PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Approved by **AICTE** and Affiliated to **Anna University**)

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Department of Mechanical Engineering

Regulation 2021
III Year – VI Semester
CME386 GAS DYNAMICS & JET PROPULSION

CME386 GAS DYNAMICS AND JET PROPULSION L T P C 3 0 0 3

COURSE OBJECTIVES

1. To study the fundamentals of compressible flow concepts and the use of gas tables.
2. To learn the compressible flow behaviour in constant area ducts.
3. To study the development of shock waves and its effects.
4. To study the types of jet engines and their performance parameters.
5. To learn the types of rocket engines and their performance parameters.

UNIT – I BASIC CONCEPTS AND ISENTROPIC FLOWS 9

Energy and momentum equations of compressible fluid flows, Concepts of compressible flow – Mach waves and Mach cone. Flow regimes, effect of Mach number on compressibility. Stagnation, static, critical properties and their interrelationship. Isentropic flow and its relations. Isentropic flow through variable area ducts – nozzles and diffusers. Use of Gas tables.

UNIT – II COMPRESSIBLE FLOW THROUGH DUCTS 9

Flows through constant area ducts with heat transfer (Rayleigh flow) and Friction (Fanno flow) – variation of flow properties. Choking. Isothermal flow with friction. Use of Gas tables.

UNIT – III NORMAL AND OBLIQUE SHOCKS 9

Governing equations - Rankine-Hugoniot Relation. Variation of flow parameters across the normal and oblique shocks. Prandtl – Meyer expansion and relation. Use of Gas tables.

UNIT – IV JET PROPULSION 9

Theory of jet propulsion – thrust equation – Performance parameters - thrust, power and efficiency. Operation, cycle analysis and performance of ram jet, turbojet, turbofan, turbo prop and pulse jet engines.

UNIT – V SPACE PROPULSION 9

Types of rocket engines and propellants. Characteristic velocity – thrust equation. Theory of single and multistage rocket propulsion. Liquid fuel feeding systems. Solid propellant geometries. Orbital and escape velocity. Rocket performance calculations.

TOTAL:45 PERIODS

OUTCOMES: At the end of the course the students would be able to

1. Apply the fundamentals of compressible flow concepts and the use of gas tables.
2. Analyze the compressible flow behaviour in constant area ducts.
3. Analyze the development of shock waves and its effects.
4. Explain the types of jet engines and their performance parameters.
5. Explain the types of rocket engines and their performance parameters.

TEXT BOOKS:

1. Anderson, J.D., “Modern Compressible flow”, Third Edition, McGraw Hill, 2003.
2. S.M. Yahya, “Fundamentals of Compressible Flow with Aircraft and Rocket propulsion”, New Age International (P) Limited, 4th Edition, 2012.

REFERENCES:

1. R. D. Zucker and O Biblarz, “Fundamentals of Gas Dynamics”,

2. 2nd edition, Wiley, 2011. 2. Balachandran, P., “Fundamentals of Compressible Fluid Dynamics”, Prentice-Hall of India, 2007.

3. Radhakrishnan, E., “Gas Dynamics”, Printice Hall of India, 2006.
4. Hill and Peterson, “Mechanics and Thermodynamics of Propulsion”, Addison – Wesley, 1965. 5.

UNIT-1

COMPRESSIBLE FLUID FLOW - FUNDAMENTALS

Introduction

Gas dynamics mainly concerned with the motion of gases and its effects .It differ from fluid dyna10mics .Gas dynamics deals with the study of compressible flow when it is in motion.

Applications

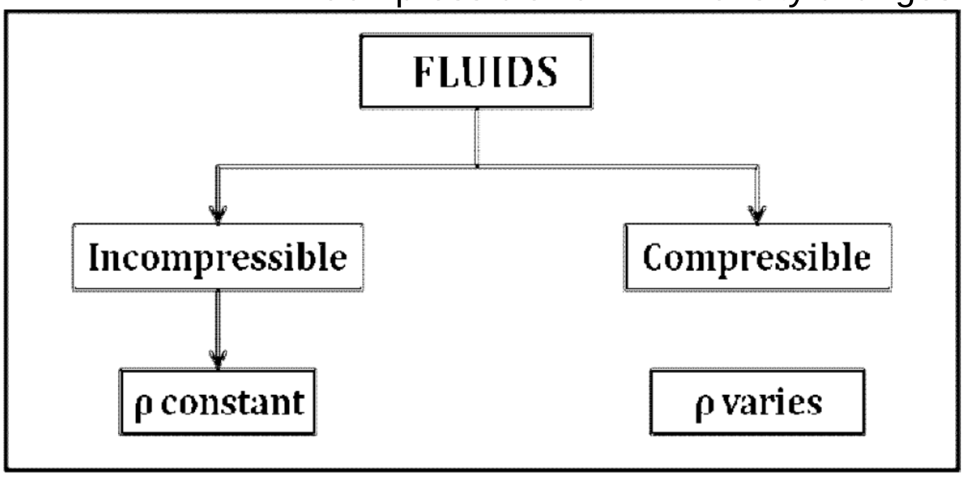
- It is used in Steam and Gas turbines
- High speed aero dynamics
- Jet and Rocket propulsion
- High speed turbo compressor

The fluid dynamics of compressible flow problems which involves the relation between forse, density, velocity and mass etc.Therfore the following laws are frequently used for solving the dynamic problems.

1. Steady flow energy equation
2. Entropy relations
3. Continity equation
4. Momentum equation

Compressible Flows

– Compressible flow - Density changes



Compressible vs. Incompressible Flow

- A flow is classified as incompressible if the density remains nearly constant.
- Liquid flows are typically incompressible.

- Gas flows are often compressible, especially for high speeds.
- Mach number, M = V/c is a good indicator of whether or not compressibility effects are important.
  - $M < 0.3$: Incompressible
  - $M < 1$: Subsonic
  - $M = 1$: Sonic
  - $M > 1$: Supersonic
  - $M >> 1$: Hypersonic

Compressibility:

A measure of the relative volume change with pressure for a given process. Consider a small element of fluid of volume v, the pressure exerted on the sides of the element is p. Assume the pressure is now increased by an infinitesimal amount dp. The volume of the element will change by a corresponding amount dv , here the volume decrease so dv is a negative quantity. An incompressible fluid cannot be compressed and has relatively constant density throughout. Liquid is an incompressible fluid. A gaseous fluid such as air, on the other hand, can be either compressible or incompressible. Generally, for theoretical and experimental purposes, gases are assumed to be incompressible when they are moving at low speeds-- under approximately 220 miles per hour. The motion of the object traveling through the air at such speed does not affect the density of the air. This assumption has been useful in aerodynamics when studying the behavior of air in relation to airfoils and other objects moving through the air at slower speeds.

In thermodynamics and fluid mechanics, compressibility is a measure of the relative volume change of a fluid or solid as a response to a pressure (or mean stress) change.

Steady Flow Energy Equation

From first law of Thermodynamics, we know that the total energy entering the system is equal to total energy leaving the system. This law is applicable to the steady flow systems.

Consider an open system-through which the working substance flows as a steady rate. The working substance entering the system at (1) and leaves the system at (2).

Let,

- P_1 - Pressure of the working substance entering the system *(N/m²)*
- V_1- Specific volume of the working substance entering the system (m^3/kg)
- C_1- Velocity of the, working substance entering the system *(m/s)*
- U_1 - Specific internal energy of the working substance entering the system *(Jlkg)*

- Z_1 - Height above the datum level for inlet in (m).

P_2, v_2 , c_2, U_2 and Z_2 - Corresponding values for the working substance leaving the system.
Q - Heat supplied to the system *(J/kg)*
W - Work delivered by the system (J/kg).

Total energy entering the system = Potential energy (gZ_1) + Kinetic energy ($c_1^2/2$)
+ Internal energy (U_1) + Flow energy ($p_1 v_1$) + Heat (Q)

Total energy leaving the system = Potential energy (gZ_2) + Kinetic energy ($c_2^2/2$)
+ Internal energy (U_2) + Flow energy ($p_2 v_2$) +

Work (W) From first law of Thermodynamics,

Total energy entering the system = Total energy leaving the system

$$gZ_1 + (c_1^2/2) + U_1 + p_1 v_1 + Q = gZ_2 + (c_2^2/2) + U_2 + p_2 v_2 + W$$

$$\mathbf{gZ_1 + (c_1^2/2) + h_1 + Q = gZ_2 + (c_2^2/2) + W}$$

[i.e h = U + pv]
The above equation is known as steady flow energy equation.

Stagnation Enthalpy

Stagnation enthalpy of a gas or vapor is its enthalpy when it is adiabatically decelerated to zero velocity at zero elevation.

$$h_1 + \frac{1}{2}C_1^2 = h_2 + \frac{1}{2}C_2^2$$

where,

- $h_1=h$,
- $c_1=c$
- for the initial state $h_2=h_0$
- for the final state $c_2=0$

By substituting this in above equation,

$$\boxed{h + \frac{1}{2}C^2 = h_0}$$

Where,
h_0 = Stagnation

enthalpy h =
Static enthalpy
c = Fluid velocity *m / s*

In an adiabatic energy transformation process the stagnation enthalpy remain constant.

Stagnation Temperature (or) Total temperature (T0)

Stagnation temperature of a gas is the temperature when it is isentropically decelerated to zero velocity at zero elevation.

We know that, Stagnation enthalpy

$$h+c^2/2=h_0$$

We have stagnation enthalpy and static enthalpy for a perfect gas is,

$$h_0=C_pT_0$$

$$h=C_PT$$

By substituting this in above equation,

$$C_pT_0 = C_PT+ c^2/2$$

Divide by Cp through out the eqn.

$$T_0 = T+ c^2/2C_p$$

$$T_0/ T= 1+C^2 /(2\gamma RT/\gamma-1)$$
$$T_0/ T= 1+(\gamma-1)/2 M^2$$

Where

- To= stagnation temperature
- T = static temperature
- M = Mach number (C/a)

Stagnation Pressure, [P_o] or total pressure

Stagnation pressure of a gas when it is isentropically decelerated to

$$\frac{P_2}{P_1}=\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

zero velocity at zero elevation. For isentropic flow.

For stagnation condition,

$$P_2 = P_0$$

$$T_2 = T_0$$

$$P_1 = P$$

$$T_1 = T$$

$$\Rightarrow \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} \times M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} \times M^2\right]^{\frac{\gamma}{\gamma-1}}$$

Stagnation velocity of sound (a_o):

We know that the acoustic velocity of sound

$a = \sqrt{\gamma RT}$

.

Various regions of flow

The adiabatic energy equation for a perfect gas is derived in terms of velocity of fluid(C) and Velocity of sound [a_0]

We have stagnation enthalpy and static enthalpy for a perfect gas is

$$h_0 = c_p T_0$$

$$h = c_p T$$

C_i Case γR

At

$_h T = 0,$

$h = o$

Put

$c = c_{max}$

$$h_0 = C_p T_0 + \frac{1}{2} C^2{}_{max}$$

$$h_0 = \frac{1}{2} C^2{}_{max}$$

Put

at

$C = 0$

$a = a_0$

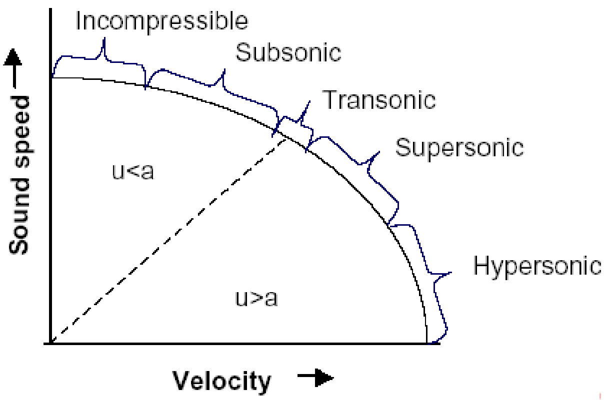
$h_0 = h$

$$h_0 = \frac{{a_0}^2}{\gamma - 1}$$

$$h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{{a_0}^2}{\gamma - 1} = \frac{1}{2} C^2{}_{max}$$

$$\boxed{h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{{a_0}^2}{\gamma - 1} = \frac{1}{2} C^2{}_{max}}$$

Flow Regime Classification



- Subsonic Flow (0.8 < M_0)
- Transonic Flow (0.8 > M_0 > 1.2)
- Supersonic Flow (M_0 > 1.2)
- Hypersonic Flow (M_0 > 5)

- **Incompressible region**

In incompressible flow region fluid velocity (c) is much smaller than
the sound velocity (a). Therefore the Mach number (M = c/a) is

very low.

Eg: flow through nozzles

- **Subsonic flow region**

The subsonic flow region is on the right of the incompressible flow region. In subsonic flow, fluid velocity (c) is less than the sound velocity (a) and the Mach number in this region is always less than unity.

i.e. $M = c/a < 1$.

Eg: passenger air craft

- **Sonic flow region**

If the fluid velocity (c) is equal to the sound velocity (a), that type of flow is known as sonic flow. In sonic flow Mach number value is unity.

$M = c/a = 1 \Rightarrow c = a$. Eg: Nozzle throat

- **Transonic flow region**

If the fluid velocity close to the speed of sound, that type of flow is known as transonic flow .In transonic flow, Mach number value is in between 0.8 and 1.2. i.e. $0.8 < M < 1.2$.

- **Supersonic flow region**

The supersonic region is in the right of the transonic flow region. In supersonic flow, fluid velocity (c) is more than the sound velocity (a) and the Mach number in this region is always greater than unity.

i.e. $M = c/a > 1$.Eg: military air crafts

- **Hypersonic flow region**

In hypersonic flow region, fluid velocity (c) is much greater than sound velocity (a).In this flow, Mach number value is always greater than 5.

i.e. $M = c/a > 5$. Eg: rockets

Reference Velocities

In compressible flow analysis it is often convenient to express fluid velocities is non dimensional forms**.**

- Local velocity of sound
- Stagnation velocity of sound
- Maximum velocity of fluid
- Critical velocity of fluid/sound. $C^* = a^*$

Maximum velocity of fluid:

From adiabatic energy equation has two components of the total energy: the enthalpy h and the kinetic energy .If kinetic energy is absent the total energy is entirely energy represented by the stagnation enthalpy h0 .the other extreme conditions which can be conceived is when the entire energy is made up of kinetic energy only=0 and C = Cmax. The fluid velocity (Cmax) corresponding to this condition is the maximum velocity that would be achieved by the fluid when it is accelerated to absolute zero temperature (T = 0, T = 0) in an imaginary adiabatic expansion process.

$$C\max = \sqrt{2h_0}$$

For a perfect gas

$$C\max = \sqrt{2C_pT_0}$$

$$= \sqrt{\frac{2\gamma}{\gamma-1}RT_0}$$

Equation $h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2}C^2 = \frac{a_0{}^2}{\gamma-1} = \frac{1}{2}C^2{}_{\max}$ and equation

$$= \sqrt{\frac{2\gamma}{\gamma-1}RT_0}$$ Yield

$$\frac{C_{\max}}{a_0} = \sqrt{\frac{2}{\gamma-1}} = 2.24\left(for\gamma = 1.4\right)$$

Critical velocity of sound

It is the velocity of flow that would exist if the flow is isentropically accelerated or decelerated to unit Mach number (critical condition).

We have
Considering the *section (where M = 1) and its

$$\Rightarrow \frac{T_0}{T^*} = \left[\frac{\gamma+1}{2}\right]$$

Multiplying both sides with γ R

$$T_0 = T^*\left[\frac{\gamma+1}{2}\right]$$

$t^2 q - {}'^2 \times \frac{+1}{2}$

$\frac{C'}{0} \quad \frac{C*}{0} \quad \sqrt{\frac{3}{\gamma+1}}$

From Equations

$\frac{C_{max}}{c} - \sqrt{\frac{2}{\gamma-1}}$ and $\frac{a^*}{ao} - \frac{C^*}{ao} - \sqrt{\frac{2}{\gamma+1}}$

$\frac{C_{max}}{a^*} = \frac{C_{max}}{a^*} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.41 \quad 01$) Fiivilly use can ufittcn the equation is

$$C_{max} = a \times \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$C^2_{max} = a^{2*} \times \frac{\gamma+1}{\gamma-1}$$

$$h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2}C^2 - \frac{1}{2}C^2{}_{max} = \frac{1}{2}a^{2*} \quad \frac{+}{\gamma-1}$$

Expressions for $\frac{T}{T'}$ $\frac{p}{p^*}$ «ud $\frac{p}{p}$

P P*

We already Seve

$\frac{'o}{T} = 1 + \frac{*}{} M'$

T y +1

Similarly

$$p_o = \left[1 + \frac{\gamma - 1}{}\right]^{\frac{}{\gamma - 1}}$$

$$\left[\quad \frac{}{} \right]^{7}$$

$$\frac{}{} \left[\quad \frac{}{} \right]$$

$$\left[\frac{y - 1}{}\right]^{y - i}$$

$$\frac{\rho_o}{p'} - \left[\frac{\gamma + 1}{2}\right]^{\frac{1}{\gamma - 1}}$$

Expressions for $\frac{T}{T^*}$ $\frac{p}{p^*}$ and p^*

$$\frac{T}{T} - + \frac{'}{2} M^2$$

At $M = 1$ $T = T^*$

$$\therefore \frac{{}_o}{T^*} = \frac{7H-}{2}$$

$$\frac{T}{T^*} \quad \frac{Tg \;\; T^*}{T_o / T}$$

$$\frac{p}{} - \frac{\left[\frac{y+1}{2}\right]^{\frac{7}{-i}}}{\left[1 + \frac{\gamma - 1}{2} M^2 \right]^{\gamma - 1}}$$

$$\frac{\rho_o / \rho}{\rho_o / \rho}$$

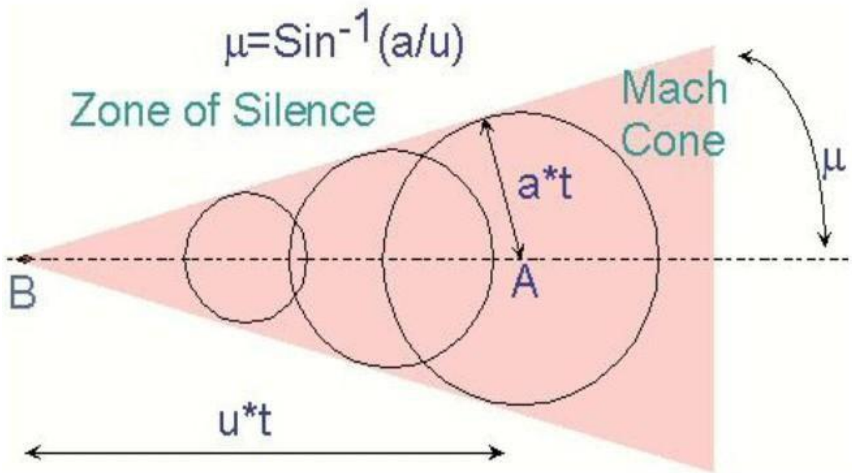
$$\frac{\rho}{\rho^*} = \frac{\left[\frac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}}$$

Mach number

In fluid mechanics, Mach number (M or Ma) is a dimensionless quantity representing the ratio of speed of an object moving through a fluid and the local speed of sound.

$$M = \frac{v}{v_{\text{sound}}}$$

Mach Cone



A conical surface that bounds the region in a supersonic flow of gas in which the sound waves (perturbations) emanating from a point source A of the perturbations are concentrated.

Reference Mach number M^*

$$M^* = \frac{C}{a^*} = \frac{C}{C^*}$$

$$M^{*2} = \frac{C^2}{a^{*2}} = \frac{C}{a^2} \times \frac{a^2}{a^{*2}} = M^2 \frac{a^2}{a^{*2}}$$

In the analysis of high speed flows, another Mach number called M^* is employed. It is defined as the non dymensionlizing the fluid velocity by the critical fluid velocity or the sound velocity.That is, Some it is more convenient to use M^* instead of M because

(i) at high fluid velocities M approaches infinity

(ii) M is not proportional to the velocity alone

It should be pointed out here that M^* does not mean M = 1this only other type of Mach number.

We have $h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2}C^2 = \frac{1}{2}C^2{}_{\max} = \frac{1}{2}a^{2*} \times \frac{\gamma+1}{\gamma-1}$

$= \frac{1}{2}a^{2*} \times \frac{\gamma+1}{\gamma-1} =$

Multiplying by 2

$$\frac{2a^2}{\gamma-1} + C^2 = \frac{\gamma+1}{\gamma-1} a^{*2}$$

Divided by a^{*2}

$\div \text{by} \frac{\gamma-1}{2}$

$$M^{*2}\left(1 + \frac{2}{\gamma-1}\frac{1}{M^2}\right) = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2}\left(\frac{2}{\gamma-1}\right)\left(\frac{\gamma-1}{2} + \frac{1}{M^2}\right) = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} = \frac{\frac{1}{2}(\gamma+1)M^2}{1+\frac{1}{2}(\gamma-1)M^2}$$

When $M^* = 0$ at M = o

$M^* = 1$ at M = 1

Eqn.we have

$$M^{*2} + \frac{2}{\gamma-1}\frac{M^{*2}}{M^2} = \frac{\gamma+1}{\gamma-1}$$

It gives

$$\frac{2}{y-1}\frac{M^{*2}}{M!} = \frac{\gamma+1}{y-1} - M^{*2}$$

$$\frac{M^{*2}}{M^2} - \frac{y+I}{2} \quad \frac{y-1}{2} M'^2$$

$$\frac{C^2}{a^{*2}} \cdot \frac{2}{\gamma-1}\frac{o^2}{a^{*2}} - \frac{y+1}{\gamma-1}$$

$$M^{*2} + \frac{2}{y-1}\frac{M'^2}{M'} \quad \frac{y+1}{y-1}$$

$$M^I \quad \frac{\left\{\frac{2}{y+1}\right\} If'}{-\left\{\frac{y+1}{>-1}\right\} V \bullet}$$

$$1 - \left\{\frac{\gamma+1}{\gamma-1}\right\} M^{*2} = 0$$

$$M^{*}_{max} \frac{C_{max}}{C^*} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45 (\text{more} — 1.4)$$

At M = m

$$1 - \left\{\frac{\gamma+1}{y-1}\right\} M^{*2} = 0$$

$$M^{*}_{max} \frac{C_{max}}{C^*} \quad \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45 (for\, \gamma = 1.4)$$

Crocco number

A non -dimensional fluid velocity can be defined by using the Maximum fluid velocity,

$$C_r = \frac{flowvelocity}{\max.fluidvelocity}$$

$$C_r = \frac{C}{C_{\max}}.$$

Problems

1) Atmospheric air at 1.1 bar and 65^{o}C is accelerated isentropically to a mach number of 1. Find final temperature, pressure and flow velocity. From gas tables, isentropic table, γ = 1.4

T = T* = 8.834 × T_o
= 0.834 × 338

T = 281.9 K

Final temperature T* = 281.9 K

$\frac{P}{P_o}$ at M =1 = 0.528

P = P* = P_o × 0.528

= 1.1 × 10^5 × 0.528

= 0.581 × 10^5 N/m^2

Q2) The pressure, temperature vrlocity of air at a point in a flow firld are 1,2 • ION/ i^2. 37‘C and 250 ms ' rtspcciix<ly, Find the stagnation pressure end stagnation temperature corresponding to given condition

Ans

Match number (Mt)

$M_1 = \frac{V_1}{C_1} = \frac{c_1}{a_1}$ Fig.

$\left.\frac{P_1}{\circ}\right]_{M \cdot 0.708} = 0.714$

$p \quad \frac{]}{0.714} \quad \frac{1.2 \times 10'}{0.714}$

= 1.681 » 10’ N/

Sts Aonp ssure, P, = 1.681x I0'N/m

Q 3) Determine x<1ocity of sound in air at 38'C.

Velocity of sound C =

= 4 2B 31

C = J53.5 ms”'

Velmity of ound at 3o°c

Q 4) Air dows through a duct with a velocity 300 ms pressure l bar, temp. 30°C. Find (i) Stagnation pressure and temperature (ii) Velocity of sound in dynamic and stagnation condition

Velocity ofso»nd »i (1)

Cj = = 20.05 Fig.

Ct =349 ms*'

$$M_1 = \frac{V}{C_1} = \frac{300}{349}$$

M; = 0.86

Stagnation properties (Pg end Tq) from Tebles

$$T_o \Big]_{M=0.86} = 0.871$$

$$T. = \frac{Tt}{0.g7I} \quad \frac{300}{0.g7I} - 347.9\ K$$

Stngnanon temperature Tp = 347.9 K

$$P_o \Big]_{M=0.86} = 0.617$$

$$P_o = \frac{Pt}{0.617} \quad \frac{1}{0.617} \quad 1.62$$

Stagnation pressure = 1.62 bar

Velocity pressure = 1.62 bar

Velocity of sound at stagnation temperature (C)

$$C_o = \sqrt{KRT_o}$$

= 20.05   47 9

Cq = 373.97 ins*[1]

Q 5) Air at stagnstioii conditioii lms a temperature of 700 K. Find the nax. possible velocity, What would be the Sonic velocity if how velocity ii l.'2 of the maximum velocity.

h + v' =@

$$\frac{V'}{2} = (h_o - h) = C_p (T_o - T)$$

V becomes maximum » hen T = 0

$$\frac{V^2}{} = C\$ T» = 1005 \bullet 700 . \frac{J}{KEK} . K$$

V_{max} = 2 1 5 7 = 1186.2 ins°'

Maximum possible flow velocity = 1186.2 ms°'

New flow velocity $V_1 = \frac{V_{max}}{2} = \frac{1186.2}{2} = 593.1\ ms^{-1}$

$h_1 + \frac{V_1^2}{2} = h_o$

c v $\frac{593\ 1^2}{}$ = c To

$T_1 + \frac{(593.1)^2}{1005 \times 2} = T$

$T_1 + 700 - \frac{(593.1)^2}{2010}$

Tt = 524.99 K (525 K)

Sonic velocity

$C_1 = \sqrt{KRT_1} = 20.05\ \sqrt{525}$

Ct - 459.4 ms '

t2 G) Air 1s discharged Tito the atmosphere trots a big vessel through a iiozzlc. Thr atmospheric pizssuic and temperature arc 1. I • 10 N"iir and 40ᵖC and thu jet velocity is 100 ixis*'. Find the chamber pirssurn and temperature.

Mahmxnbw

My - $\frac{VI}{C1}$

$C_1 = \sqrt{KRT_1}$ Fip.

= 20.05 = 354.7* me-l

$T_{A'}$ 68.9 K

M

Ten eraflue at section (A) = 368.9 K

ıp Velocity pof sotind at station (A) (CQ

$C_g = QT_Y$ = 20.05 8 9 = 385.1 me'"

M = 0

Mech munber MA- $\frac{V_\_}{C}$ $\frac{.SO}{385.1}$

Sl8

“ 'MA 0.65

i

P_o]M=υ.∠8 To find the ieinpemltue in the chamber

p_o $\frac{PQ}{o.sse}$ $\frac{1\,1\ 10}{o.9se}$ = 1 101 • 10 N.iii'

Chnniher pressure Pp - 1.lOl x 10 N. ui'

] =0.999

$T = \frac{T_1}{0.999} = \frac{313}{0.999} = 313.33$ K

C1unil›cr tmiperanuz Tq = 313.33 K

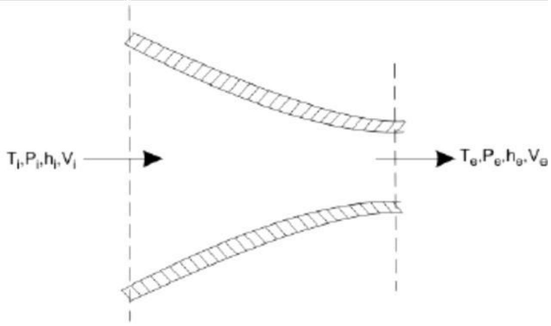
UNIT 2

FLOW THROUGH VARIABLE AREA DUCTS

A nozzle is a duct that increases the velocity of the flowing fluid at the expense of pressure drop. A duct which decreases the velocity of a fluid and causes a corresponding increase in pressure is a diffuser. The same duct may be either a nozzle or a diffuser depending upon the end conditions across it. If the cross-section of a duct decreases gradually from inlet to exit, the duct is said to be convergent. Conversely if the cross section increases gradually from the inlet to exit, the duct is said to be divergent. If the cross-section initially decreases and then increases, the duct is called a convergent-divergent nozzle. The minimum cross-section of such ducts is known as throat. A fluid is said to be compressible if its density changes with the change in pressure brought about by the flow. If the density does not changes or changes very little, the fluid is said to be incompressible. Usually the gases and vapors are compressible, whereas liquids are incompressible.

Nozzle:

A nozzle is primarily used to increase the flow velocity.



The first law reduces to

$$h_e + \frac{V_e^2}{2} = h_i + \frac{V_i^2}{2}$$

Or

$$V_e^2 - V_i^2 = 2(h_i - h_e)$$

If the inlet velocity is negligible $V_i \approx 0$ *and then*

$$V_e \sqrt{2(h_i - h_e)}$$

$$V_e \sqrt{2(h_i - h_e) + V_i^2}$$

The velocity is increased at the cost of drop in enthalpy. If an ideal gas is flowing through the nozzle, the exit velocity Ve can be expressed in terms of inlet and outlet pressure and temperatures by making use of the relations

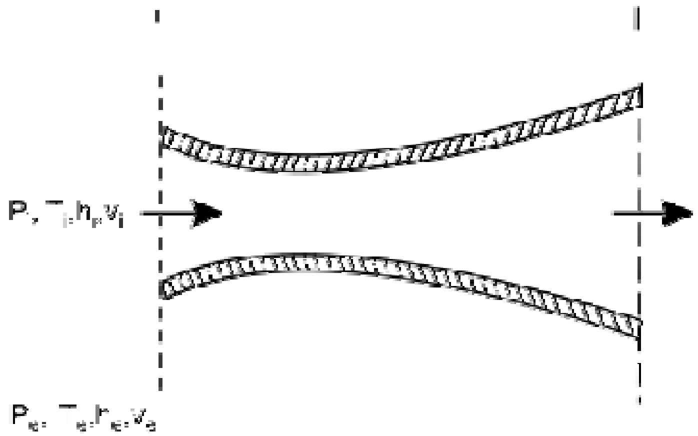
$$P\upsilon = RT$$

$$dh = c_p dT$$

$$V_e^2 = 2c_p(T_i - T_e) = 2c_p T_i \left(1 - \frac{T_e}{T_i}\right)$$

Diffuser:

A diffuser can be thought of as a nozzle in which the direction of flow is reversed.



The diffuser discharges fluid with higher enthalpy.
The velocity of the fluid is reduced.

- **Example of a reversible process:**

– Slow compression of air in a balloon does work on the air inside the balloon, and takes away energy from the surroundings - When the balloon is allowed to expand, the air inside and the surrounding air are both restored to original conditions
Example of an irreversible process:

Heat flows from hot to cold, never in the opposite direction; Most conductive and viscous processes are irreversible Stagnation point is thus when fluid is brought to stagnant state (eg, reservoir)

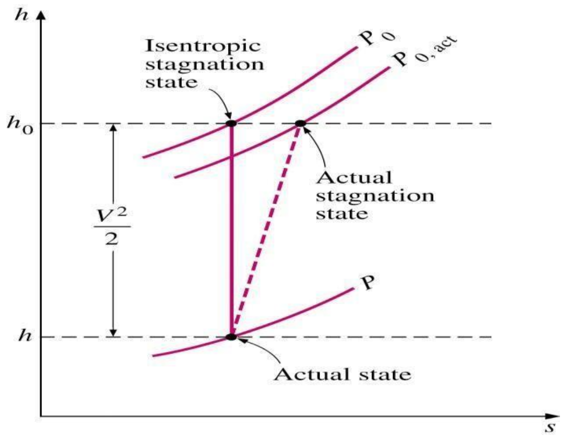
Stagnation properties can be obtained at any point in a flow field if the fluid at that point were decelerated from local conditions to zero velocity following an isentropic (frictionless, adiabatic) process

Pressure: p_0
Temperature: T_0
Density: ρ_0

- If a fluid were brought to a complete stop ($C_2 = 0$)
- Therefore, h_0 represents the enthalpy of a fluid when it is brought to rest adiabatically.
- During a stagnation process, kinetic energy is converted to enthalpy.
- Properties at this point are called **stagnation properties** (which are identified by subscript 0)

If the process is also reversible, the stagnation state is called the **isentropic stagnation state.**

Stagnation enthalpy is the same for isentropic and actual stagnation states



Actual stagnation pressure *P0,act* is lower than *P0* due to increase in entropy *s* as a result of fluid friction. Nonetheless, stagnation processes are often approximated to be isentropic, and isentropic properties are referred to as stagnation properties

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Q 8) A conical diffuser has entry diameter 20 cm, the match number temp and pressure are 0.6, 120 KN/m^2 and 340 K. The mach number at exist is 0.2. For 1 - D isentropic flow, calculate the following.

i) Pressure, temp and velocity at exist

ii) Mass flow rate, and exit diameter

iii) Change in impulse function

From the gas tables,

$$\left.\frac{T_1}{T_o}\right]_{M=0.6} = 0.933$$

$$T_o = \frac{340}{0.933} = 364.4 \text{ K}$$

Stagnation temperature $T_o = 364.4$ K

$$\left.\frac{P_1}{P_o}\right]_{M=0.6} = 0.784$$

$$\frac{1.2 \times 10}{0.784}$$

P» = 1.53 • 10 N'm^2

To find tcmperanirc at the exit

We have.

$$\left.\frac{T_2}{T_o}\right]_{M=0.2} = 0.992$$

' T2 64.4 • 0.992

T2 361.5 K

To £zad pressure at tkc csit

We have.

2 = 0.973

P = 1.S3 « 10‘ » 0.973

= 1.48 10 N/m'

To find velocity at the txit

Sonic velocity at (1)

C2' QT = 20.05 3 5

C2' 381.2 za/s

Velocity $V_2 = M_2 \times C_2$

= 0.2 « 381.2

' V2 7d.2 ms*

To find mass flow rate

Velocity at inlet

Vi Ct Mt = . JT « Mt

We have $\left.\frac{A_2}{A^*}\right]_{0.2} = 2.914$

$\left.\frac{A_1}{A^*}\right]_{0.6} = 1.188$

$$\frac{A_2}{A_1} = \frac{2.964}{1.188} = \frac{(d_2)^2}{(d_1)^2}$$

$= 20.05\ \sqrt{340} \times 0.6$

$V_1 = 221.82\ ms^{-1}$

$$r_1 = \frac{P_1}{RT_1} = \frac{1.2 \times 10^5}{287 \times 340} = 1.23\ Kg/m^3$$

Mass flow rate $= r_1\ A\ V_1$

$\dot{m} = 1.23 \times \frac{\pi}{4}(0.2)^2 \times 221.82 = 8.57\ Kg/s$

$\therefore$ Exit diameter, $d_2 = 31.5$ cm

Impulse function

$$F = PA + rAV^2 = PA\left[1 + \frac{\rho}{P}V^2\right]$$

$$= PA\left[1 + \frac{1}{RTP}V^2\right] = PA\left[1 + \frac{KV^2}{KRT}\right]$$

i.e., $F = PA\left[1 + KM^2\right]$

$DF = F_2 - F_1$

$DF = P_1A_1\left[1 + KM_1^2\right] - P_2A_2\left[1 + KM_2^{\,2}\right]$

DF = 6.503 KN

Q 9) Air is discharged from a reservoir at Po = 6.91 Kgf/cm^{2}and to = 325^{o}C through a nozzle to an exit pressure of 0.98 Kgf/cm^2 . If the flow rate is 1 kg/s, find the throat area, pressure and velocity. Also find the exit area, exit temperature and exit velocity.

Ans

Exit Mach number Me] = 1.93

$$\frac{P_e}{P_o} = \frac{0.98}{6.91} = 0.142$$

To find exit temperature

$$\left.\frac{T_e^*}{T_o}\right]_{M=1.93} = 0.573$$

$$T_e = 598 \times 0.573$$

$$= 342.65K$$

$$r_e = \frac{P_e}{R\,T_e} = \frac{0.98 \times 9.81 \times 10^4}{287 \times 342.65}$$

$$r_e = 0.975 \text{ Kg/m}^3$$

To find velocity at exit (V_e)

Sonic Velocity

$$C_e = \sqrt{KRT_e} = 20.05\sqrt{342.65}$$

$$C_e = 371.14 \text{ ms}^{-1}$$

$$V_e = M_c\,C_e = 1.93 \times 371.14 = 716.3 \text{ ms}^{-1}$$

To find exit area

We have

$$\dot{m} = r_e\,A_e\,V_e$$

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{1}{0.978 \times 716.3} = ... 0.0014 \text{ m}^2$$

To find throat pressure (P*)

$$\left.\frac{P^*}{P_o}\right]_{M=1} = 0.528$$

$$P^* = 0.528 \times 6.91 = 3.65 \text{ K gf/cm}^2$$

To find throat tempeature (T*)

$$\left.\frac{T^*}{T_o}\right]_{M=1} = 0.834$$

$$T^* = 0.834 \times 598 = 498.732 \text{ K}$$

$$\left.\frac{A}{A^*}\right]_{M=1} = 1.000$$

$$\left.\frac{A}{A^*}\right]_{M=1.93} = 1.593$$

$$A^* = \frac{A_e}{1.593} = \frac{0.0014}{1.593} = 0.00088 \text{ m}^2$$

$$V^* = \sqrt{KRT^*} = 20.05\sqrt{498.732} = 447.76 \text{ ms}^{-1}$$

Q 10) Air flows isentropically through a C.D. The inlet conditions are pressure 700 KN/m^2, temperature 320^{o}C, velocity 50 m/s. The exit pressure is 10^{5}KN/m^2 and the exist area is 6.25 cm^2. Calculate

i) Mach number, temperature and velocity at exit
ii) Pressure, temperature and velocity at throat
iii) Mass flow rate
iv) Throat area

Ans: To find the exit mach number

(M2) Sonic velocity at inlet

$$C_1 = \sqrt{KRT_1}$$

$$= 20.05\sqrt{593}$$

- 488.25 ms '

$M_1 = \frac{V}{Ct}$

$\frac{50}{488.25}$ - 0.1

$\left.\frac{P_1}{}\right]_M = 0.993$

p•- › $\overline{993}$

= 7.04 • 10 N.'ni

Exit mach number

$M_2]\frac{P_2}{}=\frac{105}{}=0.149 = 1.9$

$\left.\frac{T_t}{T_o}\right]_{M=0.1}$ - 0.998

To = $\frac{5'3}{0.998}$

- 594.2 K

$T_o\,]_{M_2=1.9}$ = 0.581

T - 594.2 • 0.581

Exit temperature Tt = 345.22 K

$C_2 = \sqrt{KRT_2}$

= 20.05 4

=3725ms-'

Vj = My • Cj

= l.S x 372.5

=707.8 "

To find throat parameters

— 0.528

Throat pressure

P* = 7.04 • 10 • 0.528

P* = 371,7 KN.'in

$\left.\frac{T^*}{T_0}\right]_{M=1} = 0.834$

T* = 594 ? • 0 834

T* = 495.5 K

Velocity

W = C*

$= \sqrt{KRT^*}$

= 0.05 49

Throat velocity V* = 446.3 nis"

$\left.\frac{A_2}{A^*}\right|_{M=1.9} = 1.555$

$$A^* = \frac{6.25 \times 10^{-4}}{1.555}$$

Throat area

$$A^* = 4.019 \times 10^{-4} \, m^2$$

Tutorial Problems:

1. The pressure, temperature and Mach number at the entry of a flow passage are 2.45 bar, 26.5° C and 1.4 respectively. If the exit Mach number is 2.5 determine for adiabatic flow of perfect gas (γ=1.3, R=0.469 KJ/Kg K).

2. Air flowing in a duct has a velocity of 300 m/s ,pressure 1.0 bar and temperature 290 k. Taking γ =1.4 and R =287J/Kg K determine: 1) Stagnation pressure and temperature, 2) Velocity of sound in the dynamic and stagnation conditions, 3) Stagnation pressure assuming constant density.

3. A nozzle in a wind tunnel gives a test –section Mach number of 2.0 .Air enters the nozzle from a large reservoir at 0.69 bar and 310 k .The cross –sectional area of the throat is1000cm² .Determine the following quantites for the tunnel for one dimensional isentropic flow 1) Pressures,temperature and velocities at the throat and test sections, 2) Area of cross- sectional of the test section , 3) Mass flow rate, 4) Power rate required to drive the compressor.

4. Air is discharged from a reservoir at Po =6.91bar and To =325°c through a nozzle to an exit pressure of 0.98 bar .If the flow rate is 3600Kg/hr determine for isentropic flow: 1)Throat area, pressure,and velocity, 2)Exit area,Mach number ,and 3)Maximum velocity.

5. Air flowing in a duct has a velocity of 300 m/s ,pressure 1.0 bar and temperature 290 K. Taking γ=1.4 and R =287J/Kg K determine: 1)Stagnation pressure and temperature, 2)Velocity of sound in the dynamic and stagnation conditions 3)Stagnation pressure assuming constant

density.

6. A conical diffuser has entry and exit diameters of 15 cm and 30cm respectively. Pressure

,temperature and velocity of air at entry are 0.69bar,340 k and 180 m/s respectively . Determine

1) The exit pressure, 2) The exit velocity and 3) The force exerted on the diffuser walls.Assume isentropic flow,γ=1.4,Cp =1.00 KJ Kg-K

UNIT – 3

FLOW THROUGH CONSTANT AREA DUCTS

Introduction

Friction is present in all real flow passages. There are many practical flow situations where the effect of wall friction is small compared to the effect produced due to other driving potential like area, transfer of heat and addition of mass. In such situations, the result of analysis with assumption of frictionless flow does not make much deviation from the real situation. Nevertheless; there are many practical cases where the effect of friction cannot be neglected in the analysis in such cases the assumption of frictionless flow leads to unrealistic influence the flow. In high speed flow through pipe lines for long distances of power plants, gas turbines and air compressors, the effect of friction on working fluid is more than the effect of heat transfer ,it cannot be neglected An adiabatic flow with friction through a constant area duct is called fanno flow when shown in h-s diagram, curves ,obtained are fanno lines. Friction induces irreversibility resulting in entropy increase. The flow is adiabatic since no transfer of heat is assumed.

Fanno Flow

A steady one-dimensional flow in a constant area duct with friction in the absence of Work and heat transfer is known as “fanno flow”.

Applications

Fanno flow occurs in many practical engineering applications of such flow includes

- Flow problems in aerospace propulsion system.
- Transport of fluids in a chemical process plants.
- Thermal and nuclear power plants.
- Petrochemical and gas industries.
- Various type of flow machineries.
- Air conditioning systems.
- High vacuum technology.
- Transport of natural gas in long pipe lines.
- Emptying of pressured container through a relatively short tube
- Exhaust system of an internal combustion engine
- Compressed air systems

When gases are transported through pipe over a long distances. It is also a practical importance when equipment handling gases are connected to high pressure reservoirs which may be located some distance away. Knowledge of this flow will allow us to determine the mass flow rate that can be candled, pressure drop etc…

In real flow, friction at the wall arises due to the viscosity of the fluid and this appears in the form of shear stress at the walls far in our discussion, we have assumed the fluid to be calorically perfect in viscid as well. Thus, strictly speaking, viscous effects cannot be accounted for in this formulation.However,in reality, viscous effects are confined to very thin region (boundary layer)near the walls. Effects such as viscous dissipation are also usually negligible.Hence, we can still assume the fluid to be inviscid and take the friction force exerted by the wall as anexternally imposed force. The origin of this force is of significance to the analysis.

The following are the main assumptions employed for analyzing the frictional flow problem. in fanno flow

- One dimensional steady flow.
- Flow takes place in constant sectional area.
- There is no heat transfer or work exchange with the surroundings.
- The gas is perfect with constant specific heats.
- Body forces are negligible.
- Wall friction is a sole driving potential in the flow.
- There is no obstruction in the flow.
- There is no mass addition or rejection to or from the flow.

In thermodynamics coordinates, the fanno flow process can be described by a curve know as Fanno line and it‟s defined as the locus of the state which satisfies the continuity and energy and entropy equation for a frictional flow is known as “fanno line”.

Fanno line or Fanno curve (Governing equation)

Flow in a constant area duct with friction and without heat transfers is described by a curve is known as Fanno line or Fanno curve

We know that,

From continuity equation,

$$m=\rho Ac$$

$$\frac{m}{A} = \rho c$$

$$G=mA=\rho c$$

Where

G- Mass flow density. c- Velocity of sound.

ρ - Density of fluid.

$$G = \rho c$$

$$c = \frac{G}{\rho}$$

$$ho=h+12\times G2\rho 2$$

$$h=ho-12\times G2\rho 2$$

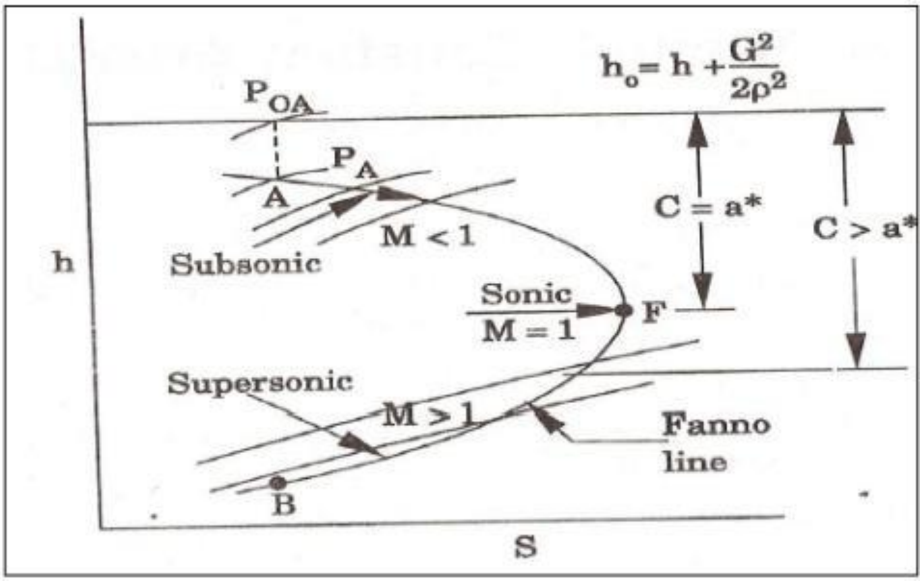
Density ρ is a function of entropy and enthalpy.

$$\rho = f(s, h)$$

Substitute the value for ρ in the equation for „h"

We get, $h=ho-12*G22\{fs,h\}2$

The above equation can be used to show a fanno-line in h-s diagram.



In the line

- Point F is the sonic point
- Point lying below are super sonic points
- Points lying above are subsonic flow

Since entropy can only increase the processes that happen

will always coverage to the sonic point F.The curve consists of two branches AF and FB. At point F the flow is sonic i,e, M=1
The flow A to F is subsonic (M<1) and B to F is Supersonic (M>1)

In subsonic flow region (A to F), the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas.

In supersonic flow region (B to F), the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas.

We know by the second law o thermodynamics that for an adiabatic flow, the entropy may increase but cannot decrease. So the processes in the direction F to A and F to B are not possible because they lead to decrease in entropy.

Fanno curves are drawn for different vales of mass flow density (G).When G increases, the velocity increases and pressure decreases in the sub sonic region. When G increases, the pressure increases and velocity decreases in the supersonic region

Important features of Fanno curve

- From the second law of thermodynamics, the entropy of the adiabatic flow increases but not decreases. Thus, the path of states along the Fanno curve must be toward the right.
- In the subsonic region, the effects of friction will be to increase the velocity and Mach number and to decrease the enthalpy and pressure of the stream .
- In the supersonic region, the effects of friction will be to decrease the velocity and Mach number and to increase the enthalpy and pressure of the stream.
- A subsonic flow can never become supersonic, due to the limitation of second law of thermodynamics, but in can approach to sonic i.e,M=1.
- A supersonic flow can never become subsonic, unless a discontinuity (shock)is present.
- In the case of isentropic stagnation, pressure is reduced whether the flow is subsonic or supersonic.

Chocking in Fanno flow

In a fanno flow, subsonic flow region, the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas. In supersonic flow region, the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas. In both cases entropy increases up to limiting state where the Mach number is one (M=1) and it is constant afterwards. At this point flow is said to be chocked flow.

Adiabatic Flow of a Compressible Fluid Through a Conduit

Flow through pipes in a typical plant where line lengths are short, or the pipe is well insulated can be considered adiabatic. A typical situation is a pipe into which gas enters at a given pressure and temperature and flows at a rate determined by the length and diameter of the pipe and downstream pressure. As the line gets longer friction losses increase and the following occurs:

- Pressure decreases
- Density decreases
- Velocity increases
- Enthalpy decreases
- Entropy increases

The question is “will the velocity continue to increasing until it crosses the sonic barrier?” The answer is NO. The maximum velocity always occurs at the end of the pipe and continues to increase as the pressure drops until reaching Mach 1. The velocity cannot cross the sonic barrier in adiabatic flow through a conduit of constant cross section. If an effort is made to decrease downstream pressure further, the velocity, pressure, temperature and density remain constant at the end of the pipe corresponding to Mach 1 conditions. The excess pressure drop is dissipated by shock waves at the pipe exit due to sudden expansion. If the line length is increased to drop the pressure further the mass flux decreases, so that Mach 1 is maintained at the end of the pipe.

The effects of friction on the properties of Fanno flow

Property	Subsonic	Supersonic
Velocity, V	Increase	Decrease
Mach number, Ma	Increase	Decrease
Stagnation temperature, T_0	Constant	Constant
Temperature, T	Decrease	Increase
Density, ρ	Decrease	Increase
Stagnation pressure, P_0	Decrease	Decrease
Pressure, P	Decrease	Increase
Entropy, s	Increase	Increase

The effect of friction in supersonic flow of the following parameters

a) velocity b) pressure c) temperature

Flow properties at $M = M^* = 1$ are used as reference values for non-dimensionalizing various properties at any section of the duct.

a) Velocity

$$\frac{dc}{c} = 12\ M2(1+\ \Upsilon-12M2)\ dM^{2}$$

Integrating between M=1 and M= M,

$$c{*}cdcc = 1M dM22M2(1+\Upsilon-12M2)$$

$$\ln\frac{c}{c^{*}} = \ln M\ [\ \Upsilon-12(1+\Upsilon-12M2)\ ]^{1/2}$$

$$\frac{c}{c^{*}} = M\ [\ \Upsilon+12(1+\Upsilon-12M2)]^{1/2}$$

Applying this equation for section 1 and 2 of the duct

$$\frac{c1}{c2} = M1M1[\ 1+\Upsilon-12M121+\Upsilon-12M22\ ]1/2$$

b) Pressure

$$p{*}pdpp = -\,-1\ M1+\Upsilon-1M22M2+\Upsilon-1M2\ dM^{2}$$

$$\ln p\ p{*} = \ln\frac{1}{M}\ [\Upsilon+12(1+\Upsilon-12M2)]$$

$$\frac{p}{p^{*}} = 1M\ [\Upsilon+12(1+\Upsilon-12M2)]1/2$$

Applying this equation for section 1 and 2 of the duct

$$\frac{p2}{p1} = \frac{M1}{M2}\ [1+\Upsilon-12M121+\Upsilon-12M22\ ]1/2$$

Temperature

$$T{*}TdTT = -1M\Upsilon-12(1+\Upsilon-12M2)\ dM^{2}$$

$$\ln TT{*} = \ln \Upsilon+12(1+\Upsilon-12M2)$$

$$\frac{T}{T^{*}} = \Upsilon+12(1+\Upsilon-12M2)$$

Applying this equation for section 1 and 2

$$\frac{T2}{T1} = \mathbf{1+\Upsilon-12M121+\Upsilon-12M22}$$

Variation of flow properties

The flow properties (P,T,ρ,C) at $M=M^*=1$are used as reference values for non- dimensionalizing various properties at any section of the duct.

Temperature

Stagnation temperature –Mach

number relation At critical state

M=1

T0 = T0*

$T = T^*$

For fanno flow T0= T0 constant*

$$\Rightarrow \boxed{\frac{T}{T^*} = \frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}}$$

Applying this equation for section 1 and 2

$$\Rightarrow \frac{T_2}{T_1} = \frac{\frac{T_{01}}{T_1}}{\frac{T_{02}}{T_2}} = \frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_1^{\,2}}$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_1^{\,2}}}$$

ach number M

$M = \frac{c}{a}$, $c = M \times \sqrt{RT\gamma}$

At critical state

M=1

C = C*

$T = T^*$

$c^* = \sqrt{R\gamma T^*}$

$$\Rightarrow \frac{C}{C^*} = \frac{M \times \sqrt{R\gamma T}}{\sqrt{R\gamma T^*}}$$

$$= M\left[\frac{T}{T^*}\right]^{\frac{1}{2}}$$

$$= M\left[\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2}M^2\right)}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{C}{C^*} = M\left[\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2}M^2\right)}\right]^{\frac{1}{2}}}$$

Applying this equation for section 1 and 2

$$\frac{C_2}{C_1} = \frac{M_2 a_2}{M_1 a_1}$$

$$= \frac{M_2}{M_1} \times \frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}}$$

$$= \frac{M_2}{M_1} \times \left[\frac{T}{T^*}\right]^{\frac{1}{2}}$$

$$= \frac{M_2}{M_1} \times \left[\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right]^{\frac{1}{2}}$$

$$\frac{C_2}{C_1} = \frac{M_2}{M_1} \times \left[\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right]$$

Density

$$\rho = \frac{1}{c}$$

$$\frac{\rho}{\rho^*} = \frac{1}{\frac{c}{c^*}}$$

$$= \frac{1}{M \left[\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M^2\right)} \right]^{\frac{1}{2}}}$$

$$= \frac{1}{M} \left| \frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right|^{\frac{1}{2}}$$

$$= \frac{1}{M} \left[\frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 + (\gamma - 1) M^2}{(\gamma + 1) M^2} \right]^{\frac{1}{2}}$$

$$\frac{\rho^*}{\rho} = \left[\frac{(\gamma + 1) M^2}{2 + (\gamma - 1) M^2} \right]^{\frac{1}{2}}$$ Applying this equation for section 1 and 2

$$\frac{\rho_2}{\rho_1} = \frac{1}{\frac{C_2}{C_1}}$$

$$= \frac{1}{\frac{M_2}{M_1}\left[\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M2^2\right)}\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{1}{2}}}$$

Pressure

Pressure=

ρRT At critical state

M=1 Expressions for $\frac{T_0}{T^*}, \frac{p_0}{p^*}$ and $\frac{\rho_0}{\rho^*}$

Considering the *section (where M = 1) and its stagnation section

We have

M⇒1

T⇒T*

p⇒p*

ρ⇒ρ*

$$P^* = \rho R T^*$$

$$\frac{P}{P^*} = \frac{\rho R T}{\rho R T^*}$$

$$= \frac{\rho}{\rho^*} \times \frac{T}{T^*}$$

$$= \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]$$

„ " $* \left(\frac{T_o^*}{T^*}\right)^{\gamma-1}$

$$\frac{P}{P} \quad \frac{\left(\frac{T_o}{T}\right)^{\gamma-1}}{*\left(\frac{T_o^*}{T'}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{'o}{P\$'} \quad \frac{P}{P'} \frac{\left(\frac{kg}{T}\right)^{y-I}}{\left(\frac{T_o^*}{T'}\right)^{\frac{7}{\gamma-1}}}$$

$$= \frac{P}{P^*} \times \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{T^*}{T_o^*}\right)^{\gamma-1}$$

$$= \frac{P}{P^*} \times \left(\frac{T_o^*}{T}\right)^{\frac{7}{\gamma-1}} \times \left(\frac{T^*}{T_0^*}\right)^{\frac{7}{\gamma-1}}$$

P T' i-'

$$= \frac{1}{M}\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right] \times \left[\frac{2\left(1+\frac{}{2}M\right.}{\gamma+1}\right]^{\frac{\gamma-1}{*}}$$

$$\frac{1}{M} \quad \frac{2\left(1q\frac{7-}{2}M^2\right)}{7*} \quad \left[\frac{2\left(1.\frac{'}{2}'M^z\right)}{y+l}\right]^{\gamma-1}$$

$$\frac{1}{M} \quad \frac{2\left()+\frac{''}{2}Ivt^*\right)}{y+1}^{\frac{1}{2}+\frac{\gamma}{\gamma-1}}$$

$$\frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{P}{P^*} = \frac{1}{M}\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}}$$

Applying this equation for section 1 and 2

$$= \frac{P_2}{P_1} = \frac{\rho_2 RT_2}{\rho_1 RT_1}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^{\,2}}{1+\frac{\gamma-1}{2}M_1^{\,2}}\right]^{\frac{1}{2}} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{P_2}{P_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}}}$$

Stagnation Pressure

$$\frac{P_0}{P} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{P_0 = P\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}$$

Considering the *section (where M = 1) and its stagnation section We have

$p_0^* = p_0$

$$P_{02} = P_1\left(\frac{T_{o2}}{T}\right)^{\gamma-1}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_2\left(\frac{o2}{T}\right)^{''''}}{P_1\left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{P_2}{P_1}\times\left(\frac{T_{o2}}{T_2}\right)^{\gamma-1}\times\left(\frac{T_{o2}}{T_2}\right)^{\gamma-1}$$

$$= \frac{P_2}{1}\times\left(\frac{T_1}{2}\right)^{\gamma-1}$$

Applying this equation for section 1 atid 2

$$P = P\left(\frac{T_{''}}{T}\right)^{i}$$

$$P_{02} = P_2\left(\frac{T_{o2}}{T_2}\right)^{\gamma-1}$$

$$-\frac{M_1}{M_2}\left[\frac{1\ \frac{7\ \ 1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2}\right]^2\times\left[\frac{1+\frac{\gamma-1}{2}\ {}^{3}_{2}}{1+\frac{72\ ,}{}Mj^2}\right]^{Y-'}$$

$$\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M^2}\right]^2 H \left[\frac{1+\frac{7-\#}{2}M_1^2}{1+\frac{Y-}{2}M\ \ 2}\right]^{\gamma-1}$$

$$-\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma-1-2\gamma}{2(\gamma-1)}}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\ 2}}{1+\frac{\gamma-1}{2}M_2^{\ 2}}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$\boxed{\frac{P_{O2}}{P_{O1}} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\ 2}}{1+\frac{\gamma-1}{2}M_2^{\ 2}}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}$$

Impulse Function

Impulse function =PA (1+ γ M^2)

Considering the *section (where M = 1) and its stagnation section We have

$F = F^*$

$p = p^*$

A =

$$\frac{F}{F^*} = \frac{PA\left(1+\gamma M^2\right)}{P^*A^*\left(1+\gamma\right)}$$

$$= \frac{\frac{\left(1+\gamma M^2\right)}{\left(1+\gamma\right)}}{\frac{P^*}{P}}$$

$$= \frac{\frac{\left(1+\gamma M^2\right)}{\left(1+\gamma\right)}}{\frac{P^*}{P}}$$

$$= \frac{\frac{\left(1+\gamma M^2\right)}{\left(1+\gamma\right)}}{M\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$= \frac{\left(1+\gamma M^2\right)}{M\left(1+\gamma\right)\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{F}{F^*} = \frac{\left(1+\gamma M^2\right)}{M\left(1+\gamma\right)\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}}$$

Applying this equation for section 1 and 2 We know that

$$\frac{F_2}{F_1} = \frac{P_2A_2}{P_1A_1} \times \frac{\left(1+\gamma M_2^{\,2}\right)}{\left(1+\gamma M_1^{\,2}\right)}$$

$$= \frac{P_2}{P_1} \times \frac{\left(1+\gamma M_2^{\,2}\right)}{\left(1+\gamma M_1^{\,2}\right)}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}} \times \frac{\left(1+\gamma M_2^{\,2}\right)}{\left(1+\gamma M_1^{\,2}\right)}$$

$$\frac{F_2}{F_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}} \times \frac{\left(1+\gamma M_2^{\,2}\right)}{\left(1+\gamma M_1^{\,2}\right)}$$

Entropy

Changing entropy is given by

$$\frac{s-s^*}{R} = -\ln\left[\frac{p_0}{P_0^{\,*}}\right]$$

$$= -\ln\left[\frac{p_0}{P_0^{\,*}}\right] \times \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}$$

Changing entropy for section 1 and 2

$$\frac{S_2-S_1}{R} = -\ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\boxed{\frac{S_2-S_1}{R} = \ln\frac{M_2}{M_1}\left[\frac{\left(1+\frac{\gamma-1}{2}M_1^{\,2}\right)}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{(\gamma+1)}{2(\gamma+1)}}}$$

Variation of Mach number with duct length

The duct length required for the flow to pass from a given initial mach number M1 to a given final mach number m2can be obtained from the following expression.

Mean friction coefficient with respect to duct length is given by

$$f = \frac{1}{L_{MAX}} \int_0^{L_{MAX}} \frac{fdx}{fdx}$$

from

$$4f\frac{dx}{D} = \frac{1-M^2}{\gamma M^2\left(1+\frac{\gamma-1}{2}\right)}\frac{dM^2}{M^2}$$

integrating limits are x=0 to x= L_{MAX}

M=M to M=1

$$\int_0^{L_{MAX}} 4f\frac{dx}{D} = \int_0^1 \frac{1-M^2}{\gamma M^2\left(1+\frac{\gamma-1}{2}\right)}\frac{dM^2}{M^2}$$

$$4f\frac{dx}{D} = \int_0^1 \frac{1-M^2}{\gamma\left(1+\frac{\gamma-1}{2}\right)}\frac{1}{M^4}dM$$

$$= 4f\frac{dx}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\frac{1}{M^4}\ln\frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)}$$

The distance (L) between two section of duct where the ach numbers M1 &M2 are given by

$$\boxed{4f\frac{L_{Max}}{D} = \left[4f\frac{L_{Max}}{D}\right]_{M_1} - \left[4f\frac{L_{Max}}{D}\right]_{M_2}}$$

Problems

1. Air flows through a pipe of 300 mm diameter. At inlet temperature is 35 ^{0}C,
pressure is 0.6 bar and stagnation pressure is 12 bar.At a location 2 m down stream, the static pressure is 0.89 bar.Estimate the average friction coefficient between two section.

Given data:

D =300 mm; T_1 = 35 ^{0}C P_{o1}= 12 bar ; P_2 =12 bar

$$\frac{P_1}{P_{O1}} = \frac{0.6}{12} = 0.05$$

From Fanno table

$$[M]_{\frac{P_1}{P_{O1}}} = 2.6$$

$$\left[4f\frac{L_{Max}}{D}\right]_{M=2.6_1} = 0.453$$

To find M_2

$$\frac{P_2}{P_1} = \frac{\left(\frac{P_2}{P^*}\right)_{M_2}}{\left(\frac{P_2}{P^*}\right)_{M_1=2.6}}$$

$$\frac{0.89}{0.62} = \frac{\left(\frac{P_2}{P^*}\right)_{M_2}}{0.275}$$

$$\left(\frac{P_2}{P^*}\right)_{M_2} = 0.275 \times \frac{0.89}{0.62} = 0.4079$$

$$[M_2]_{0.479} = 2$$

$$\left[4f\frac{L_{Max}}{D}\right]_{M=2} = 0.305$$

$$\boxed{4f\frac{L_{Max}}{D} = \left[4f\frac{L_{Max}}{D}\right]_{M1=_{26}} - \left[4f\frac{L_{Max}}{D}\right]_{M_2=2}}$$

$$= \left[4f\frac{L_{Max}}{D}\right]_{M1=_{26}} - \left[4f\frac{L_{Max}}{D}\right]_{M_2=2} = 0.453 - 0.305 = 0.148$$

$$\left[4f\frac{2}{D}\right]_{M=2} = 0.148$$

$$f = \frac{0.148 \times 3}{8} = 5.5 \times 10^{-3}$$

2 . A circular duct passes 8.25 Kg / S of air at an exit Mach number of 0.5. The entry pressure and temperature are 345 KPa and 38 ^{0}C respectively and the co – efficient of friction 0.005. If the Mach number at entry is 0.15, determine,

(i) The diameter of the duct,
(ii) Length of the duct,
(iii) Pressure and temperature at exit and

(iv) Stagnation pressure loss.

Given Data:
m = 8.25 Kg / S ; M_2 = 0.5 ; P_1 = 345 KPa ; T_1 = 311K ; f =0.005 ;

M_1= 0.15

(i) Diameter of the pipe

$$M = \rho_1 A_1 C_1$$

$$= \frac{P_1}{RT_1} A_1 M_1 \sqrt{\gamma R T_1}$$

$$m = \frac{P_1 A_1 \sqrt{\gamma} M_1}{\sqrt{RT_1}}$$

$$A_1 = \frac{m\sqrt{RT_1}}{\sqrt{\gamma} M_1 P_1} = \frac{8.25 \times \sqrt{287 \times 311}}{345 \times 10^3 \times 0.15\sqrt{1.4}}$$

$$= 0.040253m$$

$$A = \frac{\pi}{4} d^2$$

$$d = 0.226389$$

(ii) Length of the pipe

From Isentropic table M1 = 0.15,γ = $\frac{4fL}{D} = \left[\frac{4fL_{Max}}{D}\right]_{M1} - \left[\frac{4fL_{Max}}{D}\right]_{M_2}$

=28.354-1.069

$$\frac{40.005 \times L}{0.226389} = 27.285$$

$$L = 308.85m$$

Pressure and temperature at the exit

$$P_{02} = \frac{P_{02}}{P_{02}*} \times \frac{P_{01}}{P_{01}*} P_{01}$$

$$= \frac{1.34}{3.928} \times 350.609 = 231.009 \text{ кPA}$$

$$P_2 = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} \times P_1 = \frac{2.629}{4.3615} \times 700$$

$$= 100.773 \text{ K}$$

$$T_2 = \frac{T_2 T_1^*}{T_2 T_1^*}; \quad T_2 = \frac{1.161}{1.185} \times 333 = 326.2557K$$

$$= \frac{1.43}{1.1945} \times 311 = 297.59K$$

(iv) Stagnation presure loss

Question – 3 :- Air is flowing in an insulated duct with a Mach number of M1 = 0.25. At a section downstream the entropy is greater by amount 0.124 units, as a result of friction. What is the Mach number of this section? The static properties at inlet are 700KPa and 60^0C. Find velocity, temperature and pressure at exit. Fin d the
properties at the critical section.

Given Data: M1 = 0.25; (S2 – S1) = 0.124 KJ/ Kg K;
P1 = 700 KPa; T1 = 60 +273 = 333K

We know that,

$$\frac{S_2 - S_1}{R} = -\ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\frac{S_2 - S_1}{R} = \ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\ln\left[\frac{p_{01}}{P_{02}}\right] = \frac{0.124}{0.287} = 0.4320557$$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

From isentropic table $M_1 = 0.25, \gamma = 1.4$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

$\frac{1}{01} = 0.957$

$: P_{01} = 731.452 kpa$ $\overline{37 \times 333}$

$$\frac{\frac{p_{01}}{P_0 *}}{\frac{p_{02}}{P_0 *}} = 1.540421$$

le $\gamma = 1.4$

.185;

$$\frac{p_{02}}{P_{01}} = \frac{2.4065}{1.540421} = 1.5622$$

From fanno table $\frac{P_0}{P_0{}^*}$ =1.5622, the corresponding Mach number M_2=0.41

b) Velocity ,pressure and Temperature at the exit section

$$T^* = \frac{T_1}{1.185} = \frac{333}{1.185}$$

$$T^* = 281.01266K$$

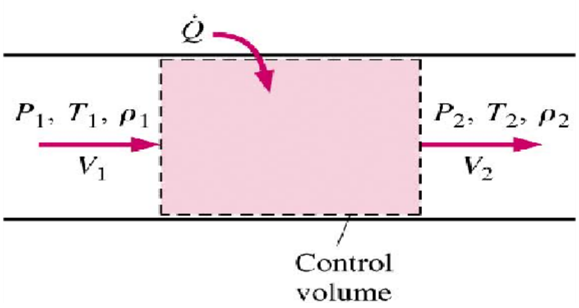
$$\frac{P}{P^*} = 4.3615$$

$$P^{*}\text{=}160.495Ka$$

$$C^* = \sqrt{\gamma RT} = 336.022m/\sec$$

Rayleigh Flow (Duct Flow with Heat Transfer and Negligible Friction)

Flow in a constant area duct with heat transfer and without friction is known as Rayleigh flow . Many compressible flow problems encountered in practice involve chemical reactions such as combustion, nuclear reactions, evaporation, and condensation as well as heat gain or heat loss through the duct wall Such problems are difficult to analyze Essential features of such complex flows can be captured by a simple analysis method where generation/absorption is modeled as heat transfer through the wall at the same rate



In certain engineering processes, heat is added either by external sources across the system boundary by heat exchangers or internally by chemical reactions in a combustion chamber. Such process are not truely adiabatic, they are called adiabatic processes.

- **Applications**

The combustion chambers inside turbojet engines usually have a constant area and the fuel mass addition is negligible. These properties make the Rayleigh flow model applicable for heat addition to the flow through combustion, assuming the heat addition does not result in dissociation of the air-fuel mixture. Producing a shock wave inside the combustion chamber of an engine due to thermal choking is very undesirable due to the decrease in mass flow rate and thrust. Therefore, the Rayleigh flow model is critical for an initial design of the duct geometry and combustion temperature for an engine.

The Rayleigh flow model is also used extensively with the Fanno flow model. These two models intersect at points on the enthalpy-entropy and Mach number-entropy diagrams, which is meaningful for many applications. However, the entropy values for each model are not equal at the sonic state. The change in entropy is 0 at M = 1 for each model, but the previous statement means the change in entropy from the same arbitrary point to the sonic point is different for the Fanno and Rayleigh flow models.

- Combustion processes.
- Regenerator,
- Heat exchangers.
- Inter coolers.

The following are the assumptions that are made for analyzing the such flow problem.

- One dimensional steady flow.
- Flow takes place in constant area duct.
- The frictional effects are negligible compared to heat transfer effects..
- The gas is perfect.
- Body forces are negligible.
- There is no external shaft work.
- There is no obstruction in the flow.
- There is no mass addition or rejection during the flow.
- The composition of the gas doesn‟t change appreciably during the flow.

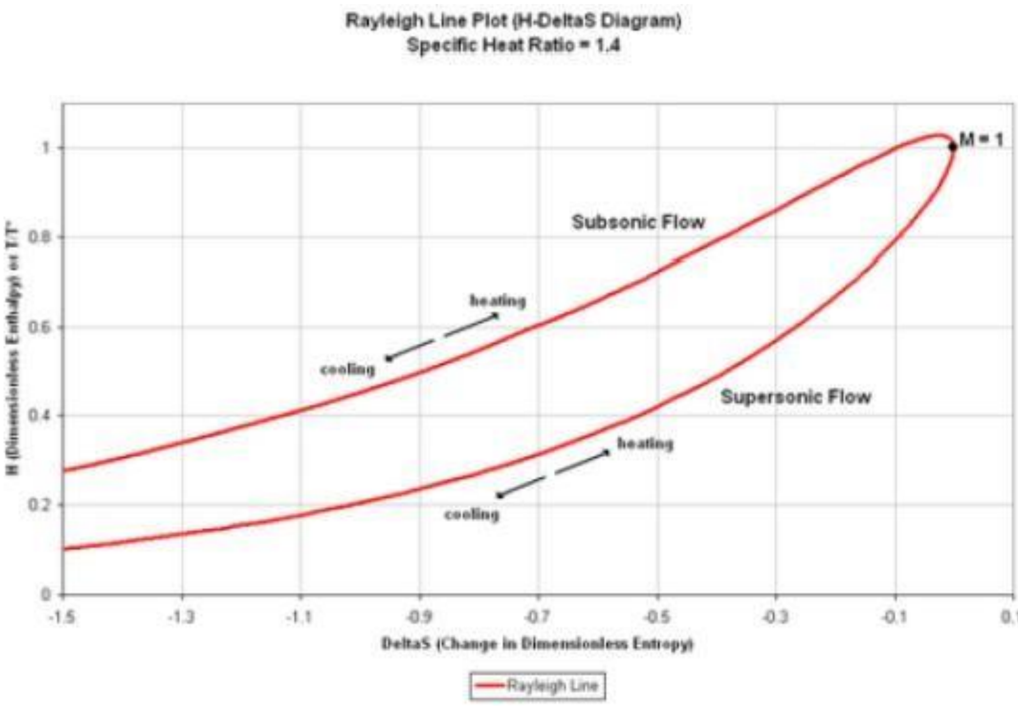
Rayleigh line (or) curve

The frictionless flow of a perfect gas through a constant area duct in which heat transfer to or from the gas is the dominant factor bringing about changes in the flow is referred to as Rayleigh flow or diabetic flow. In thermodynamic coordinates, the Rayleigh flow process can be described by a curve known as Rayleigh line and is defined as the locus of quasi- static thermodynamic state points traced during the flow. The Rayleigh line satisfies the equation of state along with simple forms of continuity and momentum equation.

Rayleigh Flow

Rayleigh flow refers to adiabetic flow through a constant area duct where the effect of heat addition or rejection is considered. Compressibility effects often come into consideration, although the Rayleigh flow model certainly also applies to incompressible flow. For this model, the duct area remains constant and no mass is added within the duct. Therefore, unlike Fanno flow, the stagnation temperature is a variable. The heat addition causes a decrease in stagnation pressure which is known as the Rayleigh effect and is critical in the design of combustion systems. Heat addition will cause both supersonic and subsonic Mach numbers to approach Mach 1, resulting in choked flow. Conversely, heat rejection decreases a subsonic Mach number and increases a supersonic Mach number along the duct. It can be shown that for calorically perfect flows the maximum entropy occurs at M = 1. Rayleigh flow is named after John Strutt, 3rd Baron Rayleigh.

Theory



A Rayleigh Line is plotted on the dimensionless H-ΔS axis.

The Rayleigh flow model begins with a differential equation that relates the change in Mach number with the change in stagnation temperature, T_0. The differential equation is shown below.

$$\frac{dM^2}{M^2} = \frac{1+\gamma M^2}{1-M^2}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{T_0}$$

Solving the differential equation leads to the relation shown below, where T_0^* is the stagnation temperature at the throat location of the duct which is required for thermally choking the flow.

$$\frac{T_0}{T_0^*} = \frac{2(\gamma+1)M^2}{(1+\gamma M^2)^2}\left(1+\frac{\gamma-1}{2}M^2\right)$$

Fundamental Equations

The following fundamental equations will be used to determine the variation of flow parameters in Rayleigh flows.

Continuity equation

We know that Mass flow rate,

Where ;-

C_1 –Velocity of fluid at inlet-m/s C_2 –Velocity of fluid at outlet-m/s

Mass flow rate; $m = \rho A c$

$$(P_1 - P_2)A = \rho A C(c_2 - c_1)$$

$$= \rho_2 A C^2{}_2 - \rho_1 A C^2{}_1$$

$$(P_1 - P_2)A = \rho_2 A C^2{}_2 - \rho_1 A C^2{}_1$$

$$(P_1 - P_2) = \rho_2 C^2{}_2 - \rho_1 C^2{}_1$$

Mach Number

The Mach number at the two sta

$$= \frac{C_2}{C_1} \times \frac{a_1}{a_2}$$

$$\boxed{\frac{P_2}{P_1} = \frac{\left[1+M^2{}_1\gamma\right]}{\left[1+M^2{}_2\gamma\right]}}$$

$$\frac{C_2}{C_1} \times \frac{\sqrt{\gamma R T_1}}{\sqrt{\gamma R T_2}}$$

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^{\frac{1}{2}}$$

- **Impulse Fuction**

$$F = P\left[1+M^2\gamma\right]$$

$$F_1 = P_1\left[1+M_1{}^2\gamma\right]$$

$$F_2 = P_2\left[1+M_2{}^2\gamma\right]$$

$$\frac{F_2}{F_1} = \frac{1+M_2{}^2\gamma}{1+M_1{}^2\gamma} \times \frac{P_2}{P_1}$$

Stagnation Pressue

Stagnation pressure-Mach number relation is given by

$$\frac{P_{01}}{P_1} = \left[1+\frac{\gamma-1}{2}M_1{}^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} = \left[1+\frac{\gamma-1}{2}M_2{}^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} \times \frac{P_1}{P_2} = \frac{\left[1+\frac{\gamma-1}{2}M_2{}^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1{}^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\boxed{\frac{P_{02}}{P_2} = \frac{P_2}{P_1} \times \frac{\left[1+\frac{\gamma-1}{2}M_2{}^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1{}^2\right]^{\frac{\gamma}{\gamma-1}}}}$$

$$\frac{P_{02}}{P_2} = \frac{1+M_1^2}{1+M_2^2} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Static Temperature

ROM EQN. $\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^2$

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^2$$

WE KNOW THAT

$$\rho_1 = \frac{P_1}{RT_1} \qquad \rho_2 = \frac{P_2}{RT_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{\frac{P_2}{RT_2}}{\frac{P_1}{RT_1}}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \frac{\rho_2}{\rho_1} \times \frac{P_1}{P_2}$$

$$\frac{T_1}{T_2} = \frac{C_1}{C_2} \times \frac{P_1}{P_2}$$

$$\frac{C_1}{C_2} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \qquad \left[\frac{\rho_2}{\rho_1} = \frac{C_1}{C_2}\right]$$

$$\frac{T_1}{T_2} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1} \times \frac{T_1}{T_2}\right]^2$$

$$\frac{T_1}{T_2} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1}\right]^2 \times \frac{T_1^2}{T_2^2}$$

$$\frac{T_1}{T_2} \times \frac{T_1^2}{T_2^2} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1}\right]^2$$

$$\frac{T_2}{T_1} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1}\right]^2$$

$$\frac{T_2}{T_1} = \left[\frac{M_2}{M_1} \times \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]$$

$$\left[\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]$$

$$\frac{T_2}{T_1} = \frac{M_2}{M_1} \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2$$

Stagnation Temperature

Stagnation Temperature – Ach Number Relation is Given by

$$\frac{T_0}{T} = \left[1 + \frac{\gamma - 1}{2} M_2^2\right]$$

$$\frac{T_{01}}{T_1} = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]$$

$$\frac{T_{02}}{T_2} = \left[1 + \frac{\gamma - 1}{2} M_2^2\right]$$

$$\frac{\frac{T_{02}}{T_2}}{\frac{T_{01}}{T_1}} = \frac{\left[1 + \frac{\gamma - 1}{2} M_2^2\right]}{\left[1 + \frac{\gamma - 1}{2} M_1^2\right]}$$

$$\frac{T_{02}}{T_{01}} = \frac{M_2^{\,2}}{M_1^{\,2}} \times \frac{\left(1+\gamma M_1^{\,2}\right)^2}{\left(1+\gamma M_2^{\,2}\right)^2} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]}{\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}$$

$$\left[\frac{T_2}{T_1} = \frac{M_2^{\,2}}{M_1^{\,2}} \times \frac{\left(1+\gamma M_1^{\,2}\right)^2}{\left(1+\gamma M_2^{\,2}\right)^2}\right]$$

$$\boxed{\frac{T_{02}}{T_{01}} = \frac{M_2^{\,2}}{M_1^{\,2}} \times \frac{\left(1+\gamma M_1^{\,2}\right)^2}{\left(1+\gamma M_2^{\,2}\right)^2} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]}{\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}}$$

Change of Entropy

$$S_2 - S_1 = C_p \ln\left[\frac{T_2}{T_1}\right] - C_p \ln\left[\frac{P_2}{P_1}\right]^{\frac{\gamma+1}{\gamma}}$$

$$S_2 - S_1 = C_p \ln\left[\frac{\left[\frac{M_2}{M_1} \times \frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right]^2}{\left[\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right]^{\frac{\gamma+1}{\gamma}}}\right]$$

$$S_2 - S_1 = C_p \ln\left[\left(\frac{M_2^{\,2}}{M_1^{\,2}}\right) \times \left(\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right)^{\left(\frac{\gamma+1}{\gamma}\right)}\right]$$

- **Heat Transfer**

We have

$Q = m c_p \left(T_{02} - T_{01}\right)$

$$Q = mc_p T_{01}\left(\frac{T_{02}}{T_{01}} - 1\right)$$

÷ BY c_p T1

$$\frac{Q}{c_p T_1} = \frac{T_{01}}{T_1}\left(\frac{T_{02}}{T_{01}} - 1\right) = = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]\left[\frac{T_2}{T_1}\frac{\left[1 + \frac{\gamma-1}{2} M_2^2\right]}{\left[1 + \frac{\gamma-1}{2} M_1^2\right]}\right]$$

$$\frac{Q}{c_p T_1} = \frac{\left(M_2^2 - M_1^2\right)\left(2 - 2\gamma M_2^2 M_1^2 + [\gamma - 1]\left(M_2^2 + M_1^2\right)\right)}{2M_1^2\left(1 + \gamma M_2^2\right)^2}$$

Expression for Heat Transfer
we have

$Q = mc_p (T_{02} - T_{01})$

the condition 1 is fixed but the value of T_{02} attains its maximum when $T_{02} = T_0^*$

Problems based on Rayleigh flow

1.The condition of gas in a combustion chamber at entry are M1=0.28, T01=380 K, P01=4.9 bar. The heat supplied in the combustion chamber is 620 kJ/kg.Determine Mach number, pressure and temperature of the gas at exit and also determine the stagnation pressure loss during heating. Take γ = 1.3, cp=1.22 kJ/Kg K.

Given,

$M_1 = 0.28$, $T_{01} = 380$ K,

$P_{01} = 4.9$ bar $= 4.9 \times 10^5$ N/m^2

$Q = 620$ kJ/kg $= 620 \times 10^3$ J/kg

Take γ = 1.3, cp=1.22 kJ/Kg K.= 1.22×10^3 J/kg K

To find

1. Mach number, pressure and temperature of the gas at exit,(M_2,P_2 and T_2)
2. Stagnation pressure loss (Δp_0)

Solution

Refer Isentropic flow table for γ =1.3 and M_1=0.28

$\frac{T_2}{}$ = 0.988 [From gas table]

$$T^*_1 = \frac{T_1}{0.342} = \frac{375.44}{0.342} = 1097.77K = T^*_2$$

$$T^*_{01} = \frac{T_{01}}{0.300} = \frac{380}{0.300} = 1266.6K = T^*_{02}$$

$$P^*_{01} = 4.09 \times 10^5 N/m^2 = P^*_{02}$$

T_{01}

$$\frac{P_2}{P_{02}} = 0.951$$

$P_1 = P_{01} \times 0.951$

$= 4.9 \times 10 \times ^5 0.951$

$P_1 = 4.659 \times 10^5$ N/m^2

$$\frac{T_1}{T^*_1} = 0.342$$

$$\frac{T_{01}}{T^*_{01}} = 0.300$$

$$P^* = \frac{P_1}{2.087} = \frac{4.659 \times 10^5}{2.087} = 2.23 \times 10^5 N/m^2$$

$$= 2.23 \times 10^5 N/m^2$$

$$P^*_1 = P^*_2$$

$$P^*_{01} = \frac{P_{01}}{1.198}$$

$T_1 = T_{01} \times 0.988$

$= 380 \times 0.988$

$T_1 = 375.44$ K

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_1 = 0.28$

2.A gas (γ=1.3 and R = 0.46 KJ / Kg K) at a pressure of 70 Kpa and temperature of 295 K enters a combustion chamber at a velocity of 75 m / sec. The heat supplied in a combustion chamber is 1250 KJ / Kg .Determine the Mach number, pressure and temperature of gas at exit.

Given: γ = 1.3: R = 0.46 KJ / Kg K $^{-1}$= 70 Kpa : T1 =295 K
C1 = 75 m/sec: Q = 1250 KJ / Kg

$$C_P = \frac{\gamma}{\gamma - 1} = 1.999333 \text{ KJ / Kg K}$$

$$M_1 = \frac{C_1}{\sqrt{\gamma R T_1}} \frac{75}{\sqrt{1.3 \times 460 \times 295}} = 01.785 \approx 0.18$$

From isentropic table M_1 = 0.18 , γ=1.3.

$$\frac{T_1}{T_0} = 0.995 : \qquad \frac{P}{P_0} = 0.979$$

$$T_{01} = 296.4824 K \qquad P_{01} = 71.5015 \; K pa$$

From Rayleigh table M1 = 0.18 , γ=1.3.

$$\frac{P}{P^*} = 2.207 \qquad \frac{P_0}{P^*_0} = 1.23 \quad \frac{T}{T^*} = 0.138$$

$$T_0{}^* = \frac{T_{01}}{0.138} = 2148.4231 K$$

We know that

Heat transfer

$$T_0{}^* = \frac{T_{01}}{0.138} = 2148.4231 K$$

$$Q = C_P \; (T_{01} - T_{01})$$

$$T_{02} = \frac{Q}{C_p} + T_{01} = \frac{1250}{1.92333} + 296.4824$$

$$= 923.5727\,K$$

From Rayleigh table corresponding to $M_2 = 0.35$

$$\frac{P}{P*} = 1.9835 \qquad \frac{T}{T*} = 0.482$$

$$P_2 = \frac{P_2}{P*} \times \frac{P*}{P_1} \times P_1$$

$$= \frac{1.9835}{2.207} \times 70 \quad = 62.911\,\text{K}\,pa$$

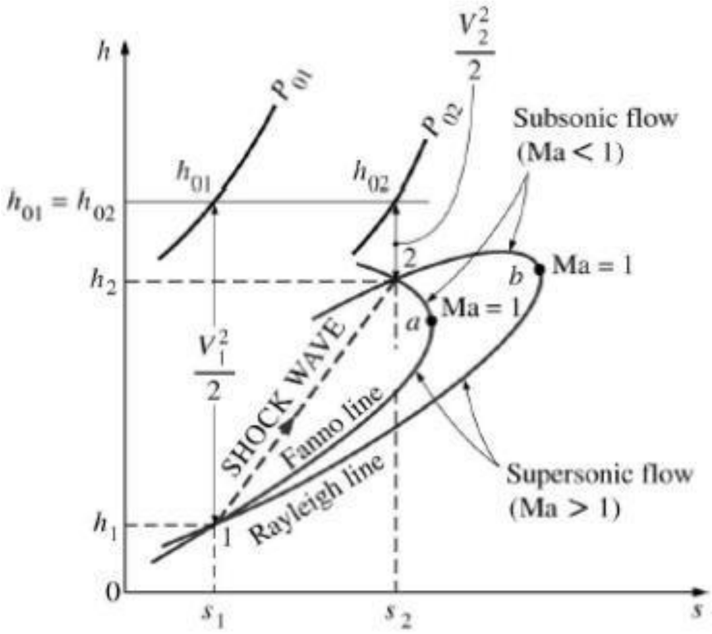
$$T_2 = \frac{0.4802}{0.158} \times 295 \quad = 900\,K$$

Result:

1. Mach number at the exit M_2 = 0.35
2. Pressure of the gas at the exit P_2 = 62.99111Kpa
3. Temperature of the gas at the exit T_2 = 900 K

Intersection of Fanno and a Rayleigh Line

Fanno and Rayleigh line, when plotted on h-s plane, for same mass velocity G, intersect at 1 and 2.as shown in fig. All states of Fanno line have same stagnation temperature or stagnation enthalpy, and all states of Rayleigh line have same stream thrust F / A. Therefore, 1 and 2 have identical values of G, h_0 and F / A. from 1 to 2 possible by a compression shock wave without violating Second Law Thermodynamics. A shock is a sudden compression which increases the pressure and entropy of the fluid but the velocity is decrease from supersonic to subsonic.



A change from states 2 to 1 from subsonic to supersonic flow is not possible in view Second Law Thermodynamics. (Entropy can not decrease in a flow process)

Tutorial Problems:

1.A circular duct passes 8.25Kg/s of air at an exit Mach number of 0.5. The entry pressure temperature are 3.45 bar and 38°C respectively and the coefficient of friction 0.005.If the Mach number at entry is 0.15, determine : I. The diameter of the duct,
II. Length of the duct, III. Pressure and temperature at the exit, IV. Stagnation pressure loss, and V. Verify the exit Mach number through exit velocity and temperature.

2) A gas (γ =1.3,R=0.287 KJ/KgK) at p1 =1bar, T1 =400 k enters a 30cm diameter duct at a Mach number of 2.0.A normal shock occurs at a Mach number of 1.5 and the exit Mach number is1.0, If the mean value of the friction factor is 0.003 determine: 1)Lengths of the duct upstream and downstream of the shock wave, 2)Mass flow rate of the gas and downstream of the shock.

3) Air enters a long circular duct (d =12.5cm,f=0.0045) at a Mach number 0.5, pressure 3.0 bar and temperature 312 K.If the flow is isothermal throughout the duct determine (a) the length of the duct required to change the Mach number to 0.7,(b) pressure and temperature of air at M =0.7 (c) the lengthof the duct required to

attain limiting Mach number, and (d) state of air at the limiting Mach number. compare these values with those obtained in adiabatic flow.

1. Show that the upper and lower branches of a Fanno curve represent subsonic and supersonic flows respectively . prove that at the maximum entropy point Mach number is unity and all processes approach this point .How would the state of a gas in a flow change from the supersonic to subsonic branch ? Flow in constant area ducts with heat transfer(Rayleigh flow)

5) The Mach number at the exit of a combustion chamber is 0.9. The ratio of stagnation temperature at exit and entry is 3.74. If the pressure and temperature of the gas at exit are 2.5 bar and 1000°C respectively determine (a) Mach number, pressure and temperature of the gas at entry, (b) the heat supplied per kg of the gas and (c) the maximum heat that can be supplied. Take γ= 1.3, Cp= 1.218 KJ/KgK

6) The conditions of a gas in a combuster at entry are: P1=0.343 bar ,T1 = 310K ,C1=60m/s.Detemine the Mach number ,pressure ,temperature and velocity at the exit if the increase in stagnation enthalpy of the gas between entry and exit is 1172.5KJ/Kg. Take Cp=1.005KJ/KgK, γ=1.4

7) A combustion chamber in a gas turbine plant receives air at 350 K ,0.55bar and 75 m/s .The air –fuel ratio is 29 and the calorific value of the fuel is 41.87 MJ/Kg .Taking γ=1.4 and R =0.287 KJ/kg K for the gas determine.

a) The initial and final Mach numbers

b) Final pressure, temperature and velocity of the gas

c) Percent stagnation pressure loss in the combustion chamber, and

d) The maximum stagnation temperature attainable.

UNIT 4

NORMAL SHOCK AND OBLIQUE SHOCKS

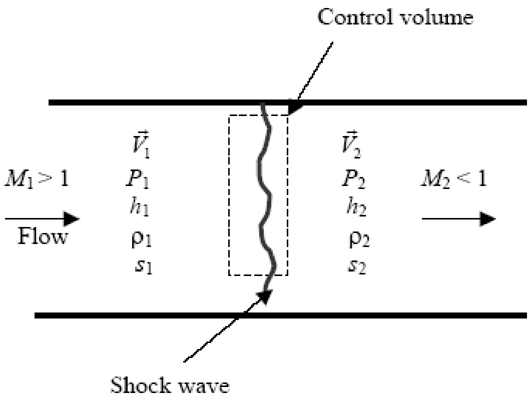
Normal Shocks

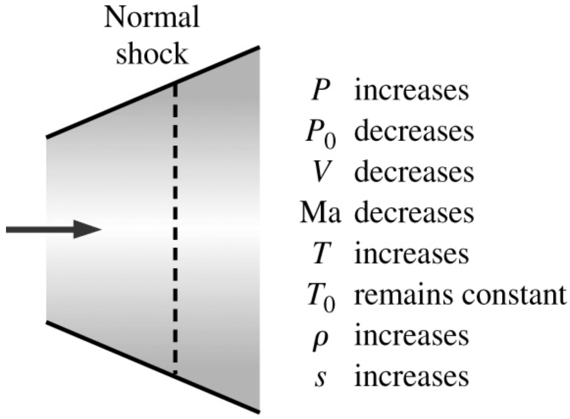
When there is a relative motion between a body and fluid, the disturbance is created if the disturbance is of an infinitely small amplitude, that disturbance is transmitted through the fluid with the speed of sound. If the disturbance is finite shock waves are created.

Shock Waves and Expansion Waves Normal Shocks

Shocks which occur in a plane normal to the direction of flow are called **normal shock waves.** Flow process through the shock wave is highly irreversible and *cannot* be approximated as being isentropic. Develop relationships for flow properties before and after the shock using conservation of mass, momentum, and energy.

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock*. The normal shock causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript 1) and after (subscript 2) a shock are related by





Assumptions

- Steady flow and one dimensional
- dA = 0, because shock thickness is small
- Negligible friction at duct walls since shock is very thin
- Zero body force in the flow direction
- Adiabatic flow (since area is small)
- No shaft work
- Potential energy neglected

Governing Equations:

(i) Continuity

$\dot{m}_x = \dot{m}_y$

(ii) Energy equation

$$\text{SFEE: } \dot{q} - \dot{\omega}_{sh} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2}$$

Across the shock

$$h_y + \frac{V_y^2}{2} = h_x + \frac{V_x^2}{2}$$

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy}$$

T_o remains constant across the shock.

(iii) Momentum

Newton‟s second law

$$\Sigma F_{xx} = \frac{\partial}{\partial t}(\dot{m}V_{xx})_{cv} + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$P_x A - P_y A = 0 + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$= \dot{m}(V_y - V_x) = \rho A V(V_y - V_x) = \rho_y A V_y^2 - \rho_x A V_x^2$$

Momentum gives

$$P_x A - P_y A = \rho_y A V_y^2 - \rho_x A V_x^2$$

$$P_x A + \rho_x A V_x^2 = P_y A + \rho_y A V_y^2$$

$$\therefore F_x = F_y$$

Impulse function remains constant across the shock.

Property relations across the shock.

(1) $\frac{T_y}{T_x}$

Energy

h_{ox} □ h_{oy}

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy} \quad (1)$$

For the isentropic x – ox

$$\frac{T_{ox}}{T_x} = 1 + \frac{K-1}{2} M_x^2$$

$$T_{ox} = T_x \left[1 + \frac{K-1}{2} M_x^2 \right] \qquad \text{..........} \ (2)$$

Similarly

$$T_{oy} = T_y \left[1 + \frac{K-1}{2} M_y^2 \right] \qquad \text{...........} \ (3)$$

Combining (1), (2) and (3)

$$T_x \left[1 + \frac{K-1}{2} M_x^2 \right] = T_y \left[1 + \frac{K-1}{2} M_y^2 \right]$$

$$\frac{T_y}{T_x} = \left[\frac{1 + \frac{K-1}{2} M_x^2}{1 + \frac{K-1}{2} M_y^2} \right]$$

(ii) $\frac{P_y}{P_x}$

Momentum

$$F_x = F_y$$

$$P_x A + \rho_x A V_x^2 = P_y A + \rho_y A V_y^2$$

$$P_x\left[1+\frac{\rho_x V_x^2}{P}\right] = P_y\left[1+\frac{\rho_y V_y^2}{P}\right]$$

$$P_x\left[1+KM_x^2\right] \quad P_y\left[1+KM_y^2\right]$$

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Equation of state P = pRT

$$P_x = \rho_x R T_x$$

$$P_y = \rho_y R T_y$$

$$\frac{\rho_x}{\rho_y} = \frac{\left(\frac{P_x}{RT_x}\right)}{\left(\frac{P_y}{RTC}\right)} = \frac{P_x T_y}{T_x P_y}$$

$$\frac{\rho_x}{\rho_y} = \frac{1+KM_y^2}{1+KM_x^2}\left[\frac{1+\frac{K-}{2}Mg}{1+\frac{K-1}{2}M_y^2}\right]$$

(iv) $\frac{V_y}{V_x}$

Continuity equation

Equation of state

P=pRT

Pg = pgRTg

$$P_y = \rho_y R T_y$$

$$\frac{V_y}{V_x} = \left[\frac{1+KM_y^2}{1+KM_x^2}\right]\left[\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}\right]$$

(v) P_{oy}

For the isentropic x — ox

$$\frac{P_{ox}}{P_x} = \left(1+\frac{K-1}{2}M_x^2\right)^{\overline{K-1}}$$

For the isentropic (y – oy)

$$\frac{P_{oy}}{P_y} = \left(1 + \frac{K-1}{2} M_y^2\right)^{\frac{K}{K-1}}$$

$$\frac{P_{oy}}{P_{ox}} = \frac{P_y}{P_x}\left[\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

But $\frac{P_y}{P_x} = \frac{1+KM_x^2}{1+KM_y^2}$

$$\therefore \frac{P_{oy}}{P_{ox}} = \left[\frac{1+KM_x^2}{1+KM_y^2}\right]\left[\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

Governing relations for a normal shock

(1) Continuity

$$\rho_x V_x = \rho_y V_y \qquad \text{....... (1)}$$

(2) Energy

$$h_{ox} = h_{oy}$$

$$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} = h_0 \qquad \text{............ (2)}$$

(3) Momentum

From (2) we have

$$)_{out} - (\dot{m}V_{xx})_{in}$$

$$h_x + \frac{V_x^2}{2} = h_0$$

$$C_p T_x + \frac{V_x^2}{2} = C_p T_o$$

$$\frac{KR}{K-1}T_x + \frac{V_x^2}{2} = \frac{KR}{K-1}T_0 \qquad \text{........}$$

$$\frac{C_x^2}{K-1} + \frac{V_x^2}{2} = \frac{C_0^2}{K-1}$$

$$\therefore C_x^2 = C_0^2 - \frac{K-1}{2}V_x^2 \qquad \text{.......... (4)}$$

Similarly $C_y^2 = C_0^2 - \frac{K-1}{2}V_y^2$ .... (5)

Equation (3) $\Rightarrow P_x - P_y = \rho V\left(V_y - V_x\right)$

$$\frac{P_x - P_y}{\rho V} = (V_y - V_x)$$

$$\frac{P_x}{\rho_x V_x} - \frac{P_y}{\rho_y V_y} = (V_y - V_x)$$

$$\frac{RTC}{V_x} \quad \frac{RTC}{V.} —(v —v)$$

$$\frac{KRT_x}{V_x} - \frac{KRT_y}{V_y} = K(V_y - V_x)$$

$$\frac{C_x^2}{V_x} \quad \frac{c^2}{V_y} -KJV,—V,) \qquad \ldots\ldots\ldots\ldots (6)$$

Using (4) and (5) in (6)

$$\frac{1}{V_x}\left[C_0^2 - \frac{K-1}{2}V_x^2\right] - \frac{1}{Vy}\left[c2 \quad \frac{K}{} 'V \right]' K(Vy \quad V)$$

$$'0 \left[\frac{1}{V_x} - \frac{1}{V_y}\right] + \frac{K-1}{2}(V_y - V_x) = K(V_y - V_x)$$

$$\frac{ct(v,—‘*.)}{V_x V_y} \quad \frac{x—i}{2}(V_y - V_x) = K(V_y - V_x)$$

$$\frac{T_O}{V_x V_y} —x— \frac{K—I}{2}$$

$$'0 \quad \left(\frac{K+1}{2}\right) V_x V_y \qquad \ldots\ldots\ldots (7)$$

$$\frac{C^2}{C\$} \quad \frac{KRT‘}{KRT_0} \quad \frac{T‘}{0} \quad \frac{2}{K+1} \qquad \left[\frac{T_0}{T} = 1 + \frac{K-1}{2}M^2\right]$$

$$\left[\frac{Tp}{T'} \_ \frac{K+!}{2}\right]$$

$$C_0^2 = \left(\frac{K+1}{2}\right) C^{*2} \qquad \ldots\ldots\ldots\ldots (8)$$

Substitution (8) in (7) we have

$$\left(\frac{K+1}{2}\right) C^{*2} = \left(\frac{K+1}{2}\right) V_x V_y$$

$$C^{*2} = V_x V_y \qquad \frac{V_x V_y}{C^{*2}} = 1$$

for a shock, $C_x^* = C_y^* = C^*$

$$\frac{V_x}{C_x^*} \frac{V_y}{C_y^*} = 1$$

$$M_x^* . M_y^* = 1 \qquad \text{By definition} \quad M^* = \frac{V}{C^*}$$

Strength of a Shock Wave

It is defined as the ratio of difference in down stream and upstream shock pressures (p_y - p_x) to upstream shock pressures (p_x). It is denoted by ξ.

$$\xi = \frac{p_y - p_x}{p_x}$$

$$\boxed{\xi = \frac{p_y}{p_x} - 1}$$

Substituting for *py/px*

$$\xi = \left[\frac{2\gamma}{\gamma+1} M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right] - 1$$

$$= \frac{1}{\gamma+1} \left[2\gamma M_x^2 - (\gamma-1) - (\gamma+1)\right]$$

$$= \frac{1}{\gamma+1} \left[2\gamma M_x^2 - \gamma + 1 - \gamma - 1\right]$$

$$= \frac{1}{\gamma+1} \left[2\gamma M_x^2 - 2\gamma\right]$$

$$\boxed{\xi = \frac{2\gamma}{\gamma+1} [M_x^2 - 1]}$$

From the above equation;

$$\boxed{\xi \propto M_x^2 - 1}$$

PROBLEMS

1.The state of a gas (γ =1.3,R =0.469 KJ/KgK.) upstream of normal shock wave is given by the following data: M_X =2.5, P_X =2 bar. T_X =275 K calculate the Mach number,pressure,temperatureand velocity of a gas down stream of shock: check the calculated values with those given in the gas tables.**Take K = γ.**

Ans

$$M_y^2 = \frac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1} M_x^2 - 1} = \frac{\frac{2}{1.3-1} + 2.5^2}{\frac{2\times1.3}{1.3-1}\times 2.5^2 - 1} = \frac{12.92}{53.19} = 0.243$$

$$M_Y = 0.4928$$

$$\boxed{\frac{P_Y}{P_X} = \frac{2\gamma}{\gamma+1} M_X{}^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{2\times1.3}{1.3+1}\times 1.5^2 - \left(\frac{1.3-1}{1.3+1}\right)$$

$$\frac{}{X} = 7.0d5 — 0.130 = 6.935$$

Pp —— 6.935x2 =13.870 bnr

$$\frac{y}{T_x} - \frac{\frac{}{2}M\$ \quad \frac{}{» -i}Mg — l}{\frac{(K+1)^2}{2(K-1)}M_x^2}$$

$$\frac{\left(i + \frac{1 3 \ \ l}{2} \quad z,x^2 \quad \frac{2 \ \ 1 3}{1.3 — l} \quad i' — i\right.}{\frac{(Le + i)^2}{?(1.3-1)}2.5}$$

$$\frac{1 + \frac{03}{2} \times Q,J\S \quad H\ 53.19}{\frac{(2.3)^2}{z(0.3)} \times 6.25} \qquad \frac{1,937 \times 53.19}{55.104}$$

$$\frac{T_y}{T_x} = 1.$$

T =L869x275=511975K

$$\frac{C_Y}{C_x} = \frac{3}{1.3+1} \times \frac{1}{6.25} + \frac{0.3}{2.3} = 0.269$$

$$C_Y = 0.269C_x \quad .0269 M_x a_x$$

C'g ——0 2693f, JT

Cg ——0.269 2.5, 3 469 275

$$C_y = M_y \sqrt{KRT_y}$$

0.4928 3 469 5 3 975

U —275.16 ni / s.

2. An Aircraft flies at a Mach number of 1.1 at an altitude of 15,000 metres.The compression in its engine is partially achieved by a normal shock wave standing at the entry of the diffuser. Determine the following for downstream of the shock.

1. Mach number
2. Temperature of the air
3. Pressure of the air
4. Stagnation pressure loss across the shock.

Given

$M_X = 1.1$

Altitude, Z 15, 000 m

Re *fer gas tables for Altitude*, $Z = 15,000\,m$

$$T_x = 216.6\,K$$

$$P_x = 0.120\,bar$$

$$P_x = 0.120 \times 10^5\,N/m^2$$

Re *fer Normal shocks gas tables for* $M_x = 1.1$ *and* $\gamma = 1.4$

$$M_y = 0.911$$

$$\frac{P_y}{P_x} = 1.245$$

$$\frac{T_y}{T_x} = 1.065$$

— 0.998

0›

— 2.133

$P,$ —— 1.24 x SPD

= 1.24 x 0.120 x 10' N 1ill'

Pp -—0.149 xlv N' in'

T —L067 x T

=l.Od5x216d

Tp ——230.67N

P - 2.133 x Pg

= 2.133 x 0,120 x l

Pg = O.259 x I0°N/ iii'

$P_{0-} = \frac{P_{0y}}{\text{o.A}}$

0.2S9x1O'

$P_{0x} = 0.2564 \times 10^5\, N/m^2$

Pressure loss

$\Delta P_0 = P_{0x} - P_{0y}$

$= 0.2564 \times 10^5 - 0.259 \times 10^5$

$\Delta P_0 = 50\, N/m^2$

3. Supersonic nozzle is provided with a constant diameter circular duct at its exit. The duct diameter is same as the nozzle diameter. Nozzle exit cross ection is three times that of its throat. The entry conditions of the gas (γ=1.4, R= 287J/KgK) are P_0 =10 bar, T_0 =600K. Calculate the static pressure, Mach number and velocity of the gas in duct.

(a) When the nozzle operates at its design condition. (b) When a normal shock occurs at its exit. (c) When a normal shock occurs at a section in the diverging part where the area ratio, A/A* =2.

Given:

A_2 =3A*

Or A_2/A*=3

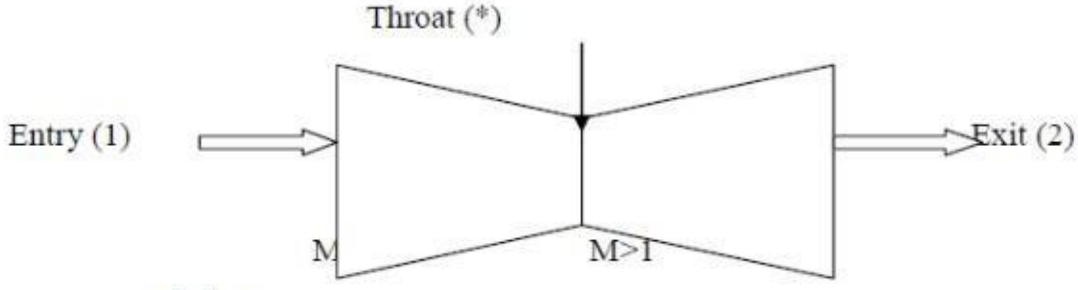
γ = 1.4

R= 287 J/KgK

P_0= 10 bar = 10^6

Pa T_0= 600K

For nozzle



Solution:

Case (i)

Refer isentropic flow table for $A_2/A^*=3$ and γ= 1.4

$M_2 = 2.64$

$T_2/T_{02} = 0.417$

Tutorial Problems:

1) The state of a gas (γ=1.3,R =0.469 KJ/Kg K) upstream of a normal shock is given by the following data: Mx =2.5, px= 2bar ,Tx =275K calculate the Mach number, pressure,temperature and velocity of the gas downstream of the shock;check the calculated values with those give in the gas tables.

2) The ratio of th exit to entry area in a subsonic diffuser is 4.0 .The Mach number of a jet of air approaching the diffuser at p0=1.013 bar, T =290 K is 2.2 .There is a standing normal shock wave just outside the diffuser entry. The flow in the diffuser is isentropic . Determine at the exit of the diffuser.a) Mach number , b) Temperature, and c) Pressure d) What is the stagnation pressure loss between the initial and final states of the flow ?

3) The velocity of a normal shock wave moving into stagnant air (p=1.0 bar, t=17^{0}C) is 500 m/s . If the area of cross- section of the duct is constant determine (a) pressure (b) temperature
(c) velocity of air (d) stagnation temperature and (e) the mach number imparted upstream of the wave front.

4) The following data refers to a supersonic wind tunnel:
Nozzle throat area =200cm^2,Test section cross- section =337.5cm^2, Working fluid ;air (γ
=1.4, Cp =0.287 KJ/Kg K) Determine the test section Mach number and the diffuser throat area if a normal shock is located in the test section.

5) A supersonic diffuser for air (γ =1.4) has an area ratio of 0.416 with an inlet Mach number of 2.4 (design value). Determine the exit Mach number and the design value of the pressure ratio across the diffuser for isentropic flow. At an off- design value of the inlet Mach number (2.7) a normal shock occurs inside the diffuser .Determine the upstream Mach number and area ratio at the section where the shock occurs, diffuser efficiency and the pressure ratio across the diffuser. Depict graphically the static pressure distribution at off design.

6) Starting from the energy equation for flow through a normal shock obtain the following relations (or) prandtl – meyer relation Cx Cy =a*^{2}M*x M*y =1

UNIT – 5

PROPULSION

Jet Propulsion System

It is the propulsion of a jet aircraft (or) Rocket engines which do not use atmospheric air other missiles by the reaction of jet coming out with high velocity. The jet propulsion in used when the oxygen is obtained from the surrounding atmosphere.

Jet propulsion is based on Newton‟s second and third low of motion. Newton‟s second law states that „the range of momentum in any direction is proportional to the force acting in that direction‟. Newton‟s third low states that for every action there is an equal and opposite reaction.

In propulsion momentum is imparted to a mass of fluid in such a manner that the reaction of the imparted momentum furnishes a propulsive force. The jet aircraft draws in air and expels it to the rear at a markedly increased velocity; the rocket greatly changes the velocity of its fuel which it ejects rearward in the form of products of combustion. In each case the action of accelerating the mass of fluid in a given direction created a reaction in the opposite direction in the form of a propulsive
force. The magnitude of this propulsive force is defined as thrust.

Types of Jet Propulsion System:

The jet propulsion engines are classified basically as to their method of operation. The two main categories of jet propulsion engines are the atmospheric jet engines and the rockets. The atmospheric jet engines require oxygen from the atmospheric air for the combustion of fuel. As a result, their performance depends to a great degree on the forward speed of the engine and upon the atmospheric pressure and temperature.

The rocket engine differs from the atmospheric jet engines in that the entire mass of jet is generated from the propellants carried within the engine, i.e., the rocket engine carries its own oxidant for the combustion of the fuel and is therefore, independent of the atmospheric air. The performance of this type of power plant is independent of the forward speed and affected to a maximum of about 10% by changes in altitude.

Air Breathing Engines

Air breathing engines can further be classified as follows: 1.Reciprocating engines (Air screw)

2. Gas Turbine engines

(i) Turbojet

(ii) Turbojet with after burner (also known as turbo ramjet, turbojet with tail pipe burning and turbojet with reheater)

(iii) Turboprop (also known as propjet).

3. Athodyds (Aero Thermodynamics Ducts)

(i) Steady combustion system, continuous air flow – Ramjet (also known as Lorin tube)

Intermittent combustion system, intermittent air flow – Pulse jet (also known as aero pulse, resojet, Schmidt tube and intermittent jet).

The reciprocating engine develops its thrust by accelerating the air with the help of a propeller driven by it, the exhaust of engine imparting almost negligible amount of thrust to that developed by the propeller.

The turbojet, turbojet with afterburner and turboprop are modified simple open cycle gas turbine engines. The turbojet engine consists of an open cycle gas turbine engine (compressor, combustion chamber and turbine) with an entrance air diffuser added in front of the compressor and an exit nozzle added aft of the turbine. The turbojet with afterburner is a turbojet engine with a reheater added to the engine so the extended tail pipe acts as a combustion chamber. The turboprop is a turbojet engine with extra turbine stages, a reduction gear train and a propeller added to the engine. Approximately 80 to 905 of the thrust of the turboprop is produced by acceleration of the air outside the engine by the propeller and about 10 to 20% of the thrust is produced by the jet exit of the exhaust gases. The ramjet and the pulsejet are athodyds, i.e., a straight duct type of jet engine without compressor and turbine wheels.

Rocket Engines

The necessary energy and momentum which must be imparted to a propellant as it is expelled from the engine to produce a thrust can be given in many ways. Chemical, nuclear or solar energy can be used and the momentum can be imparted by electrostatic or electromagnetic force.

Chemical rockets depend up on the burning of the propellant inside the combustion chamber and expanding it through a nozzle to obtain thrust. The propellant may be solid, liquid, gas or hybrid.

The vast store of atomic energy is utilized incase of nuclear propulsion. Radioactive decay or Fission or Fusion can be used to increase the energy of the propellant.

In electrical rockets electrical energy from a separate energy source is used and the propellant is accelerated by expanding in a nozzle or by electrostatic or electromagnetic forces.

In solar rockets solar energy is used to propel spacecraft.

The Ramjet Engine

The ramjet engine is an air breathing engine which operates on the same principle as the turbojet engine. Its basic operating cycle is similar to that of the turbojet. It compresses the incoming air by ram pressure, adds the heat energy to velocity and produces thrust. By converting kinetic energy of the incoming air into pressure, the ramjet is able to operate without a mechanical compressor. Therefore the engine requires no moving parts and is mechanically the simplest type of jet engine which has been devised. Since it depends on the velocity of the incoming air for the needed compression, the ramjet will not operate statistically. For this reason it requires a turbojet or rocket assist to accelerate it to operating speed.

At supersonic speeds the ramjet engine is capable of producing very high thrust with high efficiency. This characteristic makes it quite useful on high speed aircraft and missiles, where its great power and low weight make flight possible in regions where it would be impossible with any other power plant except the rocket. Ramjets have also been used at subsonic speeds where their low cost and light weight could be used to advantage.

Principle of Operation:

The ramjet consists of a diffuser, fuel injector, flame holder, combustion chamber and exit nozzle (Ref figure 9). The air taken in by the diffuser is compressed in two stages.

The external compression takes place takes place because the bulk of the approaching engine forces the air to change its course. Further compression is accomplished in the diverging section of the ramjet diffuser. Fuel is injected into and mixed with air in the diffuser. The flame holder provides a low velocity region favourable to flame propagation, and the fuel-air mixture recirculates within this sheltered area and ignites the fresh charge as it passes the edge of the flame holder. The burning gases then pass through the combustion chamber, increasing in temperature and therefore in volume. Because the volume of air increases, it must speed up to get out of the way off the fresh charge following behind it, and a further increase in velocity occurs as the air is squeezed out through the exit nozzle. The thrust produced by the engine is proportional to this increase in velocity.

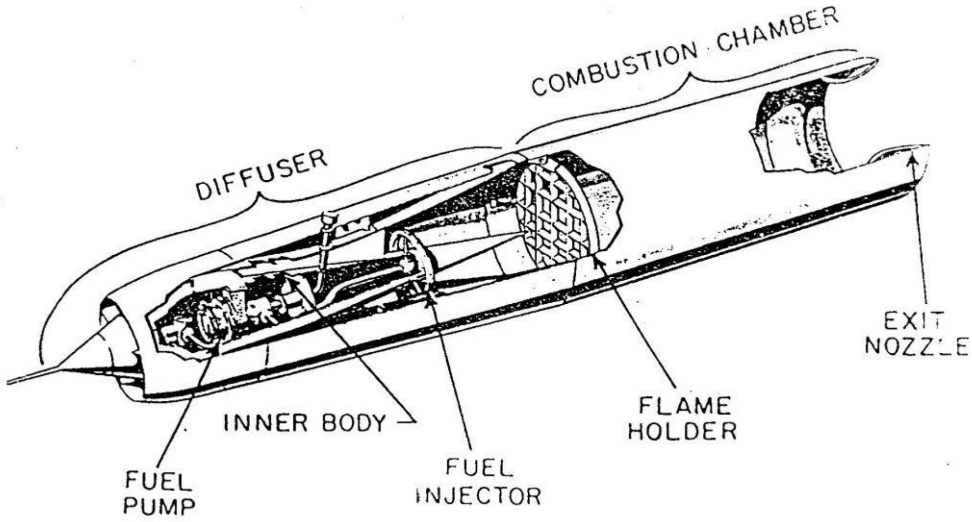


Figure 9b Supersonic ramjet, showing components.

Advantages

- Ramjet is very simple and does not have any moving part. It is very cheap and requires almost no maintenance.
- Since turbine is not used the maximum temperature which can be allowed in ramjet is very high, about 2000 ^{0}C as compared to about 1000 ^{0}C in turbojets. This allows a greater thrust to be obtained by burning fuel at A/F ratio of about 15.1 which gives higher temperatures.
- The SFC is better than turbojet engines at high speed and high altitudes.
- There seems to be no upper limit to the flight speed of the ramjet.

Disadvantages

- Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ramjet without an external launching device.
- The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.

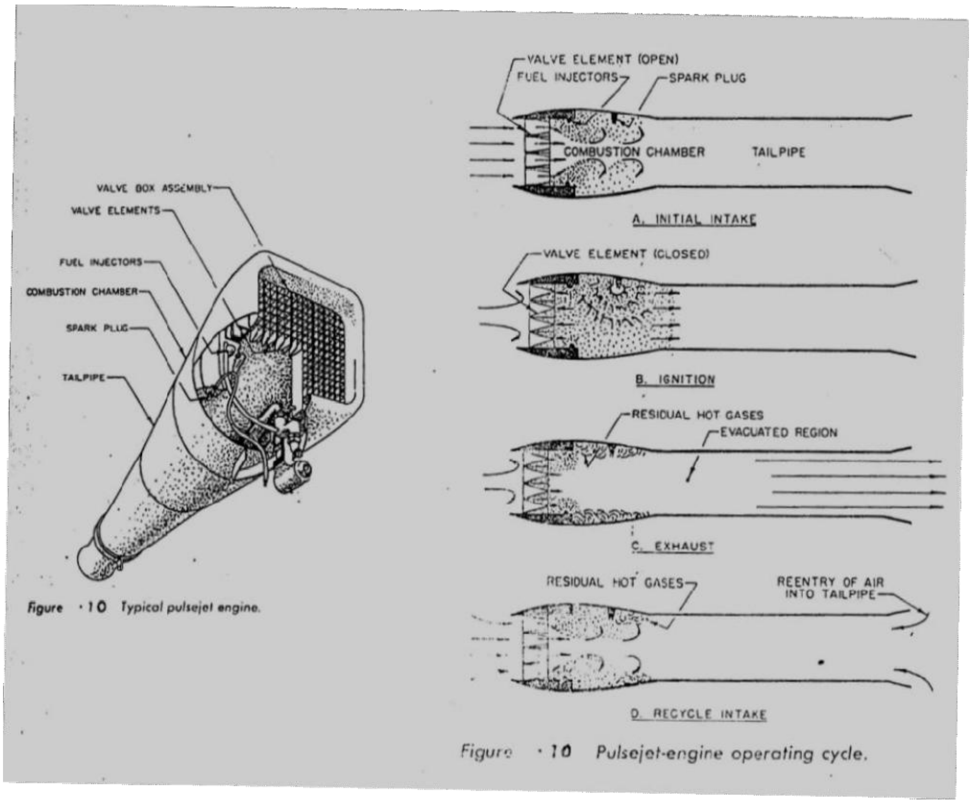


Figure ·10 Typical pulsejet engine.

Figure ·10 Pulsejet-engine operating cycle.

Application:

- Due to its high thrust at high operational speed, it is widely used in high speed aircrafts and missiles.
- Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

Pulse Jet Engine

The pulse jet engine is an intermittent, compressor less aerodynamic power plant, with few or none of the mechanical features of conventional aviation power plants. In its simplest form, the operation of the pulse jet depends only on the properties of atmospheric air, a fuel, a shaped tube and some type of flow-check valve, and not on the interposition of pistons, impellers, blades or other mechanical part whose geometry and motion are controllable. The pulse jet differs from other types of air breathing engines, in that the air flow through it is intermittent. It can produce static thrust.

Operations:

During starting compressed air is forced into the inlet which opens the spring loaded flapper valves. In practice this may done by blowing compressed air though the valve box or by the motion of the engine through the air. The air that enters the engine passes by the fuel injector and is mixed with the fuel(Fig. A)

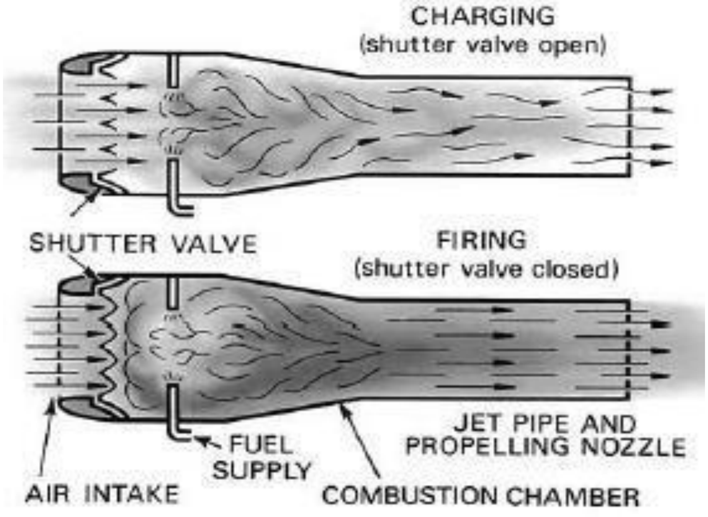


Fig. 1-7 A pulse jet engine.

When the fuel-air mixture reaches the proper proportion to burn, it is ignited by a spark plug. The burning takes place with explosive force, thus causing a very rapid rise in pressure, the increase in pressure forces the flapper valves shut and propels the charge of burned gases out of the tail pipe, as in B of the figure.

The momentum of the gases leaving the tailpipe causes the air to continue t flow out even after the pressure within the engine has reached atmospheric pressure. The pressure within the engine is therefore evacuated to below atmosphere, part C in figure.

After the pressure has reached its lowest point, atmospheric pressure (and the ram pressure if the engine is in flight) forces air into the engine through the flapper valves. At the same time, air will also be drawn in through tailpipe, since the pressure within the tailpipe is low and has nothing to prevent the entry of air, At this point, part D in figure, the engine is ready to begin another cycle. The fequency of cycles depends upon the duct shape and working temperature in V-1 rocket it was about 40 c/s which corresponds to about 2400 rpm of a two stroke reciprocating engine.

Once the engine operation has become established, the spark plug is no longer necessary. The reignition between each cycle is accomplished when the fresh charge of fuel and air is ignited by some residual flame which is left over from the preceding cycle. The air flow which reenters the tailpipe is important from both the engine operation and thrust stadpoints. Experiments have shown that the amount of air which flows into the tailpipe can be several times as much as that which flows into the inlet. This mass flow of air increases the thrust of the engine by providing additional mass for the explosion pressure to work on. It also increases the pressure within the engine at the beginning of each explosion cycle, resulting in a more efficient burning process. Reentry of air into the tailpipe is made more difficult as the airspeed surrounding the engine increases. The thrust of the engine, therefore, tends to decrease with speed. As the speed increases, the amount of reentering air flow decreases to the point where the internal pressure is eventually too low to support combustion and the engine will no longer operate.

Characteristics :

The chief advantages of the pulse jet are its simplicity, light weight, low cost and good zero speed thrust characteristic. Its particular disadvantages are its 650-800 km/h. operating speed limit, rather limited altitude range and somewhat unpredictable valve life.

One interesting and sometimes objectionable, feature of the pulse jet engine is the sound it makes when in operation. The sound is a series of loud reports caused by the firing of the individual charges of fuel and air in the combustion chamber. The frequency of the reports depends upon the length of the engine form the inlet valves to the end of the tailpipe and upon the temperature of the gases within the engine. The resulting sound is a continuous, loud, and vibratory note that can usually be heard for several kilometers.

The pulse jet has low thermal efficiency. In early designs the efficiency obtained was about 2 to 3% with a total flight life of 30 to 60 minutes. The maximum operating speed is seriously limited by tow factors: (i) It is possible to design a good diffuser at high speeds. (ii) The fiepper valves, the only mechanical part in the pulse jet, also have certain natural frequency and if resonance with the cycle frequency occurs then the valve may remain open and no compression will take place. Also, as the speed increases it is difficult for air to flow back. This reduces total compression pressure as well as the mass flow of air which results in inefficient combustion and lower thrust. The reduction in thrust and efficiency is quite sharp as the speed increases.

Advantages :

- This is very simple device only next to ramjet and is light in weight. It requires very small and occasional maintenance.
- Unlike ramjet, it has static thrust because of the compressed air starting, thus it does not need a device for initial propulsion. The static thrust is even more than the cruise thrust.
- It can run on an almost any type of liquid fuel without much

effect on the performance. It can also operate on gaseous fuel with little modifications.

- Pulse jet is relatively cheap.

Disadvantages :

- 1.The biggest disadvantage is very short life of flapper valve and high rates of fuel consumption. The SFC is as high as that of ramjet.
- The speed of the pulse jet is limited within a very narrow range of about 650-800 km/h because of the limitations in the aerodynamic design of an efficient diffuser suitable for a wide range.
- The high degree of vibrations due to intermittent nature of the cycle and the buzzing noise has made it suitable for pilotless crafts only.
- It has lower propulsive efficiency that turbojet engine.
- The operational range of the pulse jet is limited in altitude range.

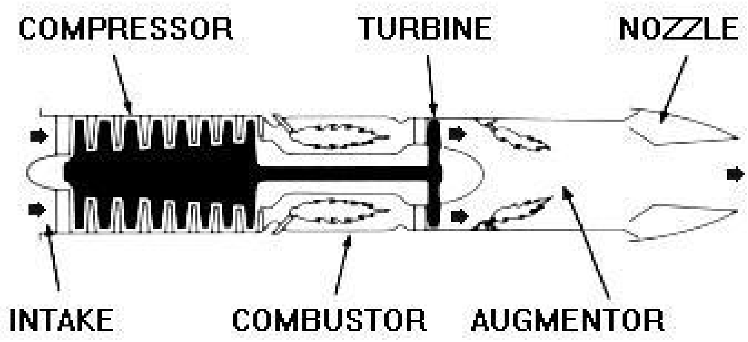
Applications:

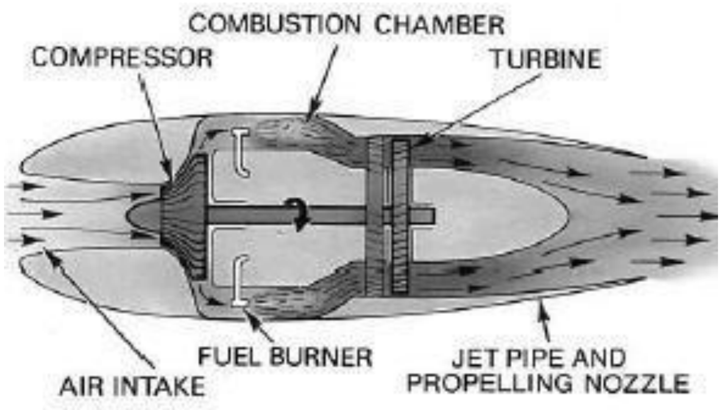
- German V-1 buzz bomb,
- American Helicopter company"s Jet Jeep Helicopter,
- Auxiliary power plant for sail planes.

The Turbojet Engine

The turbojet engine consists of diffuser which shows down the entrance air and thereby compresses it, a slows down the entrance air and thereby compresses it, a simple open cycle gas turbine and an exist gas into kinetic energy. The increased velocity, of air thereby produces thrust.

Figure 2 shows the basic arrangement of the diffuser, compressor, combustion chamber, turbine and the exhaust nozzle of a turbojet engine. Of the total pressure rise of air, a part is obtained by the rain compression in the diffuser and rest in the compressor. The diffuser converts kinetic energy of the air into pressure energy. In the ideal diffuser, the air is diffused isentopically down to zero velocity. In the actual diffuser the process is irreversible adiabatic and the air leaves the diffuser at a velocity between 60 and 120 m/s.

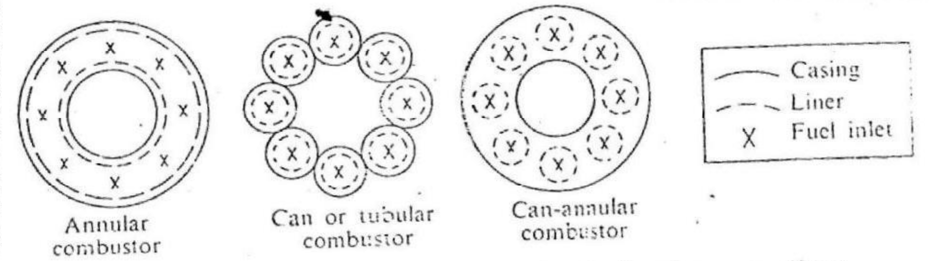




The centrifugal compressor gives a pressure ratio of about 4:1 to 5:1 in a single stage and usually a double-sided rotor is used. The turbojet using centrifugal compressor has a short and sturdy appearance. The advantages of centrifugal compressor are high durability, ease of manufacture and low cost and good operation under adverse conditions such as icing and when sand and small foreign particles are inhaled in the inlet duct. The primary disadvantage is the lack of straight-through airflow. Air leaves compressor in radial direction and ducting with the attendant pressure losses is necessary to change the direction. The axial flow is more efficient than the centrifugal type and gives the turbojet a long slim, streamlined appearance. The engine diameter is reduced which results in low aircraft drag. A multistage axial flow compressor can develop a pressure ratio as high as 6:1 or more. The air handled by it is more than that handled by a centrifugal compressor of the same diameter.

A variation of the axial compressor, the twin-spool (dual spool, split spool or coaxial) compressor has two or more sections, each revolving at or near the optimum speed for its pressure ratio and volume of air. A very high-pressure ratio of about 9:1 to 13:1 is obtained. The use of high-pressure ratio gives very good specific fuel consumption and is necessary for thrust ratings in the region of 50000 N or greater.

In the combustion chamber heat is added to the compressed air nearly at constant pressure. The three types being „can", „annular" and „can-annular" (ref.fig.3). In the can type individual burners, or cans, are mounted in a circle around the engine axis with each one receiving air through its own cylindrical shroud. One of the main disadvantages of can type burners is that they do not make the best use of available space and this results in a large diameter engine. On the other hand, the burners are individually removable for inspection and air-fuel patterns are easier to control than in annular designs. The annular burner is essentially a single chamber made of concentric cylinders mounted co-axially about the engine axis. This arrangement makes more complete use of available space, has low pressure loss, fits well with the axial compressor and turbine and form a technical viewpoint has the highest efficiency, but has a disadvantage in that structural problems may arise due to the large diameter, thin-wall cylinder required with this type of chamber. The problem is more severe for larger engines. There is also some disadvantage in that the entire combustor must be removable from the engine for inspection and repairs. The can- annular design also makes good use of available space, but employs a number of individually replaceable cylindrical inner liners that receive air through a common annular housing for good control of fuel and air flow patterns. The can-annular arrangement has the added advantage of greater structural stability and lower pressure loss than that of the can type.



Combustion chamber, longitudinal and cross sections.

Figure 3

The heated air then expands through the turbine thereby increasing its velocity while losing pressure. The turbine extracts enough energy to drive the compressor and the necessary auxiliary equipments. Turbines of the impulse, reaction and a combination of both types are used. In general, it may be stated that those engines of relatively low thrust and simple design employ the impulse type, while those of large thrust employ the reaction and combination types.

The hot gas is then expended through the exit nozzle and the energy of the hot gas is converted into as much kinetic energy as is possible. This change in velocity of the air passing through the engine multiplied by the mass flow of the air is the change of momentum, which produces thrust. The nozzle can be a fixed jet or a variable area nozzle. The variable area nozzle permits the turbojet to operate at maximum efficiency over a wide range of power output. Clamshell, Finger or Iris, Centre plug with movable shroud, annular ring, annular ring with movable shroud are the various types of variable area nozzle for turbojet engines. The advantage of variable area nozzle is the increased cost, weight and complexity of the exhaust system.

The needs and demands being fulfilled by the turbojet engine are

- Low specific weight – ¼ to ½ of the reciprocating engine
- Relative simplicity – no unbalanced forces or reciprocating engine
- Small frontal area, reduced air cooling problem- less than ¼ th the frontal area of the reciprocating engine giving a large decrease in nacelle drag and consequently giving a greater available excess thrust or power, particularly at high speeds.
- Not restricted in power output - engines can be built with greatly increased power output over that of the reciprocating engine

without the accompanying disadvantages.

- Higher speeds can be obtained – not restricted by a propeller to speeds below 800 km/h.

Turboprop Engine (Propeller turbine, turbo-propeller, prop jet, turbo-prop)

For relatively high take-off thrust or for low-speed cruise applications, turboprop engines are employed to accelerate a secondary propellant stream, which is much larger than the primary flow through the engine. The relatively low work input per unit mass of secondary air can be adequately transmitted by a propeller. Though a ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan engine and with variable pitch, it is capable of a wider range of satisfactory performance.

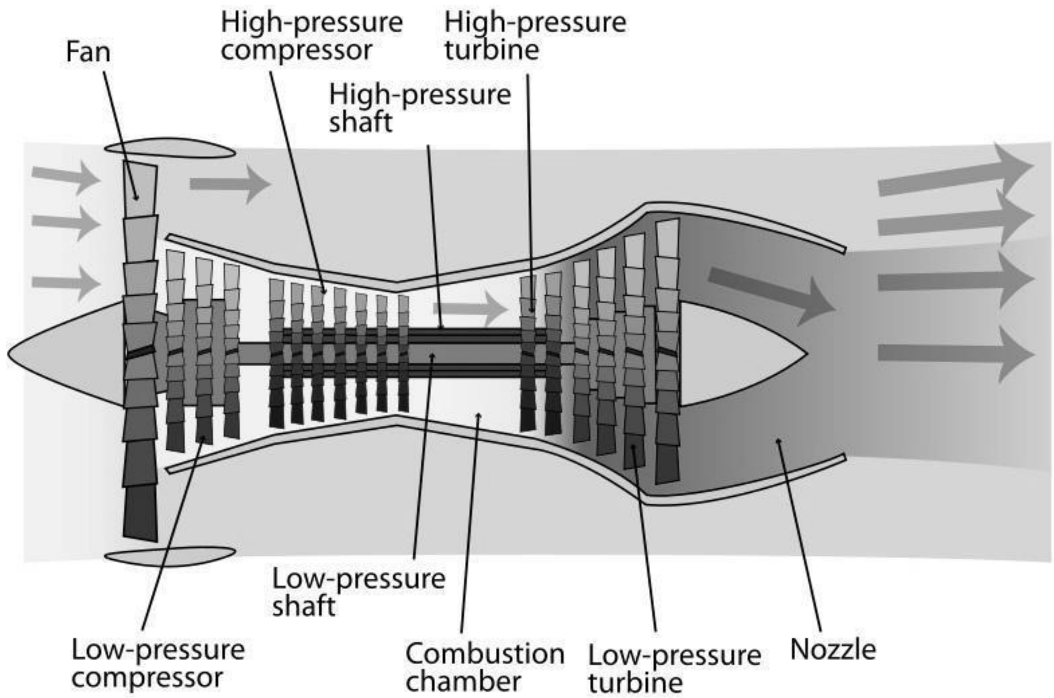
In general, the turbine section of a turboprop engine is very similar to that of a turbojet engine. The main difference is the design and arrangement of the turbines. In the turbojet engine the turbine is designed to extract only enough power from the high velocity gases to drive the compressor, leaving the exhaust gases with sufficient velocity to produce the thrust required of the engine. The turbine of the turboprop engine extracts enough power from the gases to drive both the compressor and the propeller. Only a small amount of power is left as thrust. Usually a turboprop engine has two or more turbine wheels. Each wheel takes additional power from the jet stream, with the result that the velocity of the jet is decreased substantially.

Figure 6 shows a schematic diagram of a turboprop engine. The air enters the diffuser as in a turbojet and is compressed in a compressor before passing to the combustion chamber. The compressor in the turboprop is essentially an axial flow compressor. The products of combustion expand in a two-stage or multistage turbine. One stage of the turbine drives the compressor and the other drives the propeller.

Thus the turbine expansion is used to drive both compressor as well as propeller and less energy is available for expansion in the nozzle. Due to lower speeds of propeller a reduction gear is necessary between turbine and the propeller.

The Turbofan Engine

The turboprop is limited to mach number of about 0.7 because of the sharp decrease in propeller efficiency encountered above that mach number. However, the turboprop concept of increasing mass flow rate without producing an excessive increment in exhaust velocity is valid at any mach number and the use of a ducted fan combined with a jet turbine provides more economical operation at mach numbers close to unity than does the simple jet turbine. If a duct or shroud is placed around a jet engine and air is pumped through the annular passage by means of one or more sets of compressor blades, the resulting engine is called a turbofan, and is capable of producing (under proper conditions) somewhat better thrust specific fuel consumption characteristics than the turbojet itself. Basically, the air passing through the fan bypasses the combustion process but has energy added to it by the compressor fan, so that a sizable mass flow can be shunted through the fan. The air which bypasses the combustion process leaves the engine with a lower amount of internal energy and a lower exhaust speed than the jet exhaust. Yet, the thrust is not decreased since the turbofan can pump more air per unit time than a conventional jet at subsonic speeds. Accordingly, the average exhaust velocity of the turbofan (averaging the turbine flow and the bypass flow) can be made smaller at a given flight speed than that of a comparable turbojet and greater efficiency can be obtained. In turbofan engine the fan cannot be designed for all compressor ratios which is efficient at all mach numbers, thus, the turbofan is efficient over a rather limited range of speeds. Within this speed range, however, its improved cruise economy makes it a desirable unit for jet transport aircraft.



The turbofan engine has a duct enclosed fan mounted at the front or rear of the engine and driven either mechanically geared down or at the same speed as the compressor, or by an independent turbine located to the rear of the compressor drive turbine (Ref. Figure 7). There are two methods of handling the fan air. Either the fan can exit separately from the primary engine air, or it can be ducted back to mix with the primary engine‟s air at the rear. If the fan air is ducted to the rear, the total fan pressure must be higher than the static pressure in the primary engine‟s exhaust, or air will not flow. Similarly, the static fan discharge pressure must be less than the total pressure the primary engine‟s exhaust, or the turbine will not be able to extract the energy required to drive the compressor and fan. By closing down the area of flow of the fan duct, the static pressure can be reduced and the dynamic pressure is increased.

The efficiency of the fan engine is increased over that of the pure jet by converting more of the fuel energy into pressure energy rather than the kinetic energy of a high velocity exhaust gas stream. The fan produces additional force or thrust without increasing fuel flow. As in the turboprop primary engine exhaust gas velocities and pressures are low because of the extra turbine stages needed to drive the fan, and as a result this makes the turbofan engine much quieter. One fundamental difference between the turbofan and the turboprop engine is that the air flow through the fan is controlled by design so that the air velocity relative to the fan blades is unaffected by the aircraft‟s speed. This eliminates the loss in operational efficiency at high air speeds which limits the maximum air speed of propeller driven aircraft.

Fan engines show a definite superiority over the pure jet engines at speeds below Mach 1. The increased frontal area of the fan presents a problem for high- speed aircraft which, of course require small frontal areas. At high speeds air can be offset at least partially by burning fuel in the fan discharge air. This would expand the gas, and in order to keep the fan discharge air at the same pressure, the area of the fan jet nozzle is increased. This action results in an increase in gross thrust due to an increase in pressure times an area (PA), and an increase in gross thrust specific fuel consumption.

Nozzle and diffuser efficiencies

In ideal case, flow through nozzle and diffuser is isentropic. But in actual case, friction exists and affects in following ways:
i) Reduces the enthalpy drop reduces the final velocity of steam iii) Increases the final dryness fraction iv) Increases specific volume of the fluid v) Decreases the mass flow rate

1. A converging, nozzle operating with air and inlet conditions of P = 4 Kg/cm^2, T0 = 450^{o}C and T = 400^{o}C is expected to have an exit static pressure of 2.5 Kgf/cm^2 under ideal conditions. Estimate the exit temperature and mach number, assuming a nozzle efficiency = 0.92 when the expansion takes place to the same back pressure.

$$M\Big]_{\frac{T}{T_0}=\frac{673}{723}=0.93} = 0.61$$

$$\left.\frac{P}{P_0}\right]_{M=0.61} = 0.778$$

$$P_0 = \frac{4}{0.778} = 5.14 \text{ Kgf / cm}^2$$

$$M_{2i}\Big]_{\frac{P_{2i}}{P_0}=\frac{2.5}{5.14}=0.486} = 1.07$$

$$\left.\frac{T_{2i}}{T_0}\right]_{M=1.07} = 0.814$$

$$T_{2i} = 723 \times 0.814 = 588.5 \text{ K}$$

Nozzle efficiency

$$12E\ \eta_N = \frac{T_1 - T_{2a}}{T_1 - T_{2i}}$$

$$0.92 = \frac{673 - T_{2a}}{673 - 588.5}$$

$$T_{2a} = 595.3\ K$$

$$M_{2a} \Big] \frac{p_{2a}}{p_{20}} = \frac{595.3}{7.23} = 0.823 \quad = 1.04$$

2. An aircraft flies at a speed of 520 kmph at an altitude of 8000 m. The diameter of the propeller of an aircraft is 2.4 m and flight to jet speed ratio is 0.74. Find the following:

(i) The rate of air flow through the propeller

(ii) Thrust produced

(iii) Specific thrust

(iv) Specific impulse

(v) Thrust power Given:

Air craft speed (or) Flight speed = 520 kmph

$$= \frac{520 \times 10^3}{3600\ s}$$

$$= 144.44\ m/s$$

Altitude $z = 8000$ m

Diameter of the propeller $d = 2.4\ m$

Flight to jet speed ratio $\sigma = \frac{u}{c_j} = 0.74$

Where c_j – *jet speed (or) Speed of exit gases from the engine*

Solution : Area of the propeller disc $A = \frac{\pi}{4} d^2$

- 4 (z.4)'

—· A 4 2

K own gas nif›er at Z —- 8000in

$\rho = 0.525\ kg/m^3$

Effective speed ratio $,\sigma = \frac{\cdot\cdot}{}$

$0.74 = \frac{}{c_j}$

Vefociy of yet .r, = 195.19 in /r

Yelocin of cir fl 'af the propeller

$$c = \frac{1}{2}\left[u + c_j\right]$$

= [144.44 + 195,19

$c = 169.81\ m/s$

Mass flow rate of air – fuel mixture

$m^{\bullet} = \rho A c$

= 0,525 > 4.52 I6S.81

$m^{\bullet} = 402.96\ kg/s$

We know that

$m = m_a + m_f$

since trims fleet rate of fiiel(m“y) is itot green. let its take

Mass noir rofe of air ri‘ =4O2.96 /s

Tltrust pi'odiiceḑ F — m‘ c —v

= 402.9d[l95.t9 — 144.44a

F = 2O.45xl0³ N

ƒ›erific thrust F -- ——

20.41x10'

Fp — 50.75 *H I* (fig/*)

Spec!fir impulse (/q) = —

$= \frac{F}{m^{\bullet} g}$

$m_a \times g$

$\frac{20.45 \quad l}{402.96x9.81}$

= 5 1 *s*

$Thrust\ power, P = Thrust(F) \times Flight\ speed\ (u)$

= 20.45 < l x 144.44

$P = 2.95 \times 10^6\ W$

Re *slllf*,“ (/) Ill". - 402.9d @/ *s*

(ii) $F = 20.45 \times 10^3\ N$

(iii) $\boxed{F_{sp} = 50.75\ N/(kg/s)}$

(iv) $\boxed{I_{sp} = 5.17\ s}$

(v) $\boxed{p = 2.95 \times 10^6 W}$

Tutorial Problems:

1.The diameter of the propeller of an aircraft is 2.5m; It flies at a speed of 500Kmph at an altitude of 8000m. For a flight to jet speed ratio of 0.75 determine (a) the flow rate of air through the propeller, (b) thrust produced (c) specific thrust, (d) specific impulse and
(e) the thrust power.

2. An aircraft flies at 960Kmph. One of its turbojet engines takes in 40 kg/s of air and expands the gases to the ambient pressure .The air –fuel ratio is 50 and the lower calorific
value of the fuel is 43 MJ/Kg .For maximum thrust power determine (a)jet velocity (b) thrust (c) specific thrust (d) thrust power (e) propulsive, thermal and overall efficiencies and (f) TSFC

3. A turboprop engine operates at an altitude of 3000 meters above mean sea level and an aircraft speed of 525 Kmph. The data for the engine is given below

Inlet diffuser efficience =0.875,Compressor efficieny =0.790. Velocity of air at compressor entry =90m/s Properties of air :_ =1.4, Cp =1.005 KJ/kg K

4. A turbo jet engine propels an aircraft at a Mach number of 0.8 in level flight at an altitude of 10 km .The data for the engine is given below: Stagnation temperature at the turbine inlet =1200K Stagnation temperature rise through the compressor =175 K Calorific value of the fuel =43 MJ/Kg Compressor efficiency =0.75. Combustion chamber efficiency =0.975, Turbine efficiency =0.81,Mechanical efficiency of the power transmission between turbine and compressor =0.98, Exhaust nozzle efficiency=0.97, Specific impulse =25 seconds.Assuming the same properties for air and combustion gases calculate,

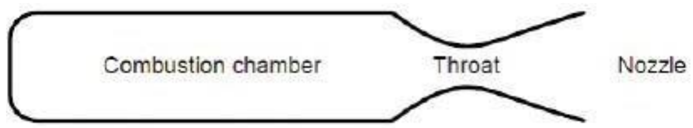
i. Fuel –air ratio
ii. Compressor pressure ratio,
iii. Turbine pressure ratio
iv. Exhaust nozzles pressure ratio ,and
v. Mach number of exhaust jet

Rocket Propulsion

In the section about the rocket equation we explored some of the issues surrounding the performance of a whole rocket. What we didn‟t explore was the heart of the rocket, the motor. In this section we‟ll look at the design of motors, the factors which affect the performance of motors, and some of the practical limitations of motor design. The first part of this section is necessarily descriptive as the chemistry, thermodynamics and maths associated with motor design are beyond the target audience of this website.

General Principles of a Rocket Motor

In a rocket motor a chemical reaction is used to generate hot gas in a confined space called the combustion chamber. The chamber has a single exit through a constriction called the throat. The pressure of the hot gas is higher than the surrounding atmosphere, thus the gas flows out through the constriction and is accelerated.



Propellants

The chemical reaction in model rocket motors is referred to as an “exothermal redox” reaction. The term “exothermal” means that the reaction gives off heat, and in the case of rocket motors this heat is mainly absorbed by the propellants raising their temperature.

The term “redox” means that it is an oxidation/reduction reaction, in other words one of the chemicals transfers oxygen atoms to another during the reaction. The two chemicals are called the oxidising agent and the reducing agent.

The most popular rocket motors are black powder motors, where the oxidising agent is saltpetre and the reducing agents are sulphur and carbon. Other motors include Potassium or ammonium perchlorate as the oxidising agent and mixtures of hydrocarbons and fine powdered metals as the reducing agents. Other chemicals are often added such as retardants to slow down the rate of burn, binding agents to hold the fuel together (often these are the hydrocarbons used in the reaction), or chemicals to colour the flame or smoke for effects. In hybrid motors a gaseous oxidiser, nitrous oxide, reacts with a hydrocarbon, such as a plastic, to produce the hot gas.

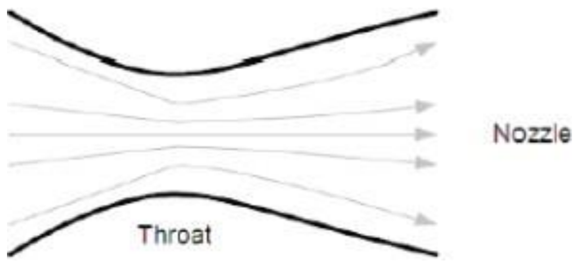
Energy Conversion

This reaction releases energy in the form of heat, and by confining the gas within the combustion chamber we give it energy due to its pressure. We refer to the energy of this hot pressurised gas as its “enthalpy”. By releasing the gas through the throat the rocket motor turns the enthalpy of the gas into a flow of the gas with kinetic energy. It is this release of energy which powers the rocket. So the energy undergoes Two conversions:

- Chemical energy to enthalpy
- Enthalpy to kinetic energy

The conversion from chemical energy to enthalpy takes place in the combustion chamber. To obtain the maximum enthalpy it is clearly important to have a reaction which releases lots of heat and generates lots of high energy molecules of gas to maximise pressure here is clearly a limit to the temperature & pressure, as the combustion chamber may melt or split if these are too high. The designer has a limitation placed on his choice of reagents in that the reaction must not heat the combustion chamber to a point where it is damaged, nor must the pressure exceed that which the chamber can survive.

Changing enthalpy to kinetic energy takes place in the throat and the nozzle. Our mass of hot gas flows into the throat, accelerating as the throat converges. If we reduce the chameter of the throat enough, the flow will accelerate to the speed of sound, at which pint something unexpected occurs. As the flow diverges into the nozzle it continues to accelerate beyond the speed of sound, the increase in velocity depending on the increase in area. This type of nozzle is called a De Laval nozzle.



Kinetic energy of a body:

If we consider a small volume of gas, it will have a very low mass. As we accelerate this gas it gains kinetic energy proportional to the square of the velocity, so if we double the velocity we get four times the kinetic energy. The velocity of the supersonic flow increases proportional to the increase in area of the nozzle, thus the kinetic energy increases by the fourth power of the increase in nozzle diameter. Thus doubling the nozzle diameter increases the kinetic energy by 16 times! The De

Laval nozzle make rocket motors possible, as only such high velocity flows can generate the energy required to accelerate a rocket.

In model rockets the reaction is chemical generally short lived, a few seconds at most, sothe amount of heat transferred to the structural parts of the motor is limited. Also, the liner of the motor casing acts to insulate the casing from the rapid rise in temperature which would result from a reaction in direct contact with the metal casing. Model rocket motors also run at quite low pressure, well below the limits if the motor casing, further protecting the casing. It can be seen that the enthalpy of a model rocket motor is thus quite low. In large launch vehicles such as Ariane, the pressure and temperature are high, the burn may last several minutes, and the mass budget for the designer is very tight. Designing motors for these purposes is highly complex.

Solid propellant:

In ballistics and pyrotechnics, a propellant is a generic name for chemicals used for propelling projectiles from guns and other firearms. Propellants are usually made from low explosive materials, but may include high explosive chemical ingredients that are diluted and burned in a controlled way (deflagration) rather than detonation. The controlled burning of the propellant composition usually produces thrust by gas pressure and can accelerate a projectile, rocket, or other vehicle. In this sense, common or well known propellants include, for firearms, artillery and solid propellant rockets: Gun propellants, such as:

- Gunpowder (black powder)
- Nitrocellulose-based powders
- Cordite
- Ballistite
- Smokeless powders

Composite propellants made from a solid oxidizer such as ammonium perchlorate or ammonium nitrate, a rubber such as HTPB, or PBAN (may be replaced by energetic polymers such as polyglycidyl nitrate or polyvinyl nitrate for extra energy) , optional high explosive fuels (again, for extra energy) such as RDX or nitroglycerin, and usually a powdered metal fuel such as aluminum.

Some amateur propellants use potassium nitrate, combined with sugar, epoxy, or other fuels / binder compounds.

Potassium perchlorate has been used as an oxidizer, paired with asphalt, epoxy, and other binders.

A solid rocket motor:

Solid rocket propellants are prepared as a mixture of fuel and oxidizing components called 'grain' and the propellant storage casing effectively becomes the combustion chamber. Liquid-fueled rockets typically pump separate fuel and oxidiser components into the combustion chamber, where they mix and burn. Hybrid rocket engines use a combination of solid and liquid or gaseous propellants. Both liquid and hybrid rockets use injectors to introduce the propellant into the chamber. These are often an array of simple jets- holes through which the propellant escapes under pressure; but sometimes may be more complex spray nozzles. When two or more propellants are injected the jets usually deliberately collide the propellants as this breaks up the flow into smaller droplets that burn more easily.