \frac{D_{\text{H}_2}}{L} [C_{\text{H}_1} - C_{\text{H}_2}]$
- (iii) Mass flux of hydrogen - molar flux \times molecular weight

$\text{H}_2 = 2$

ϕ

Solved Problem on concentration:

① A vessel contains a binary mixture of O_2 & N_2 with partial pressure in the ratio $0.21^{V.P.}$ & $0.79^{X.P.}$ at $25^\circ C$. If the total pressure of mixture is 1.1 bar , calculate following:

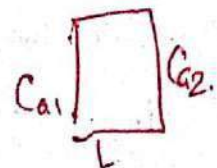
- (i) Molar concentration, (C) $\frac{P_{O_2}}{P_{T}} = \frac{P_{O_2}}{C R T}$ kg-mole/m^3
 (ii) Mass densities $P_{O_2} = C_{O_2} \times M_{O_2}^{32}$ $\left. \begin{array}{l} C = \rho/M \\ \rho = C \times M \end{array} \right\}$ Overall density $(\rho) = P_{O_2} + P_{N_2}$
 $P_{N_2} = C_{N_2} \times M_{N_2}^{28}$ $\parallel C_{O_2} + C_{N_2} = C$
 (iii) Mass fraction $m_{O_2} = \rho_{O_2} / \rho$ $m_{N_2} = \frac{P_{N_2}}{P}$
 Molar fraction of each species $x_{O_2} = \frac{C_{O_2}}{C}$ $x_{N_2} = \frac{C_{N_2}}{C}$

② A mixture of O_2 & N_2 with their partial pressure in the ratio 0.21 & 0.79 in a container at $25^\circ C$. Calculate the molar concentration, the mass density, and the mass fraction of each species for a total pressure of 1 bar . What would be the average molecular weight of the mixture?
 $M = P_{O_2} M_{O_2} + P_{N_2} M_{N_2}$

Solved Problems on members

① Helium diffuses through a plane membrane of 2 mm thick. At the inner side the concentration of helium is $0.025 \text{ kg mole/m}^3$. At the outer side " " " " is $0.007 \text{ kg mole/m}^3$. What is diffusion flux? of helium through the membrane. Assume diffusion co-efficient of helium with respect to plastic $1 \times 10^{-9} \text{ m}^2/\text{s}$

diffusion flux of helium $\frac{m_A}{A} = \frac{D_{AB}}{L} [C_{A1} - C_{A2}] = \frac{1 \times 10^{-9} \text{ kg-mole}}{\text{s-m}^2}$



$\frac{\text{kg-mole}}{\text{m}^2 \text{ s}}$

molar flux $\frac{\text{kg-mole}}{\text{s-m}^2}$

mass flux $\frac{\text{kg}}{\text{s-m}^2}$

Oxygen at 0.5°C & Press of 2 bar is flowing through a rubber pipe of inner dia 25mm & wall thickness 2.5mm. The diffusivity of O_2 through rubber is $0.21 \times 10^{-9} \text{ m}^2/\text{s}$ & the solubility of O_2 in rubber is $3.12 \times 10^{-3} \frac{\text{kg} \cdot \text{m}^{-3}}{\text{bar}}$

Find the loss of O_2 by diffusion per m length of pipe.

C_{a1} = Solubility in
 C_{a2} = Solubility

$$\text{molar flux} = \frac{(m)}{A} = \frac{D_{ab} (C_{a1} - C_{a2})}{L}$$

$$\text{And } \frac{\text{kg} \cdot \text{mol}}{\text{s}}$$

$$r_1 = r_2 = 12.5 \text{ mm}$$

$$r_2 = \text{Inner radius} + \text{thickness}$$

$$L = r_2 - r_1$$

$$A = \frac{2\pi L (r_2 - r_1)}{\ln r_2/r_1}$$

$$L = 1 \text{ m}$$

CO_2 and air experience equimolar counter diffusion in a circular tube whose length & diameters are 1.2 m & 60 mm respectively. The system is at a total pressure of 1 atm & a temp of 273 K. The ends of the tube are connected to large chambers. Partial pressure of CO_2 at one end is 20 mm Hg while other end is 40 mm Hg. Calculate

- ① mass flow of CO_2 kg/s
- ② mass flow of air kg/s

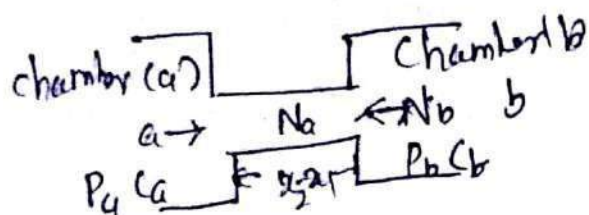
$$P_{\text{CO}_2} = \frac{20}{760} = 0.0263 \text{ atm} = 0.0263 \times 101.325 \text{ kPa}$$

$$P_{\text{air}} = \frac{90}{760} \text{ atm} = 0.1184 \text{ atm}$$

$$\text{D}_{\text{CO}_2} = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$$

Steady state Equimolar counter diffusion

two chamber (large) @ a & b connected by a passage



Nety steel stn molar diffu

$$\text{molar flux } N_a = \frac{m_a}{A} = \frac{D}{L T} \left[\frac{P_{a1} - P_{a2}}{x_2 - x_1} \right]$$

$$\text{Molar flux } N_b = \frac{m_b}{A} = \frac{D}{L T} \left[\frac{P_{b1} - P_{b2}}{x_2 - x_1} \right]$$

$$\frac{m_a}{A} - \text{molar flux } \text{kg-mole} / \text{s-m}^2$$

$$D - \text{Diffusion Co-efficient } \text{m}^2/\text{s}$$

$$G - \text{Universal Constant } 8314 \text{ J/kg-mole-K}$$

$$\begin{matrix} P_{a1} \\ P_{a2} \end{matrix} \left. \begin{matrix} \text{Partial pressure of constituent 1 \& 2} \end{matrix} \right\} \text{ in N/m}^2$$

Problems for equimolar counter diffusion.

Q Ammonia and air are in equimolar counter diffusion in a cylindrical tube of 2.5 cm dia. & 1.5 m length. The total pressure is 1 atm & the temp is 25°C. One end of the tube is connected to a large reservoir of ammonia & the other end of tube is open to atmosphere. If the mass diffusivity for the mixture is $22.8 \times 10^{-4} \text{ m}^2/\text{s}$.

Calculate the following

- 1) Mass rate of ammonia in kg/h
- 2) mass rate of air in kg/h.

$$\text{Total pres } P = P_a + P_b$$

$$A = \pi/4 d^2$$

$$1) \text{ find } m_a - \text{kg-mol/s}$$

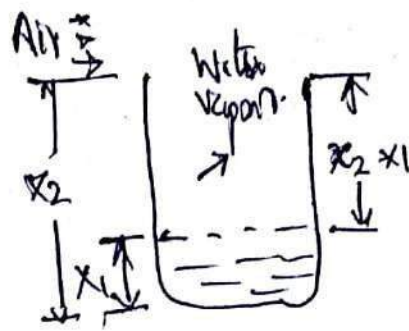
$$2) m_a \times \text{molecular weight of ammonia}$$

$$\text{kg-mol} \times 18.2$$

$$57.6 \times 10^{-2} \text{ kg/h}$$

$$1) m_a = m_b \quad \text{mass rate of air}$$

isothermal evaporation of water in to air.



Assumptions:-

- 1) The system is isothermal & total pressure remains constant
- 2) Syst is steady state condition.
- 3) Air & water vapour behave as ideal gases.

From Fick's law diffusion.

$$\text{molar flux } \frac{m_a}{A} = \frac{D_{ab}}{GT} \cdot \frac{P}{(x_2 - x_1)} \ln \left(\frac{P_{a2}}{P_{a1}} \right)$$

$$\text{molar flux } \frac{m_a}{A} = \frac{D_{ab}}{GT} \cdot \frac{P}{(x_2 - x_1)} \ln \left(\frac{P - P_{w2}}{P - P_{w1}} \right)$$

$$\frac{m_a}{A} \text{ molar flux kg-mole/s-m}^2$$

P - Total pressure

P_{w1} \rightarrow Partial pressure of water vapour corresponding to saturation temp at 1 mbar

P_{a2} \rightarrow Partial pressure of dry air at 20 N/m².

Solved Problem on Fick's law of diffusion of water into air.

① Determine the diffusion rate of water from the bottom of a test tube of 25mm diameter and 35mm long into dry air at 25°C . Take diffusion co-efficient of water in air is $0.28 \times 10^{-4} \text{ m}^2/\text{s}$.
 $25^{\circ}\text{C} = 298\text{K}$ $R = 8.314 \text{ J/mol}\cdot\text{K}$

②. An open pan 20cm in diameter and 8cm deep contains water at 25°C and is exposed to dry atmospheric air. If the rate of diffusion of Water Vapour is $8.64 \times 10^{-4} \text{ kg/h}$, estimate the diffusion co-efficient of water in air.

③ Estimate the diffusion rate of water from the bottom of a test tube 10mm in diameter and 15cm long into dry atmosphere air at 25°C . Diffusion co-efficient of water into air is $0.255 \times 10^{-4} \text{ m}^2/\text{s}$.

(water - moving)
water - steam

HEAT EXCHANGERS:-

Introduction:-

A heat exchanger is defined as an equipment which transfers the heat from a hot fluid to a cold fluid.

Types of Heat Exchanger :-

- ① Nature of heat exchange process
- ② Relative direction of fluid motion
- ③ Design & constructional features
- ④ Physical state of fluids.

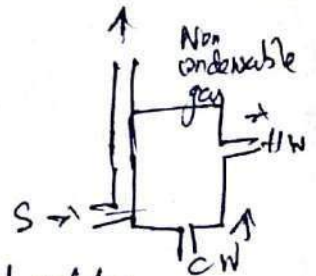
① Nature of heat exchange process:-

On the basis of the nature of heat exchange process heat exchangers are classified as:

- ① Direct contact heat exchanger or Open HE
- ② Indirect C HE

① OHE

The heat exchange takes place by direct mixing of hot & cold fluid. This H transfer is usually accompanied by mass transfer.
Ex: cooling tower, Direct contact heat exchangers.



② I Contact HE

The transfer of heat b/w 2 fluids could be carried out by transmission through a wall which separates the 2 fluids.

- ① Regenerators
- ② Recuperators or Surface heat exchangers

(i) Regenerators: In the type of HE, hot & cold fluids flow alternately through the same space.
Ex: IC engine, gas turbine.

(ii) Recuperators ① Surface heat exchanger.

This is the most common type of HE in which the hot & cold fluid do not come into direct contact with each other but are separated by a tube wall or a surface.
Automobile radiator, Air preheater, Economiser

(3-1)