PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Approved by **AICTE** and Affiliated to **Anna University**) sky is the limit

Department of Mechanical Engineering

Regulation 2021

II Year – IV Semester

ME3491 THEORY OF MACHINES

ME3491	THEORY OF MACHINES	L	T	P	C
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COURSE OBJECTIVES:

1 To study the basic components of mechanisms, analyzing the assembly with respect to the displacement, velocity, and acceleration at any point in a link of a mechanism and design cam mechanisms for specified output motions.
2 To study the basic concepts of toothed gearing and kinematics of gear trains
3 To Analyzing the effects of friction in machine elements
4 To Analyzing the force-motion relationship in components subjected to external forces and analyzing of standard mechanisms.
5 To Analyzing the undesirable effects of unbalances resulting from prescribed motions in mechanism and the effect of dynamics of undesirable vibrations.

UNIT – I KINEMATICS OF MECHANISMS 9

Mechanisms – Terminology and definitions – kinematics inversions of 4 bar and slide crank chain – kinematics analysis in simple mechanisms – velocity and acceleration polygons– Analytical methods – computer approach – cams – classifications – displacement diagrams - layout of plate cam profiles – derivatives of followers motion – circular arc and tangent cams.

UNIT – II GEARS AND GEAR TRAINS 9

Spur gear – law of toothed gearing – involute gearing – Interchangeable gears – Gear tooth action interference and undercutting – nonstandard teeth – gear trains – parallel axis gears trains – epicyclic gear trains – automotive transmission gear trains.

UNIT – III FRICTION IN MACHINE ELEMENTS 9

Surface contacts – Sliding and Rolling friction – Friction drives – Friction in screw threads – Bearings and lubrication – Friction clutches – Belt and rope drives – Friction aspects in brakes– Friction in vehicle propulsion and braking.

UNIT – IV FORCE ANALYSIS 9

Applied and Constrained Forces – Free body diagrams – static Equilibrium conditions – Two, Three and four members – Static Force analysis in simple machine members – Dynamic Force Analysis – Inertia Forces and Inertia Torque – D'Alembert's principle – superposition principle – dynamic Force Analysis in simple machine members

UNIT – V BALANCING AND VIBRATION 9

Static and Dynamic balancing – Balancing of revolving and reciprocating masses – Balancing machines – free vibrations – Equations of motion – natural Frequency – Damped Vibration – bending critical speed of simple shaft – Torsional vibration – Forced vibration – harmonic Forcing – Vibration isolation. (Gyroscopic principles)

TOTAL: 45 PERIODS

OUTCOMES: At the end of the course the students would be able to

1. Discuss the basics of mechanism.
2. Solve problems on gears and gear trains.
3. Examine friction in machine elements.
4. Calculate static and dynamic forces of mechanisms.
5. Calculate the balancing masses and their locations of reciprocating and rotating masses. Computing the frequency of free vibration, forced vibration and damping coefficient.

REFERENCES:

1. AmitabhaGhosh and Asok Kumar Mallik, “Theory of Mechanisms and Machines”, Affiliated East-West Pvt. Ltd., 1988.
2. Rao.J.S. and Dukkipati.R.V. “Mechanism and Machine Theory”, New Age International Pvt. Ltd., 2nd edition,2014.
3. Rattan, S.S, “Theory of Machines”, McGraw-Hill Education Pvt. Ltd., 5th edition 2019.
4. Robert L. Norton, Kinematics and Dynamics of Machinery, Tata McGraw-Hill, 2013.
5. Wilson and Sadler, Kinematics and Dynamics of Machinery, Pearson, 2008.

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Low (1) ; Medium (2) ; High (3)															

UNIT-01 - Kinematic of Mechanics

Theory of machine:

- The study of relative motion between the parts of a machine
- The study of the forces which act on the parts.

KOM deals with the study of relative motion between the various parts of the machines

DOM deals with the study of various forces acting on various machine elements.

KOM is working without considering a force.

→ displacement [distance travel / time taken]

→ velocity

→ Acceleration [changes in velocity with respect to time]

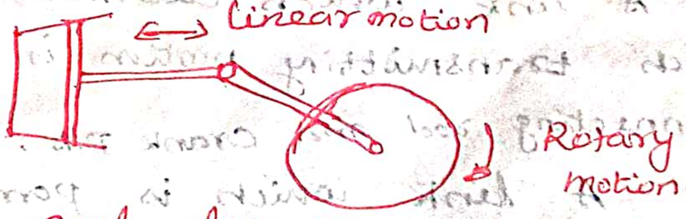
Dynamics of Mechanism:

It involves the calculation of force impressed upon parts of a mechanism.

→ IC Engine [Internal Compression]

→ Piston

→ Connecting rod.

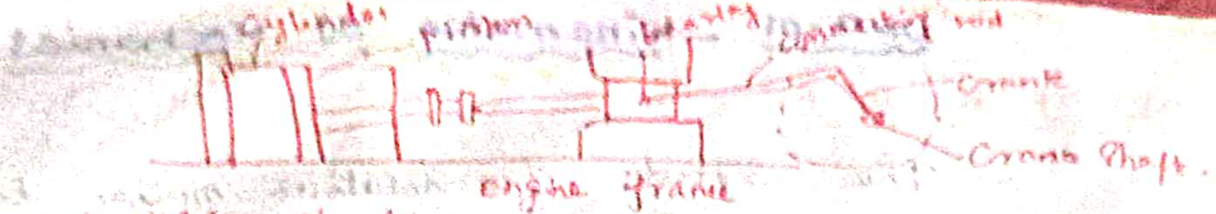


Terminology and definition of mechanism:

I) Kinematic link: [To transfer energy]

A kinematic link or element is defined as a part of the machine that has relative motion with regard to another part of the same machine.

Each part of a machine which moves relative to some other part is known as kinematic link or element. Ex: Reciprocating steam engine



Characteristic of links

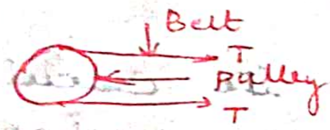
→ It should be capable of relative motion

Rigid body:

If the distance b/w A & B doesnot change then the body is called rigid body

Resistant body:

Body which is rigid for a purposes they have to serve. But naturally they are non-rigid body.



when Tension given it is rigid body other wise it is non-rigid body.

Types of link

- Rigid link
- Flexible link
- fluid link

1. Rigid link – A link which doesnot undergo any deformation which transmitting motion is known as rigid link. eg: connecting rod and crank pin.
2. Flexible link – A link which is partially deformed in such a way that it doesnot affect the transmission of a motion is known as flexible link
   (eg) – chain drive, belt drive & rope.
3. Fluid link – In this motion is transmitted through the fluid by applying pressure
   (eg) – hydraulic press, hydraulic jack

Machine vs Mechanism:

1. Machine	Mechanism
a) Machine may have any mechanism for transmitting mechanical work or power	Mechanism is the skeleton outline of the machine to produce motion between the various link
b) A basic fn of machine is to obtain mechanical output in most cases	Mechanism transmit & modifies the motion alone.
c) eg : Washing machine	eg - spring, door

Types of constrained motion

1. **Completely constrained motion:**

When the relative motion between two link is limited to a definite direction, then the motion is said to be completely constrained motion.

a) DOF = 1

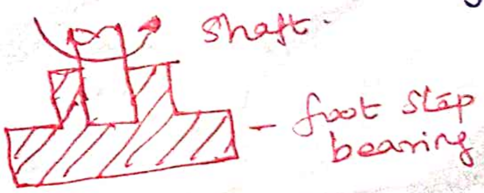
2. **Incompletely constrained motion:**

When the relative motion between two link can take place in ~~action~~ more than one direction then the motion is called as incompletely constrained motion.

DOF = 2

3. **Successfully constrained motion:**

When the relative motion bet the link is not completely by itself, but it is achieved by some other means, then it said to be successfully (or) partially constrained motion.



matic pair :

A kinematic pair is a joint of two link that permits relative motion

→ When any two link are connected in such a way that their relative motion is completely or successfully constrained they form a kinematic pair.

Ex: In a reciprocating steam engine

(a) crank and connecting rod

(b) connecting rod and piston rod

(c) Piston and engine cylinder

The crank and connecting rod of the steam engine are said to form a kinematic pair, because (i) they are in contact and (ii) they have relative motion bet them.

Types of pair :

1. Type of contact between elements.

a) Lower pair

A pair of link having surface or area contact between the member is known as lower pair

eg → Nut turning on a screw

→ ball bearing

→ Shaft rotating in bearing.

b) Higher pair :

When a pair has a point (or) line contact between the links it is known as higher pair

eg : → wheel rolling on a surface

× Cam & follower pair

× ball or roller bearing

* Types of relative motion :

(i) sliding pair :

When two link have a sliding motion relative to each other

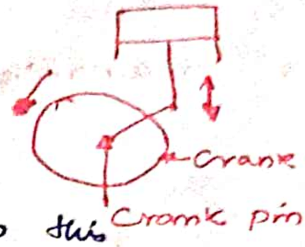
eg – Piston & cylinder.

(ii) Turning pair :

when one element revolves around another element it form a turning pair.

eg : Shaft & bearing

→ Rotating crank



(iii) Screw pair :

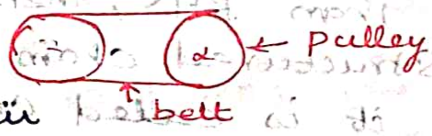
It is also known as helical. In this type of pair two mating elements have threads on it.

eg : Nut & bolt

→ lead screw of a lathe with nut.

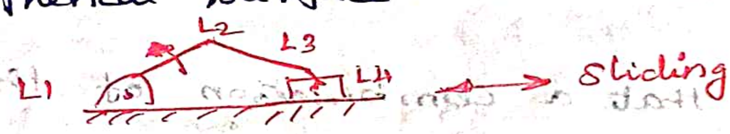
(iv) Rolling pair :

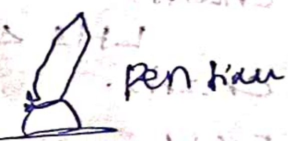
when one element is free to roll over the other one



(v) Spherical pair

when one element move relative to the other along a spherical surface.





Kinematic chain

A kinematic chain is defined as the combination of kinematic pair in which each link form a part of two kinematic pair and the relative motion between the link is either completely constrained or successfully constrained.

A chain be locked, constrained and unconstrained.

A kinematic chain having four link is known as simple kinematic chain.

A kinematic chain having more than four link is known as compound kinematic chain.

→ when link are connected in a sequence with first link is connected to the last then the chain is called closed kinematic chain

→ when the link are not connected to the last, then the chain is called as open kinematic chain

Conditions to form a kinematic chain.

The required equation / condition to form a kinematic chain are

$$\left.\begin{array}{l} n = 2p - 4 \\ j = \frac{3}{2}n - 2 \end{array}\right\} \text{lower pair}$$

$$j + \frac{h}{2} = \frac{3}{2}n - 2 \Rightarrow j = \frac{3}{2}n - 2 \} \text{ higher pa}$$

where

n = Number of link

p = No of pair

j = Number of binary joint and

h = number of higher pairs

LHS > RHS

LHS is greater than RHS, then the given chain is called locked (or) structured chain

LHS = RHS → then it is called constrained kinematic chain.

LHS < RHS → then it is called unconstrained kinematic chain.

EX: 1 State that a combination of three links cannot form a kinematic chain

Soln: consider an assemblage of three link AB, BC and CA which are pin joined at A, B and C as in fig

$n = 3$

$P = 3$

$j = 3$

$h = 0$

$n = 2P - 4 = 2(3) - 4$

$6 - 4 = 2$

$\boxed{n = 2}$

$j + \frac{h}{2} = \frac{3}{2}n - 2$

$3 + 0 = \frac{3}{2} \times 2 - 2$

$3 > 1$

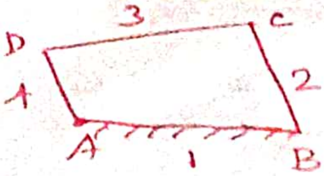
LHS > RHS

$j = \frac{3}{2}n - 2$

$= \frac{3}{2} \times 2 - 2$

Locked or structured chain

Four link chain



$n = 4 \quad h = 0$
$p = 4$
$j = 4$

$n = 2p - 4$
$= 2 \times 4 - 4$
$= 8 - 4$
$\boxed{n = 4}$

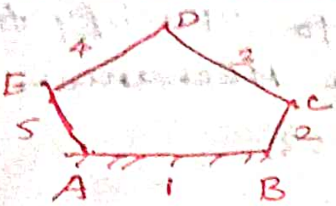
eqn $\quad j + h/2 = 3/2\, n - 2$

$4 + 0 = \frac{3}{2} \cdot 4 - 2$

$\boxed{j = 6 - 2}$

LHS = RHS Constrained kinematic chain

Five link chain



$n = 5,\ P = 5,\ j = 5$ and $h = 0$

$n = 2P - 4 = 2 \times 5 - 4 = 10 - 4 = 6$

$\boxed{n = 6}$

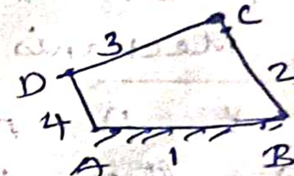
$j = 3/2\, n - 2$

$= 3/2 \times 6 - 2$

$5 < 7$

LHS < RHS Unconstrained chain

Types of joints in chain



1. Binary joint:

If two link are joined at the same connection, the joint is known as binary joint

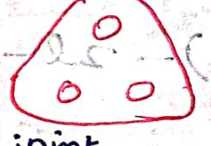
 → binary link

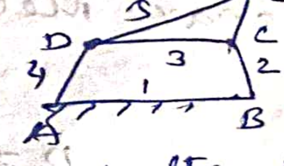
2. Ternary joint

$j =$ No of binary joint $+ 2 \times$ No of ternary joint

$= 3 + (2 \times 2) = 7$

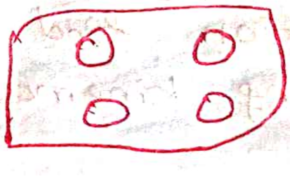
If three link are joined at the same connection, then the joint is known as Ternary joint.
one ternary joint is equivalent to two binary joint

 → Ternary link

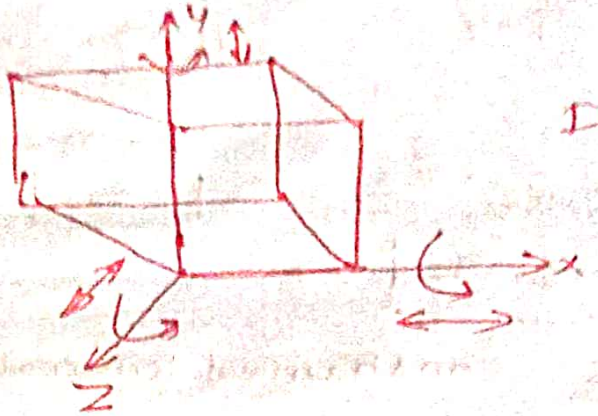


3. Quaternary joint

If four link are joined at the same connection, then the joint is known as Quaternary joint

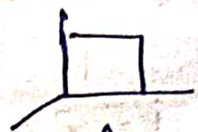
quaternary link → 

Degree of freedom (or) mobility



Degree of freedom is the number of independent parameter required to specify the location of every link with in a Mechanism.

Planar Mechanism:



* A mechanism formed when all the links of the mechanism lie in the same plane is known as planar mechanisms
* It has max of 3 degree of freedom

Spatial Mechanism:

* A Mechanism formed when all the links of the Mechanisms lie on the different plane is known as spatial mechamism.
* It has max [3 - translatory motion in x, y, z axis & 3 - Rotary motion in x y z axis] of 6 degree of freedom.

To find degree of freedom:

Kutzbach criterion to find DOF in planar Mechanism.

$$DOF = 3(n-1) - 2l - h$$

n = No of link
l - No of lower pair
h - No of higher pair

Using Kutzbach criterion find the degree of freedom for the following kinematic chains.

(i) Three bar chain
ii) Four bar chain
iii) Cam with roller
(iv) Cam with knife edge

(i) Three bar chain

$n = 3,\ l = 3$

$DOF = 3(3-1) - 2(3) = 0$

$\boxed{DOF = 0}$

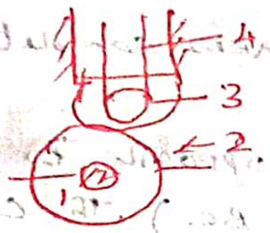
(ii) Four bar chain

$n = 4,\ l = 4,\ h = 0$

$DOF = 3(4-1) - 2(4) - 0$

$\boxed{DOF = 1}$

(iii) Cam with roller

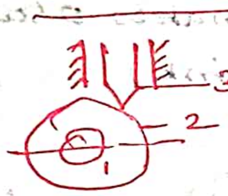


$n = 4,\ l = 3,\ h = 1$

$DOF = 3(4-1) - 2(3) - 1$

$\boxed{DOF = 2}$

(iv) Cam with knife edge

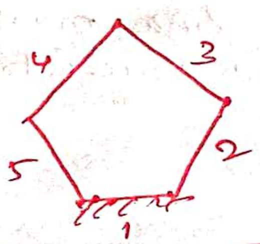


$n = 3 \quad l = 3 \quad h = 1$

$DOF = 3(3-1) - 2(3) - 1$

$\boxed{DOF = -1}$

(v)

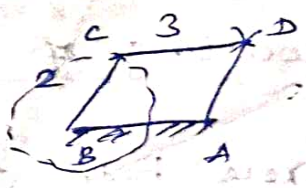


$n = 5,\ l = 5\ ;\ h = 0$

$= 3(n-1) - 2l - h$

$= 3(5-1) - 2(5) - 0$

$\boxed{DOF = 2}$



$2+3 \le 1+4$

Grashof's law (2marks)

The sum of the shortest and longest link should not be greater than the sum of the other two link length. If there is to be continuous relative motion between the two links.

Inversion of Mechanism:

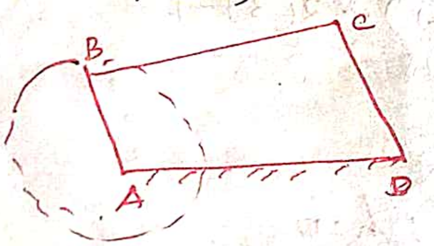
* If one link of kinematic chain is fixed then it is called a Mechanism
* The process of fixing different link one at a time to get different mechanism is called inversion of mechanism.

Kinematic chain

- Four bar chain (All four turning pair)
- Slider crank pin (Three turning pair and one sliding pair)
- Double slider crank chain (Two turning & 2 sliding pair)

Four bar chain

1. Frame - The fixed link is known as frame (link AD).
2. Crank (or driver) - A link that makes complete revolution is called crank (AB)
3. Coupler (or connecting rod) - A link opposite to the fixed link is known as coupler (link BC). The coupler is the link which is not connected to the frame.
4. Lever (or follower) - The link which makes oscillation (a partial rotation) is known as lever. (CD link)



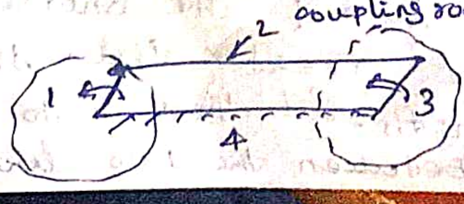
Application

The electric motor are used to drive the mechanism. In such case one of the link must be crank to recieved power from motor.

Types of 4 bar chain

Coupling of locomotive wheels

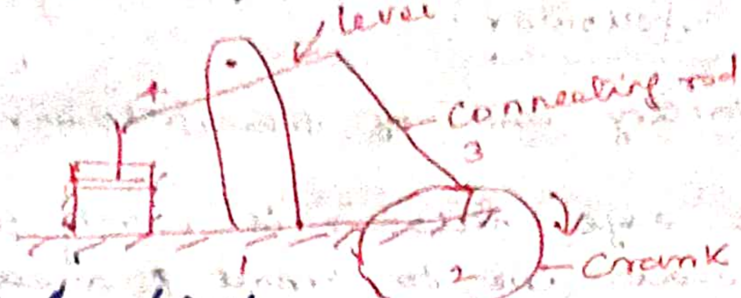
It is example of a double crank mechanism where both crank about the point in the fixed link. It consist of four link and the opposite link are equal in length



pair - 4
Turning pair

Beam Engine

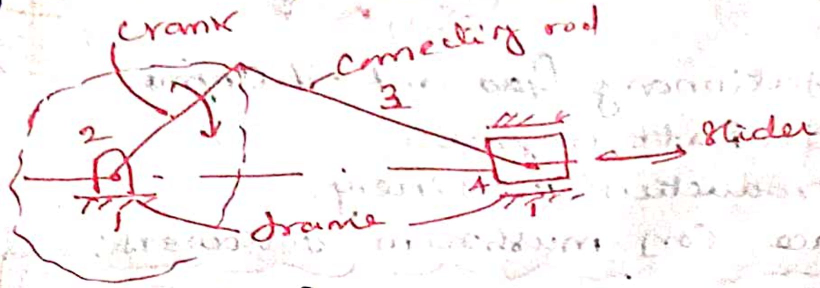
Example of crank - rocker mechanism. where one link oscillates, while other rotates about the fixed link.



Inversion of single slider crank chain

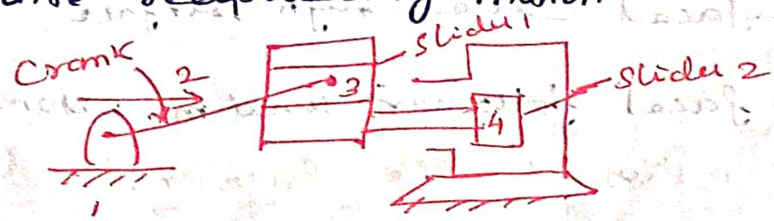
A single slider crank is a modification of a basic four bar chain. It consist of one sliding pair and the three turning pair.

* This used to convert reciprocating motion into rotary motion and vice versa.

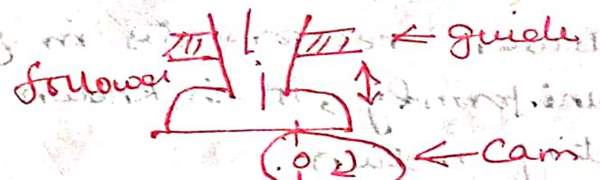


Scotch Yoke mechanism:

The inversion is used for converting rotary motion into reciprocating motion.



CAM:

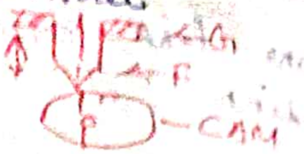


Cam is a rotating machine element which gives reciprocating or oscillating motion to another machine element called follower.

App: Internal combustion engines [inlet & outlet valve]

Classification of followers:

According to the Surface in Contact



1. Knife-edge follower:

→ When conducting end of the follower has a sharp knife-edge

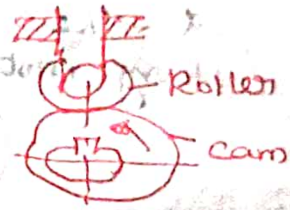
→ The knife edge follower is rarely used because of excessive wear due to small area of contact

→ Side thrust (force)

2. Roller follower:

→ less wear compared to knife edge follower.

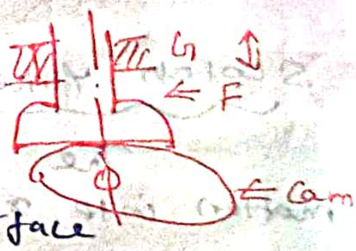
→ Side thrust, more space



Application

* Stationary Gas and oil engine
* Aircraft engine
* Production machinery.

3. Flat Place (or) mushroom followers:



→ less side thrust compared to knife edge follower.

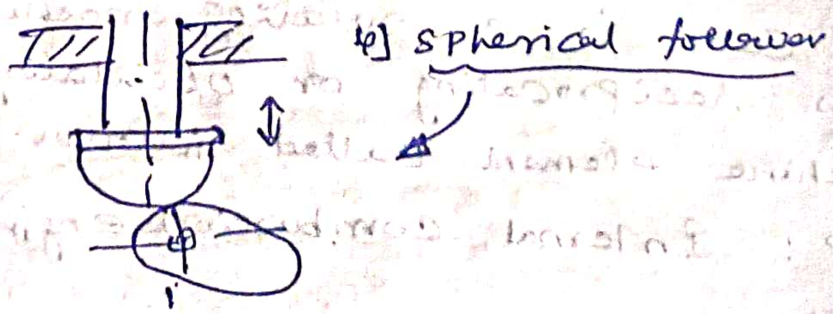
→ The flat-faced caused high surface stresses.

→ It is flat faced follower used in automobile engine

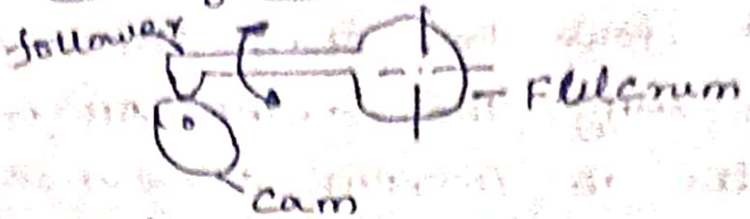
According to the motion of the follower.

1. Reciprocating or translating motion:

When a follower reciprocates in guide as the cam rotates uniformly, it is known as reciprocating (or) translating motion.

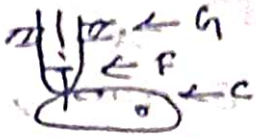


2. oscillating / rotating follower:



When the uniform rotary motion of a cam is converted into pre-determined oscillating motion of the follower.

→ According to the location of line of movement of the follower passes through the centre of the rotation of the CAM.



3. Off-set follower:

When the motion of the follower is along an axis away (offset) from the axis of cam centre, it is called off-set follower.

→ In order to reduce the side thrust in guide of followers.

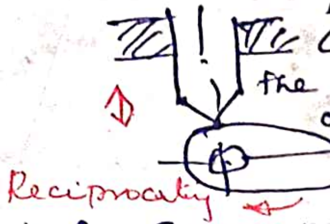
Classification of cam:

CAM

According to shape.

1. wedge / flat cam
2. Radial / plate cam / disc
3. cylindrical cam
4. spiral cam
5. spherical cam

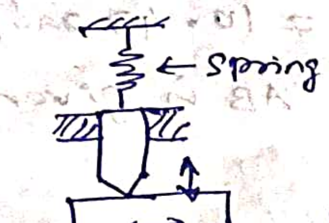
Radial / plate / disc



of The surface of the cam is so shaped that the follower reciprocates or oscillates in a plane at right angle to the axis of the cam

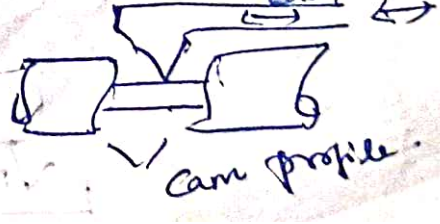
* A cam made out of from plate, such follower moves radially from the centre of rotation is known as plate cam

1) wedge / flat cam:



→ It is an translation movement
→ follower can either translate or oscillate

cylindrical cam:



Problem:

1. In a four bar chain ABCD, AD is fixed and is 120mm long. The crank AB is 30mm long and rotates at 100rpm clockwise. While the link CD = 60mm oscillates about D, BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°

Given data

A = 120mm (fixed) AB = 30mm; N_{BA} = 100rpm (cw)

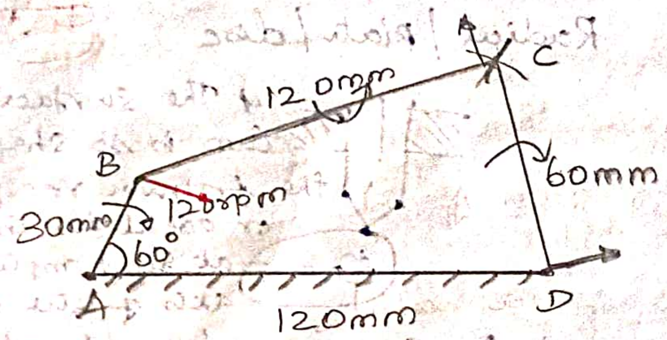
CD = 60mm, BC = AD; ∠BAD = 60°

Soln: Relative velocity method.

Procedure

scale 1:2)

Step 1 : Configuration diagram (Say 1cm = 20mm)



Step 2 : Velocity of input link

Speed of input link, N_{BA} = 100rpm (given)

$$\omega_{BA} = \frac{2\pi N_{BA}}{60}$$

$$= \frac{2\times\pi\times 100}{60} = 10.47 \text{ rad/s}$$

Velocity of the input link AB is given by

$$V_{BA} = r \times \omega_{BA}$$

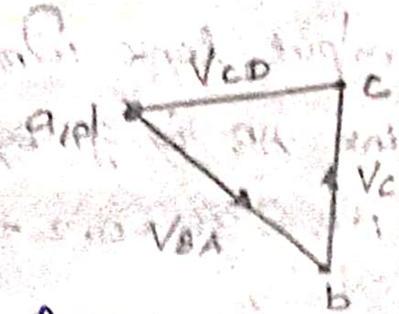
30cm = 0.03m

$$= AB \times \omega_{BA}$$

$$= 0.03 \times 10.47$$

$$\boxed{V_{BA} = 0.3141 \text{ m/s}}$$

Step 3 : Velocity diagram scale (1 cm = 0.1 m/s)



Step 4 Angular velocity of link CD

Velocity of link CD, $V_{CD} = 0.2387$ m/s

The angular velocity of link CD is gn by

$$\omega_{CD} = \frac{V_{CD}}{CD} = \frac{0.2387}{0.06}$$

$\boxed{\omega_{CD} = 4 \text{ rad/s}}$ (clockwise about D)

2. In a four link mechanism the crank AB rotates at 36 rad/s. The length of the links AB = 200 mm, BC = 400 mm, CD = 450 mm and AD = 600 mm. AD is the fixed link. At the instant when AB is at right angle to AD, Det the velocity of

(i) the mid point of link BC.

(ii) a point on link CD, 100 mm from the the points connecting the link CD and AD

Data :

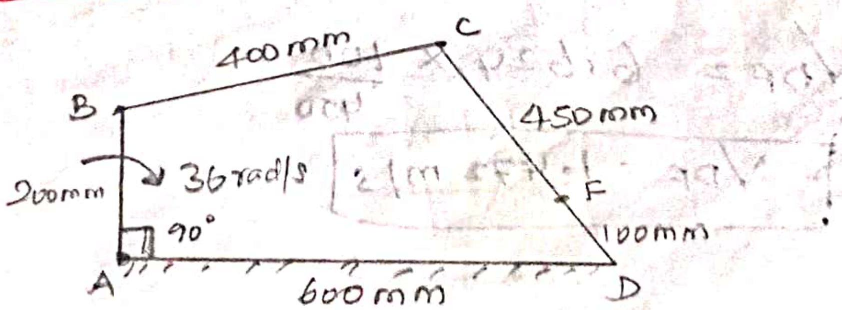
$\omega_{BA} = 36$ rad/s AB = 200 mm, BC = 400 mm,

CD = 450 mm AD = 600 mm (fixed) ; $\angle BAD = 90°$

Soln : Relative velocity method :

Procedure

Step 1 : Configuration diagram Scale 1:2 (1 cm = 100 mm)



Step 2 Velocity of input link

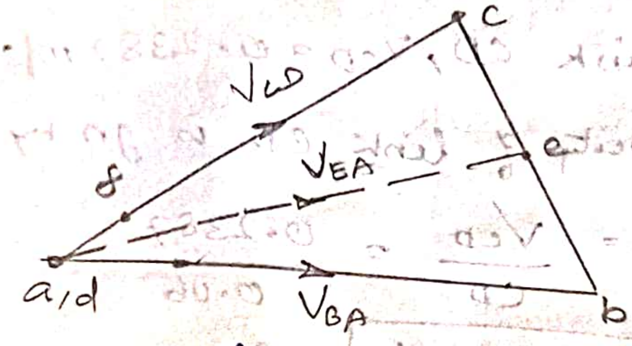
Angular velocity of input link $\omega_{BA} = 36\,rad/s$

Velocity of the input link AB is gn by

$$V_{BA} = \omega_{BA} \times AB = 36 \times 0.2 = 7.2\,m/s$$

Step 3 Velocity diagram (Say 1cm = 1.5 m/s)

5cm = 0.712 m/s



Step 4 Velocities of various links

(i) Velocity of point E on link BC:

Locate the point e at the centre of vector to be fond velocity of mid point of link BC. Join e and a. ae give mid point E.

~~Measure velocity of~~ By measurement from the velocity dia.

Velocity of mid-point E of link BC with respect A

$$A = V_{EA} = \overline{ea} = 6.552\,m/s$$

ii) Velocity of point F on link CD

$$\therefore \frac{V_{DF}}{V_{DC}} = \frac{df}{dc} = \frac{DF}{DC}$$

$$\text{Vector } \overline{df} = \overline{dc} \times \frac{DF}{DC}$$

Velocity dia we get $= V_{CD} = 6.624\,m/s$

$$V_{DF} = 6.624 \times \frac{100}{450}$$

$$\boxed{V_{DF} = 1.472\,m/s}$$

5) The crank of a slider-crank mechanism is 15cm and the connecting rod is 60cm long. The crank makes 300rpm in the clockwise direction. When it has turned 45° from the inner dead centre position, determine:

(i) velocity of slider

(ii) Angular velocity of connecting rod.

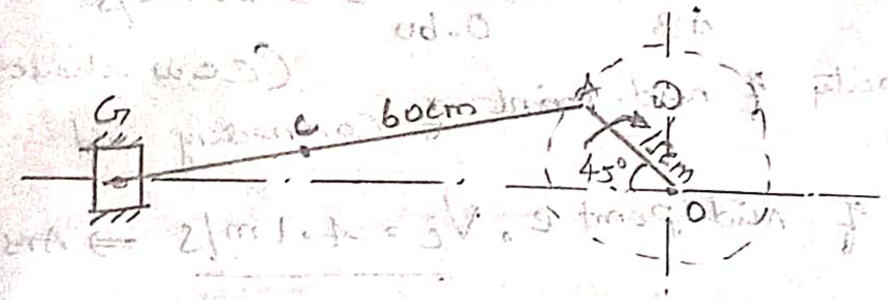
(iii) Linear velocity of mid-point of the connecting rod.

Given data: $OA = 15cm$; $AB = 60cm$; $N_{OA} = 300rpm(c)$

$\angle AOB = 45°$

Soln: Relative velocity method.

Step 1 Configuration diagram (1cm = 25mm)



Step 2 velocity of input link

Speed of input in link $N_{OA} = 300rpm$ (Given)

$$\omega_{OA} = \frac{2\pi N_{OA}}{60} = \frac{2\times\pi\times300}{60}$$

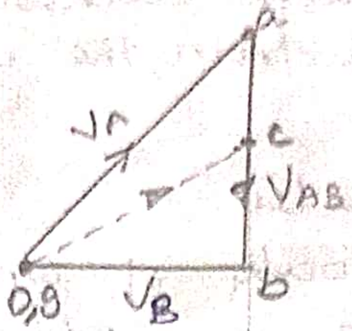
$$\boxed{\omega_{OA} = 31.42 \text{ rad/s}}$$

Velocity of crank $O_A = V_{OA} = \omega_{OA} \times OA$

$= 31.42 \times 0.15$

$$\boxed{V_{OA} = 4.713 \text{ m/s}}$$

Step 3 velocity diagram



Step 4: Velocity of various links

(i) Velocity of slider B:

$$V_B = V_{ob} = 4\,m/s$$

(ii) Angular velocity of connecting rod AB

Velocity of connecting rod $V_{AB} = 3.35\,m/s$

The angular velocity of connecting rod AB

$$\omega_{AB} = \frac{V_{AB}}{AB} = \frac{3.35}{0.60} = 5.58\ rad/s$$

(ccw about A)

(iii) Linear velocity of mid-point of connecting rod

Velocity of mid-point c, $V_c = 4.1\,m/s$ ⇒ Ans.

4) In the fig angular velocity of the crank OA is 500 rpm. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimension of various link are BD = 46 mm, OA = 28 mm, AB = 44 mm and BC = 49 mm.

The centre distance bet the centre of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point c. The slider moves along a horizontal path and OC is vertical.

data:

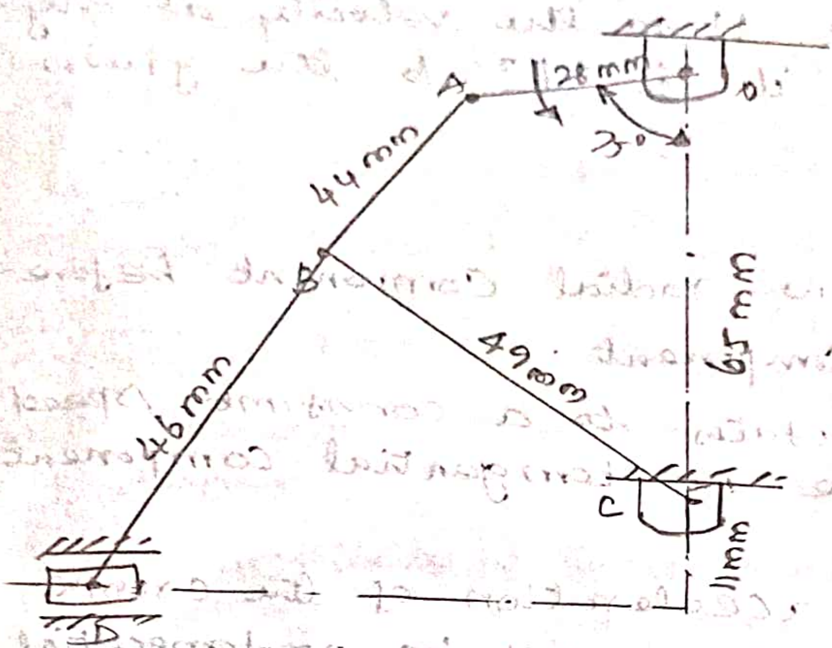
$N_{OA} = 500$ rpm $\angle BOA = 75°$ $BD = 46$ mm $OA = 28$ mm

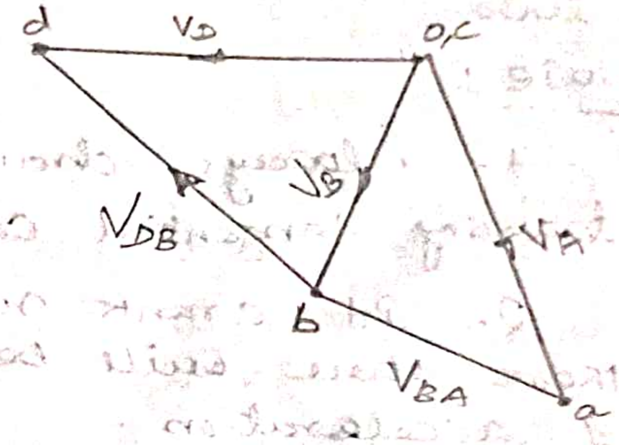
$AB = 44$ mm $BC = 49$ mm $OC = 65$ mm

Soln : Relative velocity method

procedure

Step 1 : Configuration diagram (Full scale)





Step 2 Velocity of input link

Speed of input link $N_{OA} = 500$ rpm

$$\omega_{OA} = \frac{2\pi N_{OA}}{60} = \frac{2\pi \times 500}{60}$$

$$\boxed{\omega_{OA} = 52.36 \text{ rad/s}}$$

Step 4 : Velocity of various links

(i) linear velocity of slider D :

$$v_D = V_{OD} = \text{Vector } od = 1.32 \text{ m/s}$$

(ii) Angular velocity of link BD

$$V_{BD} = 1.374 \text{ m/s}$$

The angular velocity of link BD

$$\omega_{BD} = \frac{V_{BA}}{BD} = \frac{1.374}{0.046} = 29.87 \text{ rad/s}$$

(ccw about D)

Acceleration diagram.

* Radial Component:

which is $\perp^r$ to the velocity at any given instant and it is parallel to the given link

* Tangential Components

which is $\parallel$ to the velocity at any given instant and it is $\perp^r$ to the given link

Note:

1. Always draw radial component before drawing tangential component.

2. If crank rotates to a constant speed there there will be no tangential component of acceleration.

3. If angular acceleration of the crank is not given, There there will be no tangential component of the acceleration

$$\left.\begin{aligned} \text{radial } a^r &= \frac{V^2}{r} \\ \text{tangential} &= a^t = \alpha \times r \end{aligned}\right\} m/s$$

1) ABCD is a four-bar chain with link AD fixed. The length of the link are AB = 62.5 mm BC = 175 mm CD = 112.5 mm and AD = 200 mm. The crank AB rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram. when the angle BAD = 60° and B and C lie on the same side of AD. find the angular velocity and angular acceleration of link BC and CD.

Given data:

$AB = 62.5\,mm$ $BC = 175\,mm$ $CD = 112.5\,mm$ $AD = 200\,mm$

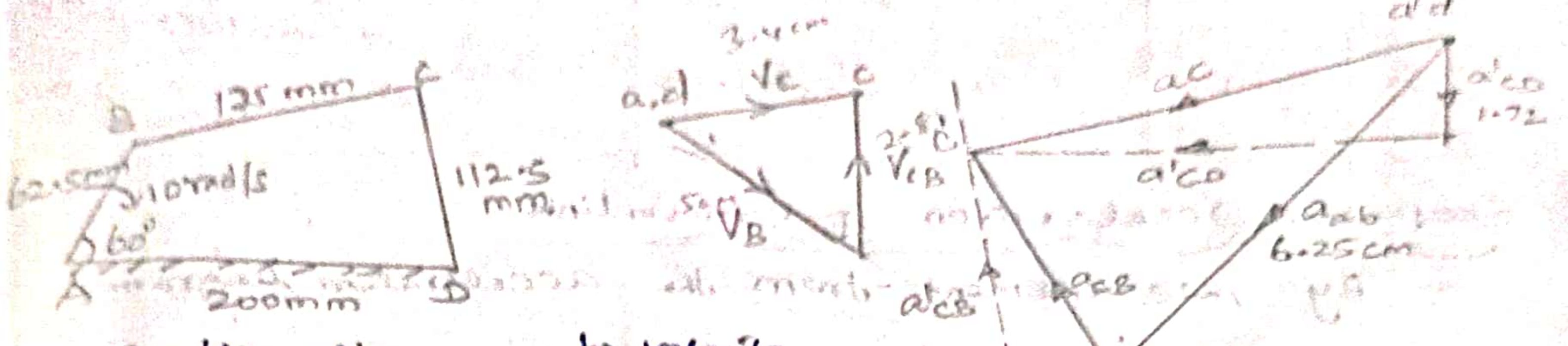
$\omega_{BA} = 10\,rad/s\,(CW)$ $\angle BAD = 60°$

Soln: Relative velocity method

Procedure:

Step 1 : Configuration diagram Scale = 1:2 (Conventional)

$L_{AD} = \frac{200}{2} = 100\,mm$



a) configuration diagram b) velocity diagram c) Acceleration diagram

Step 2 : velocity of input link AB

Velocity of Input link AB, $V_{BA} = \omega_{BA} \times AB$

$= 10 \times 0.0625$

$= 0.625\,m/s \longrightarrow V_{BA} = 0.625\,m/s = 5\,cm$

Step 3 : velocity diagram $(1\,cm = 0.4\,m/s)$ $1\,cm = \frac{0.625}{5} = 0.125\,m/s$

Step 4 : velocity of various link : from velocity diagram

$V_{CB} = \bar{bc} = 0.35\,m/s \Rightarrow 2.8 \times 0.125 = 0.35\,m/s$

$V_{CD} = \bar{dc} = 0.44\,m/s$ — $V_{CD} = 3.4 \times 0.125 = 0.425\,m/s$

The angular velocity of link BC and CD

$$\omega_{BC} = \frac{V_{CB}}{BC} = \frac{0.35}{0.175} = 2\,rad/s \text{ (counter clockwise B)}$$

$$\omega_{CD} = \frac{V_{CD}}{CD} = \frac{0.44}{0.1125} = 3.9\,rad/s \text{ (clockwise D)}$$

Radial and tangential components of acceleration of various links

Link	length of link	Velocity	Radial component of acceleration $a^r = v^2/\text{length of link}$ Magnitude	Direction	Tangential component of acceleration Magnitude	Direct.
—	m	m/s	m/s^2		m/s^2	
AD (fixed)	0.2	0	0	—	0	—
AB	0.0625	0.625	$a^r_{BA} = 6.25$	∥ to BA	0 ; $\omega_{AB} = C$ hence $\alpha = 0$	—
BC	0.175	0.35	$a^r_{BC} = 0.7$	∥ to BC	a^t_{CB} can't be calculated	⊥ to BC
CD	0.1125	0.44	$a^r_{CD} = 1.72$	∥ to CD	a^t_{CD} cannot be calculated	⊥ to CD

Step 6 : Acceleration of various link.

By measurement from the acceleration diagram

$$a^t_{CB} = 4.2 \, m/s^2$$

$$a^t_{CD} = 5.5 \, m/s^2$$

The angular acceleration of link BC and CD

$$\alpha_{BC} = \frac{a^t_{CB}}{BC} = \frac{4.2}{0.175} = 24 \, rad/s^2 \text{ (clockwise about B)}$$

$$\alpha_{CD} = \frac{a^t_{CD}}{CD} = \frac{5.5}{0.1125} = 48.89 \, rad/s^2 \text{ (counter clockwise about D)}$$

Ex: 2 In a small steam engine running at 600 rad/min clockwise, length of crank is 80 mm and ratio of connecting rod length to crank radius is 3. For the position when crank makes 45° to horizontal determine
(i) velocity & acceleration of the piston
(ii) angular velocity and angular acceleration of the connecting rod (Apr/May - 2023)
(iii) The linear velocity and acceleration of a point X on connecting rod 80mm from crank pin

data: $\omega_{OA} = 600 \, rad/min = \frac{600}{60} = 10 \, rad/s$ (ccw)

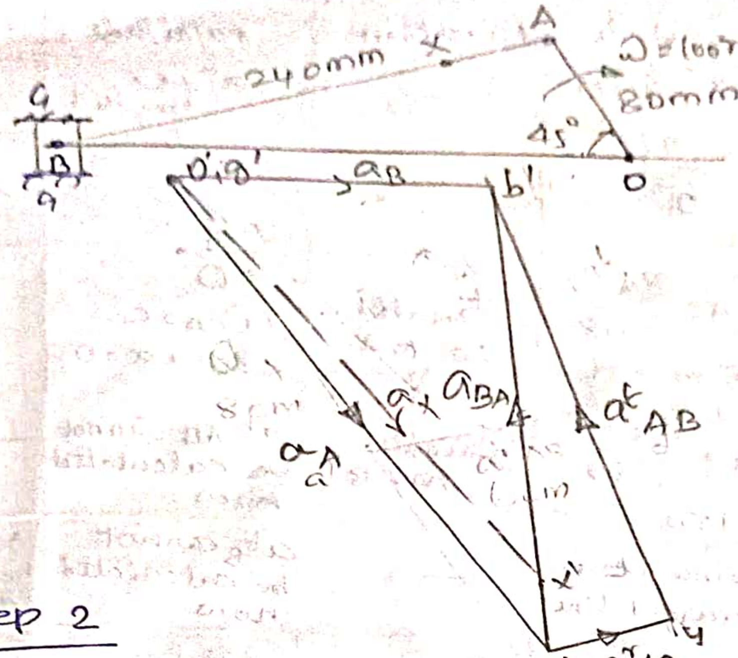
$OA = 80 \, mm \quad n = l/r = 3$

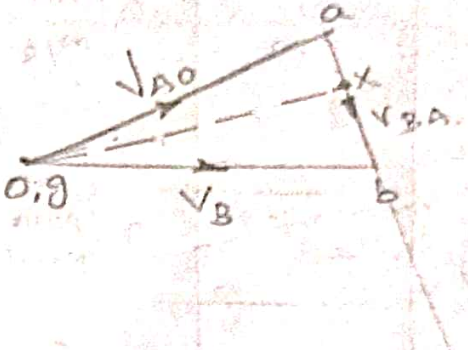
or $l = 3r = 3(80) = 240\,mm$; $\angle BOA = 45°$

Soln: Relative velocity method.

Procedure :

Step 1 : Configuration diagram. (Scale 1:5) (1cm = 40mm)





Step 2

Velocity of crank OA

$$V_{OA} = \omega_{OA} \times OA$$
$$= 100 \times 0.08 = 8\,m/s$$

Step 2 : Velocity diagram

1cm = 2m/s.

Step 4 Velocity of various link

Velocity of piston V_B = Vector ob = 7m/s

Velocity of connecting rod V_{BA} = Vector ab = 6m/s

Velocity of point x on AB, V_x = vector x = 7m/s

Note velocity diagram, the position of poin x on the connecting rod can be obtained as

$$\frac{AX}{AB} = \frac{ax}{ab} \quad \text{or} \quad ax = \frac{Ax}{AB} \times ab$$

$$= \frac{80}{240} \times 6 = 2\,m/s$$

Step 5: Acceleration

Now the angular velocity of the connecting rod AB is given by

$$\omega_{AB} = \frac{V_{AB}}{AB} = \frac{6}{0.24} = 25 \text{ rad/s (Counter Clockwise about A)}$$

Step 5 : Acceleration diagram : The value of radial & tangential components of acceleration of various link

(cm = 100 m/s²)

Link	length	velocity of link	Radial component of acceleration		Tangential Components of acceleration	
			magnitude	Direction	magnitude	direction
—	m	m/s	v^2/length m/s²		$a^t = \alpha \times$ length of link	
OG fixed	—	0 (Since it is fixed)	0	—	0	—
OA	0.080	8	$a^r_{OA} = \frac{8^2}{0.108}$ $= \frac{800}{100}$ 8	Parallel to OA	0 $\omega_{OA} = C$ α hence $\alpha = 0$	—
AB	0.240	6	$a^r_{AB} = \frac{6^2}{0.24}$ $= \frac{150}{100}$ 1.5	Parallel to AB	a^t_{AB} Cannot be calculated now	⊥ to AB
Slider B	—	—	0 (Since it is straight line	—	a^t_B cannot be calculated now	Acts in horizontal direction

1CM = 400

Step 6 : Acceleration of various link.

$= \frac{80}{...}$ $2.3 \times 100 = \frac{60}{835}$ m/s²

Acceleration of piston B, a_B = Vector $o'b'$ = 8.35 cm

Tangential component of acceleration of connecting rod AB, a^t_{AB} = ~~8.4 cm~~ $6.5 \times 100 = \cancel{650}$ 560 m/s²

Acceleration of point x, a_x = Vector $o'\cancel{a}x'$ = 7.4 cm $\times 100$ = 700 m/s

$$\alpha_{ob} = \frac{8}{0.26}$$

In the acceleration dia the position of point x on the connecting rod a'b'

$$\frac{Ax}{AB} = \frac{a'x'}{a'b'} \text{ or } a'x' = \frac{Ax}{AB} \times a'b'$$

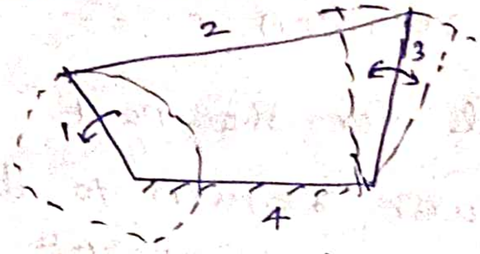
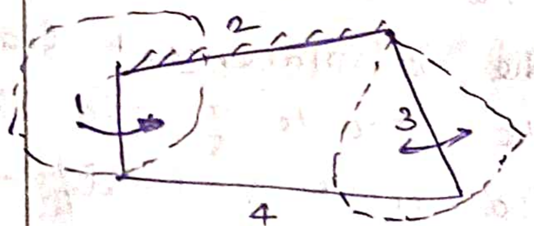
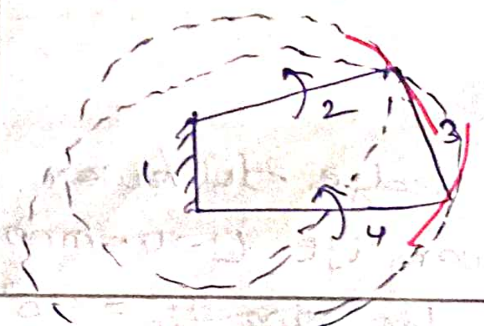
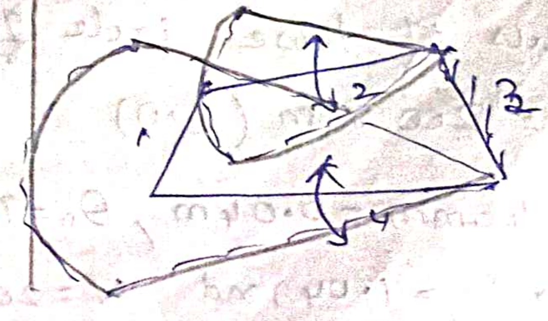
$$= \frac{80}{240} \times 560 = 186.67 \text{ m/s}^2$$

angular acceleration of the connecting rod AB.

$$\alpha_{AB} = \frac{a'_{AB}}{AB} = \frac{80}{240} \times \cancel{560 = 186.67 \text{ m/s}^2}$$

$$\alpha_{AB} = \frac{\alpha^t_{AB}}{AB} = \frac{560}{0.24} = 2.33 \ rad/s^2$$ (Clockwise about B)

Inversion of four-bar Chain

Sl.No	Inversion	Mechanism	Practical Application
1.	First inversion	Crank-rocker mechanism  Link 4 fixed link 1 rotates; link 2 and 3 oscillate	1. Beam engine 2. All rotary oscillating converters.
2.	Second inversion	Crank-rocker mechanism Link 2 is fixed link 1 rotates link 3 and 4 oscillate 	1. Beam engine 2. All rotary oscillating converters
3.	Third inversion	Double-crank mechanism Link 1 fixed link 2 and 4 rotate link 3 oscillates 	1. Coupled wheels of a locomotive 2. All rotary-rotary converters.
4.	Four inversion	Double rocker mechanism Link 3 is fixed, link 2 and 4 oscillate 	1. Watt's indicator mechanism 2. Pantograph 3. Ackermann steering.

Cam profile with knife edge follower:

Ex: A cam is to be designed for a knife edge follower with the following data.

- Follower lift is 40 mm with SHM during 90° of cam rotation
- Dwell for the next 30°
- Follower return to its original position with SHM during the next 60° of cam rotation
- Dwell for the remaining cam rotation

The line of stroke of the follower passes through the axis of the camshaft. Radius of the base circle of the cam is 40mm

i) Draw the displacement diagram

ii) Draw the profile of the cam

iii) Det the max velocity and acceleration of the follower during forward and return stroke if the cam rotates at 200rpm in the clockwise direction.

data:

Knife edge follower

Follower lift $L = 40mm$

Angle for rise $\theta_0 = 90°$

Angle for dwell $\theta_d = 30°$

Angle for return $\theta_r = 60°$

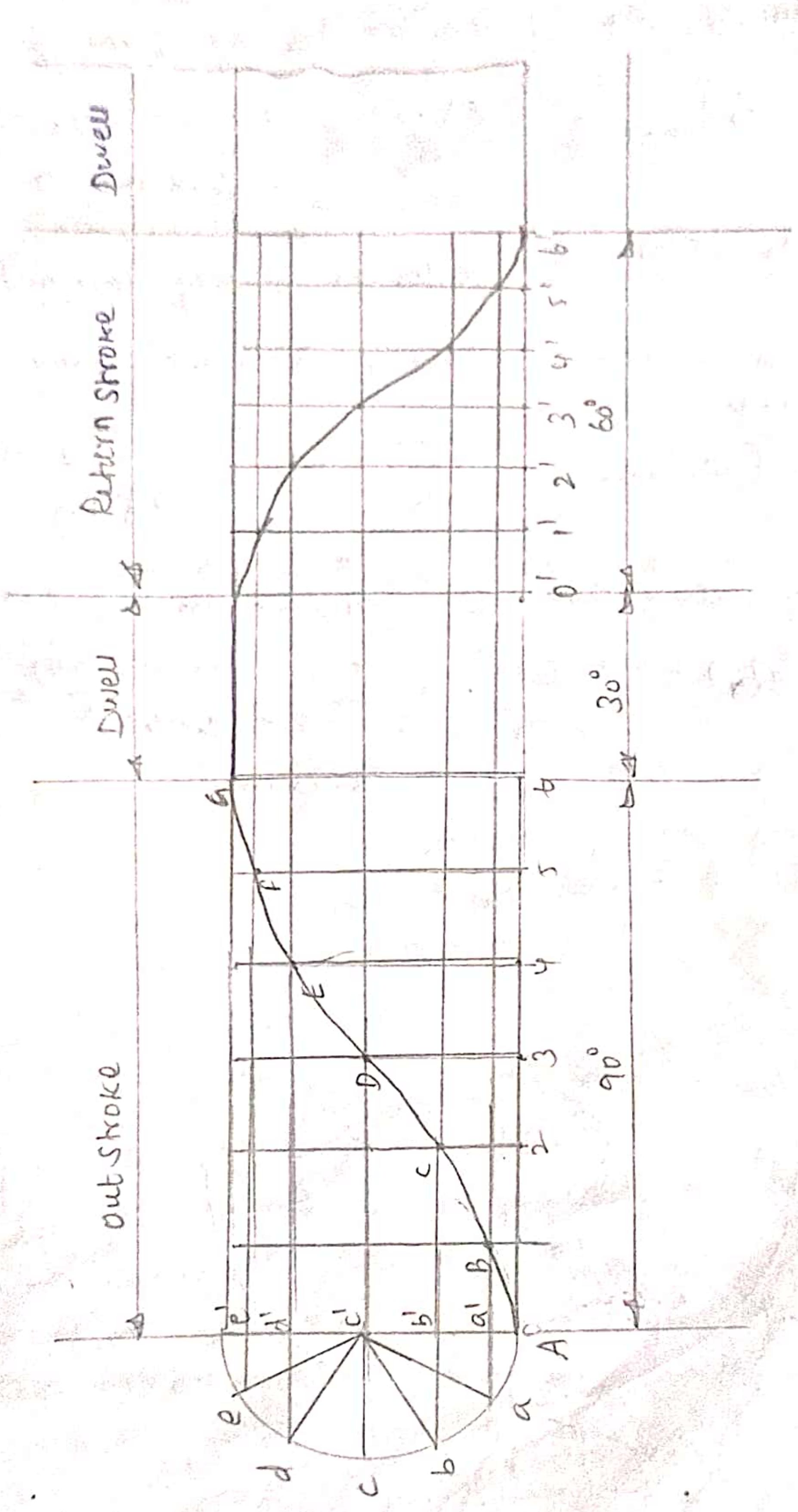
Radius of base circle of cam $= r_b = 40mm$

$N = 200 rpm$ (CW)

Given: $L = 40mm = 0.04m$, $\theta_0 = 90° = 90 \times \frac{\pi}{180} = 1.57 rad$

$\theta_r = 60° \times \frac{\pi}{180} = 1.047 rad$ $N = 200 rpm$

Angular velocity $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94\ rad/s$



max velocity of follower during forward and return stroke

SHM Forward stroke $(v_0)_{max} = \frac{\pi L \omega}{2\theta_0} = \frac{\pi \times 0.04 \times 20.94}{2 \times 1.57} = 0.83 m/s$

max velocity of follower having SHM during return stroke.

SHM return stroke. $(V_r)_{max} = -\frac{\pi L \omega}{2\theta_r} = -\frac{\pi \times 0.04 \times 20.94}{2 \times 1.047} = -1256 mm/s$

$= -1.256 m/s$

max acceleration of follower during forward and return stroke.

~~max~~ max acceleration of follower having SHM during forward stroke

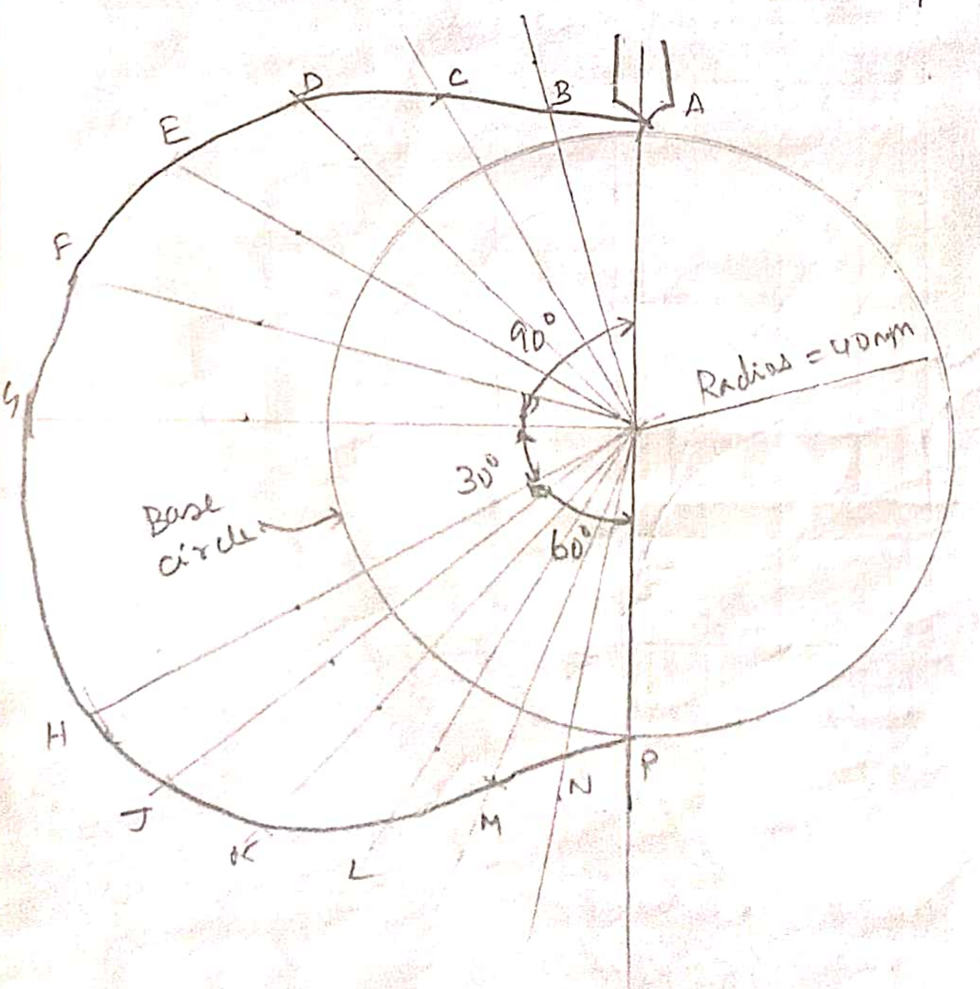
$$(a_0)_{max} = \frac{\pi^2 L \omega^2}{2\theta_0^2} = \frac{\pi^2 \times 0.04 \times 20.94^2}{2 \times (1.57)^2} = 35114.3 mm/s^2$$

$$= 35.114 m/s^2$$

max acceleration of follower having SHM during Return stroke.

$$(a_r) = \frac{\pi^2 L \omega^2}{2\theta_r^2} = \frac{\pi^2 \times 0.04 \times (20.94)^2}{2 \times (1.047)^2} = 78956.8 mm/s^2$$

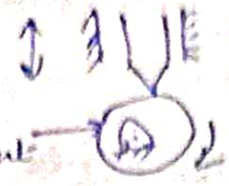
$$= 78.95 m/s^2$$



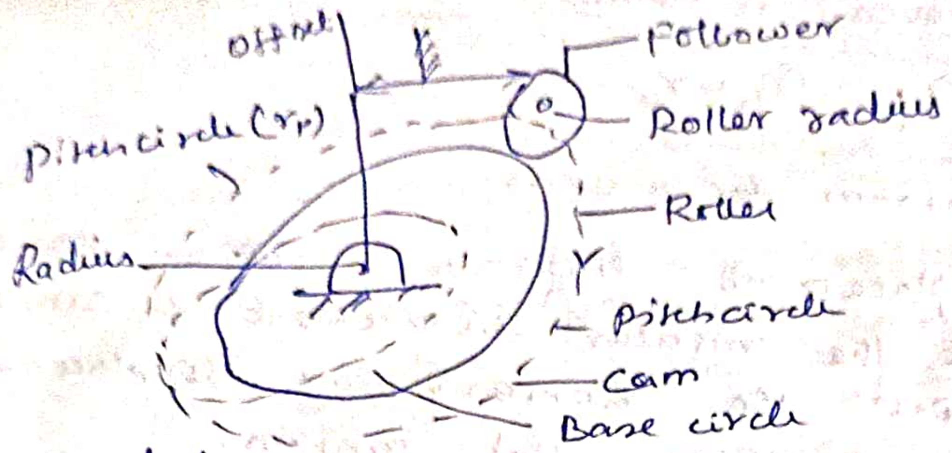
Cam and follower:

Cam ⇒ Rotating motion
follower ⇒ Translatory motion

Cam ⇒ Input
follower ⇒ Output



Layout of radial / plate cam



Nomenclature:

1. Trace point (Theoratical)

It describe, the follower motion
(~~The trace point is roller center~~) Trace point is the theoretical refelence point on the follower which is used to generate pitch curve.

× Location of trace point

→ For a knife edge follower, the trace point is at the knife edge.

→ For a roller follower, the trace point is at the roller centre.

→ For a flat faced follower, the trace point is at the point of contact at which the follower is in contact with cam on base circle.

Base circle : Base circle is the smallest circle that can be drawn tangential to cam profile from the centre of rotating of cam.

Cam profile

The working surface of cam which comes into contact with follower is known as cam profile

Prime circle: The smallest circle that can be drawn tangent to the pitch curve is called as prime circle

Pitch circle: pitch circle is the circle passing the pitch point and concentric distance through which the follower moves (or) rotates.

$$\boxed{r_p = r_b + r_r}$$

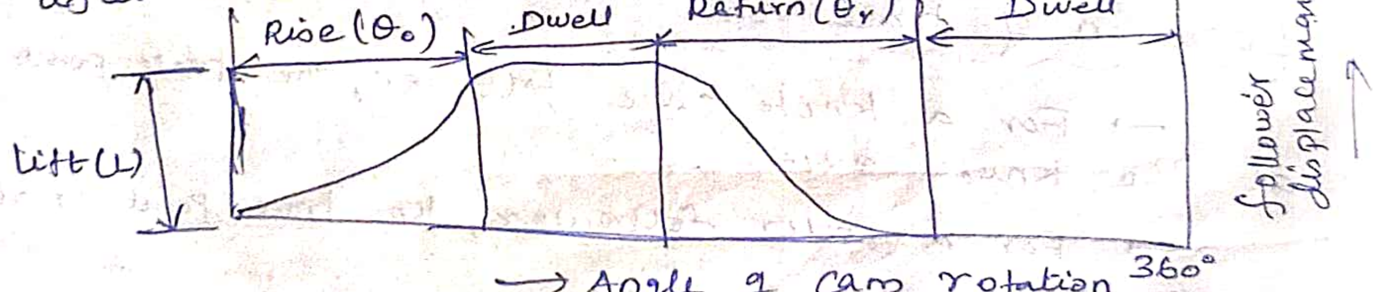
Displacement diagram for a cam

→ The displacement diagram is one in which x-axis represent the angular displacement of cam and Y-axis represent the corresponding displacement of the follower from its initial position.

→ The displacement diagram consist of three distinct part rise, return and dwell

→ Dwell is the period during which the follower remains at rest.

→ Dwell may be between rise, and return and after return also

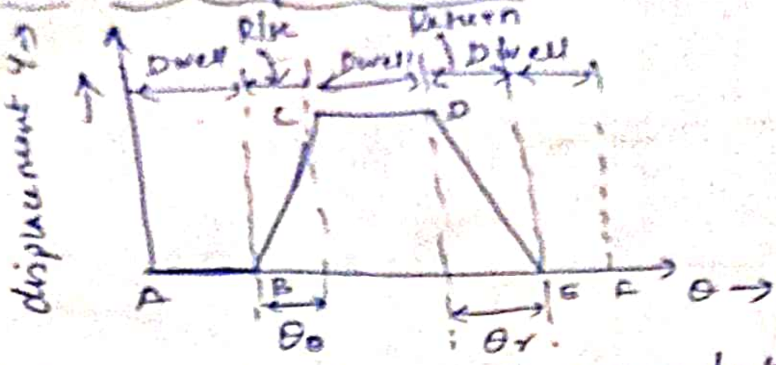


It may be noted that the point of inflexion during rise and return portion correspond to pitch points

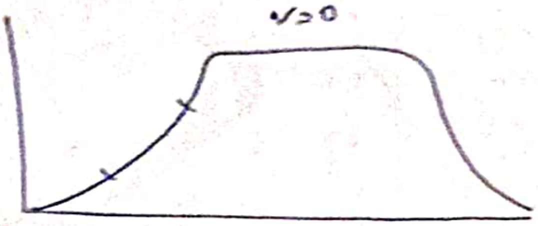
Basic follower motion:

1. uniform follower motion
2. Simply harmonic motion
3. uniform Acceleration & retardation motion / parabolic motion
4. cycloid motion

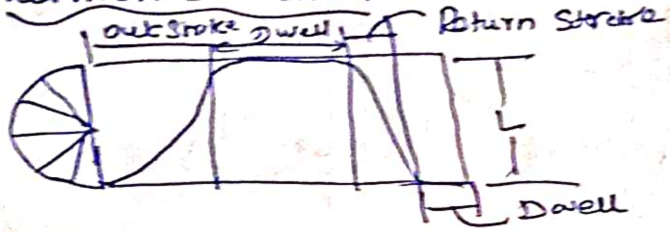
uniform velocity motion



uniform acceleration and retardation



Simple harmonic motion



Cam profile with knife-edge profile.

Pb: 2 A Cam is to be designed for a knife-edge follower with the following data

(i) Cam lift = 40 mm during 90° of cam rotating with simple harmonic motion

(ii) Dwell for the next 30°

(iii) During the next 60° of Cam rotates, the follower returns to its original position with simple harmonic motion

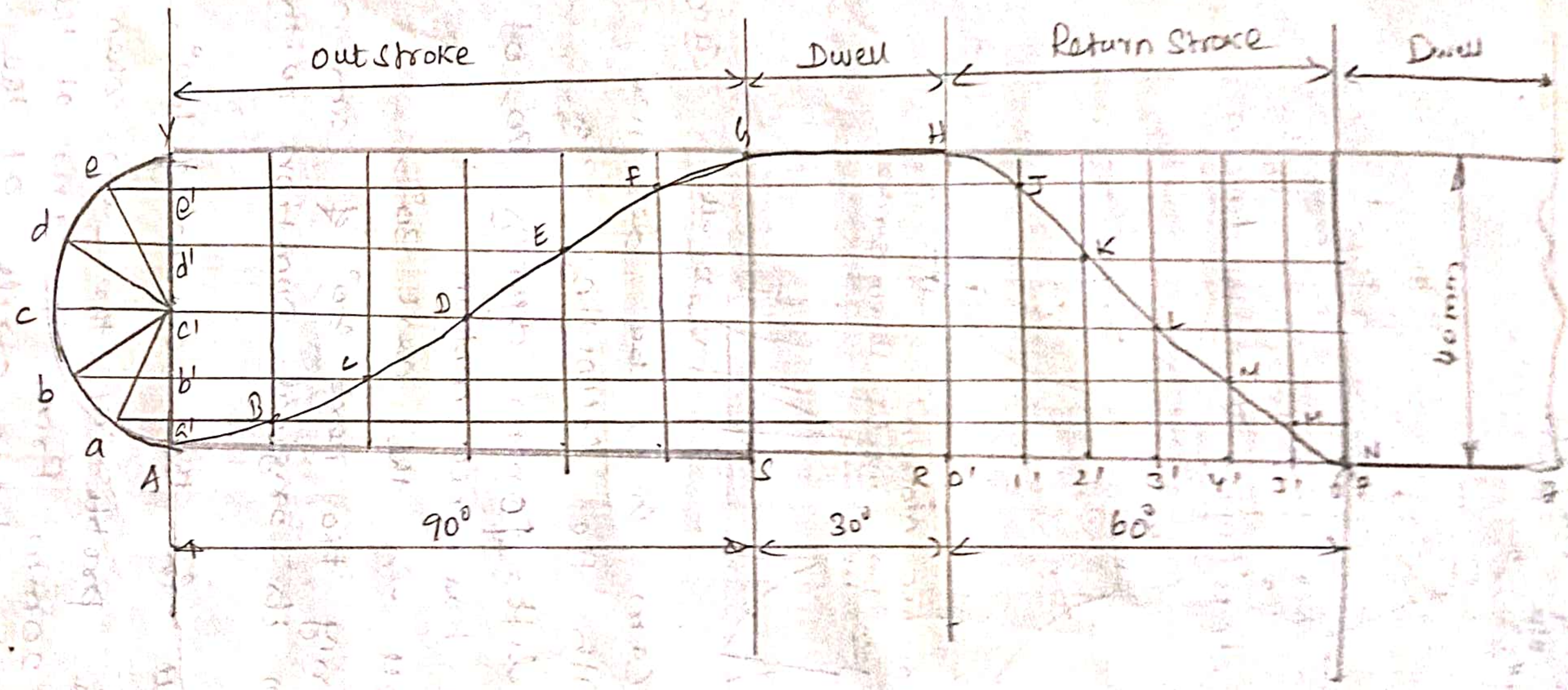
(iv) Dwell for the remaining 180°

Draw the profile of the cam when the line of stroke is offset 20 mm from the axis of the camshaft. The radius of the base circle of the cam is 40 mm

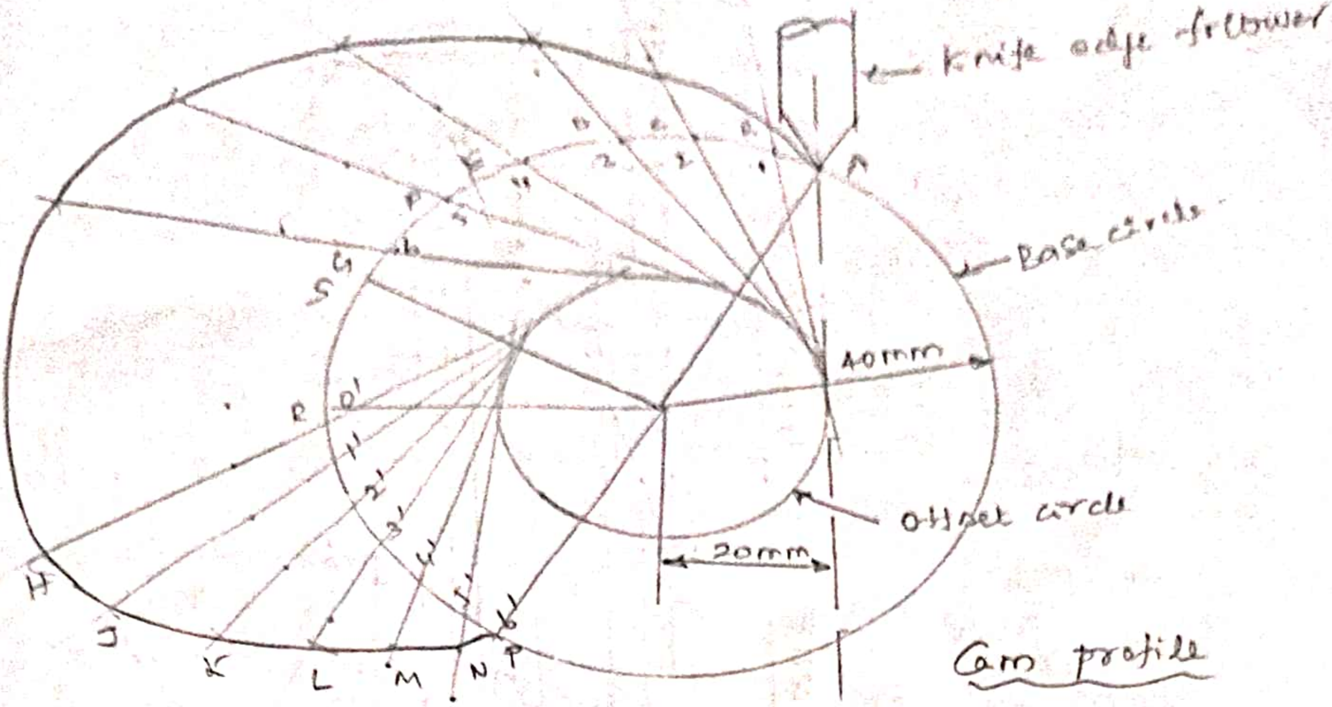
data: knife edge follower, Follower lift L = 40 mm

Angle for rise $\theta_o = 90°$ Angle for dwell $\theta_d = 30°$

Angle for return $\theta_r = 60°$; Radius of base circle of cam $r_b = 40$ mm.



Displacement diagram.



Cam profile

Cam profile with roller follower.

Pb: A cam rotating clockwise with a uniform speed is to give the roller follower of 20mm dia with the following motion.

a) Follower to move outward through a distance of 30mm during 120° of cam rotation.

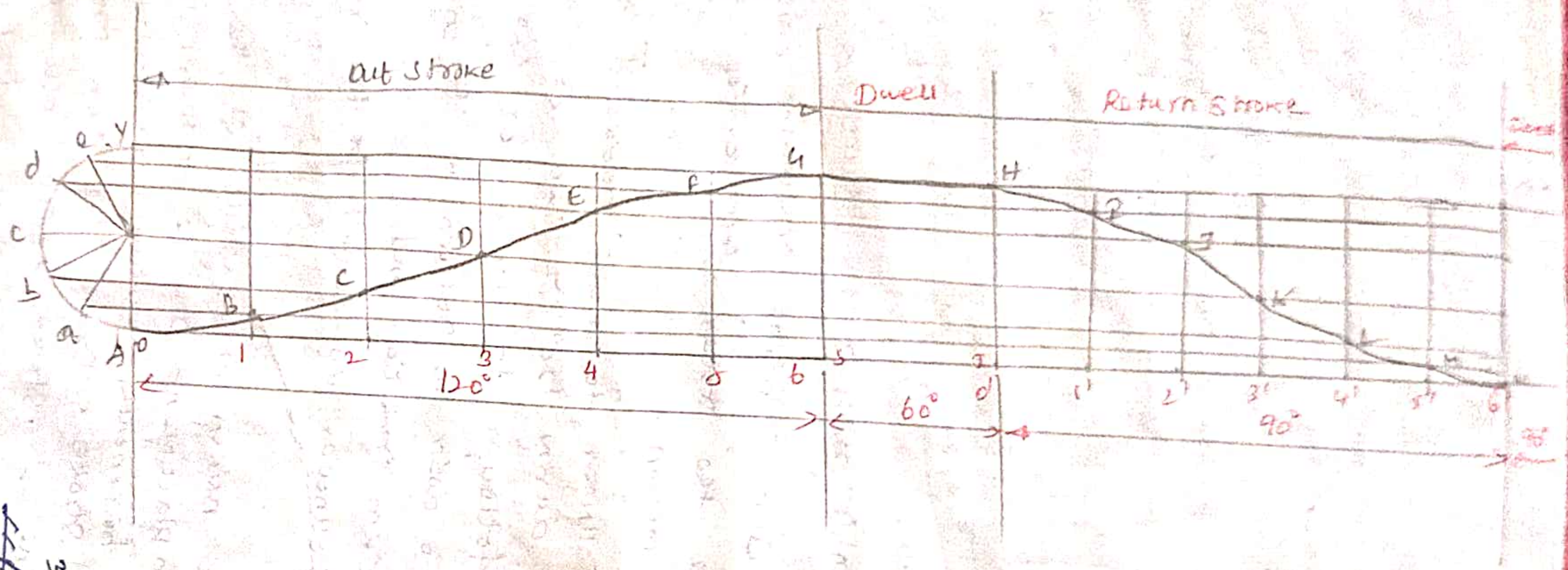
b) follower to dwell for 60° of cam rotation

c) Follower to return to its initial position during 90° of cam rotation and

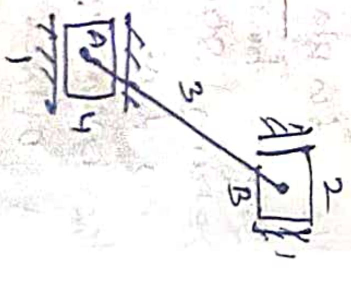
d) Follower to dwell for the remaining 90° of cam rotation.

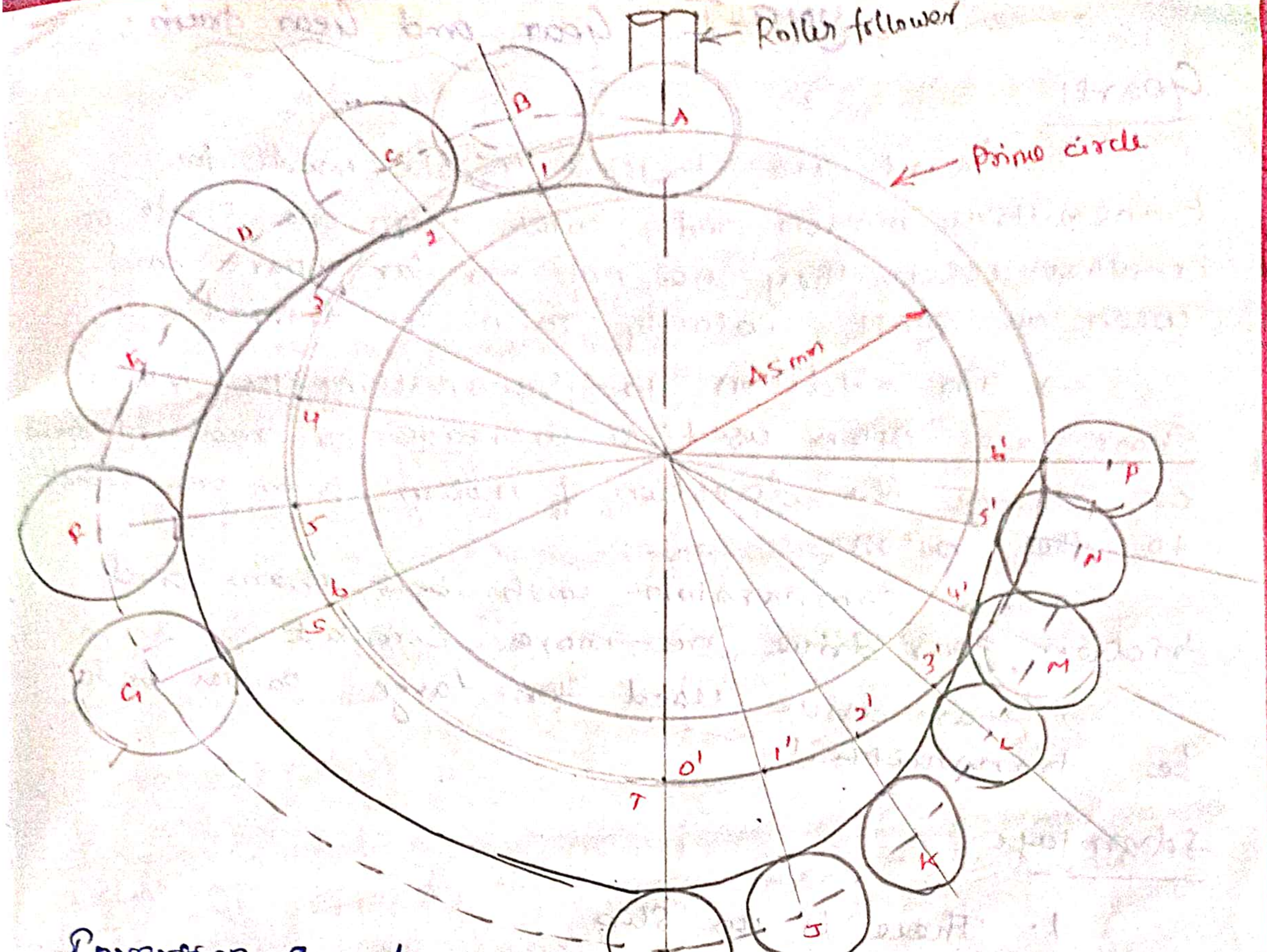
The minimum radius of the cam is 45mm and the displacement of the follower is to take place with simple harmonic motion on both the outward and return stroke. Draw the cam profile.

(i) line of stroke of the follower passes through the axis of the camshaft and (ii) the line of stroke of offset by 15mm from the axis of the cam.



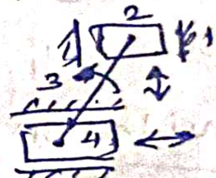
Out Stroke
Dwell
Return Stroke
120°
60°
90°

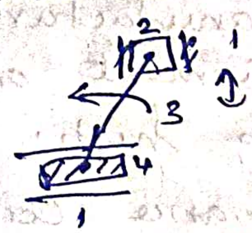




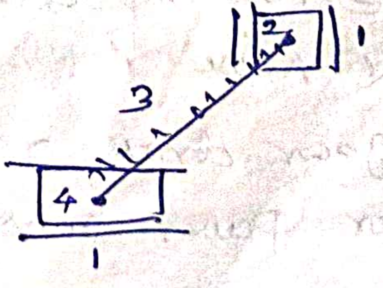
Inversion of double slider crank chain

Inversion	Mechanism	Application
1. First Inversion	Link 1 is fixed link, 3 rotates, links 2 and 4 reciprocates	Elliptical trammel
2. Second Inversion	Link 4 is fixed, link 3 rotates, link 2 reciprocates	Scotch Yoke Mechanism.
3. Third Inversion	Link 3 is fixed, link 1 rotates, links 2 & 4 reciprocate	Oldham's Coupling





Link 1 - fixed link
2 & 4 - Slider
3 - Crank
Pair 2-3 & 3-4 = Turning pairs
Pair 1-2 & 1-4 = Sliding pairs



UNIT-II Gear and Gear train:

Gears:

Gears are toothed wheels used for transmitting motion and power from one shaft to another when they are not too far apart and when a const velocity ratio is desired.

→ In addition to transmitting the motion gears are often used to increase or reduced speed or change the direction of motion from one shaft to the other.

→ In comparison with belt, chain and friction, gear drive are more compact

→ gear drive used for large power is to be transmitted.

Advantage:

1. There is no slip
2. It is capable of transmitting larger power
3. It is more efficient (upto 99%)
4. It require less space

Limitation:

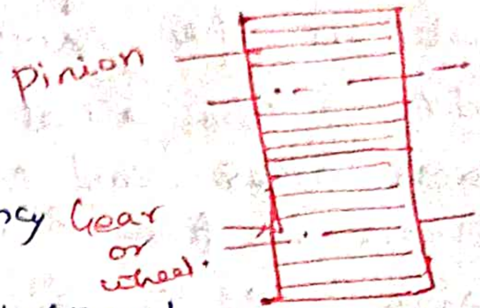
1. The manufacture of gear require special tools and equipment and hence the manufacturing cost of gear is high.
2. Maintenance cost of gear drive is comparatively high.
3. The error in cutting teeth may cause vibration & noise
4. The gear drive require precise alignment of shaft for power transmission

1. Classification of Gears based on relative position of two Shaft carrying gears:

A. Parallel Axis Gears

1. Spur Gear:

Spur gear have tooth parallel to the axis of rotation and are used for transmitting motion bet two parallel shaft



Adv

1. Simple in construction
2. They have highest efficiency

Application : They are used in high speed and high load application

2. Helical Gears

Helical gear have teeth inclined to the axis of rotation and are used for transmitting motion bet two parallel Shaft.

Adv: 1. Helical gear operate smoother and quieter
2. Helical gear have greater load carrying capacity.

Application : Because of smoother action, the helical gear gears are prefered in high speed and high load application such as automobile.

Axial circular pitch (P_c)
normal circular pitch [P_n]

3. Rack and pinion Gears:

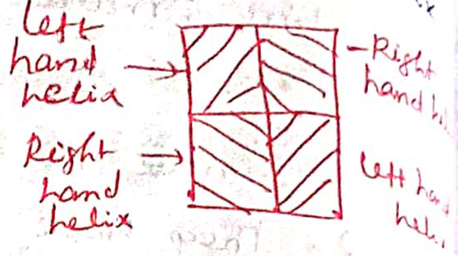
Rack is a segment of a gear of infinite diameter.

→ When a straight line gear (called rack) meshes with the circular wheel (called pinion), then the combination are formed rack and pinion arrangement

Application It is found in lathe where rack gives motion to Saddle.

4. Herringbone Gears:

It is double helical gears consisting of teeth having a right and left handed helix cut on the same blank.

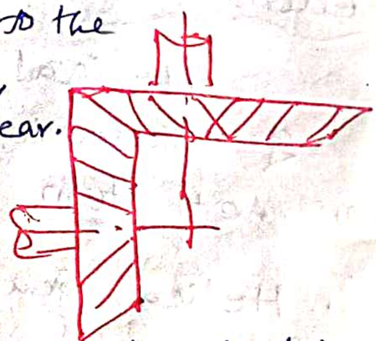


Adv : Two axial thrust oppose each other and nullify, hence the shaft is free from any axial force

Appli It is limited to high load carrying capacity Such as cement mills and crushes.

B. Straight bevel gears.

Bevel gear are parallel to the line generating the pitch cones, then they are Straight bevel gear.

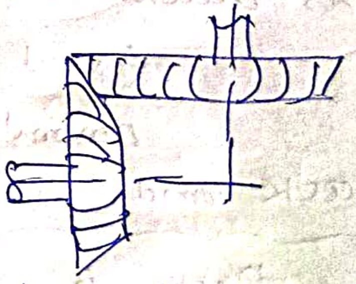


App : It is used in automobile differential

Adv : It can operate under high speed and high load.

2. Spiral bevel gears:

When the teeth of a bevel gear are inclined at an angle to the face of the bevel they are known as spiral bevel gear.

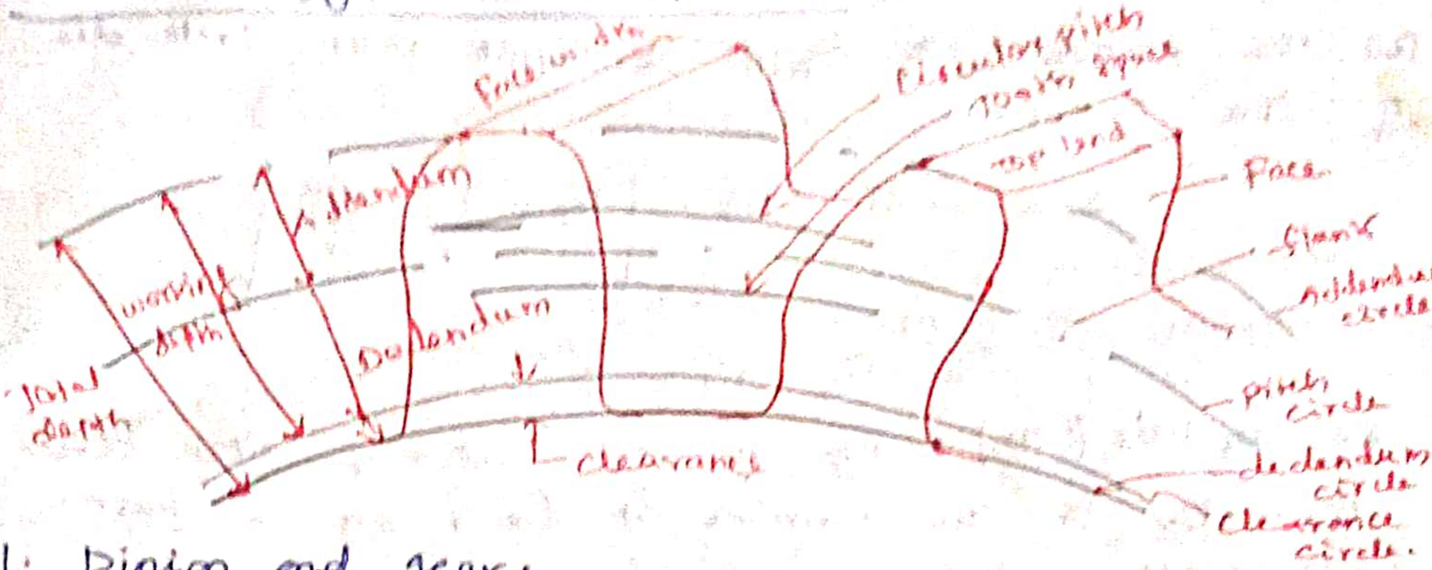


Appl :- Differential of an automobile.

Adv :

The spiral tooth, the contact length and Contact ratio are more.

Terminology used in Gears:



1. Pinion and gear:

A pinion is the smallest of two mating gears.

2. pitch surface:

pitch surface is the surface of the imaginary rolling cylinder that replaces the toothed gear.

3. pitch circle: pitch circle is an imaginary circle on gear, by which pure rolling action would be given the same motion as the actual gear.

4. Pitch circle diameter (D) ÷ pitch circle diameter is the diameter of the pitch circle.

5. pitch point ÷ pitch point is a point on line joining centre of two mating gears at which the two pitch circles meet.

6. circular pitch (P_c) → It is distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding on the adjacent tooth

D = pitch circle dia
T = No of teeth on gear

$$P_c = \frac{\pi D}{T} \qquad P_d = \frac{T}{D} = \frac{\pi}{P_c}$$

7. Diametral pitch (P_d) :- Diametral pitch is defined as the number of teeth per unit pitch circle dia of the gear.

$$\text{Diametral pitch} = \boxed{P_d = \frac{T}{D} = \frac{\pi}{P_c}}$$

$$P_c \times P_d = \pi$$

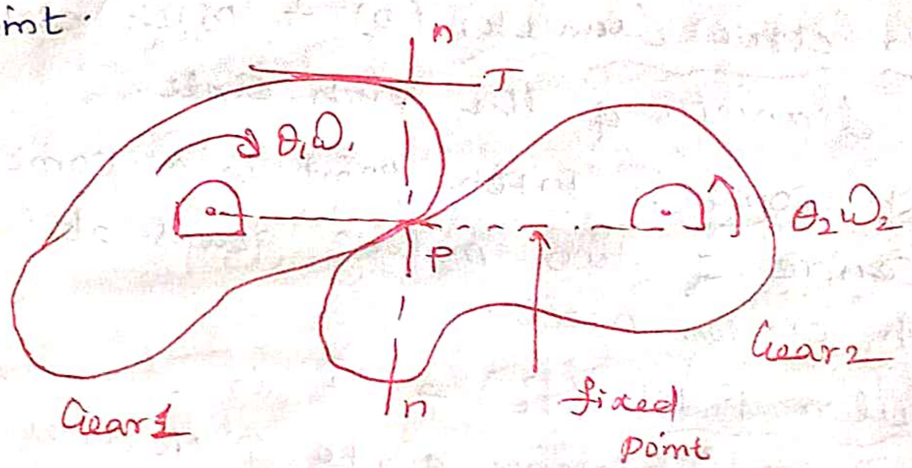
8. <u>module (m)</u> : It is the ratio of pitch circle diameter to the number of teeth on the gear.

$$\boxed{m = \frac{D}{T}}$$

<u>Fundamental law of gearing</u> :

The law of gearing states that for maintaining constant angular velocity ratio between two ~~machine gears~~ meshing gears through out the mesh.

The common normal to the tooth profile at all contact point with in mesh, must always pass through a fixed point on the lines of centres called pitch point.



$$\omega_1 \times O_1 P = \omega_2 \times O_2 P$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P}} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

$$\text{length of arc of contact} = \frac{\text{length path of contact}}{\cos\phi}$$

$$\text{Contact ratio} = \frac{\text{length of arc of contact}}{\text{circular pitch } (P_c)}$$

Note:

1. $$\text{Angle turned through by Pinion} = \frac{\text{length of arc of contact}}{\text{Circumference of pitch circle of pinion}} \times 360$$

$$\left.\begin{array}{l}\text{Circumference of pitch circle of}\\ \text{pinion}\end{array}\right\} = 2\pi r$$

2. $$\left.\begin{array}{l}\text{Angle turned through by}\\ \text{gear wheel}\end{array}\right\} = \frac{\text{length of arc of contact}}{\text{Circumference of pitch circle of wheel}} \times 360$$

$$\left.\begin{array}{l}\text{Circumferences of pitch circle}\\ \text{of wheel}\end{array}\right\} = 2\pi R$$

Formulae Summary:

m = module.

T_P and T_G = No of teeth in pinion and gear wheel.

r and R = Pitch circle radii of pinion and gear wheel.

r_A and R_A = Addendum circle radii ~~of~~ pinion and gear wheel

a_p and a_w = Addendum on pinion and gear

~~vi~~. ω_P and ω_G = Angular velocity of pinion and gear

(i) $r = \frac{m T_P}{2}$ and $R = \frac{m T_G}{2}$

(ii) $r_A = r + a_p$ and $R_A = R + a_w$

(iii) Length of path of approach $KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$

(iv) Length of path of recess $PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$

(v) Length of path of contact $KL = KP + PL$

(vi) $$\text{Length of arc of Contact} = \frac{\text{length of path of contact}}{\cos\phi}$$

$$= KL/\cos\phi$$

(vii) Contact ratio = $\dfrac{\text{Length of arc of Contact}}{\pi m}$

(viii) Angle turned through by pinion } = $\dfrac{\text{length of arc of Contact}}{\text{Circumference of Pitch circle of pinion}} \times 360°$

(ix) Angle turned through by wheel } = $\dfrac{\text{length of arc of Contact}}{\text{circumference of pitch circle of wheel}} \times 360°$

x) Velocity of Sliding at the point of Engagement (K) } $V_{SK} = (\omega_p + \omega_g) KP$

(xi) Velocity of Sliding at the point of disengagement (L) } $V_{SL} = (\omega_p + \omega_g) PL$

(xii) Velocity of Sliding at the pitch Point P, $V_{SP} = 0$

pb:

1) A pair of gears having 40 and 20 teeth respectively are of 20° involute form. The addendum length is 5mm and module pitch is 5mm. If the Smaller wheel is driver and rotates at 2000rpm. Find the velocity of Sliding (i) at the point of engagement, (ii) at the point of disengagement and (iii) the pitch point.

data

No of teeth on pinion } $T_P = 20$

No of teeth on gear } $T_G = 40$

Pressure angle $\phi = 20°$ Involute

Addendum $= a_p = a_w = 5$

$m = 5$

Speed of pinion $N_p = 2000$ rpm

Find : Velocity of sliding
(i) engagement (ii) Disengagement iii) Pitch point

Soln:

Angular velocity of pinion $\omega_P = \frac{2\pi N_P}{60}$

$= \frac{2\pi \times 2000}{60} = 209.44$ rad/s

Gear ratio, $\frac{\omega_P}{\omega_G} = \frac{T_G}{T_P}$ → formula.

$\frac{209.44}{\omega_G} = \frac{40}{20}$

$\boxed{\omega_G = 104.72 \text{ rad/s}}$

Pitch circle radii of pinion and gear.

$r = \frac{mT_P}{2} = \frac{5 \times 20}{2} = 50mm$

$R = \frac{mT_G}{2} = \frac{5 \times 40}{2} = 100mm$

Addendum circle radii of pinion and gear are given by

$r_A = r + a_p = 50 + 5 = 55mm$

$R_A = R + a_w = 100 + 5 = 105mm$

Length of path of approach $KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{(105)^2 - (100)^2\cos^2 20^\circ} - 100 \sin 20^\circ$

$\boxed{KP = 12.65mm}$

Length of path of recess $PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$PL = \sqrt{(55)^2 - (50)^2\cos^2 20^\circ} - 50\sin 20^\circ$

$\boxed{PL = 11.49mm}$

(i) Velocity of sliding at the point of enengagement (V_{SK})

$V_{SK} = (\omega_P + \omega_G)KP$

$= (209.44 + 104.72) \times 12.65$

$$\boxed{V_{SK} = 3974.12 \text{ mm/s or } 3.97 \text{ m/s}}$$

(ii) Velocity of sliding of point of disengagement (V_{SL})

$$V_{SL} = (\omega_1 + \omega_2) \times PL$$

$$= (209.44 + 104.72) \times 11.49$$

$$\boxed{V_{SL} = 3609.7 \text{ mm/s or } 3.61 \text{ m/s}}$$

(iii) Velocity of sliding at the pitch point $(V_{SP}) = 0$

At the pitch point, the length of path of Contact is zero

formula. Pitch line speed $V_p = \omega_p \times r_p$

" " " $V_g = \omega_g \times R_g$

Pb 2: Two involute gear of 20° pressure angle are in mesh. The number of teeth and pinion is 20mm and the gear ratio is 2. If the pitch expressed in module is 5mm and the pitch line speed is 1.2m/s, assuming addendum as standard and equal to one module.

Find 1. The angle turned through by pinion, when one pair of teeth is in mesh

2. The max velocity of sliding.

data:

Pressure $\phi = 20°$

$T_p = 20$

$G = 2$

$m = 5$

$V_p = V_g = 1.2 \text{ m/s}$

Addendum; $a_p = a_w = 1$ module $= 5$mm

Find

(i) Angle turned through by pinion

(ii) max velocity of sliding ω_p & $\omega_G = ?$

Soln:

$$G = \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 2$$

$$T_G = G \times T_P = 2 \times 20 = 40$$

$$\boxed{T_G = 40}$$

1. Angle turned through by pinion

$$r = \frac{m T_P}{2} = \frac{5 \times 20}{2} = 50mm$$

$$R = \frac{m T_G}{2} = \frac{5 \times 40}{2} = 100mm$$

Addendum circle radii of pinion and gear

$$r_A = r + a_p = 50 + 5 = 55mm$$

$$R_A = R + a_w = 100 + 5 = 105mm$$

Length of path of approach $KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$$= \sqrt{(105)^2 - (100)^2 \cos^2 90^\circ} - 100 \sin 20^\circ$$

$$= 12.65mm$$

Length of path of recess $PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20} - 50\sin 20^\circ = 11.5mm$$

Length of path of contact $KL = KP + PL$

$$= 12.65 + 11.5$$

$$\boxed{KL = 24.15mm}$$

length of arc of contact $= \frac{KL}{\cos\phi} = \frac{24.15}{\cos 20^\circ}$

$$= 25.7 mm$$

Angle turned by pinion = $\frac{\text{length of arc of contact}}{\text{Circumference of pinion } 2\pi r} \times 360°$

$$= \frac{25.7}{2\pi \times 50} \times 360°$$

$$= 29.45°$$

2. max velocity of sliding

pitch line speed $V = \omega_P r = \omega_G R$

$$\omega_P = V/r = \frac{120}{5} = 24 \text{ rad/s}$$

$$\omega_G = \frac{V}{R} = \frac{120}{10} = 12 \text{ rad/s}$$

$\therefore$ max velocity of sliding

$$V_{SK} = (\omega_P + \omega_G) KP$$

$$= (24+12) \times 12.65$$

$$\boxed{V_{SK} = 455.4 \text{ mm/s or } 0.455 \text{ m/s}}$$

Pb 3 Two mating involute spur gear of 20° pressure angle have a gear ratio of 2. The number of teeth on pinion is 20 and its speed is 250 revolutions per minute. The module pitch of the teeth is 12mm. If the addendum of each wheel is such that the path of approach and path of recess on each side are half the max possible length.

Find (i) addendum for pinion and gear wheel.

ii) Length of arc of contact

iii) The max velocity of sliding during approach and recess.

Assume pinion to be driver.

data

$\phi = 20^\circ$ | $N_P = 250$ rpm
$T_P = 20$ | $m = 12$ mm
$G = 2$ | $K_P = \frac{1}{2} MP$ $[MP = r\sin\theta]$

soln:

Angular velocity of pinion

$$\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 250}{60} = 26.16 \text{ rad/s}$$

Gear ratio $G = \frac{T_G}{T_P} = 2$

$$T_G = 2 \times T_P = 2 \times 20 = 40$$

$$G = \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P}$$

$$\omega_G = \frac{\omega_P}{2} = \frac{26.16}{2} = 13.08 \text{ rad/s}$$

or
pitch circle radii of pinion and gear wheel.

$$r = \frac{mT_P}{2} = \frac{12 \times 20}{2} = 120\text{mm}$$

$$R = \frac{mT_G}{2} = \frac{12 \times 40}{2} = 240\text{mm}$$

(i) Addendum for pinion and gear wheel.

$$KP = \frac{1}{2} MP = \frac{r\sin\phi}{2}$$

$$\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi = \frac{r\sin\phi}{2}$$

and
$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

substitute the value R & r in eqn.

$$\sqrt{R_A^2 - 240^2\cos^2 20^\circ} - 240\sin 20^\circ = \frac{120\sin 20^\circ}{2}$$

$$\boxed{R_A = 247.77 \text{ mm}}$$

Addendum of gear wheel $a_W = R_A - R$

$= 249.22 - 240 = 9.22$ mm

Substitute the values of R and r in eqn (i)

$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

$$\sqrt{r_A^2 - 120^2\cos^2 20} - 120\sin 20^\circ = \frac{240\sin 20^\circ}{2}$$

$$\boxed{r_A = 139.5\text{ mm}}$$

Addendum of pinion $= a_p = r_A - r$

$= 139.5 - 120$

$$\boxed{a_p = 19.5\text{ mm}}$$

(ii) <u>length of arc of contact:</u>

length of path of contact $KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$$= \sqrt{(249.22)^2 - (240)^2\cos^2 20^\circ} - 240\sin 20^\circ$$

$= 20.52$ mm.

length of path of recess $PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$$= \sqrt{(139.5)^2 - (120)^2\cos^2 20^\circ} - 120\sin 20^\circ$$

$= 41.08$ mm.

Length of path of Contact $KL = KP + PL$

$= 20.52 + 41.08$

$= 61.6$ mm

length of arc of contact $= \dfrac{\text{length of path of contact}}{\cos\phi}$

$$= \frac{61.56}{\cos 20^\circ} = 65.51\text{ mm}$$

(iii) max Velocity of sliding during approach and recess

Velocity of sliding during approach.

$$V_{SK} = (\omega_1 + \omega_2) \times \text{length of path of approach}$$

$$= (\omega_1 + \omega_2) K_P$$

$$= (26.16 + 13.08) \times 20.52$$

$$= 805.2 \text{ mm/s (or } 0.805 \text{ m/s}$$

Velocity of sliding during recesses

$$V_{SL} = (\omega_1 + \omega_2) PL$$

$$= (26.16 + 13.08) \times 41.08$$

$$\boxed{V_{SL} = 1611.97 \text{ mm/s or } 1.612 \text{ m/s}}$$

4) A pinion having 20 teeth engages with an internal gear having 80 teeth. If the gear having involute profiled teeth with 20° pressure angle, module of 10mm and addendum of 10mm. Find the path of contact, arc of contact and contact ratio.

data

$T_P = 20$ $\quad \phi = 20°$ $\quad$ Addendum $= a_p = a_w = 10mm$

$T_G = 80$ $\quad m = 10mm$

Soln:

(i) Length of path of Contact

$$r = \frac{mT_P}{2} = \frac{10 \times 20}{2} = 100mm$$

$$R = \frac{mT_G}{2} = \frac{10 \times 80}{2} = 400mm$$

Addendum circle radii of pinion and gear wheel

$$r_A = r + a_p = 100 + 10 = 110mm$$

$$R_A = R - a_w = 400 - 10 = 390mm$$ (gear wheel is an internal gear

Internal gearing length of path of approach

$$KP = R\sin\phi - \sqrt{R_A^2 - R^2\cos^2\phi}$$

$$= 400 \times \sin 20^\circ - \sqrt{(390)^2 - (400)^2(\cos 20^\circ)^2}$$

$$= 32.8\ mm$$

Length of path of recess $PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$$= \sqrt{(110)^2 - (100)^2(\cos 20^\circ)^2} - 100\sin 20^\circ = 23\ mm$$

Length of path of contact $KL = KP + PL$

$$= 32.8 + 23$$

$$= 55.8\ mm$$

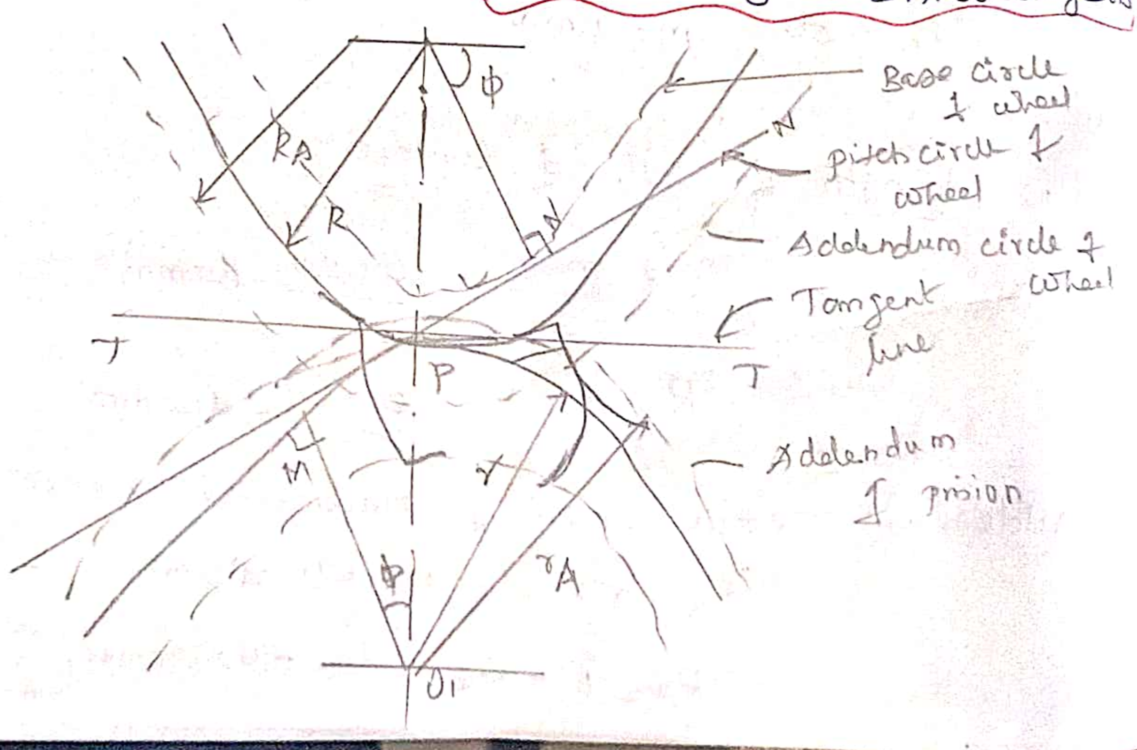
(ii) Length of arc of contact $= \dfrac{\text{length of path of contact}}{\cos\phi}$

$$= \frac{55.8}{\cos 20} = 59.38\ mm$$

(iii) Contact ratio $= \dfrac{\text{length of arc of contact}}{\pi m}$

$$= \frac{59.38}{\pi \times 10} = 1.89 \text{ pairs.}$$

Interference & ~~undercutting~~ under cutting in Involute gears:



To avoid interference the length of path of contact [KL] ~~than~~ should always be less than or equal to the maximum length of path of contact (MN)

mathematically the condition to avoid interference is given by

length of path of contact ≤ max length of path of contact

$$KL \le MN$$

KL = length of path of contact = KP + PL and
MN = max length of path of contact = MP + PN

from geometry we get.

max length of path of approach $MP = r\sin\phi$
min length of path of recess $PN = R\sin\phi$

max length of path of contact

$$MN = MP + PN = (r+R)\sin\phi.$$

(i) length of path of approach ≤ max length of path of approach.

$$\boxed{KP \le MP}$$

(ii) length of path of recess ≤ max length of path of recess

$$\boxed{PL \le PN}$$

Method to avoid interferences.

→ By modifying addendum of gear teeth
→ By increasing the pressure angle.
→ By modifying tooth profile (or) profile shifting
→ By increasing the number of teeth on the mating ~~pair~~ pinion.

Minimum formula

* minimum No of teeth on gear in order to avoid interference

$$T_G(\min) = \frac{\cancel{2A_G}\ 2A_W}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

A_W = Addendum coefficient of gear wheel

G_W = Gear ratio.

* minimum No of teeth on pinion to avoid interferences.

$$T_P(\min) = \frac{2A_G}{G\left(\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right)} \rightarrow (1)$$

(condition)

in term of addendum coefficient of gear wheel (a_w)

$$T_P(\min) = \frac{2A_P}{\sqrt{1+G(G+2)\sin^2\phi}-1} \rightarrow (2)$$

Condition

addendum coefficient of pinion (a_p)

Condition 3.

Minimum No of teeth on pinion in order to avoid interference with rack

$$T_P(\min) = \frac{2A_R}{\sin^2\phi}$$

A_R = Addendum coefficient of rack.

Pb: Two gear wheel mesh externally to give a velocity ratio of 3 to 1. The involute teeth has 6 mm module and 20° pressure angle. Addendum is equal to one module. Det the number of teeth on pinion to avoid interference and the corresponding number on the wheel.

data:

Gear ratio 3 to 1

module = 6

pressure angle = 20°

$$\text{Gear ratio} = \frac{\omega_p}{\omega_g} = \frac{T_g}{T_p}$$

$$G = \frac{3}{1} = 3$$

To find

Number of teeth on Pinion to avoid interference

Soln:

$$T_{p\,(min)} = \frac{2A_g}{G\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$= \frac{2(1)}{3\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20} - 1}$$

$$\boxed{T_p = 44.94 \simeq 45}$$

Number of teeth on pinion

$$T_p = \frac{T_g}{G} = \frac{45}{3} = 15$$

$$\boxed{T_p = 15}$$

Pb: Two gear wheel mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6. The std addendum is 1 module. If the pressure angle 20° and pinion rotates at 90rpm find

(i) The no of teeth on each wheel so that interference is just avoided.

(ii) The length of the path of contact.

(iii) The max velocity of sliding bet the teeth

data

$$\frac{\omega_p}{\omega_G} = \frac{T_G}{T_P} = \frac{3}{1} = 3 \quad m = 6mm$$

$$a_p = a_w = 1 \text{ module}.$$

$$\phi = 20°$$

$$N_p = 90 rpm.$$

Soln:

$$T_G(min) = \frac{2A_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

$$= \frac{2(1)}{\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20} - 1}$$

$$\boxed{T_{G(min)} = 44.94 \simeq 45}$$

No of teeth on pinion

$$T_P = \frac{T_G}{G} = \frac{45}{3} = 15$$

ii) Length of path of contact

$$r = \frac{mT_P}{2} = \frac{6 \times 15}{2} = 45$$

$$R = \frac{mT_G}{2} = \frac{6 \times 45}{2} = 135mm$$

Addendum circle radii in pinion and gear wheel.

$r_A = r + \text{Addendum} = 45 + 6 = 51\,mm$

$R_A = R + \text{Addendum} = 135 + 6 = 141\,mm$

length of path $\Big]$ $KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{(141)^2 - (135)^2\cos^2 20^\circ} - 135\sin 20^\circ$

$= 15.37\,mm$

length of path of recess $PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{(51)^2 - (45)^2\cos^2 20^\circ} - 45\sin 20^\circ$

$= 13.12\,mm$

length of path of contact $KL = KP + PL$

$= 15.37 + 13.12$

$\boxed{KL = 28.49\,mm}$

(iii) max velocity of sliding bet the teeth

$\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 90}{60} = 9.42\,rad/s$

Velocity ratio $= \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 3$

$\omega_G = \frac{9.42}{3} = 3.14\,rad/s$

max velocity of sliding bet teeth

$V_S = (\omega_P + \omega_G)\,KP$ as $KP > PL$

$= (9.42 + 3.14) \times 15.37$

$V_S = 193.05\,mm/s$ (or) $0.193\,m/s$

pb: A pair of involute spur gear with 20° pressure angle and pitch of module 6 mm is in mesh. The number of teeth in pinion is 16 and its rotational speed is 240 rpm. The gear ratio is 1.75. Find the to avoid the interference det
(i) addendum on pinion and wheel
(ii) length of path of contact
(iii) max velocity of sliding on either side of pitch point

data

find: a_p and a_w
a_p

$\phi = 20°$

module = 6 mm

No of teeth of pinion $T_p = 16$

$N_p = 240$ rpm

gear ratio = 1.75

$G = \frac{T_g}{T_p} = 1.75$

$\frac{\omega T}{\omega G} = \frac{T_G}{T_P}$ $T_G = T_p \times G$
$= 16 \times 1.75$
$= 28$

Soln:

(i) Addendum of pinion of wheel.

$\cancel{\frac{mT_p}{2}} =$

$$T_p = \frac{2A_p}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$$

$$16 = \frac{2 \times A_p}{\sqrt{1+1.75(1.75+2)\sin^2 20°} - 1}$$

$A_p = 2.636$

$a_p = A_p \times m$

$= 2.636 \times 6$

$= 15.82$ mm

Addendum on gear wheel (a_w)

$$T_4 = \frac{2A_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$28 = \frac{2A_w}{\sqrt{1+\frac{1}{1.75}\left(\frac{1}{1.75}+2\right)\sin^2 20}-1}$$

$A_w = 1.156$

$a_w = A_w \times m = 1.156 \times 6 = 6.936\,mm$

(iii) length of path of Contact

Pitch circle radii of pinion and gear wheel

$$r = \frac{mT_P}{2} = \frac{6\times 16}{2} = 48\,mm$$

$$R = \frac{mT_4}{2} = \frac{6\times 28}{2} = 84\,mm$$

Addendum radii of pinion and gear wheel.

$r_A = r +$ Addendum on pinion $= r + a_p$

$= 48 + 15.82 = 63.82\,mm$

$R_A = R + a_w$

$= 84 + 6.936 = 90.936\,mm$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(90.936)^2 - 84^2\cos^2 20} - 84\sin 20^\circ = 16.42\,mm$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(63.82)^2 - (48)^2\cos^2 20} - 48\sin 20^\circ$$

$= 28.73$

$KL = KP + PL = 16.42 + 28.73 = 45.14\,mm$

(iii) max velocity of sliding on either side of pitch point

$$\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

$$G = \frac{\omega_p}{\omega_q} = 1.75$$

$$\omega_q = \frac{\omega_p}{1.75} = \frac{25.13}{1.75} = 14.36 \text{ rad/s}$$

max velocity of sliding during approach

$$V_{SK} = (\omega_p + \omega_q) KP$$

$$= (25.13 + 14.36) \times 16.42$$

$$= 648.42 \text{ mm/s or } 0.648 \text{ m/s}$$

max velocity of sliding during recess

$$V_{SL} = (\omega_p + \omega_q) PL$$

$$= (25.13 + 14.36) \times 28.73$$

$$\boxed{V_{SL} = 1134.55 \text{ mm/s (or) } 1.134 \text{ m/s}}$$

Gear Train

A gear train is defined as a combination of gears that is used for transmitting motion from one shaft to another.

Speed ratio : Speed ratio may be defined as the ratio of the speed of the driving gear to the speed of driven gear.

$$\text{Speed ratio} = \frac{\text{Speed of driving gear}}{\text{Speed of driven gear}}$$

Train Value of a gear train may be defined as the ratio of the speed of the driven gear to the speed of driving gear

$$\text{Train value} = \frac{1}{\text{Speed ratio}} = \frac{\text{Speed of the driven gear}}{\text{Speed of the driving gear}}$$

Sign Convention: A positive sign of the velocity ratio indicates that the driver and driven are rotating in same direction (+) same direction (+)

→ Negative indicates that they are rotating in the opposite direction (−)

Train value of a simple Gear Train

let N_1 = Speed of driving gear in rpm

N_2 = Speed of driven gear in rpm

d_1 = pitch circle dia of driving gear

d_2 = pitch circle dia of driven gear

T_1 = No of teeth on driving gear

T_2 = No of teeth on driven gear

pitch line velocity of gear 1 = pitch line velocity of gear 2

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

$$\left(\frac{N_2}{N_1}\right) = \left(\frac{d_1}{d_2}\right) \longrightarrow (1)$$

Pitch (p) of both mating gear are same.

$$\frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}$$

$$\left(\frac{d_1}{d_2}\right) = \left(\frac{T_1}{T_2}\right) \longrightarrow (2)$$

eqn (i) & (ii)

$$\text{Train Value } \left(\frac{N_2}{N_1}\right) = \left(\frac{d_1}{d_2}\right) = -\left(\frac{T_1}{T_2}\right)$$

ii) Compound Gear Train:

→ When two gear are fixed on the same shaft, then the gears form a compound gear.

→ A gear train one or more compound gears is known as compound gear train

Adv: It can provide higher speed reduction for the given centre distance between the input and output shaft using smaller gears.

Application: The compound gear train are used to achieve the required larger speed reduction

N_1 = Speed of driving gear 1

T_1 = No of teeth on driving gear 1

N_2, N_3, N_4, N_5 & N_6 = Speed of gear 2,3,4,5 and 6

T_2, T_3, T_4, T_5 & T_6 = No of teeth on gears 2,3,4,5,6 & 6 respectively.

gear 1 and 2 $\quad \dfrac{N_2}{N_1} = -\left(\dfrac{T_1}{T_2}\right) \rightarrow (1)$

3 and 4 = $\dfrac{N_4}{N_3} = -\left(\dfrac{T_3}{T_4}\right) — (2)$

5 and 6 = $\left(\dfrac{N_6}{N_5}\right) = -\left(\dfrac{T_5}{T_6}\right) \rightarrow (3)$

Multiplying the eqn (i), (ii) and (iii) we get

$$\left(\frac{N_2}{N_1}\right)\left(\frac{N_4}{N_3}\right)\left(\frac{N_6}{N_5}\right) = \left(-\frac{T_1}{T_2}\right)\left(-\frac{T_3}{T_4}\right)\left(-\frac{T_5}{T_6}\right)$$

$$\frac{N_4}{N_1} = -\left(\frac{T_1}{T_2}\right)\left(\frac{T_3}{T_4}\right)\left(\frac{T_5}{T_6}\right)$$

[∵ $N_2 = N_3$ as gear B and C are mounted on same shaft)

Train value = $\dfrac{\text{Speed of last driven}}{\text{Speed of first driver}}$

= No of teeth driver / No of teeth driven.

Pb: A compound gear train consist of 6 gears. The number of teeth on the gears are as follows.

Gear	A	B	C	D	E	F
No of teeth	60	40	50	25	30	24

The gears B and C are on one shaft while the gears D and F are on another shaft. The gear A drives gear B, gear C drives gear D and gear E drives gear F. If the speed of the gear 100 rpm. Det the speed of the gear F

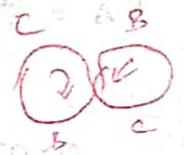
data

$T_A = 60 \quad T_B = 40 \quad T_C = 50 \quad T_D = 25 \quad T_E = 30 \quad T_F = 24$

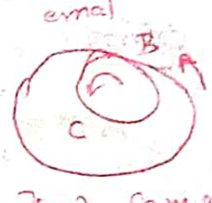
$N_A = 100$ rpm.

Soln:

$$\left(\frac{N_B}{N_A}\right)\left(\frac{N_D}{N_C}\right)\left(\frac{N_F}{N_A}\right) = \left(\frac{T_A}{T_B}\right)\left(\frac{T_C}{T_D}\right)\left(\frac{T_E}{T_F}\right)$$



– ve
Ext – Opposite
ernal



$$\frac{N_F}{100} = \left(\frac{60}{40}\right)\left(\frac{50}{25}\right)\left(\frac{30}{24}\right) = 375 \text{ rpm}$$

Int – Same
(erna)
+ve

Epicyclic gear train

Anti clock = – negative
positive = + positive.

When the axis of rotation of one or more gears is allowed to rotate about another axis then the gears train is known as epicyclic gear train.

* Epicyclic gear train one or more gear move upon and around another gear.

In case simple, compound and reversed gear train
→ The axes on which the gears are mounted are fixed relative to each other.

In case epicyclic
→ The axes of the shaft on which the geer are mounted may be relative motion between them.

Adv: To achieve high speed reduction within a very limited space

APP: * Automobile differential
* M/c tool * hoist * wrist watches

Types of epicyclic gear train

1. Simple epicyclic gear train
2. compound epicyclic gear train

Pb: In a epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rpm in the anticlockwise direction about the centre of the gear A which is fixed, det the speed of gear B.

If the gear A instead of being fixed makes 300 rpm in the clockwise direction. What will the speed of gear B.

data $T_A = 36$ $T_B = 45$ $N_C = 150$ rpm (anticlockwise)

Soln: Cullon:

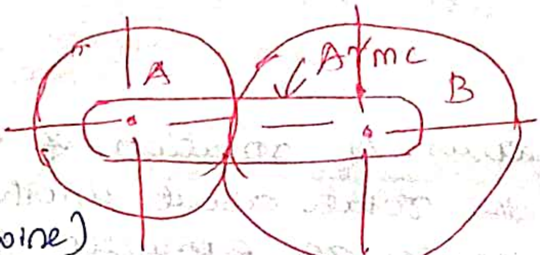
(i) arm $\Rightarrow$ 150 rpm (Gear A fixed)

ii) arm $\Rightarrow$ 150 rpm (Gear A 300 rpm) table

So Same direction +ve
opp. direction −ve

+ → A.C.W

− → C.W



Calculation
(− clockwise
+ Anticlockwise)

Condition of motion	Revolution of element Arm C	Gear A ($T_A = 42$)	Gear B ($T_B = 58$)
i) Fix the arm & give gear A +1 revolution (ccw)	0	+1	$-\frac{T_A}{T_B}$
2) multiply by x	0	+x	$-x \times T_A/T_B$
3. Add (+y)	+y	+y	+y
4. Total motion	y	x+y	$y - x \times T_A/T_B$

Speed of gear B when gear A is fixed

i) Arm rotates at 150rpm CCW $y = +150$rpm

(ii) Gear A is fixed

$$x + y = 0$$

$$x = -y = -150\text{rpm}$$

Speed of gear B, $N_B = y - x \times \frac{T_A}{T_B}$

$$= 150 - \left[-150 \times \frac{36}{45}\right] = 270\text{rpm}$$

Speed of gear B = 270 rpm (anticlockwise)

ii) Speed of gear B when gear A makes 300rpm (clockwise)

→ Arm 150rpm ↺ Gear A $\quad x + y = -300$

$N_A = 300$rpm ↻ → $N_B = ?$ $\quad x = -300 - y$

$$= -300 - 150$$

$N_B = y - x \times \frac{T_A}{T_B}$ $\quad = -450$rpm

$$= 150 - \left[\left(-450 \times \frac{36}{45}\right)\right] = 510\text{rpm}$$

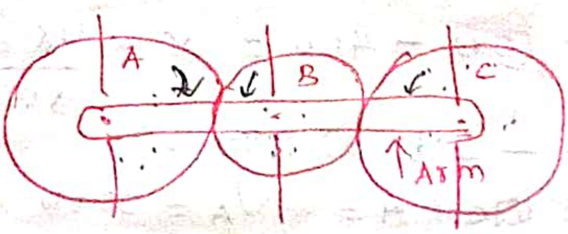
$\boxed{\text{Speed of gear } B = 510\text{ rpm}}$.

Pb 2 In an epicyclic gear train, the number of teeth on gear wheel A, B and C are 48, 24 and 50 respectively. If the arm rotates at 400rpm clockwise find

(i) Speed of gear wheel C when A is fixed

(ii) Speed of gear wheel A when C is fixed.

GD



$T_A = 48 \quad T_B = 24 \quad T_C = 50 \quad N_C = 400$rpm (clockwise)

Soln:

Condition of motion	Arm	Gear A 48	Gear B 24	Gear C 50
(i) Arm fixed Gear A +1 rev (ccw)	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_C}$ $-\frac{T_A}{T_C}$
2) multiply by x	0	$+x$	$-x\frac{T_A}{T_B}$	$-xT_A/T_C$
3) Add $(+y)$	$+y$	$+y$	$+y$	$+y$
4. Total motion	y	$x+y$	$y-x\frac{T_A}{T_B}$	$y-x\frac{T_A}{T_C}$

(i) Speed of gear wheel C when A is fixed

a) Arm rotates at 400 rpm clockwise

$$y = -400\,rpm$$

b) Gear A fixed

$$x+y=0 \qquad x=-y=-(-400)=400\,rpm$$

Speed of gear wheel C $N_C = y - x\frac{T_A}{T_C}$

$$= 400 - 400\times\frac{48}{50}$$

$$\boxed{\text{Speed of gear wheel C} = 16\,rpm \text{ (anticlockwise)}}$$

(ii) Speed of gear A when C is fixed

a) Arm rotates 400 rpm clockwise $y=-400\,rpm$

b) Gear wheel C is fixed $N_C=0$

$$N_C = y - x\frac{T_A}{T_B} = 0$$

$$y = x\frac{T_A}{T_B} \text{ (or) } -400 = x\times\frac{48}{50}$$

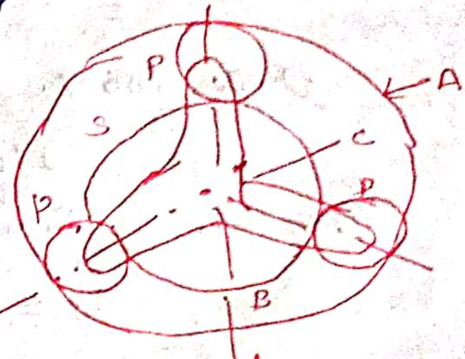
$$x = -416.67\,rpm$$

Speed of gear wheel A = $N_A = x+y$

$$= -416.67+400 = -16.67\,rpm$$

Speed of gear wheel A = 16.67 (Clockwise)

(X) Pb: 2021 An epicyclic gear train for an electric motor. The wheel S has 15 teeth and is fixed to motor shaft rotating at 1450 rpm. The planet P has 45 teeth, gear with fixed annular A and rotates on a spindle carried by an arm which is fixed to output shaft. The planet P also gear with the sun wheel S. Find the speed of output shaft. If motor is transmitting 1.5 KW. Find the torque required to fixed the annular



data

$T_S = 15$ $N_S = 1450$ rpm $T_P = 45$

$P = 1.5 KW = 1.5 \times 10^3$ W

Soln:

$d_A = d_S + 2d_P \rightarrow$ Gear wheel A, S and P

$T_A = T_S + 2T_P = 15 + (2)45 = 105$

Condition of motion	Revolution of element (N)			
	Arm C	Sun wheel $T_S = 15$	Planet wheel $T_P = 45$	Annular A $T_A = 105$
1) Fix arm A +1 rev (ACW)	0	+1	$-T_S/T_P$	$-\frac{T_S}{T_P} \times \frac{T_P}{T_A} = -\frac{T_S}{T_A}$
2) multiply by x	0	+x	$-x\, T_S/T_P$	$-x\frac{T_S}{T_A}$
3. Add +y	+y	+y	+y	+y
Total motion	y	x+y	$y - x\frac{T_S}{T_y}$	$y - x\frac{T_S}{T_A}$

Condition: i) motor shaft such wheel S rotate 1450 rpm

$x + y = 1450$ rpm $\rightarrow$ (CCW) $\rightarrow$ ①

ii) Annular wheel is fixed

$y - x\frac{T_S}{T_A} = 0$ or $y - 0.143x = 0 \rightarrow$ ②

eqn (i) & (ii)

$x = 1268.75$ rpm

$y = 181.25$ rpm

Speed of output shaft = Speed of arm = $y = 181.25$ rpm

" = 181.25 (in same direction the sun wheel S)

Torque required to fix the annular wheel A

$$T_1 = \frac{P \times 60}{2\pi N_S} = \frac{1.5 \times 10^3 \times 60}{2\pi \times 1450} = 9.879 \text{ N-m}$$

η is 100%.

$$I/P + O/P = 0$$

$$T_1 \omega_1 + T_0 \omega_0 = 0 \qquad N_1 = N_S$$

$$N_0 = N_{arm}$$

$$T_0 = -T_1 \cdot \frac{\omega_1}{\omega_0} = -T_1 \times \frac{N_1}{N_0}$$

$$= -9.879 \times \frac{1450}{181.25} = -79.03 \text{ N} \cdot \text{m}$$

for equilibrium

$$T_1 + T_0 + T_B = 0$$

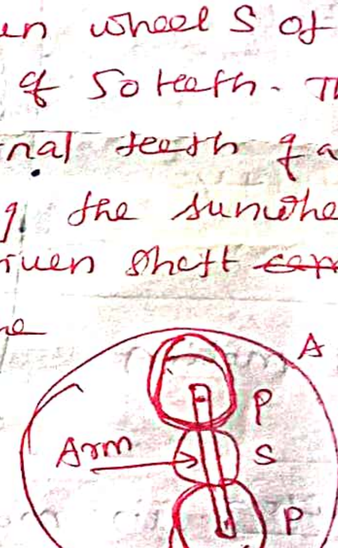
$$9.879 - 79.03 + T_B = 0$$

$$T_B = 69.15 \text{ Nm}$$

Torque required at the annular wheel A } $= 69.15$ Nm

Pb-X: Epicyclic gear train

An epicyclic gear train has a sun wheel S of 30 teeth and two planet wheel P-P of 50 teeth. The planets wheel mesh with the internal teeth of a fixed annulus. The driving shaft carrying the sunwheel transmit 4 kW at 300 rpm. The driven shaft ~~carrying~~ is connected to an arm which carries the planet wheels. Det the speed of driven shaft and torque transmitted if the overall efficiency is 95%.



UNIT - III

Friction:

Friction in machine element

Friction is a force that opposes the motion or tendency of a movement

Effect of friction

→ Beneficial effect ⇒ cannot drive automobile without the beneficial effect of friction.

→ It makes the tractive force.

Application ⇒ tires, clutch, brake and belt drive.

higher friction is beneficial.

→ Detrimental effect: Friction is undesirable in moving parts of machines.

In machine element such as bearing and gears, friction must be minimized to avoid power losses.

Types of friction:

Depending upon the condition of contacting surfaces.

Friction
- Dry
  - Sliding friction → nut & screw
  - Rolling friction → Ball & Roller bearing.
- Skin
- film

Dry friction :

The friction exist bet two unlubricated surface is known as dry friction

eg: Nut & Screw.

(i) Sliding or solid friction.

The friction takes place the one surface slides over another surface known as sliding friction

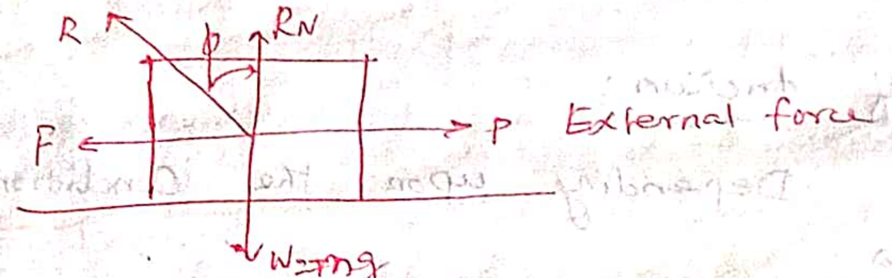
(ii) Rolling friction

The friction takes place when one surface rolles over another surface it is known as Rolling friction.

eg → Ball & Roller bearing

Note :

Rolling friction is always less than the sliding friction



P – applied force

F = frictional force

ϕ = Angle of frictio

W = wt of the body

R_N = Reacting bet body A and the plane B (Normal reaction)

R = Resultant force.

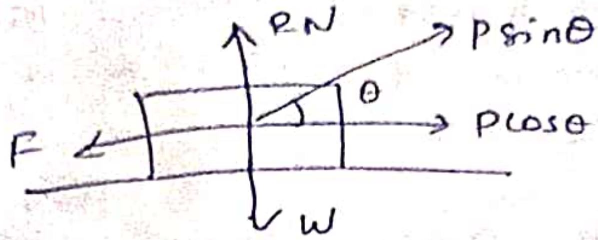
x ; $\cos = \frac{Adj}{hyp}$ $\sin = \frac{opp}{hy}$ $\tan\theta = \frac{\sin}{\cos} = \frac{opp}{Adj}$

$$\tan\phi = \frac{F}{R} = \mu \Rightarrow F = \mu R$$

μ = co-efficient of friction

$$\boxed{\tan\phi = \mu}$$

Equilibrium of a body in a horizontal plan:



(i) pulling (Horizontal)

$$P\cos\theta = F$$

$$P\cos\theta = \mu R$$

$$R = \frac{P\cos\theta}{\mu} \rightarrow (1)$$

Vertical

$$R - P\sin\theta = W$$

$$\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$$

$$\Rightarrow R = W + P\sin\theta \longrightarrow (2)$$

Eqn (1) & (2)

$$W \mp P\sin\theta = \frac{P\cos\theta}{\mu}$$

$$\frac{P\cos\theta \times \cos\phi}{\sin\phi} = W - P\sin\theta$$

$$P(\cos\theta\cos\phi + \sin\theta\sin\phi] = W\sin\phi$$

$$P[\cos\theta - \phi] = W\sin\phi$$

$$\boxed{P = \frac{W\sin\phi}{\cos\theta - \phi}}$$

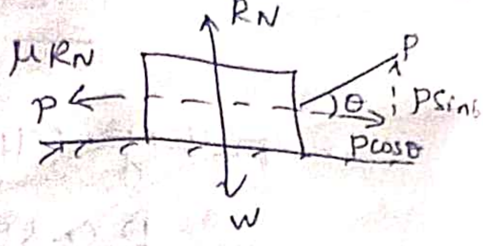
The value of P is minimum $\cos(\theta-\phi)$ is max

$$\cos(\theta-\phi) = 1 = \cos 0 \quad \text{or} \quad (\theta-\phi) = 0$$

$$\theta = \phi$$

$$\boxed{P_{min} = W\sin\theta - W\sin\phi}$$

Pb The force required to pull a body of weight 50N on a rough horizontal plan is 15N. Det the Co-efficient of friction, if the force is applied at an angle of 15° with the horizontal.



data

$W = 50N \quad P = 15N \quad \theta = 15^\circ$

Soln:

$$F = \mu R_N = 15 \cos 15^\circ \rightarrow (i)$$

resolving forces vertically we get

$$R_N + 15 \sin 15^\circ = W - 50$$

$$R_N = 50 - 15 \sin 15^\circ = 46.12N$$

Substitute the value of R_N in eqn $\rightarrow$ (i)

$$\mu \times 46.12 = 15 \cos 15^\circ$$

$$\boxed{\mu = 0.314}$$

Motion of the body down the Plane

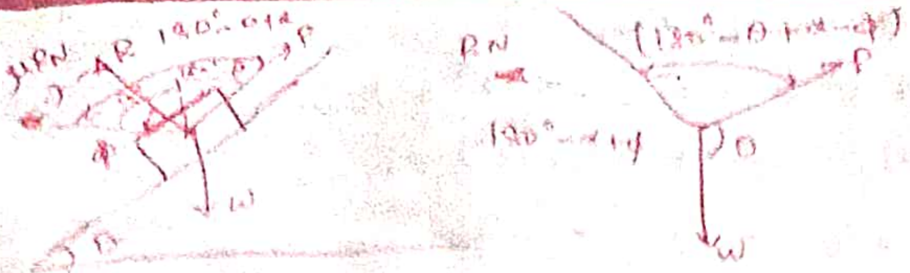
a) without considering friction:

Body moving downward without friction. Since the frictional force is not to be taken into account there will be same three force (i) weight (W) (ii) Normal reaction (R_N) and (iii) Effort (P_0) under which the body will be equilibrium position:

$$P_0 = \frac{W \sin \alpha}{\sin(\phi - \alpha)}$$

b) with considering friction

Friction is taken into account, the force of friction $F = \mu R_N$ will act up the plane and resultant reaction R with an angle ϕ with R_N toward its right



$$\frac{P}{\sin(180^\circ - \alpha + \phi)} = \frac{W}{\sin(180^\circ - \theta + \alpha - \phi)}$$

$$\frac{P}{\sin(\alpha - \phi)} = \frac{W}{\sin(\theta - \alpha + \phi)}$$ $\sin(180^\circ - \theta) = \sin\theta$

$$\boxed{P = \frac{W\sin(\alpha - \phi)}{\sin(\theta - (\alpha - \phi))}}$$ → Downward motion of body with friction

upward motion of body without considering friction

$$\boxed{P = \frac{W\sin(\alpha + \phi)}{\sin(\theta - (\alpha + \phi))}}$$

Pb Aug 2021
An effort of 1200N is required just to move a certain body up an inclined plane of angle 12°. The Force acting parallel to the Plane. If the angle of inclination of the plane is increased to 15°, then the effort required is 1400N. Find the weight of the body and co-efficient of friction.

data : $P_1 = 1200N$; $\alpha_1 = 12^\circ$ $P_2 = 1400$ $\alpha_2 = 15^\circ$

Soln :

case (i) $P_1 = 1200N$ $\alpha_1 = 12^\circ$

Revolving the forces along the plane

$$W\sin 12^\circ + F_1 = 1200$$

$$W\sin 12^\circ + \mu R_{N1} = 1200$$ $F_1 = \mu R_{N1}$

Resolving the forces normal to the plane

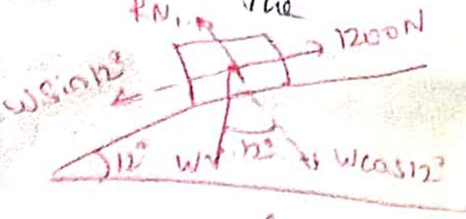
$$R_{N1} = W\cos 12^\circ$$

Sub the value of R_{N_1}

$$W\sin 12^\circ + \mu \times W\cos 12^\circ = 1200$$

$$W(\sin 12^\circ + \mu\cos 12^\circ) = 1200$$

Case ii) The body is in equilibrium under the acting of force.



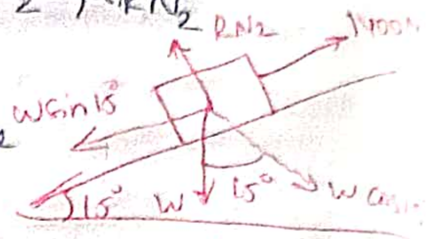
(a)

$$P_2 = 1400N \quad \alpha = 15^\circ$$

Resolving force along the plane

$$W\sin 15^\circ + F_2 = 1400N \qquad F_2 = \mu . R_{N_2}$$

$$W\sin 15^\circ + \mu R_{N_2} = 1400N$$



(b)

Resolving force normal to the plane

$$R_{N_2} = W\cos 15^\circ$$

Sub the value of R_{N_2}

$$W\sin 15^\circ + \mu \times W\cos 15^\circ = 1400$$

$$W(\sin 15^\circ + \mu\cos 15^\circ) = 1400$$

Divide

$$\frac{W(\sin 15^\circ + \mu\cos 15^\circ)}{W(\sin 12^\circ + \mu\cos 12^\circ)} = \frac{1400}{1200} = 1.167$$

$$\sin 15^\circ + \mu\cos 15^\circ = 1.167(\sin 12^\circ + \mu\cos 12^\circ)$$

$$0.259 + 0.966\mu = 1.167(0.208 + 0.978\mu)$$

$$\boxed{\mu = 0.097} \qquad 0.242 + 1.141\mu$$

$$0.259 - 0.242 = 1.141\mu - 0.966\mu$$

$$0.017 = 0.175\mu$$

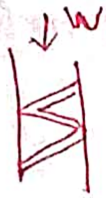
$$\mu = 0.097$$

Weight of the body (W)

$$W(\sin 12^\circ + 0.092\cos 12^\circ) = 1200$$

$$\boxed{W = 3971.47\,kN}$$

Friction in screw threads.



1) square threads
2) V-thread

$P \rightarrow$ pitch value

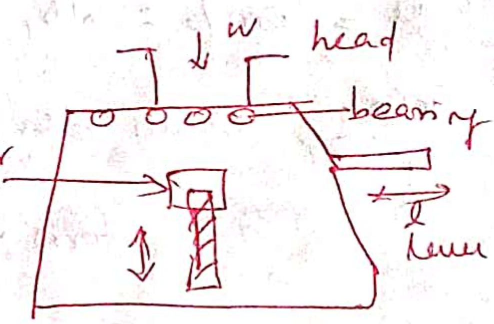
$\alpha \rightarrow$ helix angle

Formula

$$P = W \tan(\alpha + \phi)$$

→ Torque to required to overcome friction bet screw & nut. $T_1 = \frac{P \times d_m}{2}$

→ Torque required to overcome friction bet collar



$$T_2 = \mu_c P \frac{d_m}{2}$$

where d_m = mean dia

μ_c = Column coefficient

W = Load

Total torque $T = T_1 + T_2$

$T = F \times l$. $F \rightarrow$ force

l – length of lever.

Effort required to lower, the load

$$P = W \tan(\alpha + \phi) \times d/L$$

Efficiency of Screw jack

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

Self locking screw

Condition

1. If $\phi > \alpha$ T = + positive $T > 0$

over hauling screw

1. $\phi < \alpha$ T = negative $T > 0$

$$\boxed{\tan\alpha = \frac{P}{\pi \times d_m}}$$

$d_m = d_c + \frac{\phi_P}{2}$

d_c = Collar on root dia

Pb A square threaded bolt of root dia 22.5mm and pitch 5mm is tightened by screwing nut whose mean dia of bearing surface is 50mm. If coefficient of friction for nut and bolt is 0.1 and for nut and bearing surface is 0.16. Find the force required at the end of a spanner 500mm long when the load on the bolt is 10 KN.

data square thread $= d = 22.5mm = 0.0225m$

$P = 5\ mm = 0.005m$ $D = 50mm$ or $R = 25mm = 0.025m$

$\mu = \tan\phi = 0.16$, $l = 500mm = 0.5m$ $W = 10KN = 10\times10^3 N$

Soln: Mean dia of Screw $d = d_c + P/2 = 22.5 + 5/2 = 25mm$

$$\tan\alpha = \frac{P}{\pi d_m} = \frac{0.005}{\pi \times 0.025} = 0.0636$$

Circumference of Screw

$$P = W\tan(\alpha+\phi) = W\left(\frac{\tan\alpha + \tan\phi}{1-\tan\alpha \cdot \tan\phi}\right)$$

$$= 10\times10^3\left[\frac{0.0636+0.1}{1-0.0636\times0.1}\right] = 1646N$$

Torque $T = P\times\frac{d}{2} + \mu W R$

$$= 1646\times\frac{0.025}{2} + 0.16\times10\times10^3\times0.025$$

$$\boxed{T = 60.575 N\text{-}m}$$

End of spanner $F = \frac{\text{Torque}}{\text{length of lever}}$

$$= \frac{60.575}{0.5}$$

$$\boxed{F = 121.15N}$$

Formula

1. Co-efficient of friction $\mu = F/R_N$

2. $\phi = R/R_N$ R = Resultant reaction, R_N = normal reaction

3. $\phi = \mu$

4. Equilibrium of a body on an inclined plan

Motion of the body up the plane

1) $P_o = \dfrac{W \sin\alpha}{\sin(\theta-\alpha)}$

2) $P = \dfrac{W \sin(\alpha+\phi)}{\sin(\theta-(\alpha+\phi)}$

3) $\eta_{up} = \dfrac{P_o}{P} = \dfrac{\cot(\alpha+\phi) - \cot\theta}{\cos\alpha - \cot\theta}$

4) $M.A = \dfrac{W}{P} = \dfrac{\sin(\theta-(\alpha+\phi)]}{\sin(\alpha-\phi)}$

motion of the body down the plane.

$P_o = \dfrac{W \sin\alpha}{\sin(\phi-\alpha)}$

$P = \dfrac{W \sin(\alpha-\phi)}{\sin(\theta-(\alpha-\phi))]}$

$\eta_{down} = \dfrac{P}{P_o} = \dfrac{\cot\alpha - \cos\theta}{\cot(\alpha-\phi) - \cot\theta}$

$M.A = W/P = \dfrac{\sin(\theta-(\alpha-\phi)}{\sin(\alpha-\psi)}$

W = wt

α = Angle of inclination of the plane horizontal

ϕ = limiting angle of Friction for the contact surface

P_o = Effect required to move the body

Friction of Screw and nut

When the nut moves upward

$P_o = \dfrac{W \sin\alpha}{\sin(\theta-\alpha)}$

$P = W \tan(\alpha+\phi)$

$\eta_{mech} = \dfrac{\tan\alpha}{\tan(\alpha+\phi)}$

$M.A = \cot(\alpha+\phi)$

When the nut is lowered

$P_o = \dfrac{W \sin\alpha}{\sin(\theta-\alpha)}$

$P = W \tan(\alpha-\phi)$

$\eta_{mech} = \dfrac{\tan(\alpha \cancel{+} \phi)}{\tan\phi}$

$M.A = \dfrac{1}{\tan(\alpha-\phi)}$

P, W, μ, α, ϕ and θ have usually

p = pitch of the thread

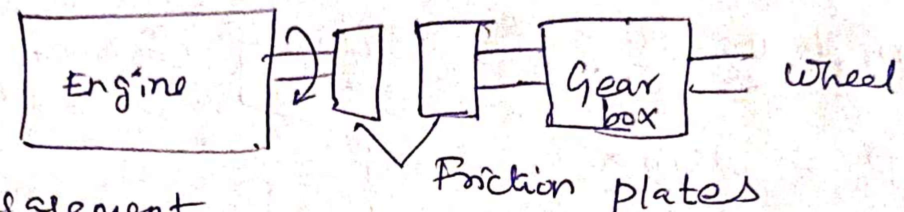
d = dia of the screw

→ max efficiency of nut and screw

$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

→ η is less than 50% then screw is said to be self locking. $\eta > 50\%$ is known as overhauling screw.

Friction in clutches



→ Engagement
↳ Power transmission occur

Cluthes: It is a machine member which makes or break the motion bet engine and Gear box.

It also used to transmit power from the engine to the gear box & as well as shifting the gears steadly.

Types of clutches.

Clutches
- Dry
  - Single plate (or) Disc clutch
  - cone
  - centrifugal
- Wet
  - Multi-plate

Single plate clutch → Formula:

(1) Torque transmitted on the single plate clutch

$$T = \mu W R$$

$$P_{max} \cdot r_2 = C$$

$$W = 2\pi C(r_1 - r_2)$$

$$T = \frac{1}{2} n \cdot \mu \cdot W (r_1 + r_2)$$

$$P = \frac{2\pi N T}{60}$$

Where μ = coefficient of friction
W = Axial thrust exerted by the spring
R = Mean radius of friction surface.

$$R = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \rightarrow \text{considering uniform pressure}$$

$$R = \frac{r_1 + r_2}{2} \longrightarrow \text{considering uniform wear}$$

r_1 = External radius of friction surface.
r_2 = Internal radius of friction surface.

Multiplate Clutch:

(i) Torque transmitted on the multiple plate clutch

$$T = n\mu W R$$

n = No of pair of contact surface $= n_1 + n_2 - 1$
n_1 = No of disc on the driving shaft
n_2 = no of disc on the driven shaft

(ii) Axial force to engage the clutch $W = 2\pi C(r_1 - r_2)$
(iii) Average pressure on the friction surface.

$$P_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi(r_1^2 - r_2^2)}$$

iv) Total no of plates = No of pairs of contact surface + 1

Cone Clutch:

(i) Torque transmitted on the cone clutch

$$T = \mu W R \operatorname{cosec}\alpha$$

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \rightarrow \text{uniform pr}$$

(ii) Axial force required at the engagement of clutch

$$W_e = W_n(\sin\alpha + \mu\cos\alpha)$$

Axial Force required at the disengagement

$$W_d = W_n(\sin\alpha - \mu \cdot \cos\alpha)$$

Pb: A single plate friction clutch with both sides of the plate being effective is used to transmit power at 1440 rpm. It has outer and inner radii 80mm and 60mm resp. The max intensity of pressure is limited to 10×10^4 N/m². If the co-efficient of friction is 0.3 det (i) total pressure exerted on the plate ii) power transmitted.

data.

$$n = 2 \quad N = 1440 \text{rpm} \quad r_1 = 80\text{mm} \; r_2 = 60\text{mm}$$

$$P_{max} = 10 \times 10^4 \text{N/m}^2 \quad \mu = 0.3$$

Soln:

(i) Total pressure exerted on the plate

$$P_{max} \cdot r_2 = C$$

$$C = 10 \times 10^4 \times 0.06 = 6000 \text{N/m}$$

Axial thrust

$$W = 2\pi c(r_1 - r_2) = 2\pi\, 6000(0.08 - 0.06)$$

$$\boxed{W = 754 \text{ N}}$$

(ii) power transmitted

$$T = \frac{1}{2} n \cdot \mu \cdot W (r_1 + r_2)$$

$$= \frac{1}{2} \times 2 \times 0.3 \times 754 (0.08 + 0.06)$$

$$= 31.67 \text{ N.m}$$

$$P = \frac{2\pi N T}{60} = \frac{2\pi\, 1400 \times 31.67}{60}$$

$$\boxed{P = 4.643 \text{ kW}}$$

Pb A car engine develop max torque at 15kW and 2400 rpm. The data provided for the clutch design are the following.

(i) Intensity of pressure on the friction surface. not to exceed 0.7 bar

ii) Provision is to be made for the loss of torque to wear as 30% of the engine torque.

iii) Co-efficient of friction for the mating lining riveted on both side of the plate is 0.35
iv) Inside dia of the friction plate is 0.6 times the outside dia. Det the suitable dimension of the clutch plate.

data $P = 15\,kW$ $N = 2400\,rpm$ $P_{max} = 0.7\,bar = 0.7\times10^5\,N/m^2$
$n = 2$ $\mu = 0.35$ $d_2 = 0.6 d_1$ $r_2 = 0.6 r_1$

Soln:

(i) max torque developed by the engine at 2400 rpm

$$T_{max} = \frac{P\times60}{2\pi N} = \frac{15\times10^3\times60}{2\pi\times2400} = 59.7\,N.m$$

$$T_{design} = 59.7\times\frac{130}{100} = 77.6\,N.m$$

Axial forced exerted by spring

$$W = 2\pi c(r_1 - r_2) \qquad \therefore P_{max}\times r = c$$

$$= 2\pi P_{max}\, r_2 (r_1 - 0.6 r_1) \qquad r_2 = 0.6 r_1$$

$$= 2\pi\times0.7\times10^5\cdot0.6 r_1 (r_1 - 0.6 r_1)$$

$$= 105557.5 r_1^2$$

Torque transmitted $T_{design} = \frac{1}{2} n\mu W (r_1 + r_2)$

$$77.6 = \frac{1}{2}\times2\times0.35\times105557.5 r_1^2 (r_1 + 0.6 r_1)$$

$$77.6 = 59112.2 r_1^3$$

$$r_1 = \left(\frac{77.6}{59112.2}\right)^{1/3} = 0.1095\,m = 109.5\,mm$$

$$r_2 = 0.6 r_1 = 0.6(109.5) = 65.7\,mm.$$

Multiplate clutch:

Pb: A multiple friction clutch is required to transmit 75 KW at 1500 rpm. The plates are alternately of steel and phosphor bronze and they run in oil. The coefficient is 0.2 and the axial force with which they are pressed together is 2600 N. The inner and outer dia of the disc are 150 mm and 250mm. Calculate no of plates required. Assume uniform intensity of pressure.

data: $d_1 = 250mm$ or $r_1 = 125mm = 0.125m$

$d_2 = 150mm$ or $r_2 = 75 = 0.075m$; $P = 75 KW = 75\times10^3 W$

$N = 1500$ rpm $W = 2600 N$ $\mu = 0.2$

soln:

$$P = \frac{2\pi NT}{60}$$

$$75\times10^3 = \frac{2\times\pi(1500)T}{60}$$

$$T = 477.46 \text{ N.m}$$

uniform pressure, Torque transmitted

$$T = \frac{2}{3} n\mu w\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

$$\frac{1.531\times10^{-3}}{0.01}$$

$$477.46 = \frac{2}{3}\times n\times0.2\times2600\left(\frac{0.125^3 - 0.075^3}{0.125^2 - 0.075^2}\right)$$

$$N = 8.99 \simeq 9$$

∴ Total no of plate = No of pair of contact surface + 1

$= 9 + 1 = 10$ plates.

Cone clutch:

Pb : Determine the axial force required to engage a cone clutch transmitting 25 kW of power at 600 rpm. Average friction dia of the cone is 400 mm semi-cone angle 12° and coefficient of friction 0.25

data $P = 25\,kW = 25\times10^3\,W$; $N = 600\,rpm$ $D = 400\,mm$

$R = 200\,mm = 0.2\,m$ $\alpha = 12°$ $\mu = 0.25$

Soln:

$$P = \frac{2\pi NT}{60}$$

$$25\times10^3 = \frac{2\pi\, 600 \times T}{60}$$

$$T = 397.9\ N\cdot m$$

Normal load acting on friction surface.

$$T = \mu\, W_n \cdot R$$

$$397.9 = 0.25 \times W_n \times 0.2$$

$$\boxed{W_n = 7957.75\,N}$$

the axial force require to engage the cone clutch

$$W_e = W_n(\sin\alpha + \mu\cos\alpha)$$

$$= 7957.75(\sin 12° + 0.25\cos 12°)$$

$$\boxed{W_e = 3600.42\,N}$$

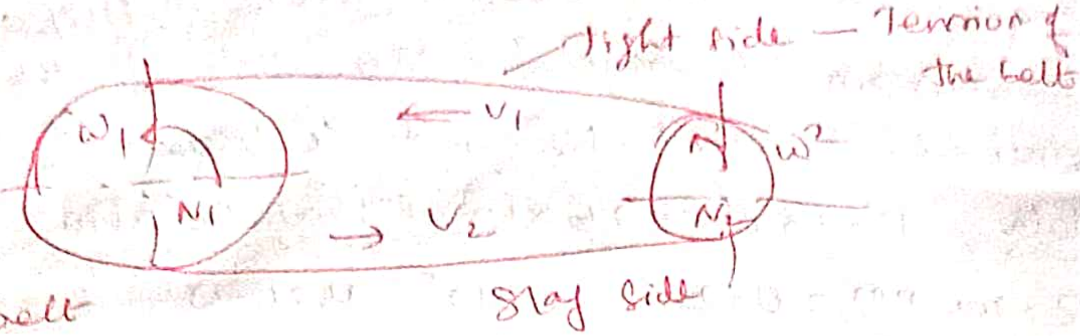
Belt and rope drives:

The flexible type of machine element are belt, rope and chain

The non-flexible type are cams, gears clutches, coupling and power screw.

It is used for power transmission purpose.

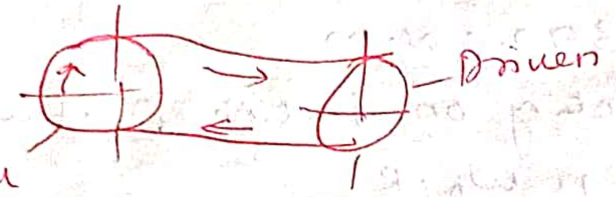
In belt & rope drive the power is transmitted due to friction between them & pulley.



Type of belt

1) Flat belt ▭ → Less than 10m — Cotton rubber
2) V – belt ▽ – lessthan 5m – Rubberised fabric & rubber

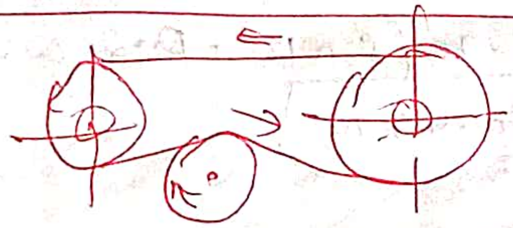
1. open belt drive:



2. cross – belt

3. Compound belt drive

4. open belt drive with idler pulley



Angular velocity ratio

It is defined as the ratio bet angular velocity of the driven pulley.

$$\omega_1 = \frac{2\pi N_1}{60} \qquad \omega_2 = \frac{2\pi N_2}{60}$$

$$\frac{\omega_2}{\omega_1} = \frac{N_2}{N_1}$$

Linear velocity – ratio of linear velocity of the driven pulley

$$V_1 = \pi d_1 N_1 \qquad V_2 = \pi d_2 N_2$$

Pb: (Pg) 2014) An open belt drive connect two pulley of 1.2m and 0.5m dia on parallel shaft 4m apart. The max tension in the belt is 1800N. The coefficient of friction is 0.3. The driven pulley of dia 1.2m run at 250rpm. Cal i) The length of the belt (ii) power transmitted (iii) The torque on each of the two shafts.

data

Open belt drive $d_1 = 1.2m$ or $r_1 = 0.6m$

$d_2 = 0.5$ or $r_2 = 0.25m$ $x = 4m$ $T_1 = 1800N$ $\mu = 0.3$

$N_2 = 250$ rpm

Soln:

(i) Length of the belt (L)

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.6 + 0.25) + 2 \times 4 + \frac{(0.6 - 0.25)^2}{4}$$

$$\boxed{L = 10.7m}$$

(ii) Power transmitted P

$T_1 = 1800N$

$d_{driven} = 1.2m$
$N_{driven} = 250rpm$

$$V = \frac{\pi d N}{60} = \frac{\pi \times 1.2 \times 250}{60} = 15.71 m/s$$

To find θ

$$\theta = (180° - 2\alpha) \times \frac{\pi}{180°} \text{ rad}$$

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.25}{4} = 0.0875$$

$$\alpha = \sin^{-1}(0.0875) = 5.02°$$

$$\theta = (180° - 5.02) \times \pi/180 = 2.966 \text{ rad}$$

To find T_2

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1800}{T_2} = e^{0.3 \times 2.996}$$

$$\boxed{T_2 = 739.39N}$$

Power transmitted

$$P = (T_1 - T_2)v$$

$$= (1800 - 739.39) \times 15.71$$

$$P = 16662.2W \text{ or } 16.66KW$$

(iii) Torque - shaft of driver

$$T_{driver} = (T_1 - T_2) r_{driver} = (1800 - 739.3) 0.25$$

$$T_{driver} = 265.15 N.m$$

Shaft of driven

$$T_{driven} = (T_1 - T_2) r_{driven} = (1800 - 739.39) 0.6$$

$$\boxed{T_{driven} = 636.37 N.m}$$

Aug 2021
2023

Pb: A leather belt is required to transmit 7.5KW from a pulley 1.2m in dia, running at 250rpm. The angle embraced is 165° and the co-efficient of friction bet the belt and pully is 0.3. If the safe working stress for the leather belt is 1.5mpa density of leather 1mg/m³ and thk of belt 10mm. Det the width of the belt taking centrifugal tension into account.

data: $P = 7.5KW = 7.5 \times 10^3 W$ $d = 1.2m$

$N = 250 rpm$ $\theta = 165° = 165° \times \frac{\pi}{180°} = 2.88 rad$

$\mu = 0.3$ $\sigma = 1.5mpa = 1.5 \times 10^6 N/m^2$ $\rho = 1 mg/m^3$

$= 1 \times 10^6 g/m^3 = 1000 kg/m^3$ $t = 10mm = 0.01m$

Soln:

velocity of belt

$$V = \frac{\pi d N}{60} = \frac{\pi \times 1.2 \times 250}{60}$$

$$V = 15.71 \, m/s$$

$$P = (T_1 - T_2) V$$

$$7500 = (T_1 - T_2) 15.71$$

$$T_1 - T_2 = \frac{7500}{15.71} = 477.6 \, N$$

$$\frac{T_1}{T_2} = e^{\mu \theta} = e^{0.3 \times 2.88} = 2.373$$

$$T_1 = 2.373 \, T_2$$

$T_1 - T_2 = 477.6 N$

$\frac{T_1}{T_2} = 2.373$

$T_1 = 2.373 T_2$

$T_1 = 477.6 + T_2$

on solving eqn

$$T_1 = 825.2 N \qquad T_2 = 347.75 N$$

mass of the belt

m = area × length × density

density = $\frac{\text{mass}}{\text{volume}}$

$$= (b \times t) l \rho$$

$$= b \times 0.01 \times 1 \times 1000 = 10b \, kg$$

$$T_c = mv^2 = 10b (15.71)^2 = 2468b \, N$$

max tension in the belt.

$$T_{max} = \sigma b t = 1.5 \times 10^6 \times b \times 0.01$$

$$= 15000 b \, N$$

$$T_{max} = T_1 + T_c \text{ or, } 15000b = 825.2 + 2468b$$

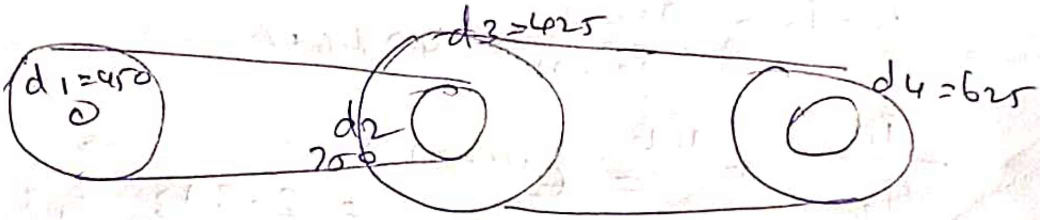
$$15000b - 2468b = 825.2$$

$$b = 0.06586 \, m$$

$$\boxed{b = 65.86 \, mm}$$

Pb: An engine running at 1200 rpm drives a line shaft by means of a belt. The engine pulley is 450 mm dia and the pulley on the line shaft being 750mm. A 425mm dia pulley on the line shaft drives a 625mm pulley keyed to a machine shaft. Find the speed of the machine shaft.

(i) There is no slip
2) There is a slip of 2.5% slip on each drive



data

$N_1 = 1200 rpm \quad d_1 = 450mm = 0.45$

$d_2 = 750mm = 0.75m \quad d_3 = 425mm = 0.425m$

$d_4 = 625 = 0.625m$

to find

(i) $$\frac{N_4}{1200} = \frac{0.450 \times 0.425}{0.750 \times 0.625} \qquad \frac{d_1 \times d_3}{d_2 \times d_4}$$

$$N_4 = \frac{229.5}{0.468} = 489.6 \text{ rpm}$$

(ii) $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - s/100\right)$$

$$s = s_1 + s_2$$

$$= 2.5 + 2.5 = 5.0\% = 5\%$$

$$\frac{N_4}{1200} = \frac{0.450 \times 0.425}{0.750 \times 0.625}\left(1 - \frac{5}{100}\right)$$

$$\boxed{N_4 = 465.12 \text{ rpm}}$$

2022
Ⓧ Screw Jack: Rog 2021

Pb: pitch of 50 mm dia threaded screw of a screw jack is 12.5 mm co-efficient of friction between screw and nut is 0.10. Des the torque to raise a load of 25 KN rotating with the screw. Also find the torque required to lower the load and efficiency of screw jack.

data $d = 50mm = 0.05 m$ $P = 12.5 mm = 0.0125 m$

$\mu = 0.1$ $W = 25 KN$

Soln $\tan\alpha = \frac{P}{\pi d} = \frac{0.0125}{\pi \times 0.05} = 0.079$ or

$\alpha = 4.55^\circ$

$\mu = \tan\phi = 0.1$ or $\phi = \tan^{-1}(0.1) = 5.71^\circ$

Torque require to raise the load (T_1)

$T_1 = W \tan(\alpha + \phi) d/2$

$= 25\times10^3 \tan(4.55^\circ + 5.71^\circ) \times \frac{0.05}{2}$

$= 113.13 N.m$

Torque require to lower the load T_2

$T_2 = W \tan(\phi - \alpha) d/2$

$= 25\times10^3 \tan(5.71^\circ - 4.55^\circ) \times \left(\frac{0.05}{2}\right)$

$= 12.65 N.m$

η screw jack

$\eta_{screw jack} = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan 4.55^\circ}{\tan(4.55^\circ + 5.71^\circ)}$

$= 0.4396$

$\boxed{\eta_{screw jack} = 43.96\%}$

Pb: Ay 2021

A rotor is driven by a co-axial motor through a single plate clutch both sides of the plate being effective. The external and internal dia of the plate are respectively 220mm and 160mm. and the total spring load pressing the plates together is 570N. The motor armature and shaft has a mass of 800kg, with an effective radius of gyration of 200mm. The rotor has a mass of 1300kg with an effective radius of gyration of 180mm. The coefficient of friction for the clutch is 0.35. The driving motor is brought to a speed of 1250rpm when the current is switched off and the clutch suddenly engaged

Det (i) The final speed of motor and rotor

ii) The time to reach this speed.

iii) The kinematic energy lost during the period of slipping.

data : $d_1 = 220mm$ or $r_1 = 110mm = 0.11m$

$d_2 = 160mm$ (or) $r_2 = 80mm = 0.08m$ $W = 570N$, $m_{motr} = 800kg$

$K_{motor} = 200mm = 0.2m$ $m_{rotor} = 1300kg$ $K_{rotor} = 180 = 0.18m$

$\mu = 0.35$ $N_1 = 1250rpm$

Soln:

Moment of Inertia $I_{motr} = m_{motr} K^2_{motor}$

$= 800(0.2)^2 = 32kg.m^2$

$I_{rotor} = m_{motor} k^2_{rotr}$

$= 1300 \times (0.18)^2 = 42.12kg.m^2$

(i) final Speed of motor and rotor.

$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi(1250)}{60} = 130.9 rad/s$

ω_2 = initial speed of rotor = 0

$$I_{motor}\,\omega_1 + I_{rotor}\,\omega_2 = (I_{motor} + I_{rotor})\,\omega_3$$

$$(32 \times 130.9) + (42.12 \times 0) = (32 + 42.12)\,\omega_3$$

$$\omega_3 = \frac{4188.8}{74.12} = 56.51 \text{ rad/s}$$

(ii) Time to reach this speed;

$$T = \frac{1}{2} n \mu W (r_1 + r_2)$$

$$= \tfrac{1}{2} \times 2 \times 0.3 \times 570\,(0.11 + 0.08) = 37.905 \text{ N.m}$$

Angular acceleration of rotor $\alpha_{rotor} = \dfrac{T}{I_{rotor}} = \dfrac{37.905}{42.12}$

$$\alpha_{rotor} = 0.9 \text{ rad/s}^2$$

Assuming α_R

$$\omega_3 = \omega_2 + \alpha_{rotor}\, t$$

$$56.51 = 0 + 0.9 \times t$$

$$t = \frac{56.51}{0.9} = 62.79 \text{ s}$$

(iii) kinetic energy lost during the period of slipping

$$KE_1 = \frac{1}{2} I_{motor}\,\omega_1^2 + \frac{1}{2} I_{rotor}\,\omega_2^2$$

$$= \tfrac{1}{2} \times 32 \times (130.9)^2 + \tfrac{1}{2} \times 42.12 \times 0^2 = 274157 \text{ N.m}$$

Angular kinematic energy after engagement.

$$KE_2 = \frac{1}{2}(I_{motor} + I_{rotor})\,\omega_3^2$$

$$= \frac{1}{2}(32 + 42.12)(56.51)^2 = 118.347 \text{ N.m}$$

kinetic energy lost during the period of slipping

$$KE_1 - KE_2$$

$$= 274157 - 118347$$

$$= 155810 \text{ N.m}$$

UNIT - IV - Force Analysis:

Static force Analysis:

When the inertia effect due to the mass of the machine components are neglected. then the analysis of mechanism is called as Static force analysis

Ex: In hydraulic lifting cranes, the magnitude of inertia force due to weight of the hoisting hook is small compared to the externally applied loads.

Dynamic force Analysis

When the inertia effect due to the mass of the components is also considered in addition to the externally applied load is called dynamic force analysis.

Ex: In high speed IC engine

Applied and Constraint forces:

Applied forces:

The external forces acting on a system of body from outside the system are called applied forces.

Ex: Electric, magnetic and gravitational forces are example of forces that can be applied without actual physical contact.

Force due to friction, force due to external load, spring force, impact force etc. are example of force that can be applied through direct physical or mechanical contact.

Classification of applied force:

(i) Active force

(ii) Reactive force.

(i) Active force: The force exerted by one body on another body is called active force.

(ii) Reactive force: When one body exerts force on another body, then the opposite force exerted by the second body on the first is called reactive force.

Constraint force:

When two or more bodies are connected together to form a group or system, the pair of action and reaction forces between any two of the connected bodies are called constraint forces.

In other words, constraint forces are the forces existing internally within the body

Static equilibrium:

- A body or group of bodies is said to be in equilibrium if all the forces exerted on the system are in balance

- A body is in static equilibrium if it remains in its state of rest or motion.

→ If the body is at rest, it tends to remain at rest and if it is in motion, it tends to keep the motion.

Condition for static equilibrium: (Equation of equilibrium)

1. The vector sum of all the external forces acting upon it is zero

$\Sigma F = 0$ → force law of equilibrium.

2. The vector sum of the moments of all forces acting about any arbitrary axis zero

$\Sigma M = 0$ → momentum law of equilibrium

These two condition are expressed as

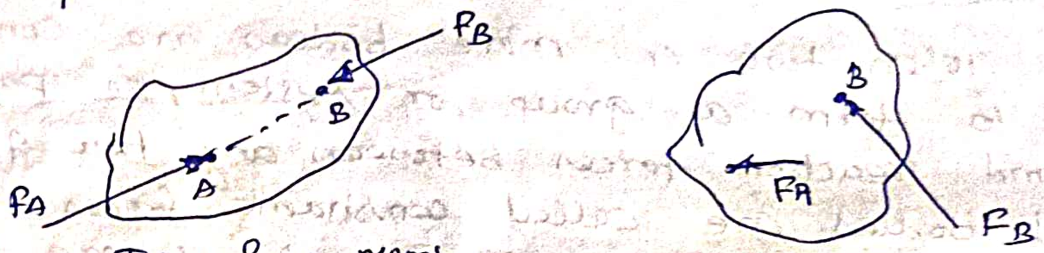
$$\sum F = 0 \quad \sum M = 0$$

The above expression is also known as equation of equilibrium.

$$\boxed{\sum F_x = 0 \quad \sum F_y = 0 \text{ and } \sum M = 0}$$

Static equilibrium of Various Members:

1. Equilibrium of a two-force memeber: which is in equilibrium



Two force member is in equilibrium

$F_A \times F_B$
Two force member not in equilibrium.

Equation of equilibrium:

$$\sum F = F_A + F_B = 0$$

F_A and F_B must have the same line of action

$$\sum M = 0$$

The member under the action of two forces will be in equilibrium

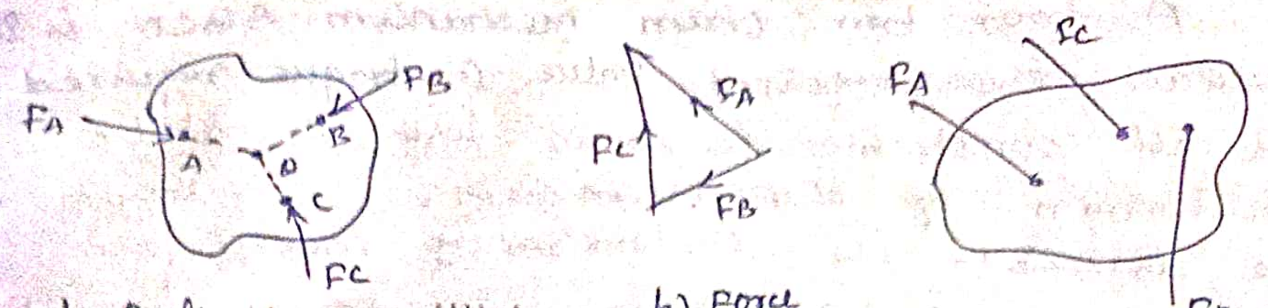
(i) The forces are in same magnitude.
(ii) The forces act along the same line
(iii) The forces are in opposite direction

Equilibrium of a Three force Member:

A body or member will be in equilibrium under the action of three forces only

(i) The resultant of the forces is zero (Force polygon should close)

(ii) the line of action of the force intersect at a point (Point of concurrency)



a) Body in equilibrium   b) Force polygon   c) Body not in equilibrium.

Equilibrium of member with two force and Torque:

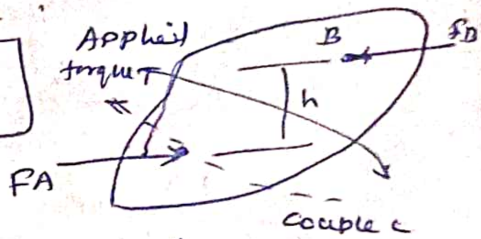
The member acted upon by the two equal force F_1 and F_2 and an applied torque T

The member under the action of two forces and an applied torque will be in equilibrium.

(i) The force are equal in magnitude, parallel in direction and opposite in sense.

(ii) The force form a couple c which is equal and opposite to the applied force.

$$\boxed{T = C = F_A \times h = F_B \times h}$$



$F_A = F_B$

T (clockwise) = c (anticlockwise)

$$\boxed{C = F_A \times h = F_B \times h}$$

Free body diagram:

A free body diagram is a sketch or drawing of the body, isolated from the rest of the machine and its surroundings, upon which the forces and moments are shown in action. In otherword a diagram which shows the forces and moments on the body free of other bodies is called free body diagram (FBD)

Static force analysis of simple Planar mechanism:

The three method used for static force analysis of mechanism

1. principle of superposition
2. principle of virtual work
3. Method of normal and radial Component.

Ex: A four bar chain mechanism ABCD is shown. Calculate the required value of torque required (T) and all constraint forces on link for static equilibrium of the mechanism. If F = 2000N in the direction. The dimensions of linkage are,

AB = 200mm BC = 370mm CD = 250 mm AD = 215mm and CE = 100mm.

Data:

F = 2000N AB = 200mm BC = 370mm CD = 250mm

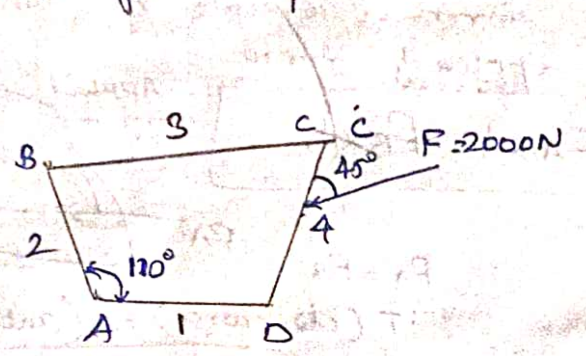
AD = 215mm CE = 100mm.

1cm = 100mm

1cm = 500N

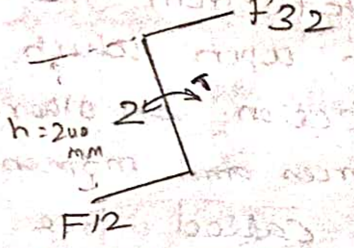
To find:

Step 1 Forces acting on various Driving Torque T.



Geomentrical construction

Step 2 Draw FBD of a link:



a) FBD of link 2

Step 3:

From force polygon by measurement.

F_{34} = scale ab x scale

= 1.9 x 500 = 950N

F_{34} = side box scale

= 2.4 x 500

= 1200N.

From link 3 $F_{43} = F_{23}$

$F_{43} = F_{23}$ = 950N

From link 2

$F_{12} = F_{32}$

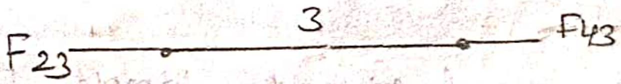
= 950

h = 200mm

T = F_{12} x 0.2 = 950 x 0.2

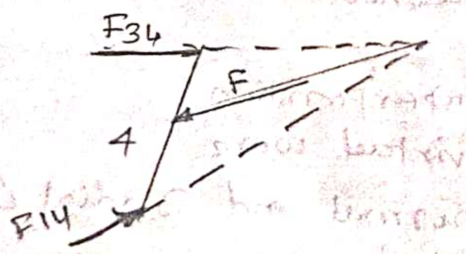
= 190 N.m

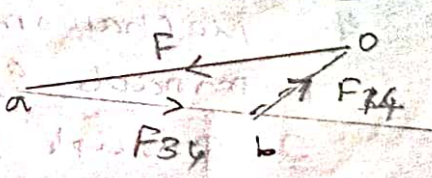
(Counter clockwise)



b) FBD of link 3

Force polygon of link 4





c) FBD of link 4

Pb: A slider - crank mechanism. The value of force applied on slider 4 is 2000N. Determine the force acting on various links and also calculate the driving torque T. The linkage dimension AB = 100mm ; BC = 300mm

∠BAC = 60°

data:

F = 3000N ; AB = 100mm BC = 300mm ∠BAC = 60°

Find:

→ Forces acting on various link

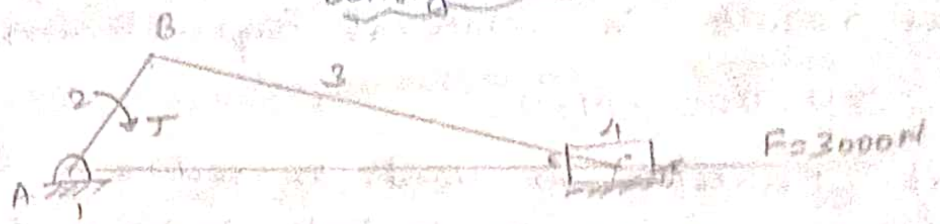
→ Driving torque T

1cm = 50 mm

1 cm = 500N.

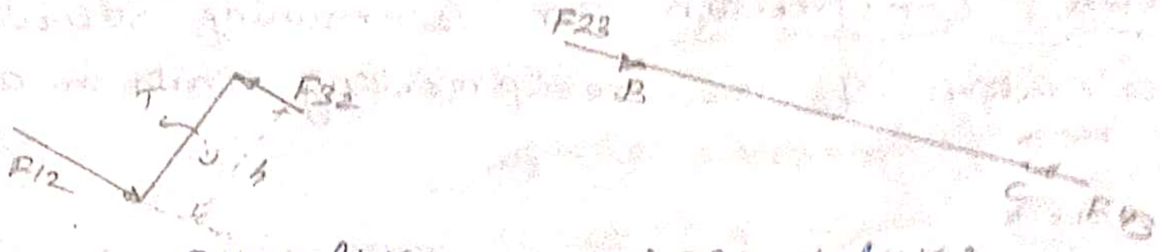
Soln:

Step 1 Configuration diagram:



Geomentrical construction:

Step 2 Draw FBD of a link.

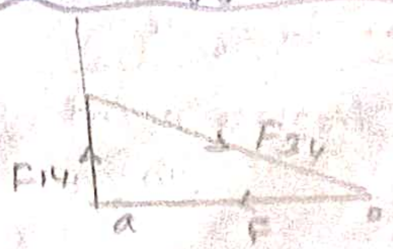


b) FBD of link 2

c) FBD of link 3

d) FBD of link 4:

e) Force polygon of slider



From force polygon

F_{14} = ab × Scale = 1.7 × 500 = 850N

F_{34} = ob × Scale = 6.4 × 500 = 3200N

From link 3 $F_{43} = F_{23} = F_{34}$ = 3200 N

link 2 $F_{32} = F_{12}$ = 3200 N

$\boxed{T = F_{32} \times h = 3200 \times 0.1 = 320 \text{ N-m (ccw)}}$

Inertia force: is a fictitious force, which when acts upon a rigid body, bring it in

$$\text{Inertia force} = -\begin{rmatrix}\text{Acceleration} \\ \text{force}\end{rmatrix} = -ma$$

Inertia torque is a fictitious torque, when applied upon the rigid body, brings it in equilibrium position

$$\text{Inertia torque} = -\begin{matrix}\text{Externally} \\ \text{applied torque}\end{matrix} = -I\alpha$$

D'Alembert's principle:

States that the inertia force and torque and the external force and torque acting on a body together result in statical equilibrium

$$\Sigma F = 0 \quad \Sigma M = 0$$

D'alembert principle is used to reduce a dynamic analysis problem into an equivalent problem of static equilibrium.

Klien's Construction For determining velocity and acceleration of the reciprocating parts in engine.

~~is abs~~

Formula

→ velocity and acceleration of the reciprocating parts in engine.

a) Displacement of the piston $x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right]$

b) velocity of piston $V_p = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$

c) Acceleration of piston $a_p = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

4) Angular Velocity of connecting rod.

$$\omega_{PC} = \frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}}$$

$$= \frac{\omega \cos\theta}{n}$$

v) Angular acceleration of connecting rod

$$\alpha_{PC} = \frac{-\omega^2 \sin\theta (n^2-1)}{(n^2 - \sin^2\theta)^{3/2}}$$

$$= \frac{-\omega^2 \sin\theta}{n}$$

→ Forces on the reciprocating parts of an engine:

a) ~~piston~~ piston effort (F_P);

1) horizontal reciprocating engine:

piston effort, $F_P = F_L \pm F_I$ — neg frictional resistance

$= F_L \pm F_I - R_F$ — considering frictional resistance)

2) vertical reciprocating engine;

piston effort $F_P = F_L \pm F_I \mp W_R$ neg frictional resistance

$= F_L \pm F_I \mp W_R - R_F$ considering frictional resistance)

3) To find net load on the piston (F_I)

a) single-acting engine $F_L = P \times \pi/4 \, D^2$

b) Double acting engine $F_L = P_1 A_1 - P_2 A_2 = P_1 A_1 - P_2(A_1 - a)$

$P_1 A_1$ = pressure and cross sectional area on the back end side of piston

$P_2 A_2$ = " " " on the crank end side of piston.

a = cross-sectional area of the piston rod.

(iii) Inertia force of the reciprocating parts (F_I)

$$F_I = m_R a_R = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

2. Force acting along the connecting rod (F_Q)

$$F_Q = \frac{F_P}{\cos\phi}$$

3. Thrust on the sides of cylinder wall (F_N)

$$F_N = F_Q \sin\phi = F_P \tan\phi$$

4. Crank-pin effort (F_T)

$$F_T = F_Q \sin(\theta+\phi) = \frac{F_P}{\cos\phi} \times \sin(\theta+\phi)$$

5. Thrust on crankshaft bearing (F_B)

$$F_B = F_Q \cos(\theta+\phi) = \frac{F_P}{\cos\phi} \times \cos(\theta+\phi)$$

6. Crank effort on crankshaft (T)

$$T = F_T \times r = \left[\frac{F_P}{\cos\theta} \sin(\theta+\phi)\right] \times r = F_P \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] \times r$$

Steps in Drawing Free body diagram:

1. A diagram of the body completely isolated from all other bodies is drawn. The free body may consist of entire system or any portion of the system.

2. All the supports (like wall, floor or any other body) are removed and replaced them by the reaction.

3. The acting force and reaction at the each body are represented in diagram with their direction and magnitude

4. The known applied load by its magnitude and directions and the unknown applied load by a symbol.

5. Appropriate dimension which are needed in defining the configuration of the force system.

6. The wt force of the free body is indicated with vertical downward arrow and is shown like an applied load

7. The fixed link has no FBD

8. If the direction of torque as input is not gn

   1) Member with two force

   2) member with three force

9. If the direction of torque on the input link

   1) member with force and a torque

   2) member with two forces.

Pb: The length of crank and connecting rod of a horizontal engine are 200 mm and 1m respectively. The crank is rotating at 400 rpm, when the crank has turned through 30° from the inner dead centre. The difference of pressure between cover and piston rod is 0.4 N/mm². If the mass of the reciprocating parts is 100kg and cylinder bore is 0.4m then calculate
(i) Inertia force 2) force on piston 3) piston effort
4) Thrust on the side of the cylinder walls
5) Thrust in the connecting rod & (Vi) Crank effort.

data: $r = 200\,mm = 0.2\,m$; $l = 1\,m$; $N = 400\,rpm$ $\theta = 30°$
$P_1 - P_2 = 0.4\,N/mm^2 = 0.4 \times 10^6\,N/m^2$; $M_R = 100\,kg$ $D = 0.4\,m$

Soln:

$$\omega = \frac{2\pi N}{60} = 2\pi(400)/60 = 41.89\ rad/s$$

$$n = l/r = 1/0.2 = 5$$

(i) Inertia force (F_i)

$$F_i = M_R\,\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 100\,(41.89)^2\; 0.2 \left[\cos 30^\circ + \frac{\cos 2\times 30^\circ}{5}\right)$$

$$F_I = 33.903\ kN$$

(ii) Net load on the piston (F_L)

$$F_L = (P_1 - P_2)A = (P_1 - P_2)\,\pi/4\, D^2$$

$$= (6.4\times 10^6)\,\pi/4\,(0.4)^2 = 50.26\ kN$$

(iii) piston effort $F_P = F_L - F_I$

$$= 50.265\times 10^3 - 33.903\times 10^3$$

$$\boxed{F_P = 16.36\ kN}$$

(iv) Thrust on the side of the cylinder wall (F_N)

$$F_N = F_P \tan\phi$$

To find ϕ

$$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 30}{5} = 0.1 \text{ or } \phi = 5.74^\circ$$

$$F_N = F_P \tan\phi = (16.36\times 10^3)\tan 5.74^\circ$$

$$= 1.644\ kN$$

v) Thrust in the connecting rod F_Q

$$F_Q = \frac{F_P}{\cos\phi} = \frac{16.36\times 10^3}{\cos 5.74^\circ} = 16.44\ kN$$

vi) Crank effort (T)

Tangential force on the connecting rod

$$F_T = F_Q \sin(\theta+\phi)$$

$$= 16.44\times 10^3 \sin(30^\circ + 5.74^\circ) = 9.60\ kN$$

Turning moment on the crank shaft.

$$T = F_T \times r$$

$$= (9.605\times 10^3)\, 0.2$$

$$= 1921.13\ N\text{-}m$$

Pb: A horizontal steam engine running at 210 rpm has a bore of 190mm and stroke of 350mm. The piston rod is 20mm in dia and connecting rod length is equivalent to a force of 350 N

Find the following when the crank is at 115° from the inner dead centre, the mean pressure being 4500 N/m² on the cover side and 1000 N/m² on the crank side.

1. Thrust on the connecting rod
2. Thrust on the cylinder wall.
3. Load on the bearings
4. Turning moment on the crankshaft.

data:

$N = 210\,rpm \quad D = 190mm = 0.19m \quad L = 350mm = 0.35m$

$r = 0.35/2 = 0.175m \quad d = 20mm = 0.02m \quad l = 950mm = 0.95m$

$M_R = 8kg \quad R_F = 350N \quad \theta = 115° \quad P_1 = 500 N/m^2 \quad P_2 = 100 N/m^2$

Soln:

$$\omega = 2\pi N/60 = 2\pi(210)/60 = 21.99\,rad/s.$$

$$n = l/r = 0.95/0.175 = 5.43$$

piston effort $F_P = F_L - F_I - R_F$

To find net load on the piston (F_L)

$$F_L = P_1 A_1 - P_2 A_2 = P_1 A_1 - P_2(A_1 - a)$$

$$A_1 = \pi/4 (D^2) = \pi/4 (0.19)^2 = 0.0283\,m^2$$

$$A_2 = A_1 - a = 0.0283 - (3.14 \times 10^{-4}) = 0.028\,m^2$$

$$a = \pi/4\, d^2 = \pi/4 (20 \times 10^{-3})^2 = 3.14 \times 10^{-4}\,m^2$$

$$F_L = (4500 \times 0.02835) - (100 \times 0.028)$$

$$\boxed{F_L = 124.77\,N}$$

To find inertia force (F_I)

$$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 8(21.99)^2 \times 0.175 \left[\cos 115^\circ + \frac{\cos 2(115^\circ)}{5.43}\right]$$

$$= -366.24\ N$$

piston effort

$$F_P = F_L - F_I - R_F = 124.77 - (-366.24) - 35\ldots$$

$$\boxed{F_P = 141.01\ N}$$

1. Thrust on the connecting rod (F_Q)

$$F_Q = \frac{F_P}{\cos\phi}$$

$$\sin\phi = \frac{\sin\theta}{\phi} = \frac{\sin 115^\circ}{5.43} = 0.1669 \text{ or } \phi = 9.61^\circ$$

$$F_Q = \frac{F_P}{\cos\phi} = \frac{141.01}{\cos 9.61^\circ} = 143.02\ N$$

2. Thrust on the cylinder wall (F_N)

$$F_N = F_P \tan\phi = 141.01 \times \tan 9.61^\circ$$

$$F_N = 23.87\ N$$

3. Load on the bearing (F_B)

$$F_B = F_Q \cos(\theta + \phi)$$

$$= 143.02 \cos(115^\circ + 9.61^\circ) = -81.23\ N$$

4. Turning moment on the crank shaft (T)

$$T = F_T \cdot r = \left[\frac{F_P}{\cos\phi} \sin(\theta + \phi)\right] r$$

$$= \frac{141.01}{\cos 9.61^\circ} \sin(115^\circ + 9.61^\circ] \, 0.175$$

$$\boxed{T = 20.6\ N\text{-}m}$$

The crank pin circle radius of a horizontal engine is 300 mm. The mass of the reciprocation parts is 250 kg. When the crank has travelled 60° from IDC, the difference between the driving and the back pressure is 0.35 N/mm². The connecting rod length between centres is 1.2 m and the cylinder bore is 0.5 m. If the engine runs at 250 rpm and if the effect of piston rod diameter is neglected calculate (i) pressure on slide bar (ii) Thrust in the connecting rod. (iii) tangential force on the crank pin and (iv) Turning moment of the crank shaft.

data:

$r = 300 mm = 0.3 m$ $\qquad (P_1 - P_2) = 0.35 N/mm^2$

$l = 1.2 m$

$M_R = 250 kg$ $\qquad D = 0.5 m = 500 mm$

$\theta = 60°$ $\qquad N = 250$ rpm

Soln:

$$\omega = \frac{2\pi N}{60} = 26.2 \text{ rad/s}.$$

$$F_L = (P_1 - P_2) \times A$$

$$= 0.35 \times \pi/4 \times (500)^2$$

$$\boxed{F_L = 68730 N} \rightarrow (1)$$

W.K.T, $\quad n = l/r = \frac{1.2}{0.3} = 4$

$$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 250 \times 26.2^2 \times 0.3 \left[\cos 60 + \frac{\cos 2\times 60}{4}\right]$$

$$\boxed{F_I = 19306 N} \rightarrow (2)$$

piston effort $F_P = F_L - F_I$

$= 68730 - 19306$

$\boxed{F_P = 49424 N}$

Now

1. pressure on slide bars

$F_N = F_P \tan\phi$ $\quad \sin\phi = \frac{\sin\theta}{n} = \frac{\sin 60}{4}$

$F_N = 49424 \times \tan 12.5°$ $\quad \phi = \boxed{12.5°}$

$\boxed{F_N = 10.96\ kN}$

2. Now force in connecting rod.

$F_Q = \frac{F_P}{\cos\phi}$

$= \frac{49424}{\cos 12.5}$

$\boxed{F_Q = 50.62 N}$

3. Tangential force on crank pin

$F_T = F_Q \sin(\theta + \phi)$

~~turning~~ $= 50.62 \sin(60 + 12.5°)$

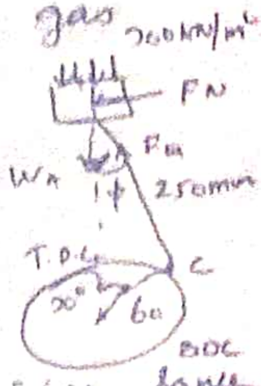
$= 48.28\ kN$

4. Turning moment

$T = F_T \times r = 48.28 \times 0.3$

$\boxed{T = 14.484\ kN.m}$

Pb: A single vertical petrol engine 100mm dia and 120mm stroke has a connecting rod 250mm long. The mass of the piston is 1.1 kg. The speed is 2000 rpm. On the expansion stroke with a crank 20° from top dead centre, the gas pressure is 700 kN/m² Det.



1. Net force on the piston
2. Resultant load on the gudgeon pin
3. Thrust on the cylinder walls and
4. Speed above which other thing remaining same

data

$D = 0.1m$ $\theta = 20°$

$L = 0.12m$ $P = 700 kN/m^2$

$l = 0.25m$ $N = 2000 rpm$.

$r = L/2 = 0.06 m$

$m_R = 1.1 kg$

Soln:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.5 rad/s$$

$$F_L = P \times \pi/4 D^2 = 700 \times \pi/4 (0.1)^2 = 5.5 kN$$

$$n = l/r = \frac{0.25}{0.06} = 4.17$$

Now

$$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 1.1 \times (209.5)^2 \times 0.06 \left[\cos 20° + \frac{\cos 2 \times 20}{4.17}\right]$$

$$\boxed{F_I = 3.254 N}$$

[illegible] $F_P = F_L - F_I + W_R$

$= 5500 - 3284 + 1.1 \times 9.81$

$\boxed{F_P = 2256.8\ N}$

Now

F_Q = Resultant load on gudgeon pin $\Big\} = \dfrac{F_P}{\cos\phi}$

$\sin\phi = \dfrac{\sin\theta}{n} = \dfrac{\sin 20^\circ}{4.17}$

$\phi = 4.7^\circ$

$= \dfrac{2256.8}{\cos 4.7^\circ}$

$\boxed{F_Q = 2265 N}$

Now Thrust force on the cylinder wall

$F_N = F_P \tan\phi = 2256.8 \times \tan 4.7$

$\boxed{F_N = 185.5 N}$

Now If the F_Q is in reversed direction i.e. it changes its sign the F_P also becomes -ve

$F_P < 0$

$F_L - F_I + W_R < 0$

$F_I > F_L + W_R$

$\cancel{F_I \geq F_L + W_1}$

Let N_1 is speed of engine above which F_Q changes its sign

$\omega_1 = \dfrac{2\pi N_1}{60}$

$M_R \omega_1^2 r \left(\cos\theta + \dfrac{\cos 2\theta}{n}\right) > 5500 + 1.1 \times 9.81$

$\omega_1^2 \times 0.66 \left[\cos 20 + \dfrac{\cos 2\times 20}{4.17}\right] > 5500 + 1.1 \times 9.81$

$\omega_1 = 273$ rad/s

$\boxed{N_1 = 2606 \text{ rpm}}$

UNIT-II

Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

When a particle or mass moving in a circular path, it experiences a centripetal force acting radially inward.

An equal and opposite force acting radially outwards on the axis of rotation and is known as centrifugal force.

This is a disturbing force, and its magnitude remains constant but the direction changes with the rotation of the mass

Centrifugal disturbing force, $F_c = m\omega^2 r$

m = mass of rotating component in kg

ω = Angular velocity $= 2\pi N/60$

N = Speed of the component

r = Distance of C.G of mass from the axis of rotation in metres.

Types of balancing:

1. Balancing of rotating masses.
2. Balancing of reciprocating masses.

Balancing of single rotating mass.

Consider a mass of m attached to shaft rotating at ω rad/s

let r be the radius of rotation of the m

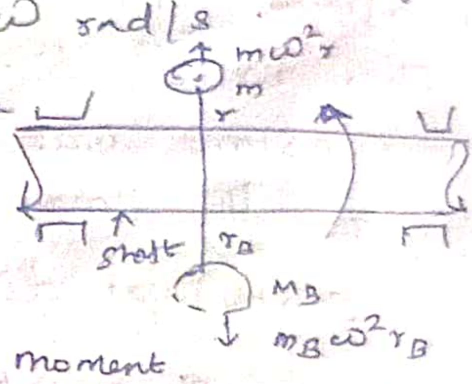
we know that the centrifugal force $F_c = m\omega^2 r$ producing out of balance effect acts radially outwards on the shaft. This out-of balance force can be balanced in any one of the following two ways.

1) single Revolving of mass in the same plane.

The disturbing mass m is balanced by introducing a counter mass or balancing mass M_B at radius of rotation r_B diametrically opposite to m in the same plane, rotating with same angular velocity ω rad/s.

$F_{c1} = m\omega^2 r \rightarrow$ Disturbing force

$F_{c2} = M_B \omega^2 r_B \rightarrow$ Balancing force



$F_{c1} = F_{c2}$ (or) $m\omega^2 r = M_B \omega^2 r_B$

$$\boxed{mr = M_B r_B}$$ — mass moment

r_B may be kept larger to reduce the value of balancing mass M_B.

2) Two revolving masses in different planes.

If the balancing mass and disturbing mass lie in different planes, disturbing mass cannot be balanced by a single mass are there will be a couple left unbalanced.

Two balancing masses are required for completed balancing and three masses are arranged in such that resultant force and couple on the shaft are zero.

$F_c = m\omega^2 r$

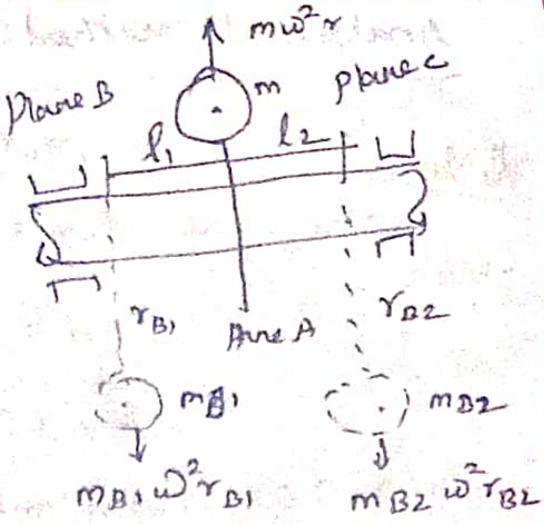
$F_{c1} = m_{B1}\,\omega^2 r_{B1}$

$F_{c2} = m_{B2}\,\omega^2 r_{B2}$

$F_c = F_{c1} + F_{c2}$ or $m\omega^2 r$

$= m_{B1}\,\omega^2 r_{B1} + m_{B2}\,\omega^2 r_{B2}$

$$\boxed{m r = m_{B1} r_{B1} + m_{B2} r_{B2}}$$



Taking moment

$F_{c1}\, l = F_c\, l_2$ (or) $m_B\,\omega^2 r_{B1}) = (m\omega^2 r) l_2$ or

$m_{B1} r_{B1} l = m r l_2$

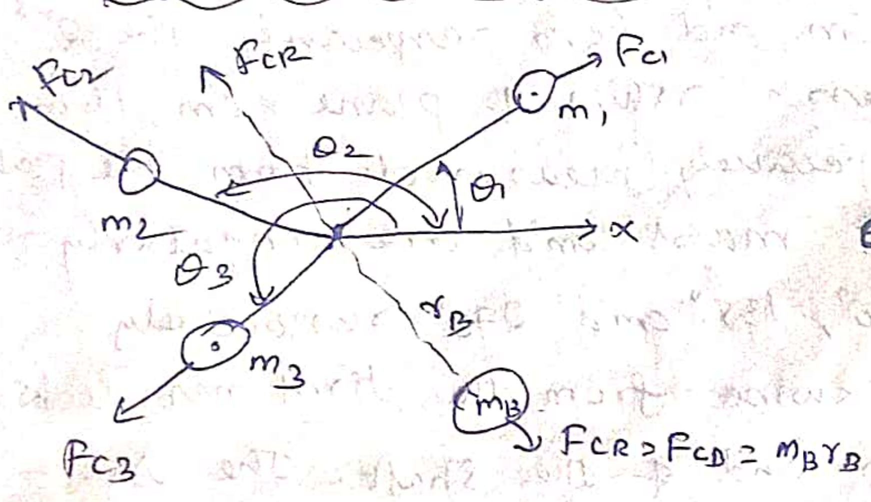
or $\quad m_{B1} \cdot r_{B1} = m r \dfrac{l_2}{l}$

$= m r \left(\dfrac{l_2}{l_1 + l_2}\right) \quad l_1 = l_1 + l_2$

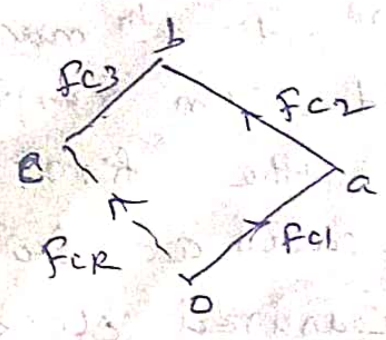
Balancing of several masses of rotating in the same plane

In several masses are rigidly attached to a shaft at different radii in one plane perpendicular to the shaft and the shaft is made to rotate, each mass will set up out of balance force on the shaft. In this case, complete balance can be obtained by placing only one balancing mass in same plane.

Angular position of planes



Force polygon (vector diagram)



Analytical method:

Step 1: $F_{C1} = m_1 r_1$ $F_{C2} = m_2 r_2$ $F_{C3} = m_3 r_3$,

Resolve
Centrifugal force horizontal & vertical ΣF_H & ΣF_V

Step 2:
$$\Sigma F_H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3$$
$$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$$

Step 3: Resultant $F_{CR} = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$

Step 4: $F_{CR} = m_B r_B$

Step 5 = $\tan\theta_R = \dfrac{\Sigma F_V}{\Sigma F_H}$ $\theta_R = \tan^{-1}\left(\dfrac{\Sigma F_V}{\Sigma F_H}\right)$

$$\boxed{\theta_B = \theta_R + 180^\circ}$$

Graphical method:

Step 1: Draw the space diagram.

Step 2: Find out the centrifugal force

Step 3: Draw the force polygon

Step 4: According to polygon law of force the closing side vector, $m_B r_B$ = Resultant force = vector

Step 5 → position of balancing mass in space diagram.

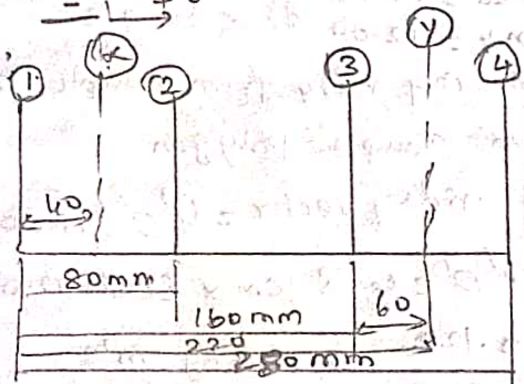
Pb: A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5 cm, 6 cm, 7 cm and 6 cm respectively. The 2nd, 3rd and 4th masses rotates of plane 8 cm, 16 cm and 28 cm respectively measured from the plane of the first mass and are angularly located at 60°, 135° and 270° respectively measured clockwise from the first mass looking from this mass end of the shaft. The shaft is dynamically balanced by two masses, both located at 5cm radii and revolving in planes mid way between those of 1st and 2nd masses and mid-way between those of 3rd and 4th masses. Det graphically or otherwise the magnitude of the masses and their respective angular position.

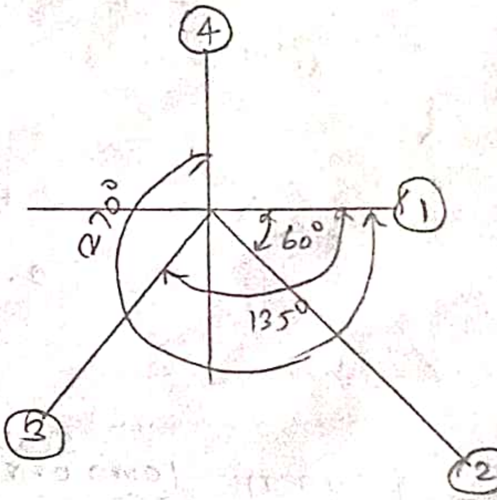
<u>data:</u> $m_1 = 18kg$ $r_1 = 5cm$ $\theta_1 = 0°$ (Assuming m_1 lies horizontal)
$m_2 = 14kg$, $r_2 = 6cm$ $\theta_2 = 60°$, $m_3 = 16kg$
$r_3 = 7cm$, $m_4 = 12kg$ $r_4 = 6cm$ $\theta_3 = 135°$, $\theta_4 = 270°$

<u>Find:</u>

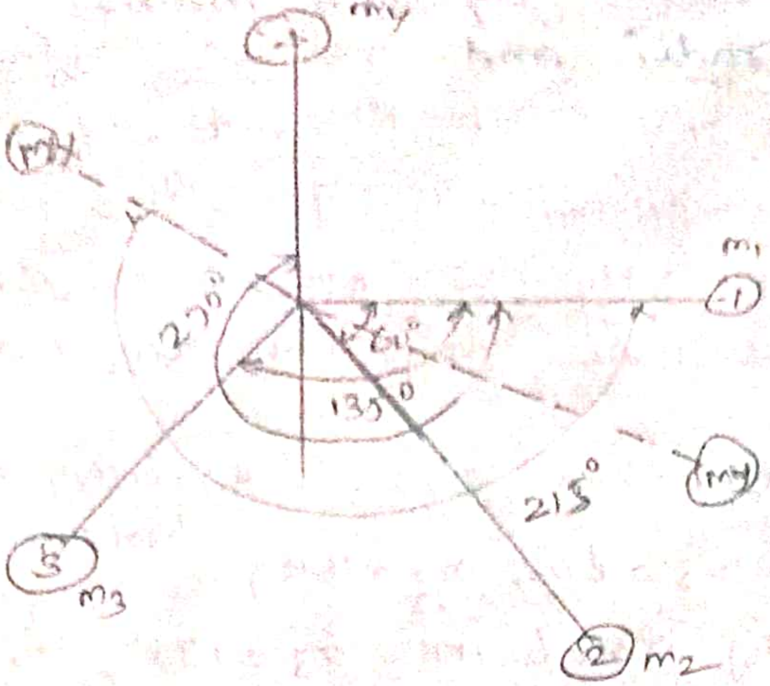
1. magnitude of two balancing m_x & m_y
2. Angular position of both balancing θ_x and θ_y

Soln:





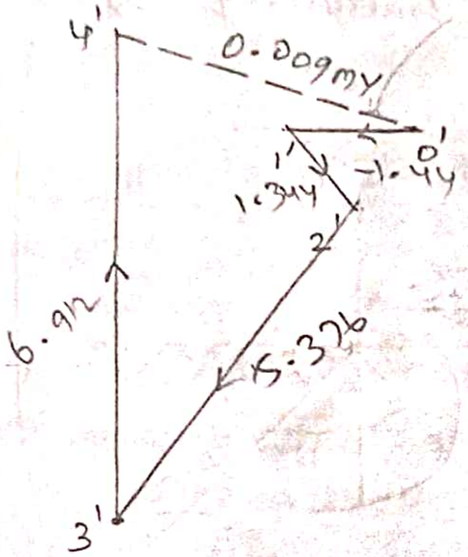
Plan	mass m (kg)	Radius r (m)	Force (Fc) $\frac{m.r}{\omega^2}$ kg.m	Distance from RP l (m)	Couple $m.r.l \div \omega^2$ kg.m²	Angle degree
1.	18	0.05	0.9	−0.04	−0.036	0°
x (RP)	m_x	0.05	0.05 m_x	0	0	θ_x
2.	14	0.06	0.84	0.04	0.0336	60°
3.	16	0.07	1.12	0.12	0.1344	135°
M y.	m_y	0.05	0.05 m_y	0.18	0.009 m_y	θ_y
4	12	0.06	0.72	0.24	0.1728	270°



1 cm = 0.01

Couple polygon Scale (1 cm = 0.025 kg.m²) = $\frac{-0.036}{Scale (0.025)}$

−0.036 = −3.6 ① = −1.44
0.0336 = 3.36
0.1344 = 13.44 ② = 1.344
0.1728 = 17.28 ③ = 5.376
0.009my = 12.00 ④ = 6.912



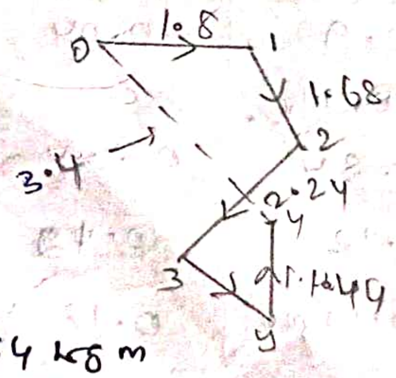
To find my & θy from couple polygon
From couple polygon
Closing side, vector = 4'0' = 0.009 my
vector 4'0' = 4.8 cm × Scale 0.025
0.09 my = 0.120 = 0.120 kg.m²
0.120 = 0.009my

$\boxed{my = 13.33 \text{ kg}}$

Force polygon Scale 1 cm = 0.5 kg.m

By measurement, angular position
θy = 25° measured by clockwise from m1

1 cm = 0.1
0.9 = 9
0.84 = 8.4
1.12 = 11.2
0.72 = 7.2 cm
0.05my = 6.65
= 0.665



1 = 1.8
2 = 1.68
3 = 2.24
4 = 1.44

0.05 mx = 5.4
0.05mx = 0.54 kg m

$\boxed{mx = 34 \text{ kg}}$

To find m_x and θ_x from force polygon

Closing side / vector $4'0' = 3.4 \times 0.5 = 1.7$ kg-m

$1.7 = 0.05\, m_x$

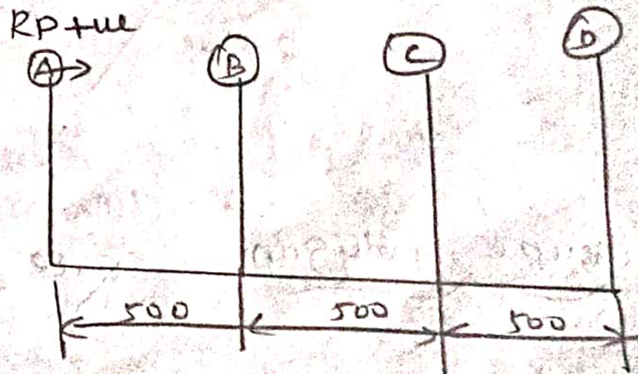
$\boxed{m_x = 34 \text{ kg}}$

By measurement, angular polygon of m_x is found

$\boxed{\theta_x = 215^\circ \text{ measured from } m_1 \text{ in cw}}$

Pb) A, B, C and D are four masses carried by a rotating shaft at radii 100mm, 150mm, 150mm and 200mm respectively. The planes in which the masses rotates are spaced at 500mm apart. and the magnitude of the masses B, C, and D are 9kg, 5 kg and 4 kg respectively. find the required mass A and the relative angular setting of the four masses so that the shaft must be complete balance.

data:



$r_A = 100\text{mm} = 0.1\text{m}$

$r_B = 150 = 0.15\text{m}$

$r_C = 150\text{mm} = 0.15\text{m}$

$r_D = 200\text{mm} = 0.2\text{m}$

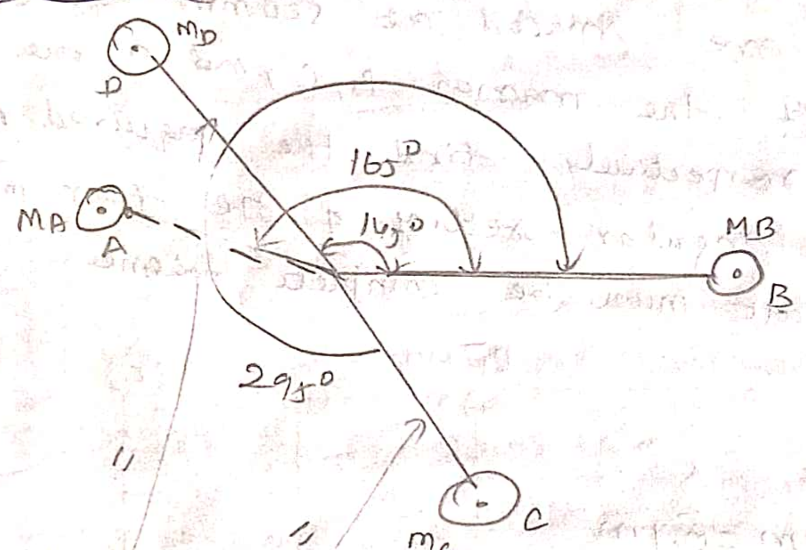
$M_B = 9$ kg

$M_C = 5$ kg

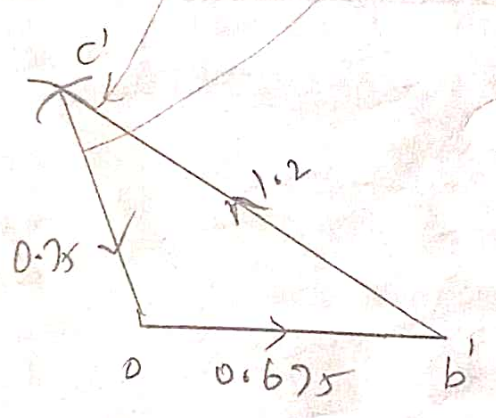
$M_D = 4$ kg.

plane	mass m kg	Radius r m	Centrifugal force m.r kg.m	Distance from R.P (l)	Couple m.r.l kg-m²
A (R.P)	M_A	0.1	0.1 M_A	0	0
B	9	0.15	1.35	0.5	0.675
C	5	0.15	0.75	1	0.75
D	4	0.2	0.8	1.5	1.2

Couple polygon: ~~Scale 1cm = 0.25 kg-m²~~



Couple polygon   Scale 1cm = 0.25 kg-m²

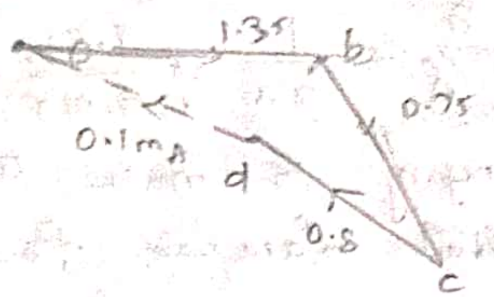


Force polygon → Angular position of planes.

By measurement $1 cm = 0.5 kg\cdot m$

$\theta_C = \angle BOC = 275°$

$\theta_D = \angle BOD = \angle 145°$



$ob = 1.35$
$bc = 0.75$
$cd = 0.8$

By measurement from force polygon.

Closing side $= dO = 1.0125 = 0.1 m_A$

$m_A = 10.12 kg$

By measurement

$$\boxed{\theta_A = \angle BOA = 165°}$$

Vibration.

1. A body of mass 20kg is suspended from a spring which defects 15mm under this load calculate the frequency of free vibration and verify that a viscous damping force amounting to approximately 1000N at a speed of 1m/s is just sufficient to make the motion a periodic

If when damped to this extent the body is subjected to a distibuting force with a max value of 125N making 8 cycles/s find the amplitude of ultimate motion.

Sol: $m = 20\,kg$ $\delta = 15\,mm = 0.015\,m$ $C = 1000\,N/m/s$
$F = 125\,N$ $f = 8$ cycle/sec.

1. Frequency of free vibration.

$$f_n = \frac{1}{2\pi}\sqrt{g/\delta} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.015}} = 4.07\,Hz$$

The critical damping to make the motion aperiodic is such that damped frequency is zero

$$c = \sqrt{\frac{s}{m} \times 4m^2} \qquad \left(\frac{c}{2m}\right)^2 = \frac{s}{m}$$

$$= \sqrt{4 \cdot s \cdot m}$$

$$= \sqrt{4 \cdot \frac{mg}{\delta} \times m}$$

$$= \sqrt{4 \times \frac{20 \times 9.81}{0.015} \times 20} = 1023\,N/m/s$$

2. Amplitude of ultimate motion.

Angular speed of forced vibration:

$$\omega = 2\pi f = 2\pi \times 8 = 50.3\,rad/s$$

$$s = \frac{mg}{\delta} = \frac{20 \times 9.81}{0.015} = 13.1 \times 10^3\,N/m$$

max amplitude of forced vibration,

$$x_{max} = \frac{F}{\sqrt{c^2\omega^2 + (s - m\omega^2)^2}}$$

$$= \frac{125}{\sqrt{1023^2 \times 50.3^2 + \left(13.1\times10^3 - 20(50.3)^2\right)^2}}$$

$$= \frac{125}{63.7\times10^3} = 1.96\times10^{-3}\, m$$

$$= 1.96\, mm$$

Pb: A vibrating system consist of a mass 5 kg spring of stiffness 3.5 N/mm and a dashpot of damping co-efficient of 100 N/m/s. Find
a) critical damping co-efficient
b) the damping factor
c) The natural frequency of damped vibration.
d) the logarithmic decrement.
e) The ratio of two consecutive amplitude and
f) the number of cycle after which the original amplitude is reduced to 20 percent.

data: $m = 5\,kg$ $S = 3.5\,N/mm = 3.5\times10^3\,N/m$ $c = 100\,N/m/s$

a) Critical damping co-efficient (c_c)

$$C_c = 2m\omega_n = 2m\sqrt{S/m} = 2\sqrt{S.m}$$

$$= 2\sqrt{3.5\times10^3\times5} = 264.57\,N/m/s$$

b) Damping factor (ξ_2) $= \frac{c}{c_c} = \frac{100}{264.57} = 0.378$

c) Natural frequency of damped vibration (f_d)

$$\omega_d = \sqrt{1-\xi_2^2}\cdot\omega_n$$

$$\omega_n = \sqrt{S/m} = \sqrt{\frac{3.5\times10^3}{5}} = 26.46\, rad/s$$

$$\omega_d = \sqrt{1-(0.378)^2}\times 26.46 = 24.49\, rad/s$$

∴ Natural frequency of damped vibration

$$f_d = \frac{\omega_d}{2\pi} = \frac{24.49}{2\pi} = 3.9\ Hz$$

d) Logarithmic decreament (δ)

$$\delta = \frac{2\pi \zeta_2}{\sqrt{1-\zeta_2^2}} = \frac{2\pi \times (0.378)}{\sqrt{1-(0.378)^2}} = 2.565$$

e) Ratio of two consecutive amplitude

$\left(\frac{x_n}{x_{n+1}}\right)$ x_n and x_{n+1}

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) \text{ or } \frac{x_n}{x_{n+1}} = e^{\delta}$$

$$= e^{2.565} = 13$$

f) No of cycle after which the amplitude is reduced to 20% (n)

$$x_n = 20\%\ x_0 = 0.2x_0$$

$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right)$$

$$2.565 = \frac{1}{n}\ln\left(\frac{x_0}{0.2x_0}\right)$$

$$\boxed{n = 0.629 \text{ cycles}}$$

Pb: A single degree damped vibrating system the suspended mass of 3.75 kg makes 12 oscillations in 7 seconds when distracted from its equilibrium position. The amplitude decrease of 0.33 of the initial value after 4 oscillations. Det (i) stiffness of spring (ii) The logarithmic decrement iii) the damping factor iv) damping coefficient.

data: $m = 3.75\,kg$ $N = 12$ $t = 7s$ $X_4 = 0.33 X_0$

Sdn: 12 oscillations are made in 7 seconds.

$$f_n = \frac{12}{7} = 1.7143\ Hz$$

Circular natural frequency.

$$\omega_n = 2\pi f_n = 2\pi(1.7143) = 10.77\ rad/s$$

(i) Stiffness of spring (S)

$$\omega_n = \sqrt{S/m} \quad \text{or} \quad (10.77) = \sqrt{\frac{S}{3.75}}$$

$$S = 435.07\ N/m$$

(ii) Logarithmic decrement (δ):

X_0 = initial amplitude.

X_4 = final amplitude $= 0.33 X_0$

$$\delta = \frac{1}{n} \ln\left(\frac{X_0}{X_n}\right) = \frac{1}{4} \ln\left(\frac{X_0}{X_4}\right) = \frac{1}{4} \ln\left(\frac{X_0}{0.33X_0}\right)$$

$$\boxed{\delta = 0.277}$$

(iii) Damping factor (ζ_2)

$$\delta = \frac{2\pi \zeta_2}{\sqrt{1-\zeta_0^2}} = 0.277$$

$$2\pi\zeta_2 = 0.277\left(\sqrt{1-\zeta_0^2}\right)$$

$$4\pi^2\zeta^2 = 0.076\left(1-\zeta_0^2\right)$$

$$\zeta_0^2 = 0.044$$

iv) Damping coefficient (c)

$$\zeta_0 = \frac{c}{C_c} = \frac{c}{2m\omega_n}$$

$$0.044 = \frac{c}{2 \times 3.75 \times 10.77}$$

$$\boxed{c = 3.566\ N/m/s}$$

3) The guns are designed so that on firing the barrel recoils against a spring. A dashpot is engaged that allow barrels to returns to its position at the end of each recoil. A gun barrel has a mass of 500 kg and a recoil spring constant of 300 N/mm. The barrel recoils 1m on firing. Det (i) initial velocity of gun barrel and (ii) critical damping coefficient of the dashpot engaged at the end of the recoil stroke.

data: $m = 500\,kg$ $S = 300\,N/mm = 300 \times 10^3\,N/m$

$x = 1\,m$

Soln: (i) Initial velocity (v)

K.E of barrel = workdone on the spring

$$\frac{1}{2} m v^2 = \frac{1}{2} \cdot S \cdot x^2$$

$$\frac{1}{2} \times 500 \times v^2 = \frac{1}{2} (300 \times 10^3) \times (1)^2$$

Initial velocity $(v) = 24.5\,m/s^2$

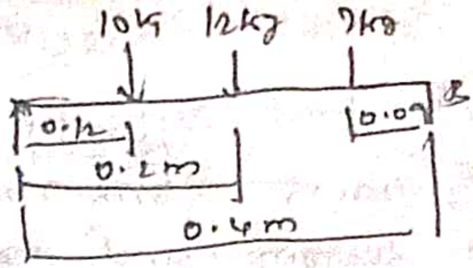
(ii) critical damping coefficient (C_c)

$$C_c = 2m\omega_n = 2m\sqrt{S/m} = 2\sqrt{S \cdot m}$$

$$= 2\sqrt{(300 \times 10^3) \times 500} = 24495\,N/m/s.$$

pb: A steel shaft 100mm in dia is loaded and supported in short bearing 0.4m apart. The shaft carries three load. First of mass 12kg at the centre, second mass of 10kg at a distance 0.12m from the left bearing and third mass of 7 kg at a distance 0.09m from the right bearing. Find the value of critical speed by using Dunkerley's method.

$d = 100\,mm = 0.1\,m \qquad l = 0.4\,m$



Sol:

moment of inertia of the shaft

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 0.1^4 = 4.91 \times 10^{-6}\,m^4$$

$$E = 2 \times 10^{11}\,N/m^2$$

deflection due to a mass of 10kg

$$\delta_1 = \frac{W a^2 b^2}{3EIl} = \frac{mg a^2 b^2}{3EIl} = \frac{10 \times 9.81 \times (0.12)^2 \times (0.28)^2}{3(2\times10^{11})(4.91\times10^{-6})0.4}$$

$$\delta_1 = 9.4 \times 10^{-8}\,m$$

Static deflection to a load of 12 kg

$$\delta_2 = \frac{W a^2 b^2}{3EIl} = \frac{(7\times9.81)(0.31)^2(0.09)^2}{3(2\times10^{11})(4.91\times10^{-6})0.4} = 4.536\times10^{-8}\,m$$

Natural frequency of transverse vibration.

$$f_n = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\delta_3}} = \frac{0.4985}{\sqrt{9.4\times10^{-8} + (1.598\times10^{-7}) + (4.53\times10^{-8})}}$$

$$f_n = 911.411\ Hz$$

$\therefore$ Critical Speed $N_{cr} = 911.41$ r.p.s

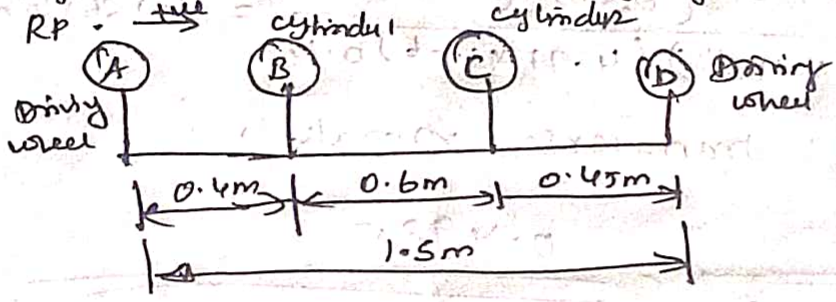
$= 911.41 \times 60$

$$\boxed{N_{cr} = 54{,}684\ rpm}$$

Reciprocating Masses:

1. A two cylinder uncoupled locomotive has inside cylinders 0.6m apart. The radius of each crank is 300mm and are at right angles. The revolving mass per cylinder is 250kg and the reciprocating mass per cylinder is 300kg. The whole of the revolving and two-third of the reciprocating masses are to be balanced and the balanced masses are to be placed, in the planes of rotation of the driving wheels @ radius of 0.8m. The driving wheel are 2m in dia and 1.5m apart.

If the speed of the engine is 80km/hr det (i) Hammer blow (ii) max variation in tractive force (c) max swaying couple.



data: $a = 0.6m$ $r = r_B = r_C = 300mm = 0.3m$

$m_1 = 250kg$ $m = 300kg$ $C = 2/3$ $b = 0.8m$ $D = 2m$

$R = 2/2 = 1m$ $L = 1.5m$ $V = 80km/hr = 22.22 m/s$

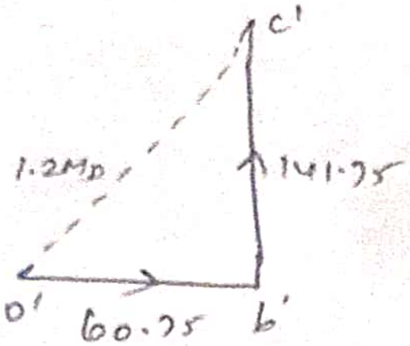
Soln:

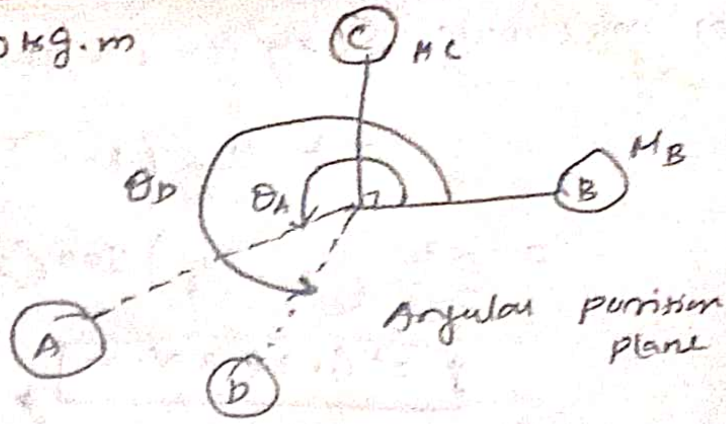
$$M_T = M_B = M_C = (m_1 + cm)$$

$$= 250 + \left(\frac{2}{3} \times 300\right) = 450kg.$$

Plane 1	Mass 2	Radius 3	Centrifugal force $(m \cdot r)$ $4 = 2 \times 3$	Distance from R.P 5	Couple $(m \cdot r \cdot l)$ $6 = 4 \times 5$
A (R.P)	M_A	0.8	$0.8 M_A$	0	0
B	450	0.3	135	0.45	60.75
C	450	0.3	135	1.05	141.75
D	M_D	0.8	$0.8 M_D$	1.5	$1.2 M_D$

couple polygon 1cm = 50 kg.m

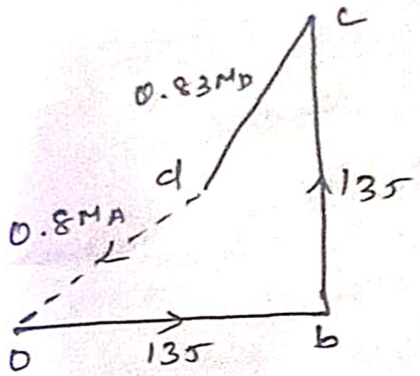




$1.2 M_D = 155 \text{ kg.m}^2$

$M_D = 155/1.2 = 129 \text{ kg}$

Force polygon 1cm = 50 kg.m



$0.8 M_A = 102.5 \text{ kg.m}$

$M_A = 102.5/0.8 = 129 \text{ kg}$

a) Hammer blow

$B = C \frac{m}{m_T} \times m_A = 2/3 \times \frac{300}{450} \times 129$

$\boxed{B = 57.33 \text{ kg}}$

$V = 22.2 \text{ m/s}$ $\omega = V/R = \frac{22.22}{1} = 22.22 \text{ m/s}$

$b = r_A = r_D = 0.8 \text{ m}$

Hammer blow

$= B\omega^2 b = 57.33 \times (22.22)^2 \times 0.8$

$= 22.65 \text{ kN}$

b) max vibration in tractive force:

$= \pm \sqrt{2}\,(1-c)\, m\omega^2 r$ $r = r_B = r_C$

$= \pm \sqrt{2}\,(1 - 2/2)\ 300 \times 22.22)^2 \times 0.3$

$= \pm 20.95 \text{ kN}$

c) maximum Swaying Couple :

$$= \pm \frac{a}{\sqrt{2}} (1-c) m\omega^2 r$$

$$= \frac{0.6}{\sqrt{2}} \left(1-\frac{2}{3}\right) 300 (22.22)^2 \times 0.3$$

$$\boxed{= \pm 6.28 \text{ kN.m}}$$