PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

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Department of Electronics and Communication Engineering

Regulation 2021

III Year – V Semester

EC3551 / Transmission lines and RF Systems

UNIT 1

TRANSMISSION LINE THEORY

1. Transmission Line Theory

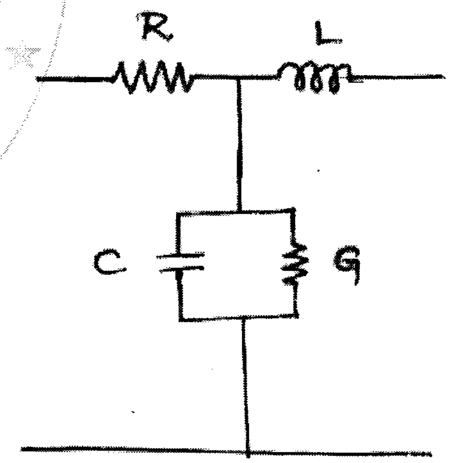
Transmission Line :

Transmission Line is a conductive method of guiding electrical signal from one end to another end.

Types :

1. Parallel lines → Open wire line / Two wire parallel line
2. Twisted Pair cable → shielded / unshielded
3. Flat Ribbon cable
4. coaxial cable
5. Strip lines

Equivalent circuit of Txn. Line :



R → Resistance (ohm/unit length)

L → Inductance (Henry/unit Length)

G → Conductance (mho/unit Length)

C → capacitance (Farad/unit Length)

R, L, G and C are called as primary constants of Txn. Lines.

Uniform

Uniform Txn. Line :

when R, L, G and C are uniformly distributed through out the line then the line is called as Uniform Txn. Line.

Secondary constants of Txn. line

1. **characteristic Impedance, 'Z_0'**

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

2. **Propagation Constant, 'γ'**

$$\gamma = P = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = \alpha + j\beta$$

3. **Wavelength, 'λ'**

$$\lambda = \frac{c}{f} \quad (\text{or}) \quad \lambda = \frac{2\pi}{\beta}$$

4. **Phase Velocity, 'V_p'**

$$V_p = \lambda f = \frac{\omega}{\beta}$$

To find 1° constants from 2° constants

$$R + j\omega L = \gamma \cdot Z_0$$

$$G + j\omega C = \gamma / Z_0$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C} \times \frac{R+j\omega L}{R+j\omega L}}$$

$$= \sqrt{\frac{(R+j\omega L)^2}{(R+j\omega L)(G+j\omega C)}}$$

$$= \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

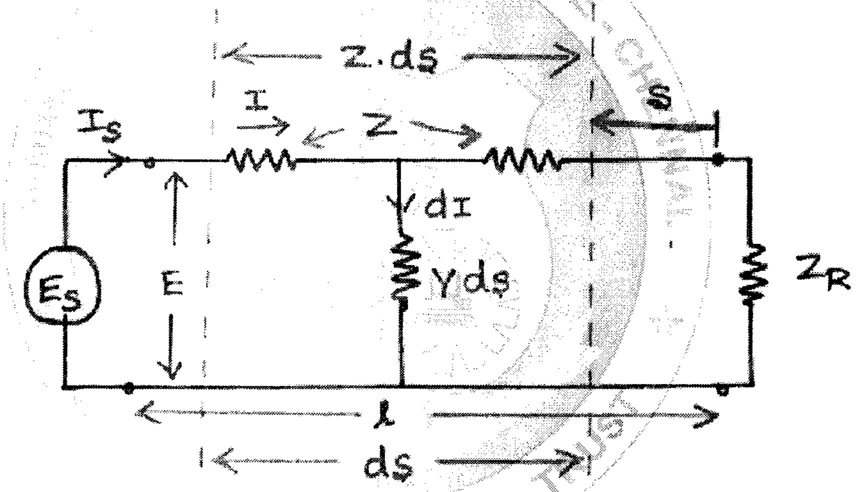
$$Z_0 = \frac{R+j\omega L}{\gamma}$$

$$\therefore R + j\omega L = Z_0 \cdot \gamma$$

General Solution Of Transmission Line :

When the voltage or current is transmitted through a transmission line, it will not be constant through out the line. There will be drop in the voltage or current.

To find the voltage and current at any point in a transmission line, Let us derive general solution of transmission line.



Let R, L, G and C be the primary constants of transmission line.

R → Series Resistance (ohms / unit length)

L → Series Inductance (Henry / unit length)

G → Shunt Conductance (mho / unit length)

C → Shunt Capacitance (Farad / unit length)

l → Total length of the Txn. line

s → Distance from load to the point of Observati-on

ds → Small section of Txn. line

Z → Series Impedance

$y \rightarrow$ Shunt Admittance

$E_S \rightarrow$ Source Voltage

$I_S \rightarrow$ Source Current

$Z_R \rightarrow$ Impedance at Receiving end

$E \rightarrow$ Voltage at any point on the line

$I \rightarrow$ Current at any Point on the line

$dE \rightarrow$ voltage drop in ds section

$dI \rightarrow$ current drop in ds section

$z.ds \rightarrow$ Impedance of small section ds

$y.ds \rightarrow$ Admittance of Small section ds

Consider a small section 'ds' having the series impedance 'zds'. Let the current flowing through this section be 'I' then the voltage drop across this section will be

$$dE = I \cdot zds$$

$$\frac{dE}{ds} = IZ \longrightarrow (1)$$

Similarly, the current drop across this section will be $dI = E \cdot yds$

$$\frac{dI}{ds} = Ey \longrightarrow (2)$$

Differentiate equs. (1) & (2) w.r.to 'S'

$$\frac{d^2E}{ds^2} = \frac{dI}{ds} \cdot z$$

$$\frac{d^2E}{ds^2} = Eyz \longrightarrow (3)$$

$E \leftrightarrow I$
$z \leftrightarrow y$

Similarly, $\frac{d^2I}{ds^2} = Iyz \rightarrow (4)$

Equations (3) and (4) are called the differential equations of Txn. Line.

To find solution for the differential equation, put $\frac{d}{ds} = m$, then equs. (3) & (4) becomes

$m^2E = Eyz$ $\qquad$ $m^2I = Iyz$

$m^2 = yz$ $\qquad$ $m^2 = yz$

$m = \pm\sqrt{yz} \longrightarrow (5a)$ $\qquad$ $m = \pm\sqrt{yz} \longrightarrow (5b)$

$\therefore$ The solution of the differential equations (3) & (4) are,

$$E = A \cdot e^{\sqrt{zy} \cdot s} + B \cdot e^{-\sqrt{zy} \cdot s} \longrightarrow (6a)$$

$$I = C \cdot e^{\sqrt{zy} \cdot s} + D \cdot e^{-\sqrt{zy} \cdot s} \longrightarrow (6b)$$

The voltage and current at the Receiver end is, put $I = I_R$, $E = E_R$ and $s = 0$ in (6a) & (6b)

$$E_R = A + B \longrightarrow (7a)$$

$$I_R = C + D \longrightarrow (7b)$$

Now, differentiate equs. (6a) & (6b) w.r.to 's'

$$\frac{dE}{ds} = A \cdot \sqrt{zy} \cdot e^{\sqrt{zy} \cdot s} - B \cdot \sqrt{zy} \cdot e^{-\sqrt{zy} \cdot s}$$

$$Iz = A \cdot \sqrt{zy} \cdot e^{\sqrt{zy} \cdot s} - B \cdot \sqrt{zy}\ e^{-\sqrt{zy} \cdot s}$$

$$I = A\cdot\sqrt{\frac{Y}{Z}}\cdot e^{\sqrt{ZY}\cdot s} - B\cdot\sqrt{\frac{Y}{Z}}\cdot e^{-\sqrt{ZY}\cdot s} \rightarrow (8a)$$

Similarly,

$$E = C\cdot\sqrt{\frac{Z}{Y}}\cdot e^{\sqrt{ZY}\cdot s} - D\cdot\sqrt{\frac{Z}{Y}}\cdot e^{-\sqrt{ZY}\cdot s} \rightarrow (8b)$$

The voltage and current at Receiver end is,

put $I = I_R$, $E = E_R$ and $s=0$ in equs. (8a) & (8b)

$$I_R = A\cdot\sqrt{\frac{Y}{Z}} - B\cdot\sqrt{\frac{Y}{Z}} \rightarrow (9a)$$

$$E_R = C\cdot\sqrt{\frac{Z}{Y}} - D\cdot\sqrt{\frac{Z}{Y}} \rightarrow (9b)$$

To find the arbitary constants A, B, C and D

Solve (7a), (7b), (9a) & (9b)

$$E_R = A + \cancel{B} \quad \text{(from equ. (7a))}$$

$$\sqrt{\frac{Z}{Y}}\cdot I_R = A - \cancel{B} \quad \text{(from equ. (9a))}$$

$$E_R + \sqrt{\frac{Z}{Y}}\cdot I_R = 2A$$

$$A = \frac{E_R}{2} + \sqrt{\frac{Z}{Y}}\cdot\frac{I_R}{2} \qquad \because \sqrt{\frac{Z}{Y}} = Z_0$$

$$A = \frac{E_R}{2} + Z_0\cdot\frac{E_R}{2Z_R}$$

$$A = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right) \rightarrow (10a) \qquad \because I_R = \frac{E_R}{Z_R}$$

Similarly,

$$B = \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right) \rightarrow (10b)$$

$$C = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right) \rightarrow (10c)$$

$$D = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right) \rightarrow (10d)$$

sub equs. (10a) → (10d) in equs. (6a) & (6b)

$$E = A \cdot e^{\sqrt{zy} \cdot s} + B \cdot e^{\sqrt{zy} \cdot s} \quad (6a)$$

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{zy} \cdot s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{zy} \cdot s} \rightarrow (11a)$$

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)\left[e^{\sqrt{zy} \cdot s} + \frac{\left(1 - \frac{Z_0}{Z_R}\right)}{\left(1 + \frac{Z_0}{Z_R}\right)} \cdot e^{-\sqrt{zy} \cdot s} \right]$$

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)\left[e^{\sqrt{zy} \cdot s} + \frac{\left(\frac{Z_R - Z_0}{\cancel{Z_R}}\right)}{\left(\frac{Z_R + Z_0}{\cancel{Z_R}}\right)} \cdot e^{-\sqrt{zy} \cdot s} \right]$$

$$E = \frac{E_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)\left[e^{\sqrt{zy} \cdot s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{zy} \cdot s} \right]$$

$$E = \frac{E_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)\left[e^{\sqrt{zy} \cdot s} + K\, e^{-\sqrt{zy} \cdot s} \right] \rightarrow (12)$$

where, $K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$; Reflection coefficient

Similarly,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_0}{Z_0}\right)\left[e^{\sqrt{zy} \cdot s} - K \cdot e^{-\sqrt{zy} \cdot s} \right] \rightarrow (13)$$

Equs. (12) & (13) are the useful equs. of Txn. Line.

Eqn. (11a) can also be solved as,

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{zy}.s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{zy}.s} \rightarrow (11a)$$

$$E = \frac{E_R}{2} e^{\sqrt{zy}.s} + \frac{E_R}{2} \cdot \frac{Z_0}{Z_R} \cdot e^{\sqrt{zy}.s} + \frac{E_R}{2} \cdot e^{-\sqrt{zy}.s} - \frac{E_R}{2} \cdot \frac{Z_0}{Z_R} \cdot e^{-\sqrt{zy}.s}$$

$$E = \frac{E_R}{2}\left(e^{\sqrt{zy}.s} + e^{-\sqrt{zy}.s}\right) + \frac{I_R . Z_0}{2}\left(e^{\sqrt{zy}.s} - e^{-\sqrt{zy}.s}\right)$$

$$E = E_R \cdot \left(\frac{e^{\sqrt{zy}.s} + e^{-\sqrt{zy}.s}}{2}\right) + I_R \cdot Z_0 \left(\frac{e^{\sqrt{zy}.s} - e^{-\sqrt{zy}.s}}{2}\right)$$

$$\therefore E = E_R \cdot \cosh . \sqrt{zy} . s + I_R \cdot Z_0 \cdot \sinh \sqrt{zy} . s \rightarrow (14)$$

Similarly,

$$I = I_R . \cosh \sqrt{zy} . s + \frac{E_R}{Z_0} . \sinh \sqrt{zy} . s \rightarrow (15)$$

Input Impedance of Transmission Line:

* The impedance measured at the i/p end of the Txn. line is the i/p impedance.

* The i/p impedance is defined as the ratio of source voltage to source current.

* It is denoted by Z_S, Z_{in}.

$$Z_S = \frac{E_S}{I_S} \rightarrow (1)$$

W.K.T the voltage and current at any point on a line is given by,

$$E = E_R \cosh\sqrt{ZY}.s + I_R Z_0 \sinh\sqrt{ZY}.s \rightarrow (2)$$

$$I = I_R \cosh\sqrt{ZY}.s + \frac{E_R}{Z_0} \sinh\sqrt{ZY}.s \rightarrow (3)$$

To find voltage & current at source end,

Put $E = E_S$, $I = I_S$ and $s = l$ in equs. (2) & (3)

$$E_S = E_R \cosh\sqrt{ZY}.l + I_R Z_0 \sinh\sqrt{ZY}.l \rightarrow (4)$$

$$I_S = I_R \cosh\sqrt{ZY}.l + \frac{E_R}{Z_0}.\sinh\sqrt{ZY}.l \rightarrow (5)$$

sub (4) & (5) in (1)

$$\therefore Z_S = \frac{E_R \cosh\sqrt{ZY}.l + I_R Z_0 \sinh\sqrt{ZY}.l}{I_R \cosh\sqrt{ZY}.l + \frac{E_R}{Z_0}.\sinh\sqrt{ZY}.l}$$

$$Z_S = \frac{I_R Z_R \cosh\sqrt{ZY}.l + I_R Z_0 \sinh\sqrt{ZY}.l}{I_R \cosh\sqrt{ZY}.l + I_R Z_R \sinh\sqrt{ZY}.l} \quad (\because E_R = I_R Z_R$$

$$Z_S = \frac{\cancel{I_R}\left(Z_R \cosh\sqrt{zy}.l + Z_0 \sinh\sqrt{zy}.l\right)}{\cancel{I_R}\left(\cosh\sqrt{zy}.l + \frac{Z_R}{Z_0} \sinh\sqrt{zy}.l\right)}$$

$$Z_S = \frac{Z_R \cosh\sqrt{zy}.l + Z_0 \sinh\sqrt{zy}.l}{\cosh\sqrt{zy}.l + \frac{Z_R}{Z_0} \sinh\sqrt{zy}.l}$$

$$\therefore Z_S = Z_0 \cdot \left[\frac{Z_R \cosh\sqrt{zy}.l + Z_0 \cdot \sinh\sqrt{zy}.l}{Z_0 \cdot \cosh\sqrt{zy}.l + Z_R \cdot \sinh\sqrt{zy}.l}\right] \rightarrow (6)$$

Case i): I/p impedance with $Z_R = Z_0$

I/p impedance of a Txn. line terminated with 'Z_0' is given as,

Substitute $Z_R = Z_0$ in equ. (6)

$$Z_S = Z_0 \cdot \left[\frac{\cancel{Z_0 \cosh\sqrt{zy}.l + Z_0 \sinh\sqrt{zy}.l}}{\cancel{Z_0 \cosh\sqrt{zy}.l + Z_0 \sinh\sqrt{zy}.l}}\right]$$

$$\therefore \boxed{Z_S = Z_0} \rightarrow (7)$$

Case ii) : I/p impedance with short circuit end ($Z_R = 0$)

I/p impedance of a Txn. line with short circuit end is given as,

put $Z_R = 0$ in equ. (6)

$$Z_S = Z_0 \left[\frac{0 + Z_0 \sinh\sqrt{zy}.l}{Z_0 \cosh\sqrt{zy}.l + 0}\right]$$

$$Z_S = Z_0 \cdot \left[\frac{\cancel{Z_0} \cdot \sinh\sqrt{zy}.l}{\cancel{Z_0} \cdot \cosh\sqrt{zy}.l} \right]$$

$$\boxed{Z_S = Z_{SC} = Z_0 \cdot \tanh.\sqrt{zy}.l} \longrightarrow (8)$$

case iii) : I/p impedance with Open circuit end ($Z_R = \infty$)

I/p impedance of a Txn. line with open circuit end is given as,

In equ. (6) take 'Z_R' as common

$$Z_S = Z_0 \cdot \frac{\cancel{Z_R}}{\cancel{Z_R}} \left[\frac{\cosh\sqrt{zy}.l + \frac{Z_0}{Z_R} \sinh\sqrt{zy}.l}{\frac{Z_0}{Z_R} \cosh\sqrt{zy}.l + \sinh\sqrt{zy}.l} \right]$$

Put $Z_R = \infty$ $\left(\because \frac{1}{\infty} = 0\right)$

$$Z_S = Z_0 \cdot \left[\frac{\cosh\sqrt{zy}.l + 0}{0 + \sinh\sqrt{zy}.l} \right]$$

$$Z_S = Z_0 \cdot \left[\frac{\cosh\sqrt{zy}.l}{\sinh\sqrt{zy}.l} \right]$$

$$\boxed{\therefore Z_S = Z_{OC} = Z_0 \cdot \coth\sqrt{zy}.l} \longrightarrow (9)$$

Transfer Impedance : 'Z_T'

It is defined as the ratio of source voltage to Receiver current. It is denoted by 'Z_T'

$$\boxed{Z_T = \frac{E_S}{\ }} \rightarrow (1)$$

W.K.T The voltage at any point on a Txn. line is given

as, $E = E_R \cosh\sqrt{zy}.s + I_R.z_0.\sinh\sqrt{zy}.s \rightarrow (2)$

To find the voltage at source end,

put $E = E_S$ and $s = l$ in equ. (2)

$$\therefore E_S = E_R.\cosh\sqrt{zy}.l + I_R z_0.\sinh\sqrt{zy}.l$$

$$= I_R Z_R \cosh\sqrt{zy}.l + I_R.z_0.\sinh\sqrt{zy}.l$$

$$E_S = I_R.(z_R.\cosh\sqrt{zy}.l + z_0.\sinh\sqrt{zy}.l)$$

$$\frac{E_S}{I_R} = z_T = z_R.\cosh\sqrt{zy}.l + z_0.\sinh\sqrt{zy}.l$$

$$\therefore \boxed{z_T = z_R.\cosh.\sqrt{zy}.l + z_0.\sinh\sqrt{zy}.l} \rightarrow (3)$$

Distortionless line :

A Transmission line which satisfies the condition

$\frac{R}{L} = \frac{G}{C}$ is called as distortionless line.

$$\boxed{RC = LG} \rightarrow (1)$$

W.K.T $\gamma = P = \alpha + j\beta = \sqrt{zy}$

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{LC\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)}$$

$$= \sqrt{LC\left(\frac{R}{L}+j\omega\right)\left(\frac{R}{L}+j\omega\right)} \quad \because \frac{R}{L} = \frac{G}{C}$$

$$\gamma = \alpha + j\beta = \left(\frac{R}{L}+j\omega\right)\sqrt{LC} \quad (\text{or}) \quad \sqrt{LC}\left(\frac{G}{C}+j\omega\right)$$

$$\alpha + j\beta = \sqrt{LC}\cdot\frac{R}{L} + j\omega\sqrt{LC} \quad (\text{or}) \quad \sqrt{LC}\cdot\frac{G}{C} + j\omega\sqrt{LC}$$

$$\alpha + j\beta = R\cdot\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} \quad (\text{or}) \quad G\cdot\sqrt{\frac{L}{C}} + j\omega\sqrt{LC}$$

$$\therefore \alpha = R\cdot\sqrt{\frac{C}{L}} \quad (\text{or}) \quad G\cdot\sqrt{\frac{L}{C}} \quad \rightarrow (2)$$

$$\beta = \omega\sqrt{LC} \quad \rightarrow (3)$$

From equ. (2) & (3) we come to know that 'α' is independent of frequency and 'β' is a constant multiplied by 'ω'.

Waveform Distortion (or) Distortion line (or) line Distortion

The signal transmitted through the Txn. line will be in complex form and has many freq. components. In ideal Txn. line, the signal received at the receive end must be same as the transmitted signal. This condition is achieved only if all the freq. component are attenuated equally and transmitted with same delay. There are 2 types of waveform distortion.

1. Frequency Distortion
2. Delay (or) Phase Distortion

1. Frequency Distortion :

It is a type of distortion in which all the Freq. Components are not attenuated at same level (equally. This distortion can be avoided if 'α' is independent of 'ω'. In Txn. line equalizers are used at the ends to reduce the distortion.

$$\alpha = \sqrt{\frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2 (RC+LG)^2}}{2}}$$

2. Delay Distortion :

It is a type of distortion in which all the Freq. components are transmitted at different time intervals. This distortion can be avoided if 'β' is a constant multiplied by ω and v_p is independent of 'ω'. coaxial cables are used to reduce this distortion

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2 (RC+LG)^2}}{2}}$$

(or)

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}}{2}}$$

Proof :-

W.K.T $\gamma = P = \alpha + j\beta = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)} \rightarrow (1)$

squaring on both sides

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = RG + j\omega RC + j\omega LG - \omega^2 LC$$

$$(\alpha^2-\beta^2) + j\,2\alpha\beta = (RG-\omega^2 LC) + j\omega(RC+LG)$$

Equating real and imaginary parts

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \rightarrow (2)$$

$$2\alpha\beta = \omega(RC+LG) \rightarrow (3)$$

Find magnitude for equ. (1)

$$\alpha + j\beta = \sqrt{(RG-\omega^2 LC) + j\omega(RC+LG)}$$

$$\sqrt{\alpha^2+\beta^2} = \sqrt{\sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+LG)^2}}$$

squaring on both sides

$$\alpha^2 + \beta^2 = \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+LG)^2} \rightarrow (4)$$

Solve equ. (2) & (4) to find 'α' and 'β'

$$\alpha^2 - \cancel{\beta^2} = RG - \omega^2 LC$$

$$\alpha^2 + \cancel{\beta^2} = \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+LG)^2}$$

$$2\alpha^2 = RG - \omega^2 LC + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+LG)^2}$$

$$\therefore \alpha = \sqrt{\frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+LG)^2}}{2}} \rightarrow (5)$$

Similarly,

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+LG)^2}}{2}} \rightarrow 6$$

From equ. 5 and 6 we come to know that, 'α' is depending on 'ω' and 'β' is not a constant multiplied by 'ω'. so, distortion takes place in line.

Reflection Co-efficient :-

It is defined as the ratio of reflected voltage or current to the incident voltage or current.

It is represented by K, ρ, r

$$\boxed{K = \frac{V_r}{V_i} = \frac{-I_r}{I_i}} \rightarrow 1$$

From general solution of Txn. line, the voltage and current can be expressed as,

$$E = \frac{E_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\sqrt{ZY}\cdot s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\cdot s}\right] \rightarrow 2$$

$$I = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right)\left[e^{\sqrt{ZY}\cdot s} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\cdot s}\right] \rightarrow 3$$

The above 2 expressions has 2 terms. The first term represented in terms of positive 's' is called as incident wave and the term represented in terms of negative 's' is called as reflected wave.

From equ. 2,

$$E = \frac{E_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right) \cdot e^{\sqrt{ZY}.s} + \frac{E_R}{2}\left(\frac{Z_R-Z_0}{Z_R}\right) e^{-\sqrt{ZY}.s} \rightarrow (4)$$

E_i (or) V_i $\qquad$ E_r (or) V_r

From equ. 4

$$E_i = \frac{E_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right) e^{\sqrt{ZY}.s} \rightarrow (5)$$

$$E_r = \frac{E_R}{2}\left(\frac{Z_R-Z_0}{Z_R}\right) e^{-\sqrt{ZY}.s} \rightarrow (6)$$

sub 5 & 6 in 1

$$K = \frac{\cancel{\frac{E_R}{2}}\left(\frac{Z_R-Z_0}{\cancel{Z_R}}\right) e^{-\sqrt{ZY}.s}}{\cancel{\frac{E_R}{2}}\left(\frac{Z_R+Z_0}{\cancel{Z_R}}\right) e^{+\sqrt{ZY}.s}} \Rightarrow \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) \cdot e^{-2\sqrt{ZY}.s}$$

At load end, $s=0$

$$\therefore \boxed{K = \frac{Z_R-Z_0}{Z_R+Z_0}} \rightarrow (7)$$

$0 < K < 1$

Similarly, from equ. 3

$$I = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right) \cdot e^{\sqrt{ZY}.s} - \frac{I_R}{2}\left(\frac{Z_R-Z_0}{Z_0}\right) \cdot e^{-\sqrt{ZY}.s} \rightarrow (8)$$

I_i $\qquad$ I_r

From equ. (8)

$$I_i = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right) \cdot e^{\sqrt{ZY}.s} \rightarrow (9)$$

$$I_r = -\frac{I_R}{2}\left(\frac{Z_R-Z_0}{Z_0}\right) e^{-\sqrt{ZY}.s} \rightarrow (10)$$

sub (9) & (10) in (1)

$$K = \frac{-\left(-\frac{\cancel{I_R}}{\cancel{2}}\left(\frac{Z_R-Z_0}{\cancel{Z_0}}\right) e^{-\sqrt{ZY}.s}\right)}{\frac{\cancel{I_R}}{\cancel{2}}\left(\frac{Z_R+Z_0}{\cancel{Z_0}}\right) e^{\sqrt{ZY}.s}}$$

$$\therefore K = \frac{Z_R-Z_0}{Z_R+Z_0} \cdot e^{-2\sqrt{ZY}.s}$$

At Load end, $s=0$.

$$\therefore \boxed{K = \frac{Z_R-Z_0}{Z_R+Z_0}} \rightarrow (11)$$

$< K < 1$

Reflection On a Line not Terminated in Z_0 :-

The Phenomenon of setting up of reflected wave in a transmission Line is called as reflection.

Reflection is maximum in open circuit ($Z_R = \infty$) or short circuit line ($Z_R = 0$).

Reflection is zero when $Z_R = Z_0$.

From general solution of txn. line, the voltage and current can be expressed as,

$$E = \frac{E_R}{2} \cdot \left(\frac{Z_R + Z_0}{Z_R}\right) \cdot \left[e^{\sqrt{ZY} \cdot s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} \cdot s} \right] \rightarrow (1)$$

$$I = \frac{I_R}{2} \cdot \left(\frac{Z_R + Z_0}{Z_0}\right) \cdot \left[e^{\sqrt{ZY} \cdot s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} \cdot s} \right] \rightarrow (2)$$

The above two expressions has 2 terms. The first term represented in terms of '+s' is called as incident wave which flows from the sending end to the receiving end.

The second term represented in terms of '−s' is called as reflected wave which flows from the receiving end to the sending end.

From equ (1),

$$E_i = \frac{E_R}{2} \left(\frac{Z_R + Z_0}{Z_R}\right) e^{\sqrt{ZY} \cdot s} \rightarrow (3)$$; Incident voltage wave

$$E_r = \frac{E_R}{2} \left(\frac{Z_R - Z_0}{Z_R}\right) e^{-\sqrt{ZY} \cdot s} \rightarrow (4)$$; Reflected Voltage wave

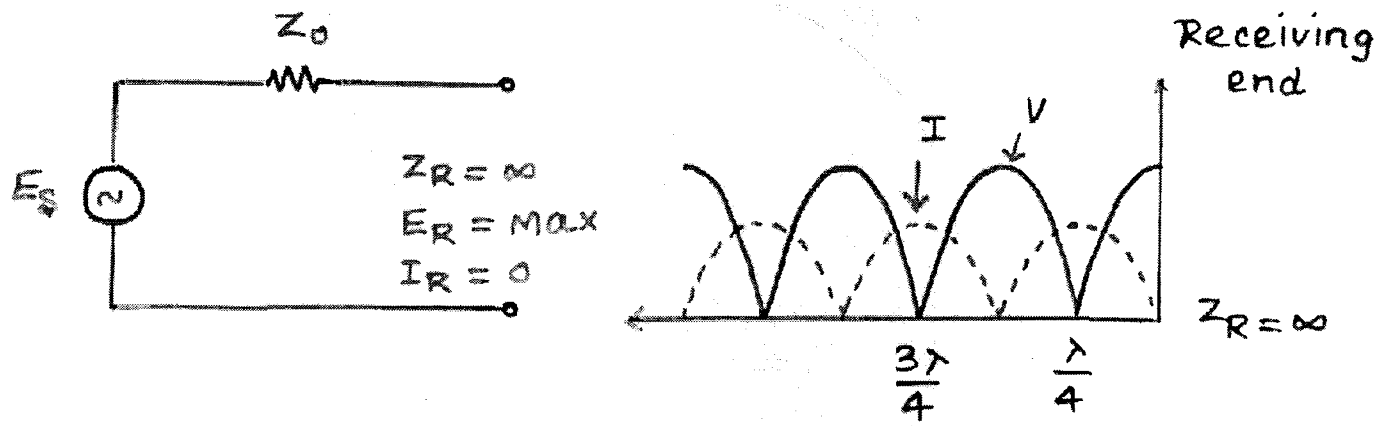
Similarly from equ (2)

$$I_i = \frac{I_R}{2} \left(\frac{Z_R + Z_0}{Z_0}\right) \cdot e^{\sqrt{ZY} \cdot s} \rightarrow (5)$$; Incident Current wave

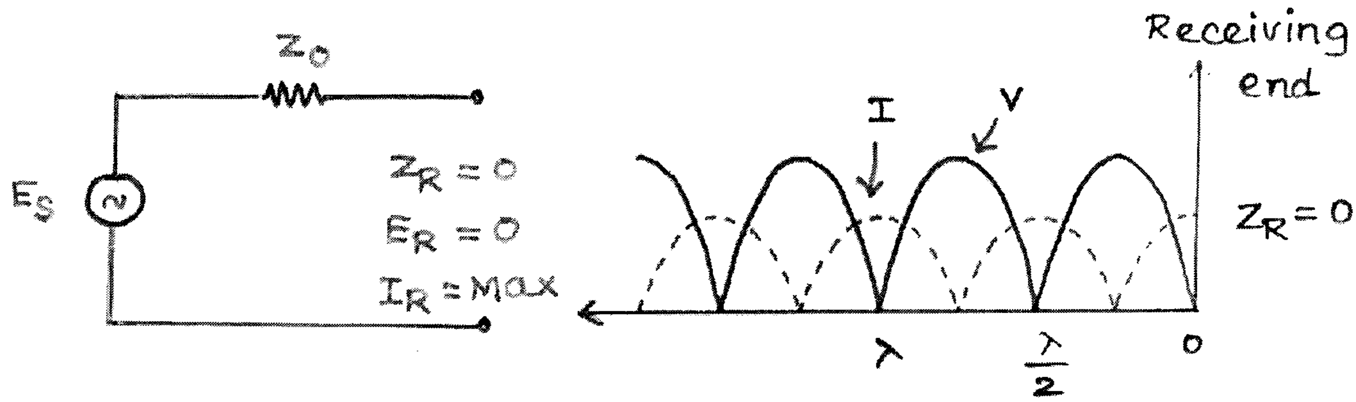
$$I_r = -\frac{I_R}{2} \left(\frac{Z_R - Z_0}{Z_0}\right) \cdot e^{-\sqrt{ZY} \cdot s} \rightarrow (6)$$; Reflected Current wave

Thus the total instantaneous Voltage or current at any point on the line is the phasor sum of voltage or current of the incident and reflected waves.

In Open circuit, the magnetic field gets collapsed at the load end and increases electric field. Due to this voltage will be maximum at load end.



In Short circuit, electric field gets collapsed at load end and increases magnetic field. Due to this, current will be maximum at load end.



Loading of Lines :

The process of increasing the inductance 'L' of a line artificially is called as Loading of a Line.

Loading is introduced in telephone cables.

There are 3 types of Loading

i) Continuous Loading

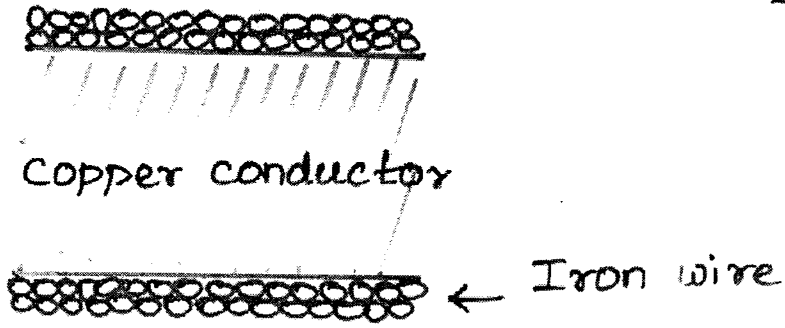
ii) Lumped Loading

iii) Patch Loading.

i) Continuous Loading :

In this method, the inductance of the line is increased uniformly along the length of the line.

In this type, iron or high permeability magnetic material in the form of a wire or tape is wound around the copper conductor as shown in figure.



Advantages :

1. Attenuation is constant over a wide range of freq.
2. Continuous Loading is used ~~only~~ on submarine cables.

Advantages :

1. Due to high toroidal cores, Large values of inductance is possible.
2. Eddy current and hysteresis losses are less.
3. cost is Less.

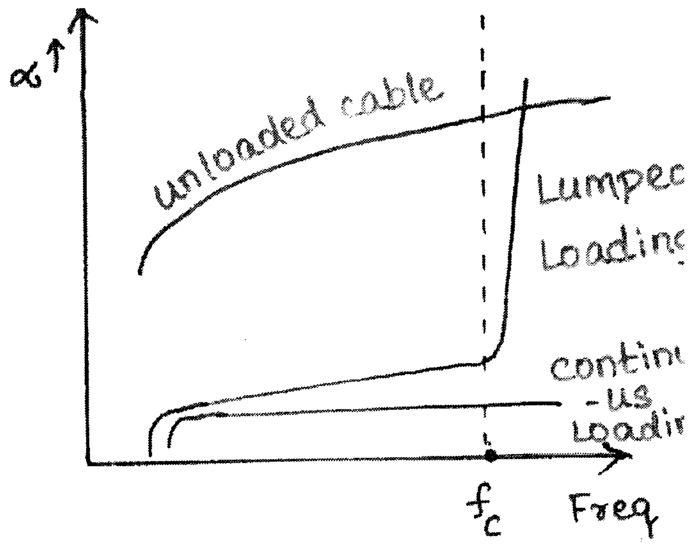
Disadvantages :-

1. Lumped Loading is useful only for voice band circuits upto 3 KHZ.
2. Inductance value is not uniform throughout the line.

iii) Patch Loading :

This type of Loading employs sections of continuously Loaded cable separated by sections of unloaded cable.

The typical length for the Patch Loading is normally 0.25 km.



Disadvantages:

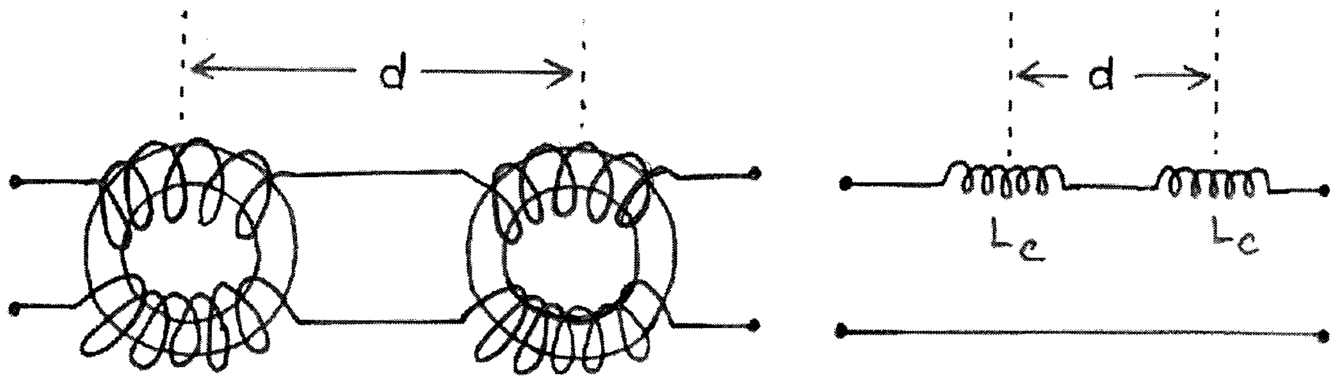
1. very expensive due to high cost of manufacture.
2. Only low inductance value is possible.
3. Since loading is done with iron wire, eddy current and hysteresis losses increases with frequency.

ii) Lumped Loading :-

In this type of loading, the inductors are introduced in lumps at uniform distances, in the line.

The inductors are introduced in both the limbs to keep the line as balanced circuit.

The lumped loading is preferred for open wire lines.



Loading coil

Telephone cable:

In the ordinary telephone cable, the wires are insulated with paper and twisted in pairs. This result in negligible values of inductance & conductance and can be neglected.

We know that,

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$(L \simeq G \simeq 0)$$

$$= \sqrt{R(j\omega C)}$$

$$= \sqrt{\omega RC}\,\sqrt{j} = \sqrt{\omega RC}\,\sqrt{\cos 90^\circ + j\sin 90^\circ}$$

$$= \sqrt{\omega RC}\,(\cos 45^\circ + j\sin 45^\circ)$$

$$= \sqrt{\omega RC}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$\alpha + j\beta = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}}$$

Equating real & imaginary terms, we get

$$\alpha = \beta = \sqrt{\frac{\omega RC}{2}}$$

$$v_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{RC}}$$

From the expressions of α, β & V_p, it is obvious that both frequency and phase distortion occurs in ordinary telephone cable. Therefore, to achive distortionless telephone cable (ie. to achieve $RC = LG$ condition), the value of Inductance has to be increased.

The process of increasing the value of inductance to achieve distortionless line is called Loading.

Loaded Telephone cable:

To understand the significance of Loading, consider a loaded telephone cable. (Inductance cannot be neglected). The primary constants of a loaded cable are R, L and C (only $G = 0$).

$$\therefore \quad Z = R + j\omega L$$

$$Y = j\omega C \qquad (\because G = 0)$$

$$\gamma = \sqrt{ZY} = \sqrt{|Z| \angle Z \; |Y| \angle Y} \quad \text{——} \quad (1)$$

$$= \sqrt{\sqrt{R^2 + \omega^2 L^2} \; \angle \tan^{-1}\left(\frac{\omega L}{R}\right) \; \omega C \angle \pi/2}$$

$$= \sqrt{\sqrt{\omega^2 L^2 \left(1 + \frac{R^2}{\omega^2 L^2}\right)} \; \angle \frac{\pi}{2} - \tan^{-1}\left(\frac{R}{\omega L}\right) \cdot \omega C \angle \pi/2}$$

$$= \sqrt{\omega L \sqrt{1 + \frac{R^2}{\omega^2 L^2}} \cdot \omega C \; \angle \pi - \tan^{-1}\left(\frac{R}{\omega L}\right)}$$

$$\therefore \gamma = \omega\sqrt{LC}\sqrt{\sqrt{1+\frac{R^2}{\omega^2L^2}}}\;\angle\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)$$

Since $R << \omega L$, $\frac{R^2}{\omega^2 L^2} \simeq 0$

$$\therefore \gamma = \omega\sqrt{LC}\;\angle\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)$$

Let $\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right) = \theta$

$$\gamma = \omega\sqrt{LC}\;\angle\theta = \omega\sqrt{LC}\; e^{j\theta}$$

$$\gamma = \omega\sqrt{LC}\,(\cos\theta + j\sin\theta) \qquad \text{—— (2)}$$

To find $\cos\theta$:

$$\cos\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\right)$$

$$= \cos\left(\frac{\pi}{2} - \frac{1}{2}\left(\frac{R}{\omega L}\right)\right)$$

$$= \sin\left(\frac{1}{2}\frac{R}{\omega L}\right) = \frac{R}{2\omega L}$$

$\left[\because \frac{R}{\omega L}\right.$ is very les

tan (smaller angle) = angl itsel

ie. If ϕ is very smal

$\cos\phi \simeq 1$

$\sin\phi \simeq \phi$

$\tan\phi \simeq \phi$

To find $\sin\theta$

$$\sin\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\right)$$

$$= \sin\left(\frac{\pi}{2} - \frac{1}{2}\cdot\frac{R}{\omega L}\right)$$

$$= \cos\left(\frac{1}{2}\cdot\frac{R}{\omega L}\right) \simeq 1$$

$\therefore$ Equation (2) can be written as

$$\gamma = \omega\sqrt{LC}\left(\frac{R}{2\omega L} + j\cdot 1\right)$$

$$\alpha + j\beta = \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

By equating real & imaginary terms, we get

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} \quad , \quad \beta = \omega\sqrt{LC} \quad \& \quad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

From the above expressions, it is obvious that by increasing the inductance value (ie. by means of Loading) distortionless line can be achieved.

Losses in a Transmission line:

The three types of losses that generally occur in a transmission line are

(1) Reflection loss & Reflection factor

(2) Return loss

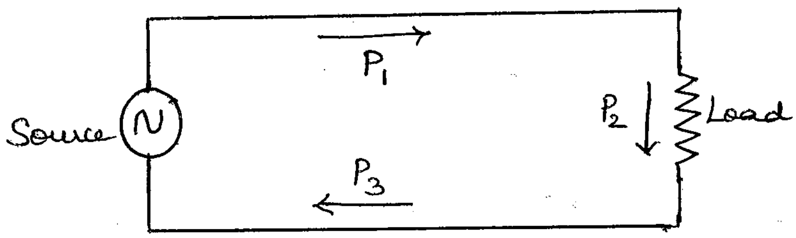
(3) Insertion loss

(1) **Reflection loss & Reflection factor:**

Reflection loss is defined as the ratio of power delivered to the load when $Z_R = Z_0$ (impedance matching condition) to power delivered to the load when $Z_R \neq Z_0$ (impedance mismatch condition).

$$\text{Reflection loss} = \frac{P_L \ (\text{when } Z_R = Z_0)}{P_L \ (\text{when } Z_R \neq Z_0)}$$

Consider the following transmission line.



P_1 — Incident power

P_2 — Actual power delivered to the load

P_3 — Reflected power.

We know that $\frac{P_3}{P_1} = K^2$ ——— (1)

$P_2 = P_1 - P_3$ ——— (2)

$$\text{Reflection loss} = \frac{P_1}{P_2}$$

$P_1 \rightarrow$ Power that must be delivered to the load

$P_2 \rightarrow$ Actual power delivered to the load

$$= \frac{P_1}{P_1 - P_3}$$

$$= \frac{P_1}{P_1 - P_1 K^2}$$

$$= \frac{1}{1 - K^2}$$

$$= \frac{1}{1 - \left(\frac{Z_R - Z_0}{Z_R + Z_0} \right)^2}$$

$$= \frac{(Z_R + Z_0)^2}{(Z_R + Z_0)^2 - (Z_R - Z_0)^2}$$

$$\text{Ref. loss} = \frac{(Z_R + Z_0)^2}{4 Z_R Z_0}$$

$$\text{Reflection loss in dB} = 10 \log \left(\frac{(Z_R + Z_0)^2}{4 Z_R Z_0} \right)$$

$$= 10 \log \left(\frac{Z_R + Z_0}{2\sqrt{Z_R Z_0}} \right)^2$$

$$\therefore \text{Reflection loss in dB} = 20 \log \left(\frac{Z_R + Z_0}{2\sqrt{Z_R Z_0}} \right)$$

$$= 20 \log \left(\frac{1}{k_f} \right)$$

Where k_f is called Reflection factor which is given as,

$$k_f = \frac{2\sqrt{Z_R Z_0}}{(Z_R + Z_0)}$$

In general, reflection factor can be defined as the ratio of geometric mean to arithmetic mean of any two impedances.

(2) Return loss:

It is defined as the ratio of incident power to reflected power.

$$\text{Return loss} = \frac{P_i}{P_r}$$

$$\text{Return loss} = \frac{1}{|K|^2} \qquad \left[\because |K|^2 = \frac{P_r}{P_i}\right.$$

$K \rightarrow$ Reflection coefficient$]$

$$\text{Return loss in dB} = 10 \log \frac{1}{|K|^2}$$

$$RL_{dB} = -20 \log |K|$$

(3) Insertion loss:

It is defined as the ratio of amount of power delivered to the load before insertion of a line to power delivered to the load after insertion of a line.

$$\text{Insertion loss} = \frac{P_L \text{ before insertion of a line}}{P_L \text{ after insertion of a line}}$$

$$\text{Insertion loss} = \frac{k_{SR}}{k_S \, k_R} e^{\alpha l}$$

Where,

k_{SR} → Reflection factor between source & receiver

$$k_{SR} = \frac{2\sqrt{Z_S Z_R}}{Z_S + Z_R}$$

k_S → Reflection factor at the sending end of a line

$$k_S = \frac{2\sqrt{Z_S Z_0}}{Z_S + Z_0}$$

k_R → Reflection factor at the receiving end of a line

$$k_R = \frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0}$$

Infinite line:

A transmission line which is infinitely long is called an infinite line (i.e. $l \to \infty$). The input impedance of an infinite line is calculated as follows;

We know that, $$Z_{IN} = \frac{Z_0 (Z_R + Z_0 \tanh \gamma l)}{Z_0 + Z_R \tanh \gamma l}$$

As $l \to \infty$ in an infinite line, sub. $l = \infty$ in the above expression.

$$\therefore Z_{IN} = \frac{Z_0 (Z_R + Z_0 \tanh \gamma \infty)}{Z_0 + Z_R \tanh \gamma \infty}$$

As $\tanh \infty = 1$,

$$Z_{IN} = \frac{Z_0 (Z_R + Z_0)}{Z_0 + Z_R}$$

$$\therefore \boxed{Z_{IN} = Z_0}$$

As the infinite line is hypothetical, a finite line equivalent to infinite line has to be derived. i.e. a finite line with input impedance equal to Z_0 has to be derived. This can be achieved by considering a transmission line which is properly terminated. ie. $Z_R = Z_0$.

If $Z_R = Z_0$ in a finite line, then its $Z_{IN} = Z_0$.

Proof:

$$Z_{IN} = \frac{Z_0 (Z_R + Z_0 \tanh \gamma l)}{Z_0 + Z_R \tanh \gamma l}$$

Sub. $Z_R = Z_0$ in the above expression,

$$Z_{IN} = \frac{Z_0 (Z_0 + Z_0 \tanh \gamma l)}{Z_0 + Z_0 \tanh \gamma l}$$

$$\therefore Z_{IN} = Z_0$$

Thus,

a properly terminated finite line (ie. $Z_R = Z_0$)	$\equiv$	Infinite line

Wavelength & velocity of propagation:

Velocity or Phase Velocity: It is defined as the velocity with which a signal of single frequency propagates along a transmission line.

ie. $$\boxed{v_p = \frac{\omega}{\beta}}$$

Wavelength: The distance travelled by a wave along a transmission medium in which the phase of the wave changes by 2π radians is called the wavelength.

$$\therefore \boxed{\lambda = \frac{2\pi}{\beta}}$$

Transmission lines:

It is defined as a physical conducting medium which transmits information/power in the form of electrical signal from one end to another.

Examples: Coaxial cable, Twisted pair cable, Open wire line etc. The type of wave propagation in these transmission lines is called guided wave propagation.

Classification of transmission lines:

Transmission lines are classified as

1. Metallic lines
2. Non metallic lines (optical fibres)
3. Strip lines
4. Waveguides

Metallic lines:

The various types of metallic lines are

(1) Single wire line (2) Two wire lines

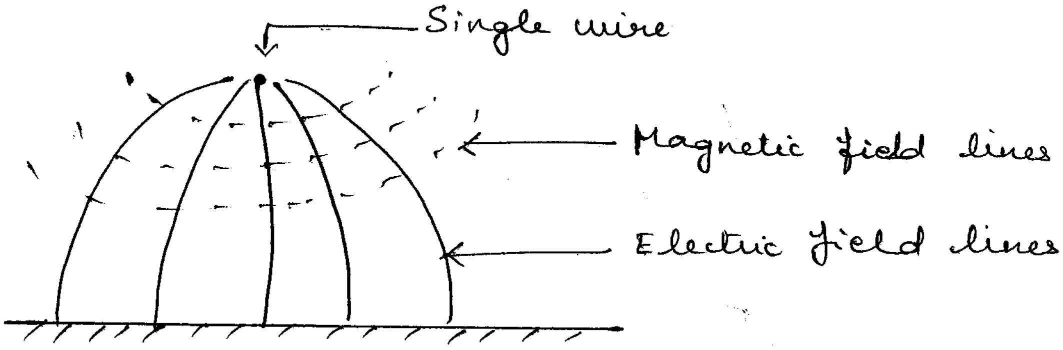
(3) Coaxial line

Single wire line :

In this type of line, a single solid conducting wire is used to connect two ends.

*. This line is acceptable at low frequencies

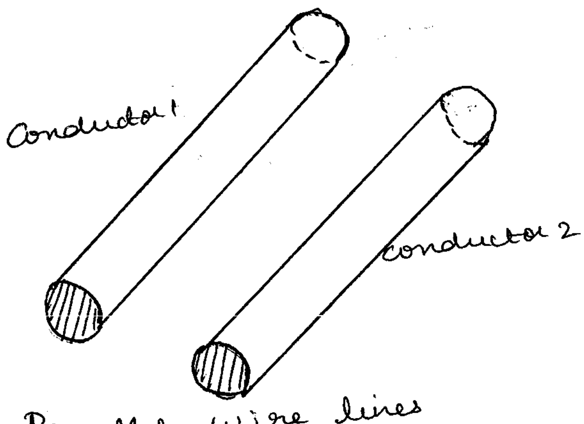
*. At high frequencies, more energy is dissipated by radiation.



Two wire lines or Parallel wire lines or open wire line :

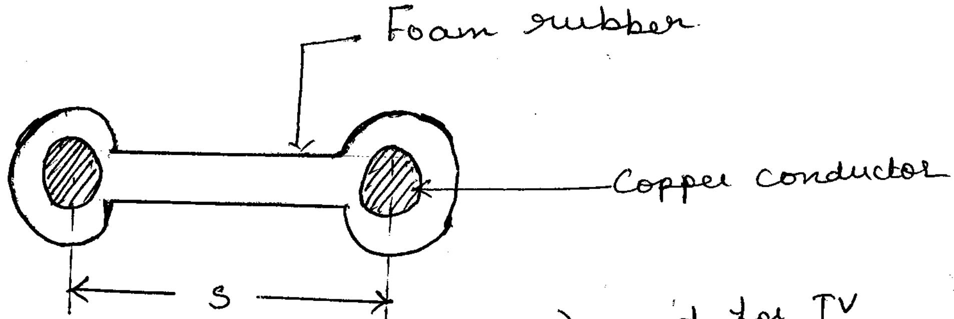
In this line, two wires or conductors are placed parallely.

*. At low frequencies (as in voice telephony or telegraphy), the wires are supported by post or buried inside the earth.



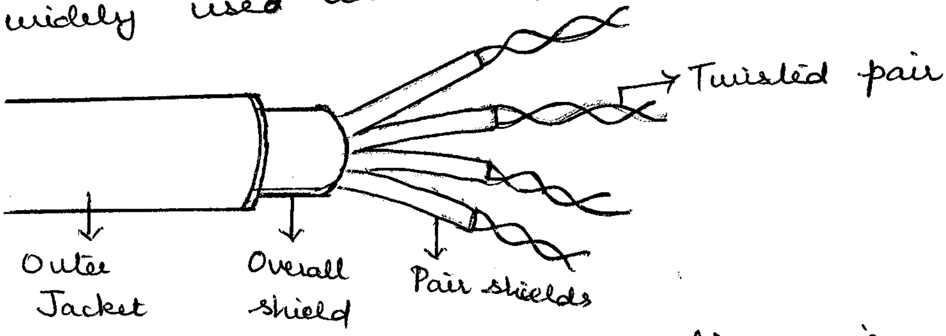
Parallel wire lines

*. One type of parallel wire line is twin lead arrangement which is used to connect an antenna to a Television set.



Twin - lead (ribbon cable) used for TV

*. Another type is twisted pair cable. This is formed by twisting 2 insulated conductors. The conductors are twisted to reduce noise interference between pairs & thus eliminate cross talk. This cable is widely used in Telephone applications.



*. Another type of parallel wire line is a Shielded pair transmission line. In this parallel wire line is placed inside a conducting pipe or metallic braid as an electromagnetic shield.

Advantages :

1. Cheaper

2. Easily installed

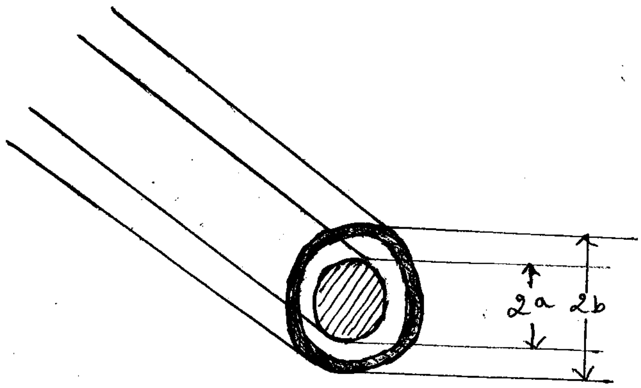
3. No dielectric between the 2 wires & hence no dielectric loss.

Disadvantages :

More radiation loss

Coaxial line :

This line consists of 2 conductors (inner conductor and outer conductor) placed coaxially.



a → Radius of the inner conductor

b → Radius of the outer conductor

The Source is connected to the inner conductor of the coaxial line. The other end of the inner conductor is connected to load Z_L. The other end of source, load & outer conductor of the coaxial line are all connected to the ground. Hence, the voltage between the inner conductor & the ground

& the outer conductor & the ground are different. Therefore, the coaxial line is an unbalanced line.

As the two conductors are at two different potentials, the fields are entirely confined to the space between the two conductors. No field exist outside the outer conductor & similarly no external radiation can penetrate the outer conductor & propagate inside.

<u>Types of coaxial cables:</u>

1. Flexible
2. Semi-rigid
3. Rigid

The flexible coaxial cable use copper braided outer conductor, a thin center conductor & a low loss solid or foam polyethylene dielectric. Semi-rigid cables have solid dielectric and thin outer conductor so that it could be bent while laying cables. The rigid cables have solid dielectric made of Teflon

<u>Advantages</u>:

1. Low dielectric & radiation loss
2. Cheaper
3. Easy to instal & to maintain

Disadvantages:

1. Can be used upto 3 GHz. Beyond this frequency, more losses occur in solid dielectric and conductor.

Strip lines:

They are transmission lines used as microwave circuits / components in conjunction with microwave semiconductor devices. They are used over the frequency range from 100 MHz to 30GHz. The commonly used dielectrics in the structure of strip line are teflon, polystyrene etc. The mode of propagation is TEM mode. Strip lines are unbalanced lines since the ground planes are kept at ground potential.

Advantages:

1. Used in microwave applications
2. Simple construction
3. Can easily be integrated with MIC components

Disadvantages:

1. Strip lines have much less power handling capability.
2. Have greater losses.

Waveguides:

It is a hollow single conductor metallic structure used to propagate high frequency signals (in the range of GHz). Waveguide dimension is inversely proportional to frequency. TE and TM are the modes of signal propagation. Depending on the cross section, it can be classified as Rectangular, Cylindrical or Elliptical waveguide.

Advantages:

1. Higher power handling capability
2. Reduces fabrication cost
3. Lower attenuation per unit length.
4. No hysteresis or eddy current loss

Disadvantages:

1. High cost due to thicker metallic structure
2. Difficult to instal & special couplings are required.

Comparison of Various transmission lines

Type	Frequency range (upper / Limit)	Losses	Usable Bandwidth	Power handling capacity	Physical size	Modes of Operation
Open wire lines	Low to VHF (500 MHz)	very high	Lowest	very high	Small	TEM
Coaxial cable	Low to Microwave (18 GHz)	Medium	High	Medium	Medium	TEM
Waveguide	Microwave (300 GHz)	Low	High	very high	Large	TE, TM
Strip line & Microstrip line	Microwave (30 GHz)	High	High	Low	Small	Quasi TEM
Optical fibres	Infra-red (0.8 μm - 2.5 μm)	Very low	very high	Very low	Very Small	TE, TM & Hybrid modes

UNIT 2

HIGH FREQUENCY TRANSMISSION LINES

2. High Frequency Transmission Lines

The standard assumptions made for the analysis of Radio Freq. lines are,

1. At very high Freq, the skin effect is considerable. Hence it is assumed that the currents may flow on the surface of conductor. Then the internal inductance becomes zero.

2. Due to skin effect, resistance R increases with $\sqrt{f}$. But the line reactance ωL increases directly with freq. 'f'. Hence the second assumption is $\omega L \gg R$. $(R \simeq 0)$

3. The third assumption is that the leakage conductance 'G' is considered as zero. (i.e., $G \simeq 0$)

$\omega C \gg G$

Line Constants for Zero Dissipation Line :

In general, the characteristic impedance 'Z_0' and propagation constant 'γ' of a Txn. line is given by,

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \longrightarrow (1)$$

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)} \quad \longrightarrow (2)$$

According to the standard assumptions for line at high Freq. $j\omega L \gg R$ and $j\omega C \gg G$. (i.e., $R = G \simeq 0$)

$$\therefore Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \rightarrow (3)$$

Since, Z_0 is real and resistive, it can be represented by symbol R_0.

$$\therefore Z_0 = R_0 = \sqrt{\frac{L}{C}} \rightarrow (4)$$

Similarly,

$$\gamma = \sqrt{(j\omega L)(j\omega C)}$$

$$= \sqrt{j^2\omega^2 LC}$$

$$\gamma = j\omega\sqrt{LC} \rightarrow (5)$$

W.K.T $\gamma = \alpha + j\beta$

$$\therefore \gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0 \rightarrow (6)$$

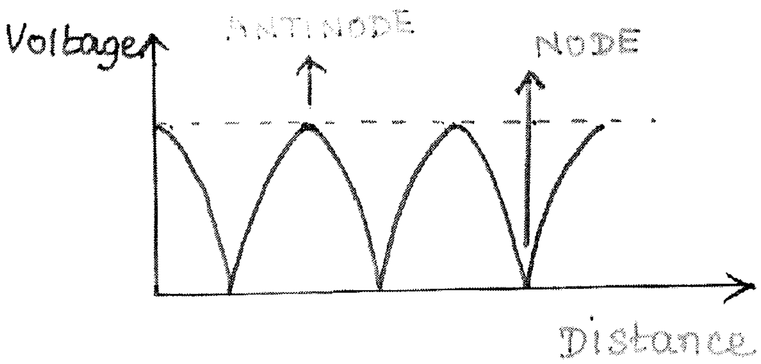
$$\beta = \omega\sqrt{LC} \rightarrow (7)$$

$$\therefore v_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}} \text{ m/sec} \rightarrow (8)$$

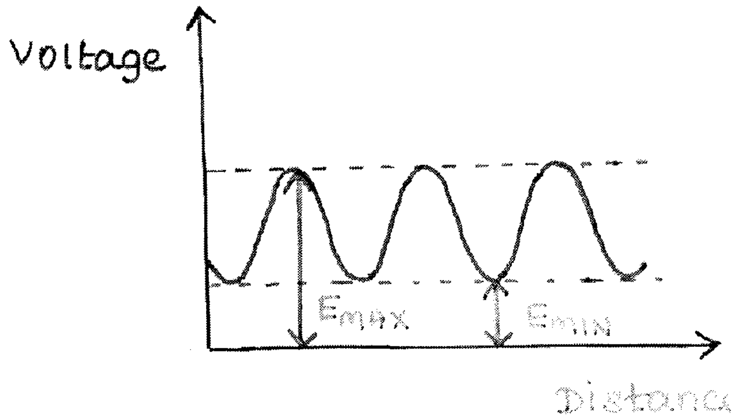
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} \text{ m} \rightarrow (9)$$

Standing wave :-

when $Z_R \neq Z_0$, some part of the transmitted signal from source to load will be reflected back towards the source. This reflected wave will combin with the incident wave which gives rise to standing wave.



* standing wave on a dissipationless line with OC or SC termination



* standing wave on a dissipationless line terminated with $Z_R \neq Z_0$

The points along the line where the magnitude of voltage or current is zero are called as Nodes.

The points along the line where the magnitude of voltage or current is maximum are called as Antinodes or Loops.

when a line is terminated in R_0, the standing waves are absent, such a line is called Smooth line

Distance B/w 2 maximum points is $\lambda/2$

Distance B/w 2 minimum points is $\lambda/2$

Distance B/w one maximum & one minimum is $\lambda/4$

Standing Wave Ratio :

* The ratio of the maximum to minimum magnitudes of voltages or currents on a line having standing waves is called as standing wave ratio.

* It is denoted by 'S' or 'SWR'

* Standing wave Ratio is given by,

$$S = SWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|E_{max}|}{|E_{min}|} = \frac{|I_{max}|}{|I_{min}|} \rightarrow ①$$

* There are two types of SWR

i) VSWR — Voltage SWR

ii) CSWR — Current SWR

Relation between 'SWR' and 'K' :-

$$S = \frac{E_{max}}{E_{min}} = \frac{|E_i| + |E_r|}{|E_i| - |E_r|} \Rightarrow \frac{\cancel{|E_i|}}{\cancel{|E_i|}} \cdot \frac{1 + |E_r/E_i|}{1 - |E_r/E_i|}$$

$$\therefore \boxed{S = \frac{1 + |K|}{1 - |K|}} \rightarrow ②$$

$$S(1 - |K|) = 1 + |K|$$

$$S - S \cdot |K| = 1 + |K|$$

$$S - 1 = S \cdot |K| + |K|$$

$$S - 1 = |K|(S + 1)$$

$$\therefore \boxed{|K| = \frac{S - 1}{S + 1}} \rightarrow ③$$

Input Impedance of Zero dissipation Line :

The i/p impedance of a line can be found using the expression

$$Z_S = \frac{E_S}{I_S} \rightarrow (1)$$

From general solution of Txn. line, the current and voltage at any point on a line is expressed as,

$$E = E_R \cosh\sqrt{zy}.s + I_R \cdot Z_0 \cdot \sinh\sqrt{zy}.s \rightarrow (2)$$

$$I = I_R \cdot \cosh\sqrt{zy}.s + \frac{E_R}{Z_0} \sinh\sqrt{zy}\cdot s \rightarrow (3)$$

For zero dissipation line,

$Z_0 = R_0$, $\gamma = j\beta$, $\alpha = 0$. so, the sending end voltage and current at a distance 's' is expressed as,

$$E_S = E_R \cdot \cosh(j\beta)s + I_R R_0 \sinh(j\beta)s$$

$$E_S = E_R \cos\beta s + j I_R R_0 \sin\beta s \rightarrow (4)$$

Similarly,

$$I_S = I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s \rightarrow (5)$$

Substitute (4) & (5) in (1)

$$Z_S = \frac{E_R \cos\beta s + j I_R R_0 \sin\beta s}{I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s}$$

$$Z_S = \frac{I_R \cdot Z_R \cdot \cos\beta s + j I_R R_0 \sin\beta s)}{I_R \cdot \cos\beta s + j \frac{I_R Z_R}{R_0} \sin\beta s)}$$

$$Z_S = \frac{\cancel{I_R}}{\cancel{I_R}} \frac{(Z_R \cos\beta s + j R_0 \sin\beta s)}{\left(\cos\beta s + j \frac{Z_R}{R_0} \sin\beta s \right)} \rightarrow (6)$$

W.K.T $\cos\beta s = \frac{e^{j\beta s} + e^{-j\beta s}}{2}$ → (7) ; $\sin\beta s = \frac{e^{j\beta s} - e^{-j\beta s}}{2j}$ → (8)

sub (7) & (8) in (6)

$$Z_S = R_0 \left[\frac{Z_R \cdot \left(\frac{e^{j\beta s} + e^{-j\beta s}}{2} \right) + \cancel{j} R_0 \left(\frac{e^{j\beta s} - e^{-j\beta s}}{2\cancel{j}} \right)}{R_0 \left(\frac{e^{j\beta s} + e^{-j\beta s}}{2} \right) + \cancel{j} Z_R \left(\frac{e^{j\beta s} - e^{-j\beta s}}{2\cancel{j}} \right)} \right]$$

$$Z_S = \frac{R_0 \times \cancel{\frac{1}{2}}}{\cancel{\frac{1}{2}}} \left[\frac{Z_R e^{j\beta s} + Z_R e^{-j\beta s} + R_0 e^{j\beta s} - R_0 e^{-j\beta s}}{R_0 e^{j\beta s} + R_0 e^{-j\beta s} + Z_R e^{j\beta s} - Z_R e^{-j\beta s}} \right]$$

$$Z_S = R_0 \cdot \left[\frac{(Z_R + R_0) e^{j\beta s} + (Z_R - R_0) e^{-j\beta s}}{(Z_R + R_0) e^{j\beta s} - (Z_R - R_0) e^{-j\beta s}} \right]$$

$$Z_s = \frac{R_0 \cdot \cancel{(Z_R+R_0)}\, \cancel{e^{j\beta s}}}{\cancel{(Z_R+R_0)}\, \cancel{e^{j\beta s}}} \cdot \left[\frac{1 + \dfrac{(Z_R-R_0)\ e^{-j\beta s}}{(Z_R+R_0)\ e^{j\beta s}}}{1 - \dfrac{(Z_R-R_0)\cdot e^{-j\beta s}}{(Z_R+R_0)\cdot e^{j\beta s}}} \right]$$

W.K.T $\dfrac{Z_R - R_0}{Z_R + R_0} = K$

$$\therefore \quad Z_s = R_0 \cdot \left[\frac{1 + K \cdot e^{-j2\beta s}}{1 - K \cdot e^{-j2\beta s}} \right]$$

$\because e^{-j\theta} = \underline{|-\theta}$

$$Z_s = R_0 \cdot \left[\frac{1 + |K| \cdot \underline{|\phi}\ \underline{|-2\beta s}}{1 - |K| \cdot \underline{|\phi}\ \underline{|-2\beta s}} \right]$$

$$Z_s = R_0 \cdot \left[\frac{1 + |K|\ \underline{|\phi - 2\beta s}}{1 - |K|\ \underline{|\phi - 2\beta s}} \right] \rightarrow (9)$$

Input impedance will be maximum if the angle is zero

$$\phi - 2\beta s = 0$$

$$2\beta s = \phi$$

$$\boxed{s = \frac{\phi}{2\beta}} \rightarrow (10)$$

$$\therefore Z_s(max) = R_0 \cdot \frac{1 + |K|}{1 - |K|}$$

$\because \dfrac{1 + |K|}{1 - |K|} = S$

$$\boxed{Z_s(max) = R_0 \cdot S} \rightarrow (11)$$

$Z_s(min)$ can be found when we move at a distance of $\frac{\lambda}{4}$ from $Z_s(max)$ point towards the source.

$$S = \frac{\phi}{2\beta} + \frac{\lambda}{4}$$

$$S = \frac{\phi}{2\beta} + \frac{\cancel{2}\pi}{{}_2\cancel{4}\beta} \qquad \because \lambda = \frac{2\pi}{\beta}$$

$$S = \frac{1}{2\beta}\left[\phi + \pi\right] \rightarrow (12)$$

$$\therefore Z_s(min) = R_0 \cdot \left[\frac{1 + |K| \cdot \underline{|\phi - \cancel{2\beta} \cdot \frac{1}{\cancel{2\beta}}(\phi + \pi)}}{1 - |K| \cdot \underline{|\phi - \cancel{2\beta} \cdot \frac{1}{\cancel{2\beta}}(\phi + \pi)}}\right]$$

$$Z_s(min) = R_0 \cdot \left[\frac{1 + |K| \cdot \underline{|\phi - \phi - \pi}}{1 - |K| \underline{|\phi - \phi - \pi}}\right]$$

$$Z_s(min) = R_0 \cdot \left[\frac{1 + |K| \underline{|-\pi}}{1 - |K| \underline{|-\pi}}\right] \qquad \because \underline{|-\pi} = -1$$

$$= R_0 \cdot \left[\frac{1 - |K|}{1 + |K|}\right]$$

$$Z_s(min) = R_0 \cdot \frac{1}{S}$$

$$\boxed{\therefore Z_s(min) = \frac{R_0}{S}} \rightarrow (13)$$

Input impedance of short Circuited Dissipationless Line

The input impedance of dissipationless line is expressed as,

$$Z_s = R_0 \cdot \left[\frac{Z_R \cos\beta s + jR_0 \sin\beta s}{R_0 \cos\beta s + jZ_R \sin\beta s} \right] \rightarrow ①$$

$$Z_s = R_0 \cdot \frac{\cancel{\cos}\beta s}{\cancel{\cos}\beta s} \left[\frac{Z_R + jR_0 \dfrac{\sin\beta s}{\cos\beta s}}{R_0 + jZ_R \dfrac{\sin\beta s}{\cos\beta s}} \right]$$

$$Z_s = R_0 \cdot \left[\frac{Z_R + jR_0 \tan\beta s}{R_0 + jZ_R \tan\beta s} \right] \rightarrow ②$$

For short circuited line $Z_R = 0$

$$\therefore Z_s = Z_{sc} = \cancel{R_0} \cdot \left[\frac{jR_0 \tan\beta s}{\cancel{R_0}} \right]$$

$$X_s = X_{sc} = R_0 \tan\beta s \longrightarrow ③$$

$$\frac{X_s}{R_0} = \frac{X_{sc}}{R_0} = \tan\beta s \longrightarrow ④$$

where, $\dfrac{X_s}{R_0}$ is Normalised i/p reactance.

$$x_s = \frac{X_s}{R_0} \quad ; \quad x_{sc} = \frac{X_{sc}}{R_0}$$

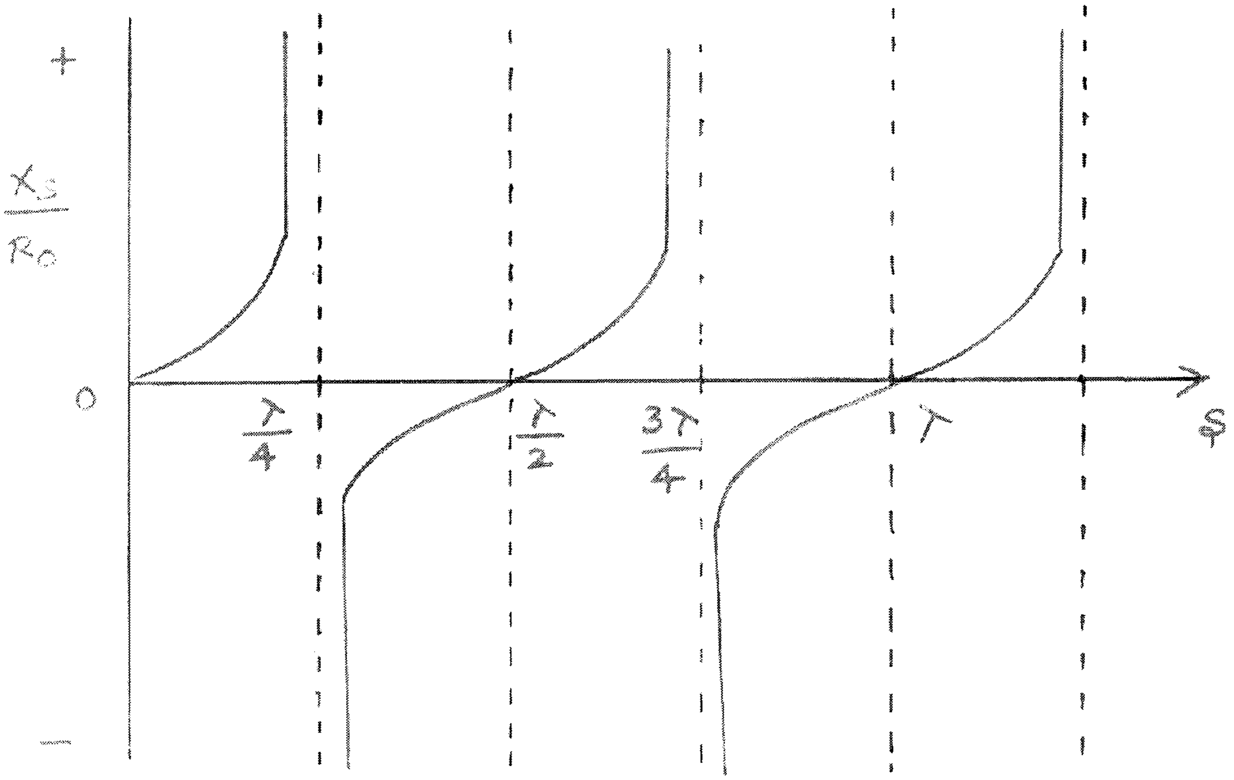
when, $s=0$, $\frac{X_s}{R_o} = \tan\left(\frac{2\pi}{\lambda} \cdot 0\right) = \tan(0) = 0$

$s = \frac{\lambda}{4}$, $\frac{X_s}{R_o} = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = \tan\left(\frac{\pi}{2}\right) = \infty$

$s = \frac{\lambda}{2}$, $\frac{X_s}{R_o} = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = \tan(\pi) = 0$

$s = \frac{3\lambda}{4}$, $\frac{X_s}{R_o} = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{3\lambda}{4}\right) = \tan\left(\frac{3\pi}{2}\right) = -\infty$

$s = \lambda$, $\frac{X_s}{R_o} = \tan\left(\frac{2\pi}{\lambda} \cdot \lambda\right) = \tan(2\pi) = 0$



Input Impedance of Open Circuited Dissipationless line :-

The i/p impedance of dissipationless line is expressed as,

$$Z_s = R_o \cdot \left[\frac{Z_R \cos\beta s + jR_o \cdot \sin\beta s}{R_o \cdot \cos\beta s + jZ_R \sin\beta s}\right] \rightarrow ①$$

$$Z_S = R_0 \cdot \frac{\cos\beta s}{\cos\beta s}\left[\frac{Z_R + jR_0 \frac{\sin\beta s}{\cos\beta s}}{R_0 + jZ_R \frac{\sin\beta s}{\cos\beta s}}\right]$$

$$Z_S = R_0\left[\frac{Z_R + jR_0 \tan\beta s}{R_0 + jZ_R \tan\beta s}\right] \longrightarrow (2)$$

For Open circuited Line $Z_R = \infty$

$$Z_S = R_0 \cdot \frac{\cancel{Z_R}}{\cancel{Z_R}}\left[\frac{1 + j\frac{R_0}{Z_R}\cdot\tan\beta s}{\frac{R_0}{Z_R} + j\tan\beta s}\right]$$

$$Z_S = R_0 \cdot \left[\frac{1 + j\frac{R_0}{Z_R}\cdot\tan\beta s}{\frac{R_0}{Z_R} + j\tan\beta s}\right]$$

put $Z_R = \infty$ in (2)

$$Z_S = R_0 \cdot \left[\frac{1 + 0}{0 + j\tan\beta s}\right]$$

$$Z_S = \frac{R_0}{j\tan\beta s}$$

$$Z_S = -jR_0\cot\beta s$$

$$\frac{Z_S}{R_0} = \frac{X_S}{R_0} = -\cot\beta s \longrightarrow (3)$$

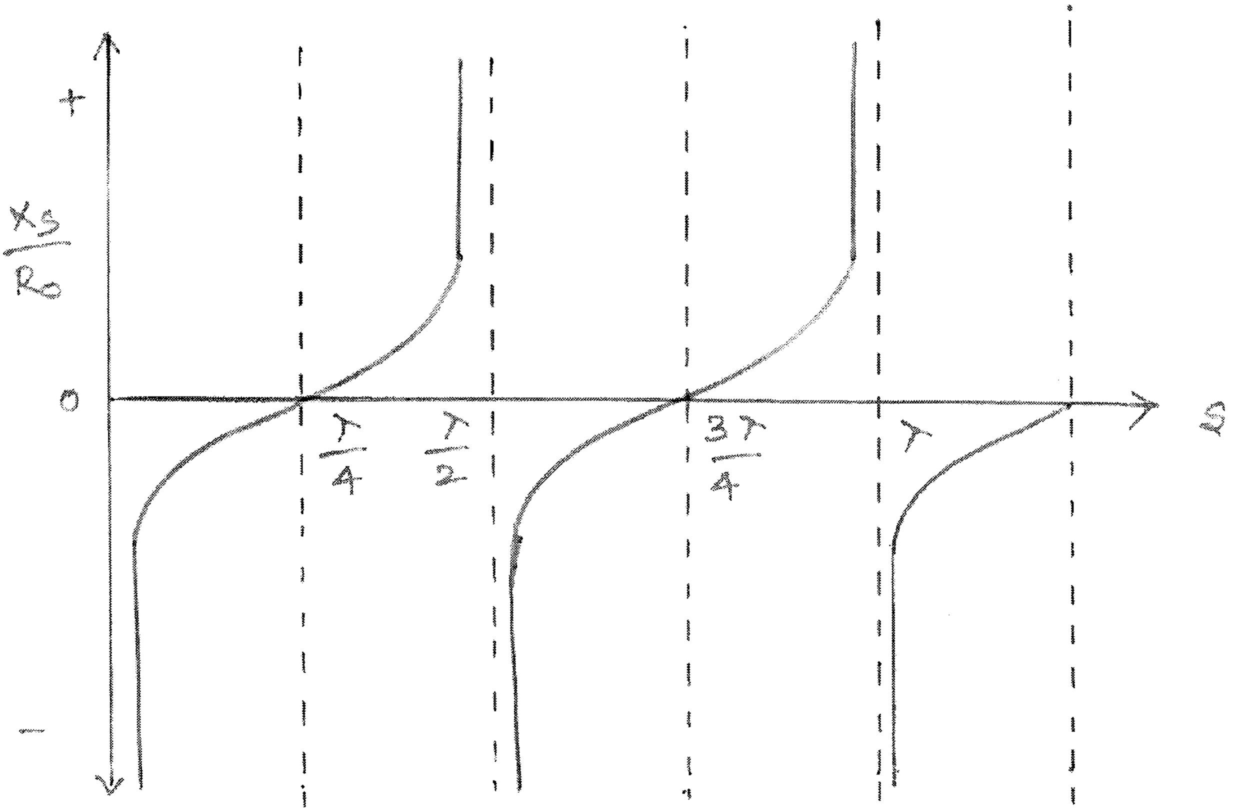
when $s = 0$, $\frac{X_S}{R_0} = -\cot\left(\frac{2\pi}{\lambda}\cdot 0\right) = -\cot(0) = -\infty$

$s = \frac{\lambda}{4}$, $\frac{X_S}{R_0} = -\cot\left(\frac{\cancel{2}\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{4}_2}\right) = -\cot\left(\frac{\pi}{2}\right) = 0$

$$s = \frac{\lambda}{2}, \quad \frac{X_s}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = -\cot(\pi) = +\infty$$

$$s = \frac{3\lambda}{4}, \quad \frac{X_s}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \cdot \frac{3\lambda}{4}\right) = -\cot\left(\frac{3\pi}{2}\right) = 0$$

$$s = \lambda, \quad \frac{X_s}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \cdot \lambda\right) = -\cot(2\pi) = -\infty$$



Voltage and Current on Dissipationless line :-

From general solution of txn. line, the voltage and current on a txn. line at any point can be expressed as,

$$E = \frac{E_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)\left[e^{\gamma s} + K \cdot e^{-\gamma s}\right] \rightarrow (1)$$

(or)

$$E = E_R \cdot \cosh \gamma s + I_R Z_0 \cdot \sinh \gamma s \rightarrow (2)$$

Similarly,

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_0}{Z_0}\right)\left[e^{\gamma s} - K \cdot e^{-\gamma s}\right] \rightarrow ③$$

(or)

$$I = I_R \cosh\gamma s + \frac{E_R}{Z_0} \sinh\gamma s \rightarrow ④$$

For dissipationless line (or) zero dissipation line

$$Z_0 = R_0 \ , \ \gamma = j\beta \ , \ R = G = \alpha = 0$$

$\therefore$ equs. ① → ④ becomes

$$E = \frac{E_R}{2}\left(\frac{Z_R + R_0}{Z_R}\right)\left[e^{j\beta s} + K \cdot e^{-j\beta s}\right] \rightarrow ⑤$$

(or)

$$E = E_R \cdot \cosh(j\beta)s + I_R R_0 \sinh(j\beta)s$$

$$E = E_R \cdot \cos\beta s + j\, I_R R_0 \sin\beta s \rightarrow ⑥ \checkmark$$

Similarly,

$$I = \frac{I_R}{2}\left(\frac{Z_R + R_0}{R_0}\right)\left[e^{j\beta s} - K \cdot e^{-j\beta s}\right] \rightarrow ⑦$$

(or)

$$I = I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s \rightarrow ⑧ \checkmark$$

For Short Circuit :- $(Z_R = 0)$; $E_R = 0$ ✓

sub $Z_R = 0$ in equs. ⑥ & ⑧

$$E_{sc} = I_R \cdot Z_R \cos\beta s + j I_R R_0 \sin\beta s$$

$$E_{sc} = 0 + j\, I_R R_0 \sin\beta s$$

$$\therefore E_{sc} = j I_R R_0 \sin\beta s \rightarrow ⑨$$

$$I_{sc} = I_R \cos\beta s + j \frac{I_R Z_R}{R_0} \sin\beta s$$

$$\therefore I_{sc} = I_R \cos\beta s \longrightarrow (10)$$

For Open Circuit : $(Z_R = \infty)$; $I_R = 0$

sub $Z_R = \infty$ in equs. (6) & (8)

$$E_{oc} = E_R \cos\beta s \longrightarrow (11)$$

$$I_{oc} = j \frac{E_R}{R_0} \sin\beta s \longrightarrow (12)$$

Power and Impedance measurement On dissipationless line :-

The expression for voltage and Current on the dissipationless line are given by

$$E = \frac{E_R}{2} \left(\frac{Z_R + R_0}{Z_R} \right) \left[e^{j\beta s} + K \cdot e^{-j\beta s} \right]$$

$$E = \frac{I_R}{2} (Z_R + R_0) \left[e^{j\beta s} + K \cdot e^{-j\beta s} \right] \longrightarrow (1)$$

lly

$$I = \frac{I_R}{2} \left(\frac{R_0 + Z_R}{R_0} \right) \left[e^{j\beta s} - K \cdot e^{-j\beta s} \right] \longrightarrow (2)$$

The Voltage and Current will be maximum when the reflected wave and incident wave are inphase.

The maximum voltage and current is expressed as,

$$E_{max} = \frac{I_R}{2}(Z_R + R_o)\left[1 + |K|\right] \rightarrow (3)$$

$$I_{max} = \frac{I_R}{2}\left(\frac{R_o + Z_R}{R_o}\right)\left[1 + |K|\right] \rightarrow (4)$$

The voltage and current will be minimum when the reflected wave and incident wave are out of phase

The minimum voltage and current is expressed a

$$E_{min} = \frac{I_R}{2}(Z_R + R_o)\left[1 - |K|\right] \rightarrow (5)$$

$$I_{min} = \frac{I_R}{2}\left(\frac{R_o + Z_R}{R_o}\right)\left[1 - |K|\right] \rightarrow (6)$$

$$\frac{E_{max}}{I_{max}} = R_o \rightarrow (7)$$

$$\frac{E_{min}}{I_{min}} = R_o \rightarrow (8)$$

The resistive impedance seen at a voltage loop is

$$\frac{E_{max}}{I_{min}} = R_{max} = R_o \cdot \left(\frac{1 + |K|}{1 - |K|}\right) = R_o \cdot S$$

$$\therefore \boxed{R_{max} = R_o \cdot S} \rightarrow (9)$$

Since the voltage and current are again inphase at a current loop, the resistive impedance seen at a current loop is

$$\frac{E_{min}}{I} = R_{min} = R_o \cdot \left(\frac{1 - |K|}{1 + |K|}\right) = \frac{R_o}{S}$$

$$\therefore \boxed{R_{min} = \frac{R_0}{S}} \rightarrow (10)$$

The power passing a voltage loop is the power effectively flowing into a resistance R_{max} at voltage E_{max}, so that

$$P = \frac{E^2_{max}}{R_{max}} \rightarrow (11)$$

The same value of power must also pass the current loop, effectively flowing into a resistance R_{min} at voltage E_{min}, so that

$$P = \frac{E^2_{min}}{R_{min}} \rightarrow (12)$$

Then, $$P^2 = \frac{E^2_{max} \cdot E^2_{min}}{R_{max}\ R_{min}} \Rightarrow \frac{E^2_{max} \cdot E^2_{min}}{\cancel{S} \cdot R_0 \cdot \frac{R_0}{\cancel{S}}}$$

$$P^2 = \frac{E^2_{max} \cdot E^2_{min}}{R_0^2}$$

$$\therefore P = \frac{|E_{max}| \cdot |E_{min}|}{R_0} \rightarrow (13)$$

The power may also be expressed as,

$$P = |I_{max}| \cdot |I_{min}| \cdot R_0 \rightarrow (14)$$

Measurement of Unknown Impedance :

The i/p impedance of a dissipationless line is given as,

$$Z_s = R_0 \cdot \left[\frac{Z_R + jR_0 \tan\beta s}{R_0 + jZ_R \tan\beta s}\right] \rightarrow (1)$$

R_{min} at a distance s' is given by,

$$Z_s(\text{min}) = R_{min} = \frac{R_0}{S} \rightarrow (2)$$

Sub (2) in (1)

$$\frac{\cancel{R_0}}{S} = \cancel{R_0} \cdot \left[\frac{Z_R + jR_0 \tan\beta s'}{R_0 + jZ_R \tan\beta s'}\right]$$

$$\frac{1}{S} = \frac{Z_R + jR_0 \tan\beta s'}{R_0 + jZ_R \tan\beta s'}$$

$$R_0 + jZ_R \tan\beta s' = SZ_R + jSR_0 \tan\beta s'$$

$$R_0 - jSR_0 \tan\beta s' = SZ_R - jZ_R \tan\beta s'$$

$$R_0 (1 - jS \tan\beta s') = Z_R (S - j\tan\beta s')$$

$$\therefore Z_R = \frac{R_0 (1 - jS\tan\beta s')}{(S - j\tan\beta s')} \rightarrow (3)$$

Reflection loss in High frequency lines:

Reflection loss is defined as the ratio of power delivered to the load to the incident power.

$$\text{Reflection loss} = \frac{P_L}{P_i}$$

$$\text{Reflection loss in dB} = 10 \log \frac{P_L}{P_i}$$

$$= 10 \log \frac{P_i - P_r}{P_i}$$

$$= 10 \log \left(1 - \frac{P_r}{P_i}\right)$$

$$= 10 \log \left(1 - |K|^2\right)$$

$$= 10 \log \left(1 - \left(\frac{S-1}{S+1}\right)^2\right) \qquad \left(\because |K| = \frac{S-1}{S+1}\right)$$

$$= 10 \log \left(\frac{(S+1)^2 - (S-1)^2}{(S+1)^2}\right)$$

$$= 10 \log \left(\frac{4S}{(S+1)^2}\right)$$

$$= 10 \log \left(\frac{2\sqrt{S}}{S+1}\right)^2$$

$$\therefore \text{Reflection loss in dB} = 20 \log \left(\frac{2\sqrt{S}}{S+1}\right)$$

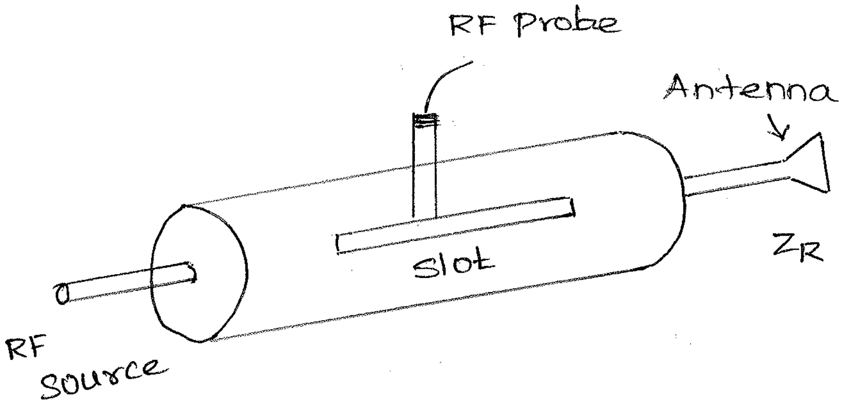
Measurement of VSWR and wavelength:

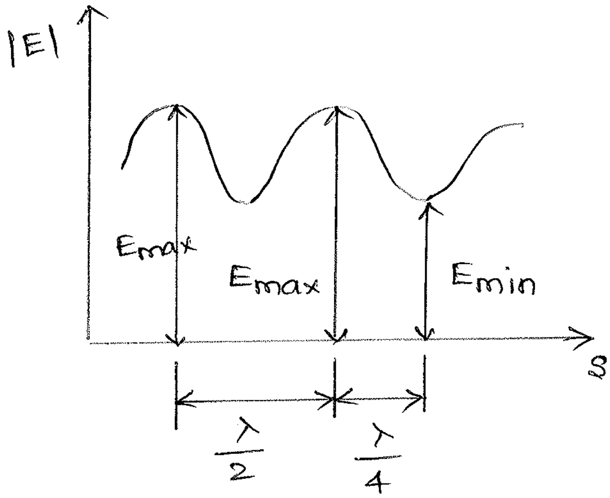
VSWR can be measured using 2 techniques.

1. Slotted line measurement
2. Directional coupler measurement

(1) Slotted line measurement:

This method is used to measure SWR and wavelength.





A longitudinal slot of length $n\lambda/2$ is cut on the coaxial line. A wire probe is inserted into the air dielectric of the line as a pickup device, a voltmeter or other detector connected between probe and sheath. Since the distance between V_{min} and V_{max} is $\lambda/4$, by placing the probe at a point which is $\lambda/4$ away from V_{min}, V_{max} can be obtained. The ratio of V_{max} to V_{min} gives the value of SWR.

$$SWR = \frac{V_{max}}{V_{min}}$$

From the value of SWR, $|K|$ can be calculated using equation,

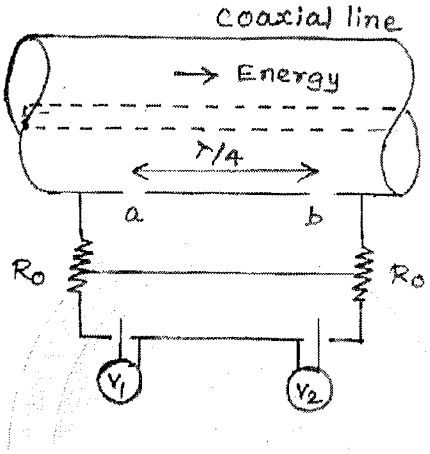
$$|K| = \frac{S-1}{S+1}$$

The same technique can also be used to measure the wavelength on the line. Wavelength can be calculated by considering the distance between two successive V_{max} or V_{min} as $\lambda/2$ or by considering the distance between V_{min} & V_{max} as $\lambda/4$. This kind of measurements are called Lecher measurements.

(2) <u>Directional coupler measurement</u>:

It consists of a section of coaxial transmission line, having two small holes in the outer sheath spaced by $\lambda/4$.

i) Directional coupler :



Clamped over these holes is a small section of line, terminated in its R_0 value at both ends to prevent reflections.

Some energy will leak through the holes, and will set up a wave traveling to both left and right in the second line.

If the main line is transmitting energy to the right, then a wave entering the secondary line through hole a and traveling to the right will be in phase, setting up a wave traveling to the right in the secondary line. This gives an indication on V_2 & is considered as V_i.

When a wave entering the secondary line through hole 'a' travels in left direction, V_1 shows indication & this can be considered as V_r (reflected voltage).

The ratio of V_r to V_i gives the value of K (Reflection coefficient). From the value of K, SWR can be calculated using the equation,

$$SWR = \frac{1 + |K|}{1 - |K|}$$

Parameters of open wire line at high frequencies:

At high frequencies, current is considered to flow on the surface of the conductor and hence internal flux and internal inductance are reduced nearly to zero.

The inductance of open wire line is given as,

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{a}\right) = 4 \times 10^{-7} \ln\left(\frac{d}{a}\right) \quad H/m$$

or $$L = 9.21 \times 10^{-7} \log\left(\frac{d}{a}\right) \quad H/m$$

The capacitance of a line is not affected by skin effect or frequency and hence it is given as,

$$C = \frac{\pi \epsilon}{\ln\left(\frac{d}{a}\right)} \quad F/m$$

$$= \frac{27.7}{\ln(d/a)} \quad \mu\mu F/m = \frac{12.07}{\log(d/a)} \quad \mu\mu F/m$$

The effective thickness of the surface layer of current may be considered as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad \text{meters}$$

where $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m , $\sigma = 5.75 \times 10^{7}$ mho/m

$$\therefore \quad \delta = \frac{0.0664}{\sqrt{f}}$$

The resistance of a round conductor of radius a meters to direct current is inversely proportional to the area as

$$R_{dc} = \frac{K}{\pi a^2}$$

While that of a round conductor with alternating current flowing in a skin of thickness δ is

$$R_{ac} = \frac{K}{2\pi a \delta}$$

Therefore the ratio of resistance to alternating current to resistance to direct current is

$$\frac{R_{ac}}{R_{dc}} = \frac{a\sqrt{\pi f \mu \sigma}}{2}$$

Which becomes, for copper,

$$\frac{R_{ac}}{R_{dc}} = 7.53\, a\sqrt{f}$$ (a → radius of the conductor)

The above equation shows that increase in resistance with increasing frequency is greater for large radius than for small radius conductors.

The resistance of an open wire line of copper, with spacing greater than 20a, is given as

$$R_{ac} = \frac{8.33 \times 10^{-8} \sqrt{f}}{a} \text{ ohms/meter of line}$$

<u>Parameters of coaxial line at high frequencies:</u>

In coaxial line, due to skin effect at high frequencies, internal flux and internal inductance can be neglected. ∴ The inductance of the coaxial line is given as

$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) = 2 \times 10^{-7} \ln\left(\frac{b}{a}\right) \text{ H/m}$$

$$L = 4.6 \times 10^{-7} \log\left(b/a\right) \text{ H/m}$$

The capacitance of the coaxial line, which is not affected by frequency, is given as

$$C = \frac{2\pi \epsilon}{\ln(b/a)} \text{ Fd/m}$$

$$= \frac{55.5\, \epsilon_r}{\ln(b/a)} = \frac{24.14\, \epsilon_r}{\log(b/a)} \ \mu\mu\text{F/m}$$

The resistance of the coaxial line is given as

$$R_{ac} = 4.16 \times 10^{-8} \sqrt{f} \left(\frac{1}{b} + \frac{1}{a} \right) \text{ ohms/m of the line}$$

where a → Outer radius of the inner conductor

b → Inner radius of the outer conductor

The shunt susceptance of the coaxial line is given as

$$y = g + j\omega c$$

The quality of the insulating material is measured in terms of power factor & is given as,

$$Pf = \frac{g}{\sqrt{g^2 + \omega^2 c^2}}$$

For good insulating material, $g << \omega c$

$$\therefore \; Pf = \frac{g}{\omega c}$$

The quality of the dielectric is also expressed in terms of the dissipation factor, which is the ratio of energy dissipated to energy stored in the dielectric per cycle.

UNIT 3

Impedance matching in High frequency lines

Impedance matching is the process of matching the load impedance to the characteristic impedance of the line or matching the line impedance to source impedance at the load or source side respectively.

Impedance matching can be achieved using

(1) Half wave line

(2) One Eighth wave line

(3) Quarter wave line

(4) Stub

(1) Half wave line or $\lambda/2$ line:

If the length of the transmission line is exactly equal to half the wavelength of the signal $(\lambda/2)$, then the line can be called as Half wave line.

To calculate its Z_{IN}, let us consider.

$$Z_{IN} = \frac{R_0 (Z_R + j R_0 \tan \beta s)}{R_0 + j Z_R \tan \beta s}$$

$s = l = \lambda/2$

$$\beta s = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$\therefore \tan \beta s = \tan \pi = 0$

$$\therefore \boxed{Z_{IN} = Z_R}$$

Since $Z_{IN} = Z_R$, half wave line is also called as one to one transformer.

(2) <u>Quarter wave line or $\lambda/4$ line:</u>

If the length of the transmission line is exactly equal to $\lambda/4$, then the line can be called as quarter wave line or $\lambda/4$ line.

$$Z_{IN} = \frac{R_0 (Z_R + j R_0 \tan \beta s)}{R_0 + j Z_R \tan \beta s}$$

$s = l = \lambda/4$

$$\beta s = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi/2$$

$\tan \beta s = \tan \pi/2 = \infty$

$$\therefore Z_{IN} = \frac{R_0 \cdot \tan \beta s \left(\frac{Z_R}{\tan \beta s} + j R_0 \right)}{\tan \beta s \left(\frac{R_0}{\tan \beta s} + j Z_R \right)}$$

If $\tan \beta s \to \infty$, then

$$\boxed{Z_{IN} = \frac{R_0^2}{Z_R}}$$

or $R_0 = \sqrt{Z_{IN} Z_R}$

A quarter wave line is used as a transformer to match a load of Z_R ohms to a source of Z_s ohms.

As $Z_{IN} \propto \frac{1}{Z_R}$, $\lambda/4$ line is also called as impedance inverter & can be widely used in achieving impedance matching.

<u>Applications of $\lambda/4$ line:</u>

1. It is used as impedance inverter
2. It is used in impedance matching at source & load of a line
3. It is used to match the impedance of a transmission line to a resistive load such as antenna.
4. It is used to match any complex load to a line with Z_0.

5. It is used as an insulator as the input impedance of a short circuited $\lambda/4$ line is infinite.

(3) <u>One-eighth wave line or $\lambda/8$ line:</u>

If the length of the transmission line is exactly equal to $\lambda/8$, then the line can be called as $\lambda/8$ line.

$$Z_{IN} = \frac{R_0 \left(Z_R + j R_0 \tan \beta s \right)}{R_0 + j Z_R \tan \beta s}$$

$$s = l = \frac{\lambda}{8}$$

$$\beta s = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \pi/4$$

$$\tan \beta s = \tan \pi/4 = 1$$

$$\therefore Z_{IN} = \frac{R_0 \left(Z_R + j R_0 \right)}{R_0 + j Z_R}$$

$$|Z_{IN}| = \frac{R_0 \sqrt{Z_R^2 + R_0^2}}{\sqrt{Z_R^2 + R_0^2}} = R_0$$

Thus, $\lambda/8$ line is used to obtain magnitude match between R_0 & source impedance & therefore called as magnitude matching line.

Stub Matching:

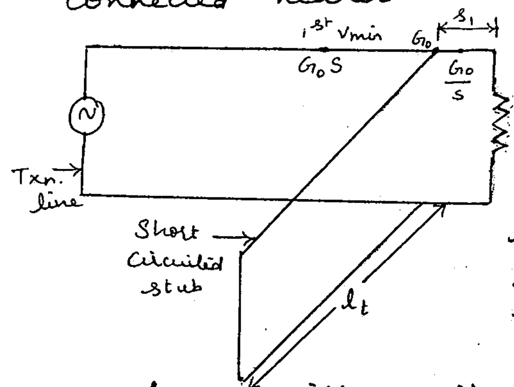
The process of achieving impedance matching using stub is called stub matching.

*. Stub is a section of transmission line which can be used in achieving impedance matching.

*. Stub has to be connected in parallel with the transmission line and nearer to the load.

How stub is used in achieving impedance matching?

Consider a transmission line which is improperly terminated ($Z_R \neq Z_0$). To match the impedance at the load, a stub has to be connected nearer to the load.



At 1st V_{min} point, $Y = G_0 S$

At 1st V_{max} point $Y = G_0 S$

$\therefore$ In between these two points, there must be a point on the line with $Y = G_0$. Exactly, at that point stub has to be connected.

Let admittance of the line be,

$$Y_{line} = G \pm jB$$

(where,
$G \rightarrow$ conductance
$G = \frac{1}{R}$
$B \rightarrow$ Susceptance)

Let at point s_1, the conductance of the line is G_0.

$\therefore$ At s_1, $Y_{line} = G_0 \pm jB$

A stub has to be connected at point s_1, such that its susceptance is chosen to be equal & opposite to that of line.

ie. $Y_{stub} = \mp jB$

Since, line and stub are connected in parallel, the total admittance is given as,

$$Y_{total} = Y_{line} + Y_{stub}$$

$$= G_0 \pm jB \mp jB$$

$$Y = G_0$$

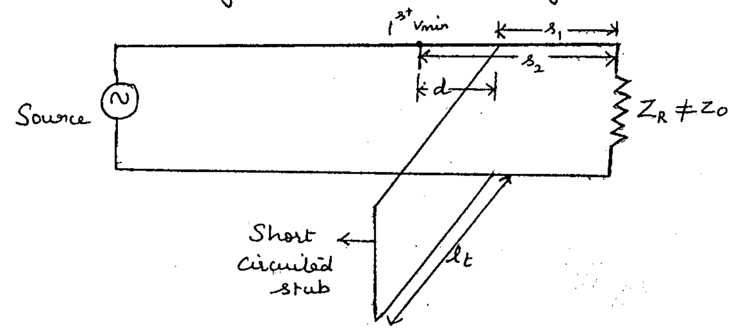
$$\Rightarrow Z = R_0$$

Thus, when stub is connected in parallel with the line at the point where conductance of the line is G_0, $Z = R_0$ & $\therefore$ impedance matching is achieved at the load.

A short circuited stub is preferred over open circuited stub because of less radiation loss & easy tuning.

Single stub matching

The process of achieving impedance matching by connecting single stub in parallel with the line is called Single stub matching.



Let $s_2 \rightarrow$ Distance between load & 1^{st} V_{min}

l_s or $s_1 \rightarrow$ Distance between load and the stub (Location of stub)

$d \rightarrow$ distance between s_2 and s_1

L or $l_t \rightarrow$ Length of the stub

WKT,

$$Z_{IN} = \frac{R_0 \left(1 + |K| \underline{|\phi - 2\beta s}\right)}{1 - |K| \underline{|\phi - 2\beta s}} \quad \text{--- (1)}$$

For the design of stub, admittance can be considered.

$$\therefore \quad Y_{IN} = \frac{1}{R_0} \frac{\left(1 - |K| \underline{|\phi - 2\beta s}\right)}{1 + |K| \underline{|\phi - 2\beta s}}$$

$$Y_{IN} = G_0 \frac{(1 - |K| \angle \phi - 2\beta s)}{(1 + |K| \angle \phi - 2\beta s)}$$

$$\frac{Y_{IN}}{G_0} = Y_{in} = \frac{1 - |K| \angle \phi - 2\beta s}{1 + |K| \angle \phi - 2\beta s} \quad \text{—— (2)}$$

[where Y_{in} is called normalised admittance]

Let $\phi - 2\beta s = \theta$

$$\therefore Y_{in} = \frac{1 - |K| \angle \theta}{1 + |K| \angle \theta} = \frac{1 - |K| e^{j\theta}}{1 + |K| e^{j\theta}}$$

$$= \frac{1 - |K| (\cos\theta + j\sin\theta)}{1 + |K| (\cos\theta + j\sin\theta)}$$

$$Y_{in} = \frac{1 - |K|\cos\theta - j|K|\sin\theta}{1 + |K|\cos\theta + j|K|\sin\theta}$$

Taking complex conjugate of RHS we get,

$$Y_{in} = \frac{1 - |K|\cos\theta - j|K|\sin\theta}{1 + |K|\cos\theta + j|K|\sin\theta} \times \frac{1 + |K|\cos\theta - j|K|\sin\theta}{1 + |K|\cos\theta - j|K|\sin\theta}$$

On simplifying the above expression, we get,

$$Y_{in} = \frac{1 - |K|^2 - 2j|K|\sin\theta}{1 + |K|^2 + 2|K|\cos\theta} \quad \text{—— (3)}$$

As $Y_{in} = \frac{Y_{IN}}{G_0} = \frac{G_{IN} + jB_{IN}}{G_0}$, equation (3) can be written as,

$$\frac{G_{IN} + jB_{IN}}{G_0} = \frac{1 - |K|^2 - 2j|K|\sin\theta}{1 + |K|^2 + 2|K|\cos\theta}$$

Equating real & imaginary terms, we get,

$$\frac{G_{IN}}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|\cos\theta} \quad \text{—— (4)}$$

&

$$\frac{B_{IN}}{G_0} = \frac{-2|K|\sin\theta}{1 + |K|^2 + 2|K|\cos\theta} \quad \text{—— (5)}$$

where $\theta = \phi - 2\beta s$.

<u>To find s_1:</u>

At point s_1 (ie. the point of connecting the stub with the line), $G_{IN} = G_0$

$$\therefore \frac{G_{IN}}{G_0} = 1$$

Sub. $\frac{G_{IN}}{G_0} = 1$ in equation (4), we get

$$1 = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|\cos(\phi - 2\beta s_1)}$$

($\because$ at $s = s_1$, $\theta = \phi - 2\beta s_1$)

$$\Rightarrow 1 + |K|^2 + 2|K|\cos(\phi - 2\beta s_1) = 1 - |K|^2$$

$$2|K|\cos(\phi - 2\beta s_1) = -2|K|^2$$

$$\cos(\phi - 2\beta s_1) = -|K|$$

$$\phi - 2\beta s_1 = \cos^{-1}(-|K|)$$

$$\phi - 2\beta s_1 = \cos^{-1}|K| - \pi$$

$$\therefore \boxed{s_1 = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}} \quad \text{—— (I)}$$

(or)

$$\therefore \boxed{l_s = s_1 = \frac{\lambda}{4\pi}(\phi + \pi - \cos^{-1}|K|)}$$

To find s_2:

At $s = s_2$, $G_{IN} = S G_0$.

$\therefore$ Sub. $\frac{G_{IN}}{G_0} = S$ in equation (4)

[At $s = s_2$
$\theta = \varphi - 2\beta s_2$)

$$S = \frac{1 - |K|^2}{1 + |K|^2 + 2|K| \cos(\varphi - 2\beta s_2)}$$

$$\frac{1 + |K|}{1 - |K|} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K| \cos(\varphi - 2\beta s_2)}$$

$$\Rightarrow \quad 1 + |K|^2 + 2|K| \cos(\varphi - 2\beta s_2) = 1 - 2|K| + |K|^2$$

$$2|K| \cos(\varphi - 2\beta s_2) = -2|K|$$

$$\cos(\varphi - 2\beta s_2) = -1$$

$$\therefore \quad \varphi - 2\beta s_2 = \cos^{-1}(-1)$$

$$\varphi - 2\beta s_2 = -\pi$$

$$\therefore \quad \boxed{s_2 = \frac{\varphi + \pi}{2\beta}} \qquad \text{—— (II)}$$

To find d:

$$d = s_2 - s_1$$

$$= \frac{\varphi + \pi}{2\beta} - \frac{\varphi + \pi - \cos^{-1}|K|}{2\beta}$$

$$\boxed{d = \frac{\cos^{-1}|K|}{2\beta}} \qquad \text{—— (III)}$$

To find l_t (length of the stub):

$$\text{Equation (5)} \Rightarrow \quad \frac{B_{IN}}{G_0} = \frac{-2|K| \sin\theta}{1 + |K|^2 + 2|K| \cos\theta}$$

[where $\theta = \varphi - 2\beta s$]

At $s = s_1$, $\cos(\varphi - 2\beta s_1) = -|K|$

Let $\theta = \varphi - 2\beta s_1$; $\cos\theta = -|K|$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\therefore \quad \sin\theta = \sqrt{1 - |K|^2}$$

Sub. $\sin\theta = \sqrt{1 - |K|^2}$ & $\cos\theta = -|K|$ in eqn (5),

$$\frac{B_{IN}}{G_0} = \frac{-2|K| \cdot \sqrt{1 - |K|^2}}{1 + |K|^2 + 2|K|(-|K|)}$$

$$= \frac{-2|K| \sqrt{1 - |K|^2}}{1 - |K|^2}$$

$$\frac{B_{IN}}{G_0} = \frac{-2|K|}{\sqrt{1 - |K|^2}}$$

$$\therefore \quad B_{IN} \text{ of a line} = G_0 \cdot \frac{-2|K|}{\sqrt{1 - |K|^2}}$$

B_{stub} (B of a stub) must be equal to $G_0 \cdot \frac{2|K|}{\sqrt{1 - |K|^2}}$ —— (6)

[Bcoz The susceptance of the stub must be equal & opposite to that of line].

For a short circuit stub, $Z_{sc} = j R_0 \tan \beta s$

$\therefore$ For a S.C stub of length 'l_t',

$$Y = \frac{1}{j R_0 \tan \beta l_t} = \frac{-j G_0}{\tan \beta l_t}$$

$$j B_{sc} = \frac{-j G_0}{\tan \beta l_t}$$

$$\therefore |B_{sc}| = \frac{G_0}{\tan \beta l_t} \quad \text{—— (7)}$$

Equating equation (6) and (7) we get,

$$G_0 \cdot \frac{2|K|}{\sqrt{1-|K|^2}} = \frac{G_0}{\tan \beta l_t}$$

$$\tan \beta l_t = \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$\therefore \beta l_t = \tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$\boxed{l_t = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)} \qquad \left[\because \beta = \frac{2\pi}{\lambda}\right]$$

where $|K| = \frac{S-1}{S+1}$

Summary:

Location of stub $\rightarrow l_s$ or $s_1 = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}$

Location of 1^{st} $V_{min} \rightarrow s_2 = \frac{\phi + \pi}{2\beta}$

$$d = \frac{\cos^{-1}|K|}{2\beta}$$

Length of the stub, $l_t = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$

or $l_t = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{S}}{S-1}\right)$

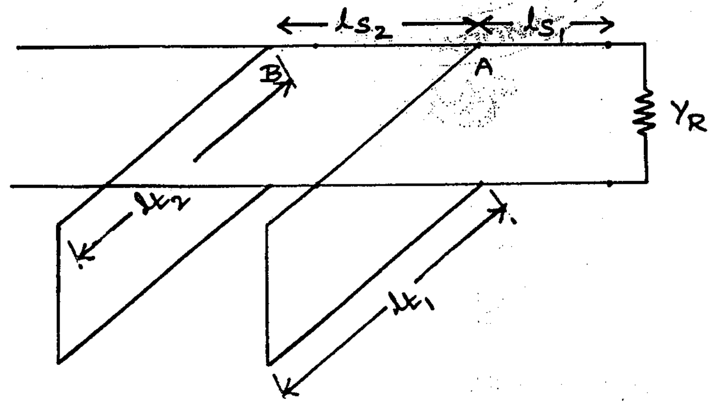
Disadvantages :-

1. The single stub matching is useful for fixed frequency only because as the frequency changes, the location of the stub will have to be changed.

2. For final adjustment, the stub has to be moved along the line slightly. This is possible only in open wire lines and not on coaxial lines.

DOUBLE STUB MATCHING :-

To overcome the disadvantages of single stub matching two short circuited stubs whose lengths are adjustable independently but whose positions are fixed.



Let the 1^{st} stub whose length is l_{t_1} be located at point 'A'. at a distance of l_{s_1} from the load end.

The input admittance at point A is given by

$$Y_A = Y_0 \cdot \left[\frac{Y_R + jY_0 \tan \beta s}{Y_0 + jY_R \tan \beta s}\right] \rightarrow (1)$$

Find the normalized admittance

$$\frac{Y_A}{Y_0} = y_A = \left[\frac{y_r + j\tan\beta s}{1 + j y_r \tan\beta s}\right] \rightarrow (2)$$

Put $s = l_{s_1}$ and rationalize equ (2)

$$y_A = \frac{y_r + j\tan\beta l_{s_1}}{1 + j y_r \tan\beta l_{s_1}} \times \frac{1 - j y_r \tan\beta l_{s_1}}{1 - j y_r \tan\beta l_{s_1}}$$

$$= \frac{y_r(1 - j y_r \tan\beta l_{s_1}) + j\tan\beta l_{s_1}(1 - j y_r \tan\beta l_{s_1})}{(1 + j y_r \tan\beta l_{s_1})(1 - j y_r \tan\beta l_{s_1})}$$

$$= \frac{y_r - j y_r^2 \tan\beta l_{s_1} + j\tan\beta l_{s_1} + y_r \tan^2\beta l_{s_1}}{1 + y_r^2 \tan\beta l_{s_1}}$$

$$y_A = \frac{y_r(1 + \tan^2\beta l_{s_1}) + j\tan\beta l_{s_1}(1 - y_r^2)}{1 + y_r^2 \tan\beta l_{s_1}}$$

Equate real and imaginary parts

$$g_A = \frac{y_r(1 + \tan^2\beta l_{s_1})}{1 + y_r^2 \tan\beta l_{s_1}} \rightarrow (3)$$

$$b_A = \frac{\tan\beta l_{s_1}(1 - y_r^2)}{1 + y_r^2 \tan\beta l_{s_1}} \rightarrow (4)$$

When a stub having a susceptance b_1 is added at this point A, the new admittance value will be,

$$y_A' = g_A + j b_A' \rightarrow (5)$$

Since only the susceptance value is altered by the addition of the stub, the conductance part remains unchanged. Here $b_A' = b_A \pm b_1$.

The Input admittance of line at point 'B' should be equal to 'G_0' so that the line appears to be terminated with Y_0. The normalized admittance at point B is

$$y_B = 1 + j b_B \rightarrow (6)$$

Finally the length of stub 2 is adjusted to produce a susceptance $-j b_B$ and the desired admittance is '1' at point B.

Typically 2 stubs are separated by fixed distances $\frac{\lambda}{4}, \frac{\lambda}{8}, \frac{\lambda}{16}, \frac{3\lambda}{8}$. The most commonly preferred is $\frac{\lambda}{4}$ and $\frac{3\lambda}{8}$.

Matching takes place between point B and the generator. So, there are chances of having reflection loss in between point B and the Load. In order to minimize the loss, the stubs are located very close to the load. Sometimes the first stub is located at load itself. But the common practice is to keep distance of 0.1λ to 0.15λ between load and 1st stub.

SMITH CHART PROCEDURE

1. Calculation of SWR:

Step 1: Locate the given normalized load impedance point and let it be 'P'.

Step 2:With '0' as centre and OP as radius draw a circle.

Step 3:The right hand intersection of the circle and the horizontal axis gives the value of SWR.

2. Calculation of Reflection coefficient, k:

Step 1:Locate the given normalized load impedance point and let it be 'P'.

Step 2:Draw the line OP and extent it to cut the 'Angle of reflection coefficient circle' at point P'. Note down the angle corresponding to this point. This gives angle of k.

Step 3:Measure the line length of OP and OP'. The ratio of OP to OP' gives the value of magnitude of k.

3. Procedure for single stub matching:

Step 1: Find the normalized load impedance and mark it as point A.

Step 2: With OA as radius draw S circle

Step 3: To find the normalized load admittance, draw a line joining OA and extend the same line till it cuts the S Circle. Mark the intersection point as B.

Step 4: To find the position of load, extend the OA line till it cuts the wavelength scale (outer circle). Mark it as point C

Step 5: Draw a unit circle (R=1)

Step 6: Select the intersection point of S circle with unit circle nearer to the load. Mark it as point D

Step 7: To find the position of stub on the main line, draw a line joining OD and extend the same line till it cuts the outer circle. Mark the point as E.

Step 8: To find the position of the stub 'l_s' from the load subtract D from E (i.e., E – D).

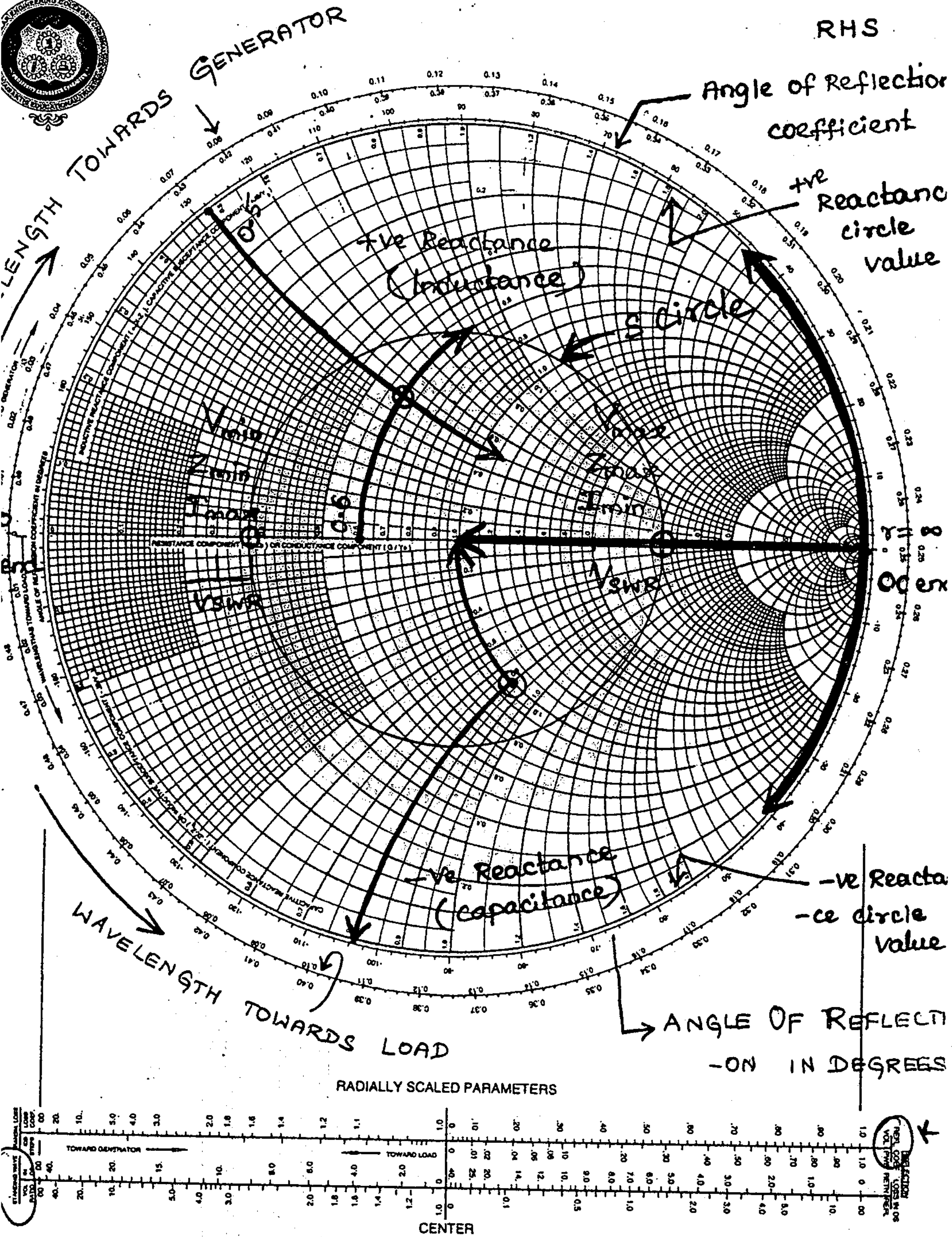
Step 9: Find the imaginary value at point D and mark the opposite value of it as F.

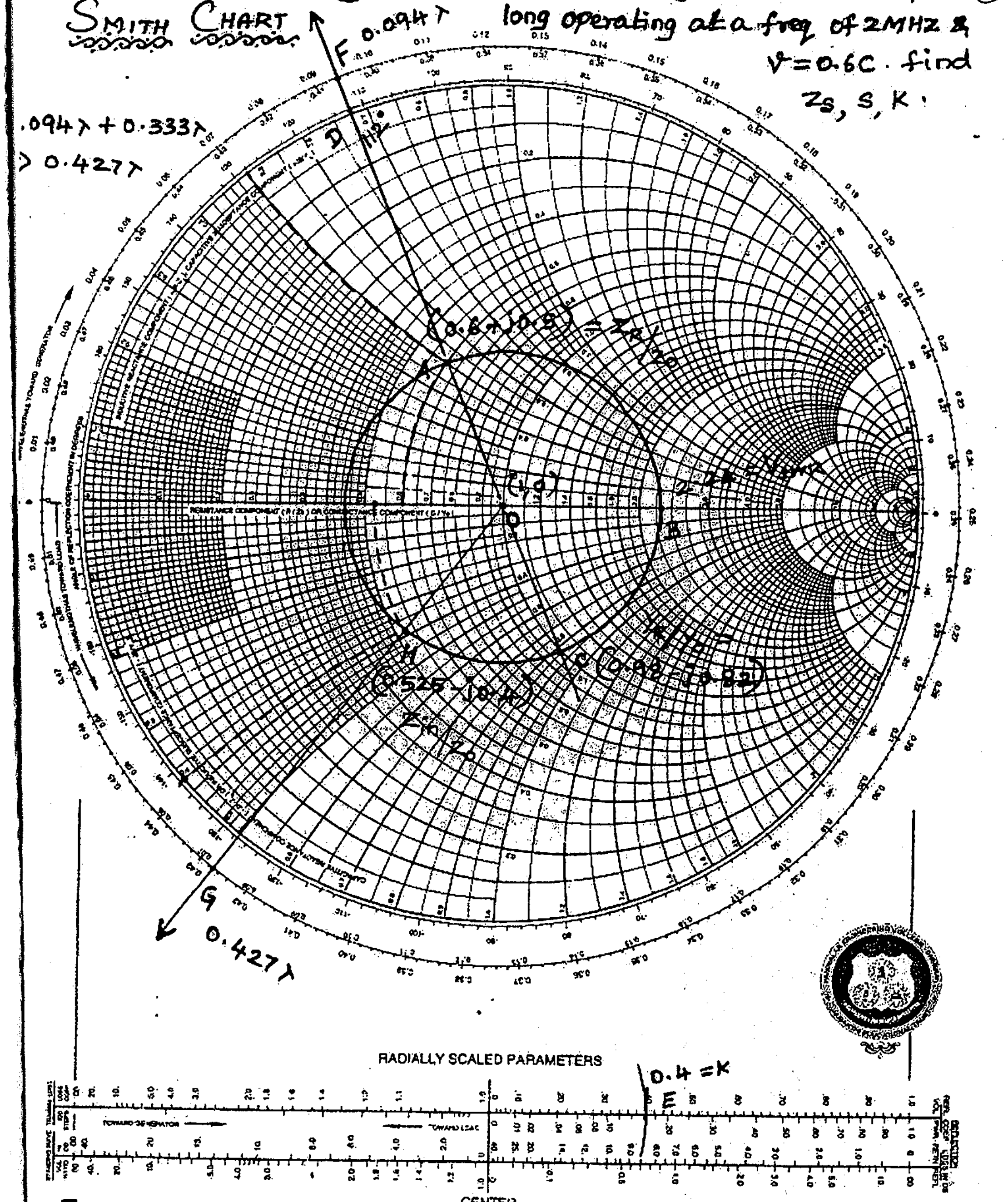
Step 10: Draw a line joining OF till it cuts the outer circle. Mark the intersection point on outer circle as G

Step 11: To find the total length of the stub 'l_t', subtract 0.25 from G point.

APPLICATIONS OF SMITH CHART

(180+j150)Ω. If the length of the line is 30m long operating at a freq of 2MHz & $v = 0.6c$ find Z_S, S, K. (10)





SMITH CHART

$$\frac{Z_S}{Z_0} = 0.525 - j0.4$$

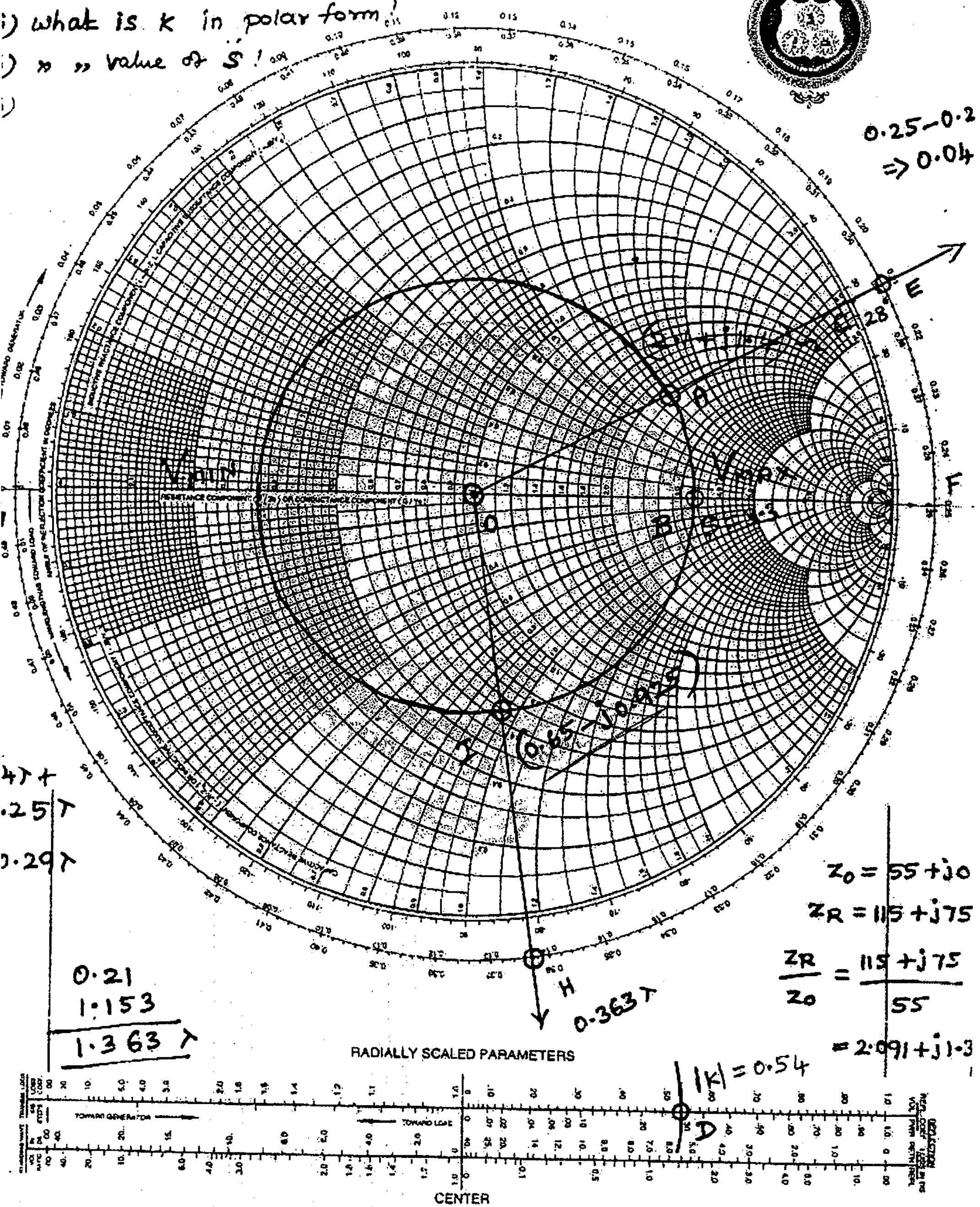
$$Z_S = 300\,(0.525 - j0.4)$$

$$= 158 - j120\ \Omega$$

a Load imp., of $(115+j75)\Omega$.

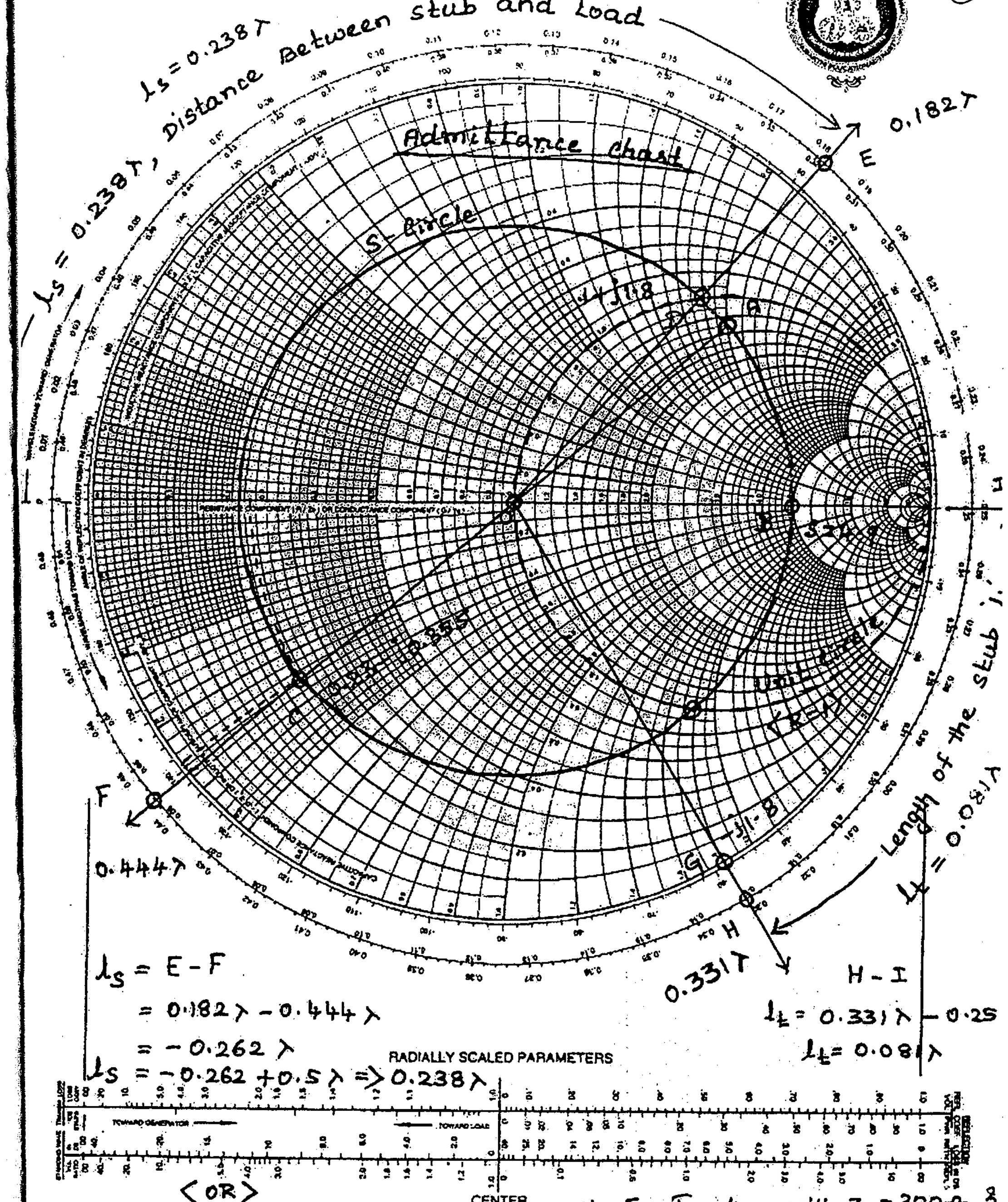
i) what is k in polar form?

ii) ,, ,, value of S?



iii) At what distance from the load will the 1st maximum and minimum of voltage will occur?

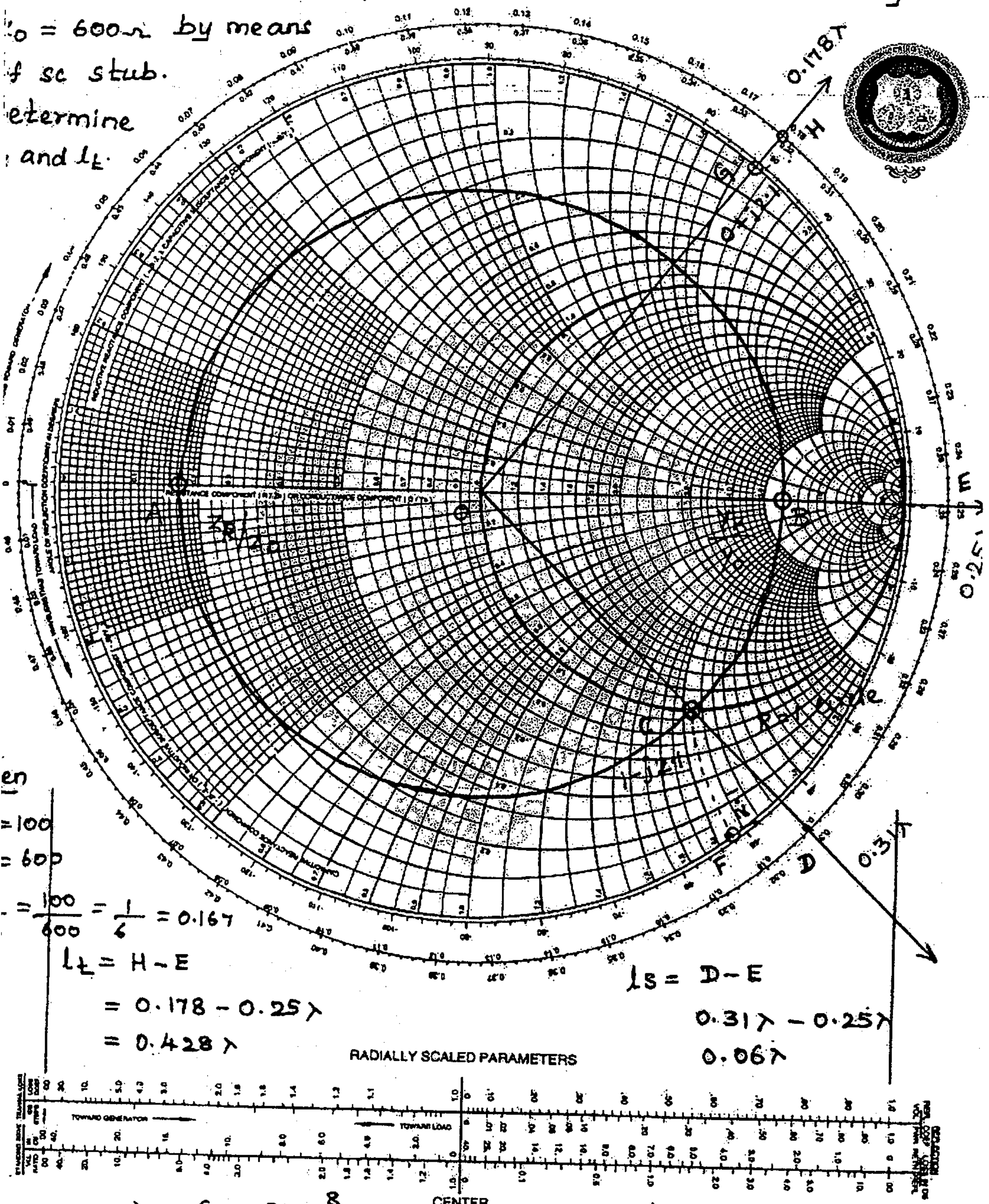
iv) If the line is 1.152λ long what will be i/p Impedance?



* For Txn. line with $z_0 = 300\Omega$ & Load Impedance $Z_L = 390+j600$.

Determine SWR, the distance a sc stub must be placed from the Load &

...be matched at a freq. of 100 MHz to a Tan. line having $Z_0 = 600\Omega$ by means of sc stub. Determine l_s and l_t.



Given

$=100$

$=600$

$= \frac{100}{600} = \frac{1}{6} = 0.167$

$l_t = H - E$

$= 0.178 - 0.25\lambda$

$= 0.428\lambda$

$l_s = D - E$

$0.31\lambda - 0.25\lambda$

0.06λ

$\lambda = \frac{c}{f} = \frac{3\times10^8}{100\times10^6}$

$= 3\,m$

$l_s = 0.06\lambda$

$= 0.06\times3$

$= 0.18\,m$

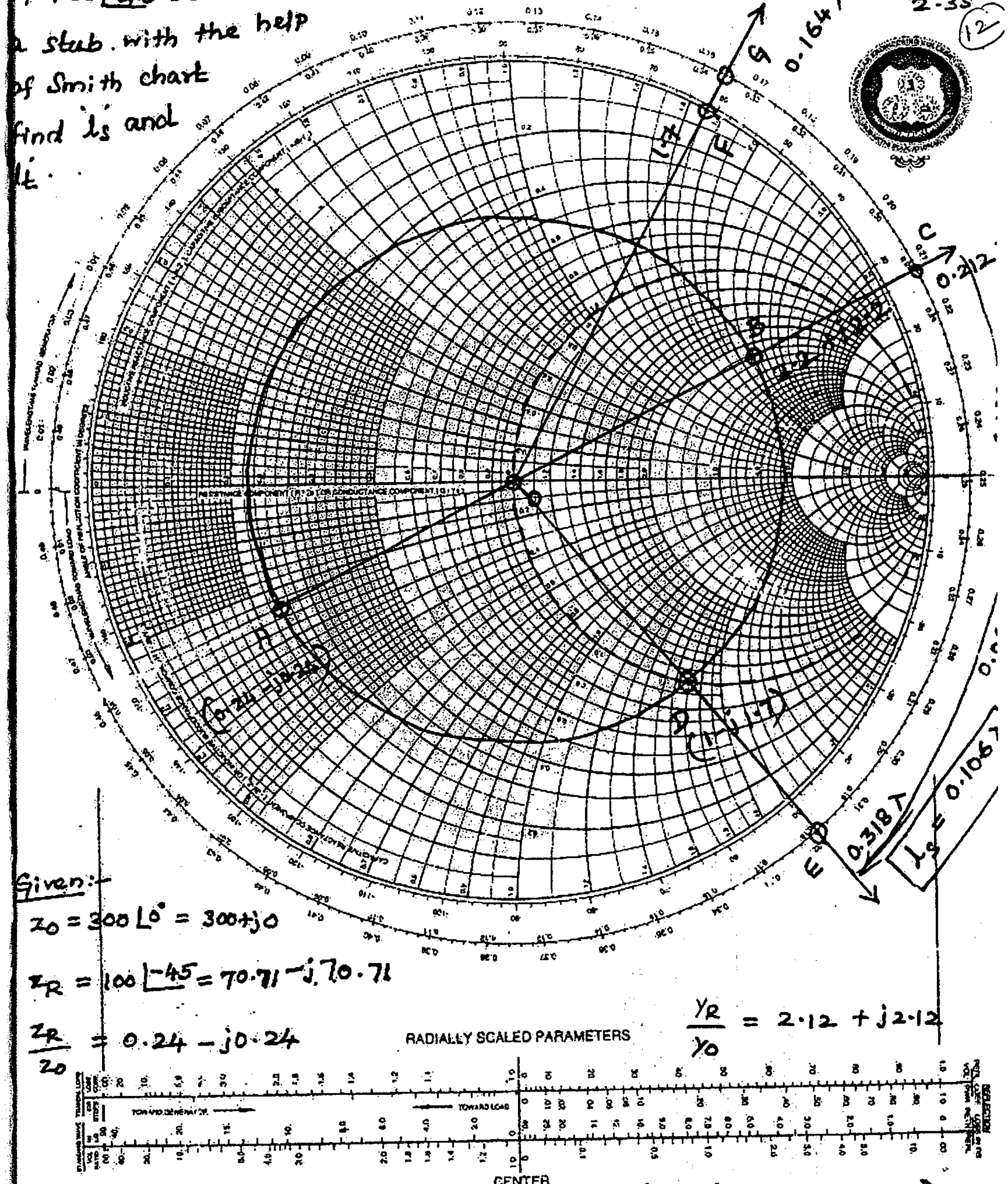
$l_t = 0.428\lambda$

$= 0.428\times3$

$= 1.284\,m$

...of $100\angle{-45^\circ}\,\Omega$. This load is to be matched to a tan. line by using a stub with the help of Smith chart. Find l_s and l_t.

2-35 (12)



Given:

$z_0 = 300\angle 0^\circ = 300 + j0$

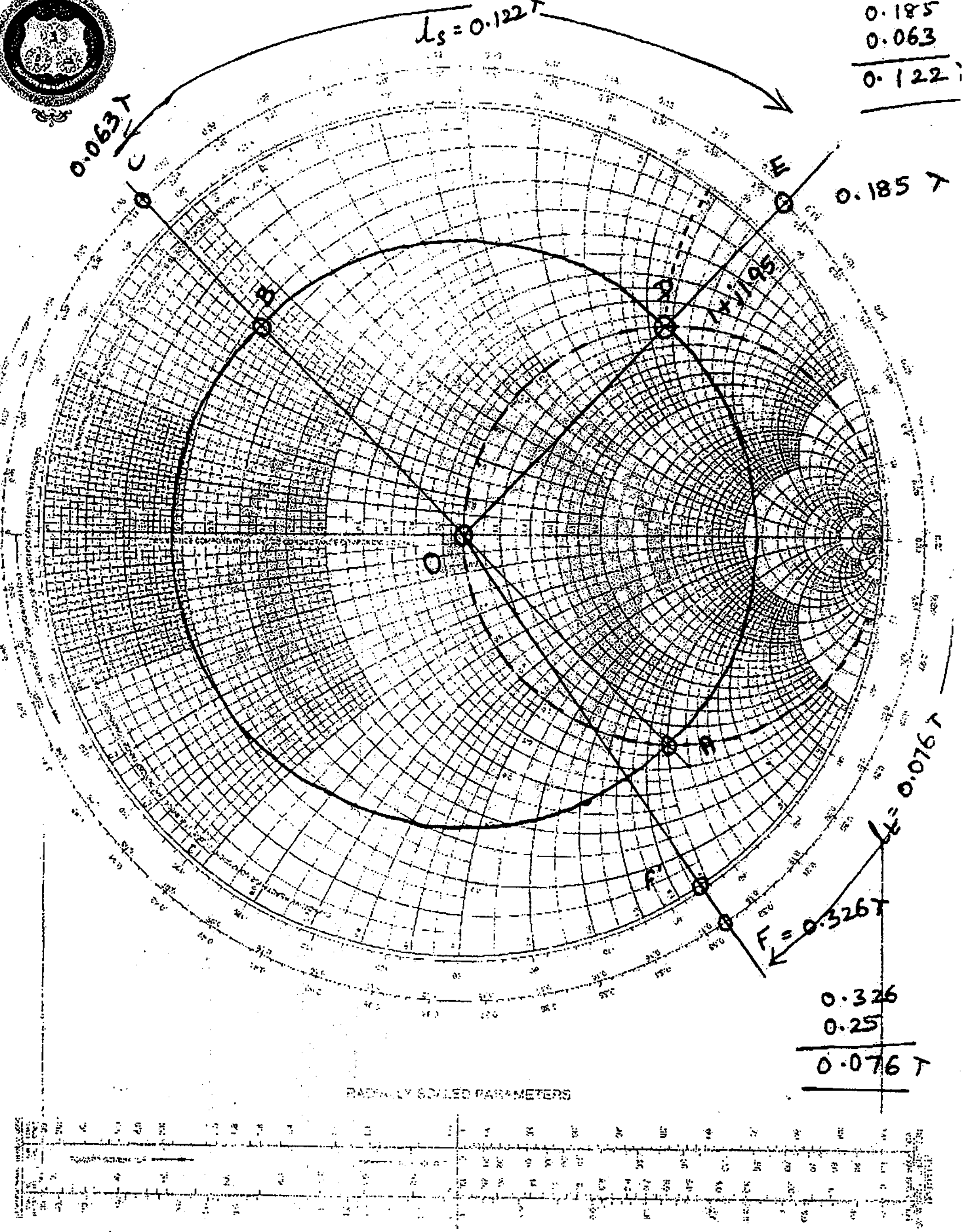
$z_R = 100\angle{-45} = 70.71 - j70.71$

$\frac{z_R}{z_0} = 0.24 - j0.24$

$\frac{y_R}{y_0} = 2.12 + j2.12$

$l_t = (0.25 + 0.164)\lambda$

$\boxed{l_t = 0.414\lambda}$

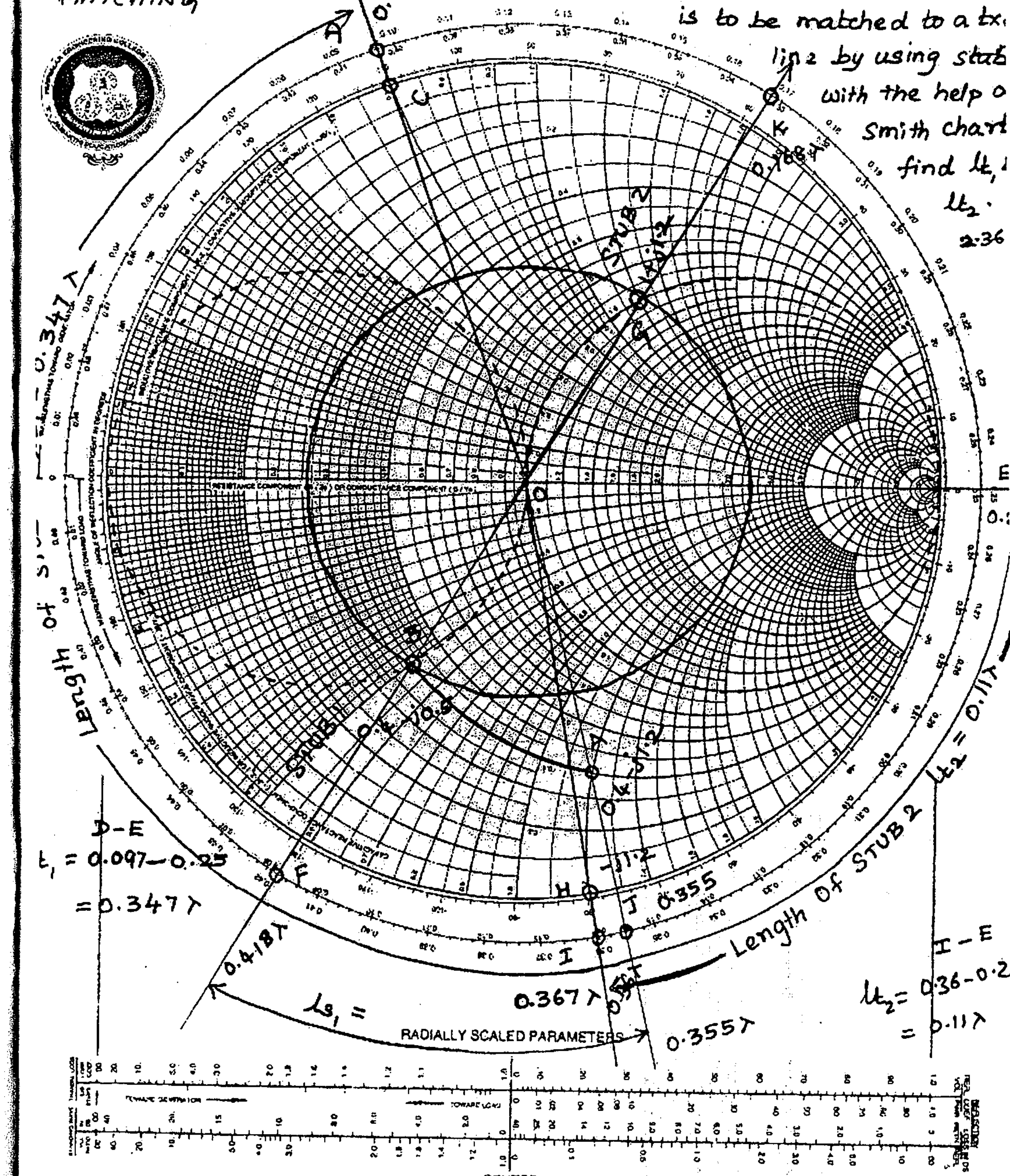


2) A Load $(50-j100)\Omega$ is connected across a 50Ω line. Design a short circuited stub to provide matching between the 2 lines at a signal Freq. of 30 MHz using Smith chart.

DOUBLE STUB MATCHING

(13)

in an Impedance of $40-j120\Omega$. This loa… is to be matched to a tx… line by using stub… with the help o… Smith chart… find l_{t_1}… l_{t_2}.



$l_{s1} = F - J = 0.418\lambda - 0.355\lambda$
$= 0.063\lambda$

$\frac{Z_L}{Z_0} = \frac{40-j120}{100} = 0.4-j1.2$

$l_{s2} = K - F = 0.168\lambda - 0.418\lambda$
$= 0.25\lambda$

Advantages of short circuited stub over open circuited stub:

1. S.C stub has lower loss of energy due to radiation
2. Greater ease in construction
3. Length of S.C stubs can easily be changed by using a sliding short circuit.

Disadvantages of Single stub matching:

1. In single stub matching, location and length of the stub depend on load impedance & signal frequency. Any change in this will require corresponding change in length and position of the stub.
2. Single stub matching requires that the stub be located at a definite point on the line. This requirement often calls for placement of the stub at an undesirable place from mechanical consideration.

UNIT 4
WAVEGUIDES

Transmission of TM waves Between Parallel Planes:

To find the field configuration or components inside the parallel planes, consider 2 plates are placed at a distance 'a' in the x-axis from '0' to 'a'. Assume the wave is propagating in z-direction and there is no boundary in 'y' and 'z' direction.

The maxwell's equation to be satisfied by the electric and magnetic field at the boundary are,

$$\nabla \times E = -j\omega\mu H \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon E \rightarrow (2)$$

equation (1) can be written as,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \rightarrow (3)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \rightarrow (4)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \rightarrow (5)$$

similarly, equ. (2) can be written as

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (6)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \rightarrow (7)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (8)$$

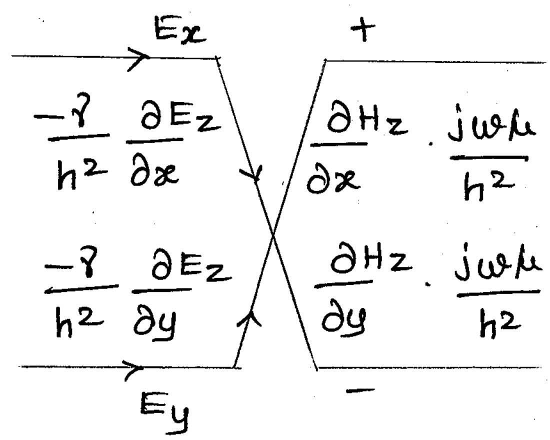
Manipulating equs. ③ → ⑧, the relation between field components inside the guide can be obtained as,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (9)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (10)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (11)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (12)$$



where, $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

For TM waves, $H_z = 0$

and for $\parallel^{el}$ plates, $\frac{\partial}{\partial y} = 0$ & $\gamma = \gamma_m$

so, equs ⑨ → ⑫ becomes

$$E_x = \frac{-\gamma_m}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (13)$$

$$E_y = 0 \rightarrow (14)$$

$$H_x = 0 \rightarrow (15)$$

$$H_y = \frac{-j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x} \rightarrow (16)$$

From equs. ⑬ → ⑯, the existing field components inside the parallel plates are,

E_x, E_z and H_y

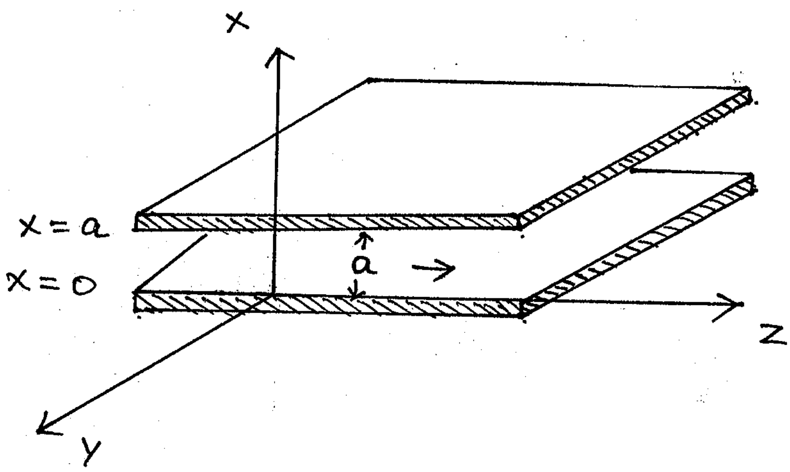
To find the field components of TM waves inside the parallel plates, we can assume a value for one of the field component and from this value, we can get the value of other field components.

Let us assume a sine wave is propagating in z-direction. Let the value of E_z be,

$$E_z = \frac{h}{j\omega\varepsilon}\left[B_1 \cosh x - B_2 \sinh x\right] \rightarrow (17)$$

where, $h = \frac{m\pi}{a}$

'a' is the distance between 2 planes in x-axis and and 'm' is the integer having the value $m = 0, 1, 2, 3 \ldots$



The Boundary conditions are,

$x = 0 \quad E_z = 0$

$x = a \quad E_z = 0$

Differentiate equ. (17) w.r.to 'x' and 'y'

$$\frac{\partial E_z}{\partial x} = \frac{-h^2}{j\omega\varepsilon}\left[B_1 \sinh x + B_2 \cosh x\right] \rightarrow (18)$$

sub equ. (18) in (13) → (16)

$$E_x = \frac{-\gamma_m}{h^2} \times \frac{-h^2}{j\omega\epsilon} \left[B_1 \sin hx + B_2 \cos hx \right]$$

$$E_x = \frac{\gamma_m}{j\omega\epsilon} \left[B_1 \sin hx + B_2 \cos hx \right] \rightarrow (19)$$

$$E_y = 0 \rightarrow (20)$$

$$H_x = 0 \rightarrow (21)$$

$$H_y = \frac{-\cancel{j\omega\epsilon}}{\cancel{h^2}} \times \frac{-\cancel{h^2}}{\cancel{j\omega\epsilon}} \left[B_1 \sin hx + B_2 \cos hx \right]$$

$$H_y = B_1 \sin hx + B_2 \cos hx \rightarrow (22)$$

At HF, $\gamma_m = j\beta_m$

so, sub $\gamma_m = j\beta_m$ and $h = \frac{m\pi}{a}$ in equs. (19) → (22), (17)

$$E_x = \frac{j\beta_m}{j\omega\epsilon} \left[B_1 \sin\left(\frac{m\pi}{a} x\right) + B_2 \cos\left(\frac{m\pi}{a} x\right) \right]$$

$$E_x = \frac{\beta_m}{\omega\epsilon} \left[B_1 \sin\left(\frac{m\pi}{a} x\right) + B_2 \cos\left(\frac{m\pi}{a} x\right) \right] \rightarrow (23)$$

$$E_y = 0 \rightarrow (24)$$

$$H_x = 0 \rightarrow (25)$$

$$H_y = B_1 \sin\left(\frac{m\pi}{a} x\right) + B_2 \cos\left(\frac{m\pi}{a} x\right) \rightarrow (26)$$

$$E_z = \frac{m\pi}{ja\omega\epsilon} \left[B_1 \cos\left(\frac{m\pi}{a} x\right) - B_2 \sin\left(\frac{m\pi}{a} x\right) \right] \rightarrow (27)$$

The field components of TM wave inside the $\parallel^{el}$

Planes can be represented interms of time and propagation variation.

$$E_x = \frac{\beta_m}{\omega \varepsilon}\left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \sin\omega t \rightarrow (28)$$

$$E_y = 0 \rightarrow (29)$$

$$E_z = -\frac{m\pi}{a\omega\varepsilon}\left[B_1 \cos\left(\frac{m\pi}{a}x\right) - B_2 \sin\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \cos\omega t \rightarrow (30)$$

$$H_x = 0 \rightarrow (31)$$

$$H_y = \left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \sin\omega t \rightarrow (32)$$

The field components of TM waves inside the Parallel plates can be summarized as follows.

$$E_x = \frac{\beta_m}{\omega \varepsilon}\left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \sin\omega t$$

$$E_y = 0$$

$$E_z = -\frac{m\pi}{a\omega\varepsilon}\left[B_1 \cos\left(\frac{m\pi}{a}x\right) - B_2 \sin\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \cos\omega t$$

$$H_x = 0$$

$$H_y = \left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \sin\omega t$$

$$H_z = 0$$

Transmission of TE Waves Between Parallel Planes:

To find the field configuration or components inside the parallel planes, consider 2 plates are placed at a distance 'a' in the x-axis from '0' to 'a'. Assume the wave is propagating in the z-direction and there is no boundary in the y and z direction.

The maxwells equation to be satisfied by the electric and magnetic field at the boundary are,

$$\nabla \times E = -j\omega\mu H \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon E \rightarrow (2)$$

x y z

equation (1) can be written as

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \rightarrow (3)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \rightarrow (4)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \rightarrow (5)$$

Similarly, equation (2) can be written as

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (6)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \rightarrow (7)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (8)$$

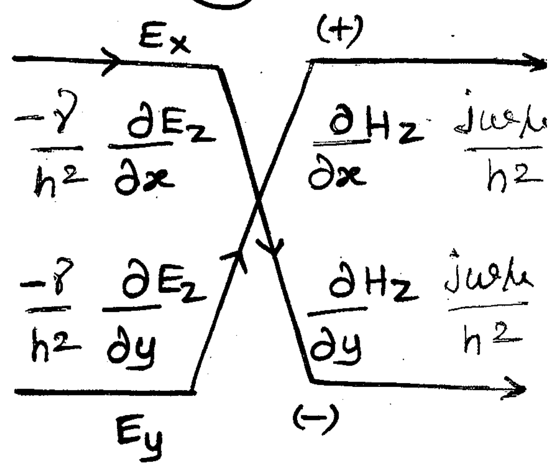
Manipulating equs. (3) → (8), the relation B/W field components inside the guide can be obtained as,

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (9)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (10)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (11)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (12)$$



where, $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

For TE Waves, $E_z = 0$

and for $\|^{el}$ plates, $\frac{\partial}{\partial y} = 0$

$$E_x = 0 \rightarrow (13)$$

$$E_y = \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial x} \rightarrow (14)$$

$$H_x = \frac{-\gamma}{h^2}\cdot\frac{\partial H_z}{\partial x} \rightarrow (15)$$

$$H_y = 0 \rightarrow (16)$$

From equs. (13) → (16), the existing field components inside the parallel plates are,

H_x, H_z and E_y

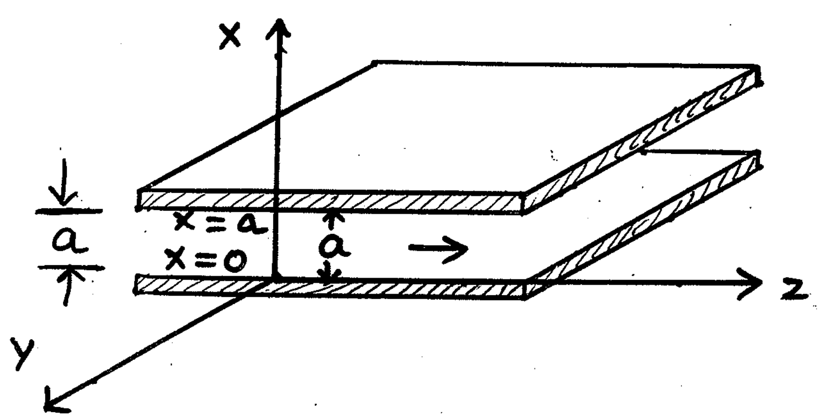
To find the field components of TE waves inside the parallel plates, we can assume a value for one of the field component and from this value, we can get the value of other field components.

Let us assume a sine wave is propagating in the z-direction. Let the value of H_z be,

$$H_z = \frac{h}{-j\omega\mu}\left[B_1 \cos hx - B_2 \sin hx\right] \rightarrow (17)$$

where, $h = \frac{m\pi}{a}$

where, 'a' is the distance between 2 planes in x-axis and 'm' is the integer having the value $m = 0, 1, 2, 3 \ldots$



The Boundary conditions are.

$x = 0 \qquad H_z = 0$

$x = a \qquad H_z = 0$

$$\frac{\partial H_z}{\partial x} = \frac{h^2}{+j\omega\mu}\left[B_1 \sin hx + B_2 \cos hx\right] \rightarrow (18)$$

sub equ (18) in (14) and (15)

$$E_y = \frac{\cancel{\gamma}\cancel{j\omega\mu}}{\cancel{h^2}} \cdot \frac{\cancel{h^2}}{\cancel{\gamma}\cancel{j\omega\mu}} \left[B_1 \sin hx + B_2 \cos hx \right]$$

$$\therefore E_y = B_1 \sin hx + B_2 \cos hx \rightarrow (19)$$

$$H_x = \frac{-\gamma}{\cancel{h^2}} \cdot \frac{\cancel{h^2}}{j\omega\mu} \left[B_1 \sin hx + B_2 \cos hx \right]$$

$$\therefore H_x = \frac{-\gamma}{j\omega\mu} \left[B_1 \sin hx + B_2 \cos hx \right] \longrightarrow (20)$$

At High Freq. $\gamma = j\beta$

$\therefore$ put $\gamma = j\beta$ and $h = \frac{m\pi}{a}$ in equs. (17), (19), (20)

$$E_y = B_1 \sin\left(\frac{m\pi}{a} \cdot x\right) + B_2 \cdot \cos\left(\frac{m\pi}{a} x\right) \rightarrow (21)$$

$$H_x = \frac{-\cancel{j}\beta}{\cancel{j}\omega\mu} \left[B_1 \sin\left(\frac{m\pi}{a} x\right) + B_2 \cos\left(\frac{m\pi}{a} x\right) \right]$$

$$H_x = \frac{-\beta}{\omega\mu} \left[B_1 \sin\left(\frac{m\pi}{a} x\right) + B_2 \cos\left(\frac{m\pi}{a} x\right) \right] \rightarrow (22)$$

$$H_z = \frac{m\pi}{-ja\omega\mu} \left[B_1 \cos\left(\frac{m\pi}{a} x\right) - B_2 \sin\left(\frac{m\pi}{a} x\right) \right] \rightarrow (23)$$

The field components of TE wave inside the $\|^{el}$ planes can be represented in terms of <u>time and propagation</u> variation.

$$E_y = \left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \sin\omega t \rightarrow (24)$$

$$H_x = \left[\frac{-\beta_m}{\omega\mu}\left(B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right)\right] e^{-j\beta_m z} \cdot \sin\omega t$$

$$\rightarrow (25)$$

$$H_z = \left[\frac{-m\pi}{ja\omega\mu}\left(B_1 \cos\left(\frac{m\pi}{a}x\right) - B_2 \sin\left(\frac{m\pi}{a}x\right)\right)\right] e^{-j\beta_m z} \cdot \sin\omega t$$

$$= \frac{\cancel{j}\, m\pi}{a\omega\mu}\left[B_1 \cos\left(\frac{m\pi}{a}x\right) - B_2 \sin\left(\frac{m\pi}{a}x\right)\right] \cdot e^{-j\beta_m z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$H_z = \frac{m\pi}{a\omega\mu}\left[B_1 \cos\left(\frac{m\pi}{a}x\right) - B_2 \sin\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \cos\omega t$$

$$\rightarrow (26)$$

The field components of TE waves inside the Parallel plates can be summarized as follows.

$$E_x = 0$$

$$E_y = \left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] \cdot e^{-j\beta_m z} \cdot \sin\omega t$$

$$E_z = 0$$

$$H_x = \frac{-\beta_m}{\omega\mu}\left[B_1 \sin\left(\frac{m\pi}{a}x\right) + B_2 \cos\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \sin\omega t$$

$$H_y = 0$$

$$H_z = \frac{m\pi}{a\omega\mu}\left[B_1 \cos\left(\frac{m\pi}{a}x\right) - B_2 \sin\left(\frac{m\pi}{a}x\right)\right] e^{-j\beta_m z} \cdot \cos\omega t$$

Characteristics (or) Properties of TE, TM Waves :-

1. Propagation Constant, γ_m :-

$\gamma_m \rightarrow$ parallel, $\gamma_{mn} \rightarrow$ Rectangular

$\gamma_{nm} \rightarrow$ circular, γ

$\gamma_{mnp} \rightarrow$ Rectangular cavity Resonator

$\gamma_{nmp} \rightarrow$ circular cavity Resonator

W.K.T $h^2 = \gamma_m^2 + \omega^2 \mu \varepsilon \rightarrow (1)$

$$\gamma_m^2 = h^2 - \omega^2 \mu \varepsilon$$

$$\gamma_m^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \varepsilon$$

$$\gamma_m = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \varepsilon} \rightarrow (2)$$

parallel, $h^2 = \left(\frac{m\pi}{a}\right)^2$

Rectangular, $h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

Rectangular cavity Resonator, $h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$

2. cut-off frequency, f_c :-

At $f = f_c$, $\gamma_m = 0$ and $\omega = \omega_c$

$\therefore$ equ. (2) becomes

$$0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu \varepsilon}$$

$$0 = \left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu \varepsilon$$

$$\left(\frac{m\pi}{a}\right)^2 = \omega_c^2 \mu \varepsilon \rightarrow (3)$$

$$\omega_c^2 = \frac{1}{\mu \varepsilon} \cdot \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c = \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2} \rightarrow (4)$$

for air medium,

$$\frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\therefore f_c = \frac{c}{2\cancel{\pi}} \cdot \frac{m\cancel{\pi}}{a}$$

$$\therefore f_c = \frac{mc}{2a} \rightarrow (5)$$

$\mu = \mu_0 \cdot \mu_r$

$\mu_0 = 4\pi \times 10^{-7}$

$\mu_r = 1$

$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

$\varepsilon_0 = 8.854 \times 10^{-12}$

$\varepsilon_r = 1$

3. Phase constant, β_m :-

At HF, $\alpha_m = 0$, $\gamma_m = j\beta_m$

From equ (2)

$$j\beta_m = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon}$$

$$\beta_m = \frac{1}{j}\sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon}$$

$$= \sqrt{\frac{1}{j^2}\left[\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon\right]}$$

$$= \sqrt{-\left[\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon\right]}$$

$$\beta_m = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2} \rightarrow (6)$$

'β_m' in terms of 'f_c'

substitute equ. ③ in ⑥

$$\beta_m = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\omega_c^2\cancel{\mu\varepsilon}}{\omega^2\cancel{\mu\varepsilon}}}$$

$$= \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\beta_m = \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2} \rightarrow ⑦$$

'β_m' in terms of 'λ_c'

$$\beta_m = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2} \rightarrow ⑧$$

$\because \lambda = \frac{c}{f}$

$\lambda_c = \frac{c}{f_c}$

4. Guide wavelength, 'λ_g' :-

$$\lambda_g = \frac{2\pi}{\beta_m} \rightarrow ⑨$$

5. Cut-off wavelength, 'λ_c' :-

$$\lambda_c = \frac{c}{f_c} \rightarrow ⑩$$

6. Phase velocity, V_p :-

$$V_p = \frac{\omega}{\beta_m} \rightarrow ⑪$$

7. Group velocity, v_g :-

$$v_p \times v_g = c^2 \rightarrow (12)$$

$$v_g = \frac{c^2}{v_p} \rightarrow (13)$$

8. Angle of Incidence, θ :-

$$\theta = \cos^{-1}(f_c/f) \rightarrow (14)$$

9. Intrinsic Impedance, η :-

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow (15)$$

10. characteristic Impedance, Z :-

$$Z_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \rightarrow (16)$$

(or) $$Z_{TE} = \frac{\omega\mu}{\beta_m} \rightarrow (17)$$

$$Z_{TM} = \eta \cdot \sqrt{1-(f_c/f)^2} \rightarrow (18)$$

(or) $$Z_{TM} = \frac{\beta_m}{\omega\varepsilon} \rightarrow (19)$$

Evanescant Mode :-

When Operating frequency is lower than cut-off frequency, the propagation constant becomes real. (i.e., $\gamma_m = \alpha_m$). The waves will not propagate. The modes that does not propagate is called as Evanescant Mode.

Propagating Mode :-

When cut-off frequency is lower than Operating frequency, the propagation constant becomes imaginary. (i.e., $\gamma_m = j\beta_m$). The waves will propagate. The modes that can propagate is called as propagating mode.

Dominant Mode :

The lowest order mode is called as dominant mode. This is the mode where wave starts to propagate.

The dominant mode of TE wave for parallel waveguide is TE_1 and TM_1.

Transmission of TE Waves Inside Rectangular Waveguide

To find the field configuration or components of TE waves inside Rectangular Waveguide, consider 2 planes are placed at a distance 'a' in x-axis and 'b' in y-axis. Assume the wave is propagating in z-direction.

The maxwell's equation to be satisfied by the electric and magnetic field at the boundary are,

$$\nabla \times E = -j\omega\mu H \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon E \rightarrow (2)$$

Equation (1) can be written as,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \rightarrow (3)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \rightarrow (4)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \rightarrow (5)$$

similarly equation (2) can be written as,

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (6)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \rightarrow (7)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (8)$$

Manipulating equs. (3) → (8) the relation between field components inside the guide can be obtained as,

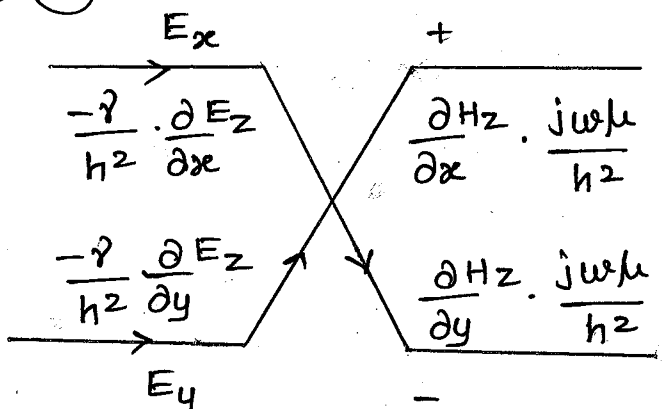
$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (9)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (10)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (11)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (12)$$

where, $h^2 = \gamma^2 + \omega^2\mu\varepsilon \rightarrow (13)$



For TE waves, $E_z = 0$ and for Rectangular waveguide $\gamma = \gamma_{mn}$

so, equs. (9) → (12) becomes

$$E_x = \frac{-j\omega\varepsilon}{h^2} \cdot \frac{\partial H_z}{\partial y} \rightarrow (14)$$

$$E_y = \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x} \rightarrow (15)$$

$$H_x = \frac{-\gamma_{mn}}{h^2} \cdot \frac{\partial H_z}{\partial x} \rightarrow (16)$$

$$H_y = \frac{-\gamma_{mn}}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (17)$$

From equs. (14) → (17), the existing field components inside Rectangular waveguide are,

E_x, E_y, H_x, H_y and H_z

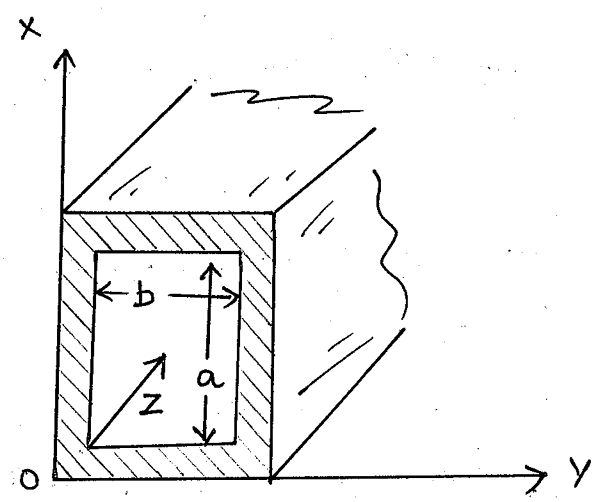
To find the field components of TE wave inside the Rectangular waveguide, we can assume a value for one of the field component and from this value, we can get the value of other field components.

Let us assume a sine wave is propagating in z-direction. Let the value of H_z component be

$$H_z = H_0 \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow (18)$$

where, 'a' and 'b' are dimensions of Rectangular waveguide and m, n are integers having the value

$m, n = 0, 1, 2, 3, \dots$



The Boundary conditions are

$$\left.\begin{array}{ll} \text{at } x = a, & H_z = 0 \\ y = b, & H_z = 0 \end{array}\right\} \rightarrow (19)$$

Differentiate equ. (18) w.r.to x and y

$$\frac{\partial H_z}{\partial x} = -H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \rightarrow (20)$$

$$\frac{\partial H_z}{\partial y} = -H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b} \cdot y\right) \rightarrow (21)$$

sub equs. (20) & (21) in (14) → (17)

$$E_x = \frac{-j\omega\varepsilon}{h^2} \times -H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b} \cdot y\right)$$

$$E_x = \frac{j\omega\varepsilon}{h^2} \cdot H_0 \cdot \frac{n\pi}{b} \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \rightarrow (22)$$

$$E_y = \frac{-j\omega\mu}{h^2} \cdot H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \rightarrow (23)$$

$$H_x = \frac{+\gamma_{mn}}{h^2} \cdot H_0 \cdot \frac{m\pi}{a} \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \rightarrow (24)$$

$$H_y = \frac{\gamma_{mn}}{h^2} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \rightarrow (25)$$

At high Frequency, $\gamma_{mn} = j\beta_{mn}$

$\therefore$ put $\gamma_{mn} = j\beta_{mn}$ and $h^2 = \omega_c^2\mu\varepsilon$ in equs. (22) → (25)

$$E_x = \frac{j\omega\cancel{\varepsilon}}{\omega_c^2\mu\cancel{\varepsilon}} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right)$$

$$E_x = \frac{j\omega}{\omega_c^2\mu} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \rightarrow (26)$$

$$E_y = \frac{-j\omega\cancel{\mu}}{\omega_c^2\cancel{\mu}\varepsilon} \cdot H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right)$$

$$E_y = \frac{-j\omega}{\omega_c^2 \varepsilon} \cdot H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} \cdot y\right) \rightarrow (27)$$

$$H_x = \frac{j\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow (28)$$

$$H_y = \frac{j\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \rightarrow (29)$$

The field components of TE wave inside the rectangular waveguide can be represented in terms of time and propagation variation. so, equs. (18), (26) → (29) becomes,

$$E_x = \frac{\cancel{j}\omega}{\omega_c^2 \mu} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$E_x = \frac{\omega}{\omega_c^2 \mu} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow (30)$$

$$E_y = \frac{-\omega}{\omega_c^2 \varepsilon} \cdot H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow (31)$$

$$H_x = \frac{\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow (32)$$

$$H_y = \frac{\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot H_0 \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow (33)$$

$$H_z = H_0 \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \sin\omega t \rightarrow (34)$$

Transmission Of TM waves Inside Rectangular Waveguide

To find the field configuration or Components inside Rectangular Waveguide, consider 2 planes are placed at a distance 'a' in x-axis and 'b' in y-axis. Assume the wave is propagating in z-direction.

The maxwell's equation to be satisfied by the electric and magnetic field at the boundary are,

$$\nabla \times E = -j\omega\mu H \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon E \rightarrow (2)$$

Equation (1) can be written as,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \rightarrow (3)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \rightarrow (4)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \rightarrow (5)$$

Similarly, equation (2) can be written as,

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (6)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \rightarrow (7)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (8)$$

Manipulating equs. (3) → (8), the relation between field Components inside the guide can be obtained as,

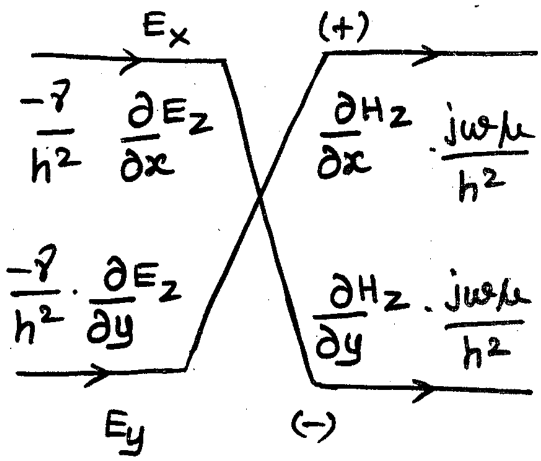
$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (9)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (10)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (11)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (12)$$

where, $h^2 = \gamma^2 + \omega^2\mu\cdot\varepsilon \rightarrow (13)$



For TM waves, $H_z = 0$

and for Rectangular Waveguide $\gamma = \gamma_{mn}$

so, equations (9) → (12) becomes

$$E_x = \frac{-\gamma_{mn}}{h^2}\cdot\frac{\partial E_z}{\partial x} \rightarrow (14)$$

$$E_y = \frac{-\gamma_{mn}}{h^2}\cdot\frac{\partial E_z}{\partial y} \rightarrow (15)$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_z}{\partial y} \rightarrow (16)$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (17)$$

From equs. (14) → (17) the existing field components inside Rectangular Waveguide are,

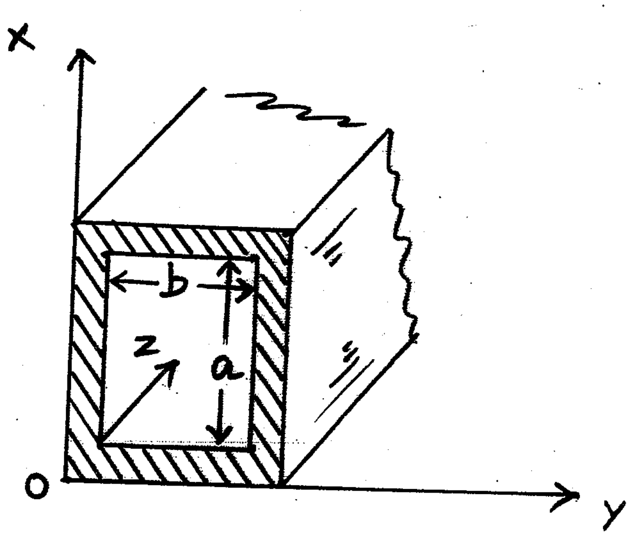
E_x, E_y, E_z, H_x and H_y

To find the field components of TM wave inside the Rectangular waveguide, we can assume a value for one of the field components and from this value, we can get the value of other field components.

Let us assume a sine wave is propagating in z-direction. Let the value of E_z component be

$$E_z = E_0 \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \cdot \rightarrow (18)$$

where, 'a' and 'b' are dimensions of Rectangular waveguide and m, n are integers having the value m, n = 0, 1, 2, 3, ...



The Boundary conditions are,

$$\left.\begin{array}{ll} \text{at } x = a, & E_z = 0 \\ y = b, & E_z = 0 \end{array}\right\} \rightarrow (19)$$

Differentiate equ. 18 w.r.to x and y.

$$\frac{\partial E_z}{\partial x} = E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \rightarrow (20)$$

Hz -Ho cos sin

$$\frac{\partial E_z}{\partial y} = E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow (21)$$

sub equs. (20) & (21) in (14) → (17)

$$E_x = -\frac{\gamma_{mn}}{h^2} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \rightarrow (22)$$

$$E_y = -\frac{\gamma_{mn}}{h^2} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow (23)$$

$$H_x = \frac{j\omega\varepsilon}{h^2} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow (24)$$

$$H_y = -\frac{j\omega\varepsilon}{h^2} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \rightarrow (25)$$

At High freq. $\gamma_{mn} = j\beta_{mn}$

$\therefore$ Put $\gamma_{mn} = j\beta_{mn}$ and $h^2 = \omega_c^2 \mu\varepsilon$ in equs. (22) → (25)

$$E_x = \frac{-j\beta_{mn}}{\omega_c^2 \mu\varepsilon} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \rightarrow (26)$$

$$E_y = \frac{-j\beta_{mn}}{\omega_c^2 \mu\varepsilon} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow (27)$$

$$H_x = \frac{j\omega\cancel{\varepsilon}}{\omega_c^2 \mu\cancel{\varepsilon}} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right)$$

$$H_x = \frac{j\omega}{\omega_c^2 \mu} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \rightarrow 28$$

$$H_y = \frac{-j\omega \cancel{\varepsilon}}{\omega_c^2 \mu \cancel{\varepsilon}} \cdot E_0 \cdot \frac{m\pi}{a} \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right)$$

$$H_y = \frac{-j\omega}{\omega_c^2 \mu} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \rightarrow 29$$

The field components of TM wave inside the Rectangular waveguide can be represented in terms of time and propagation variation. so, equs. 26 → 29 becomes,

$$E_x = \frac{-\cancel{j}\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$E_x = \frac{-\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \cdot \sin\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow 30$$

$$E_y = \frac{-\cancel{j}\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$E_y = \frac{-\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} \cdot y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow 31$$

$$H_x = \frac{\cancel{j}\omega}{\omega_c^2 \mu} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} y\right) \cdot e^{-j\beta_{mn} z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$H_x = \frac{\omega}{\omega_c^2 \mu} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cdot \cos\left(\frac{n\pi}{b} \cdot y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow 32$$

$$H_y = \frac{-\cancel{\gamma}\omega}{\omega_c^2 \mu} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \cdot e^{-j\beta_{mn} z} \cdot \frac{\cos\omega t}{\cancel{\gamma}}$$

$$H_y = \frac{-\omega}{\omega_c^2 \mu} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b} \cdot y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t \rightarrow (33)$$

The field components of TM waves inside the Rectangular Waveguide is summarized as,

$$E_x = \frac{-\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t$$

$$E_y = \frac{-\beta_{mn}}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_{mn} z} \cdot \cos\omega t$$

$$E_z = E_0 \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta_{mn} z} \cdot \sin\omega t$$

$$H_x = \frac{\omega}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_{mn} z} \cdot \cos\omega t$$

$$H_y = \frac{-\omega}{\omega_c^2 \mu \varepsilon} \cdot E_0 \cdot \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \cdot e^{-j\beta_{mn} z} \cdot \cos\omega t$$

$$H_z = 0$$

Characteristics (or) properties of TE, TM waves of Rectangular Waveguide :

1. Propagation constant, γ_{mn} :-

W.K.T $h^2 = \gamma_{mn}^2 + \omega^2\mu\varepsilon \rightarrow (1)$

$$\gamma_{mn}^2 = h^2 - \omega^2\mu\varepsilon$$

$$\gamma_{mn}^2 = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega^2\mu\varepsilon$$

$$\gamma_{mn} = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega^2\mu\varepsilon} \rightarrow (2)$$

2. Cut-off frequency, f_c :-

At $f = f_c$, $\gamma_{mn} = 0$ and $\omega = \omega_c$

$\therefore$ equ (2) becomes

$$0 = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega_c^2\mu\varepsilon}$$

Squaring on both sides

$$0 = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega_c^2\mu\varepsilon$$

$$\omega_c^2\mu\varepsilon = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \rightarrow (3)$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\therefore f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \rightarrow (4)$$

For free space, $\frac{1}{\sqrt{\mu\varepsilon}} = c$

So, equ (4) becomes,

$$f_c = \frac{c}{2\pi}\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \rightarrow (5)$$

3. <u>Phase constant</u>, β_{mn} :-

At HF, $\alpha_{mn} = 0$; $\gamma_{mn} = j\beta_{mn}$

So, equ. (2) becomes

$$j\beta_{mn} = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega^2\mu\varepsilon}$$

$$\beta_{mn} = \frac{1}{j}\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega^2\mu\varepsilon}$$

$$= \sqrt{\frac{1}{j^2}\left[\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega^2\mu\varepsilon\right]}$$

$$= \sqrt{-\left[\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \omega^2\mu\varepsilon\right]}$$

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \rightarrow (6)$$

β_{mn} in terms of cut-off frequency

from equ. (3), $\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] = \omega_c^2\mu\varepsilon$

$\therefore$ equ (6) becomes,

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c^2\mu\varepsilon}{\omega^2\mu\varepsilon}\right)}$$

$$\beta_{mn} = \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2} \rightarrow (7)$$

β_{mn} in terms of λ_c

$$\beta_{mn} = \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(\lambda/\lambda_c\right)^2} \rightarrow (8)$$

4. Guide Wavelength, λ_g :-

$$\lambda_g = \frac{2\pi}{\beta_{mn}} \rightarrow (9)$$

5. cut-off Wavelength, λ_c :-

$$\lambda_c = \frac{c}{f_c} \rightarrow (10)$$

6. Phase velocity, v_p :-

$$v_p = \frac{\omega}{\beta_{mn}} \rightarrow (11)$$

7. Group velocity, v_g :-

$$v_p \times v_g = c^2 \rightarrow (12)$$

$$v_g = \frac{c^2}{v_p} \rightarrow (13)$$

8. Angle of Incidence, 'θ' :-

$$\theta = \cos^{-1}(f_c/f) \rightarrow (14)$$

9. Intrinsic Impedance, 'η' :-

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow (15)$$

For air medium,

$$\eta = 377\,\Omega$$

10. characteristic Impedance, :-

$$Z_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \rightarrow (16)$$

$$\text{(or)} \quad Z_{TE} = \frac{\omega\mu}{\beta_{mn}} \rightarrow (17)$$

$$Z_{TM} = \eta \cdot \sqrt{1-(f_c/f)^2} \rightarrow (18)$$

(or)

$$Z_{TM} = \frac{\beta_{mn}}{\omega\varepsilon} \rightarrow (19)$$

Impossibility of TEM wave in Waveguide :

Transverse Magnetic (TM) waves and Transverse Electric (TE) waves can propagate through the rectangular waveguide. For TM waves, no component of the magnetic field exists in z-direction and TE waves has no component of the Electric field in z-direction.

Consider that TEM wave exists within a hollow guide of any shape. By the property, the lines of magnetic field intensity H must lie entirely in the transverse plane. For a non-magnetic material with condition $\nabla \cdot H = 0$, the lines of H must lie in closed loops so, to have existence of the TEM waves inside the guide, this H lines must be in a plane transverse to the axis of the guide.

According to Maxwell's first equation, the magnetomotive force (mmf) around each closed loop must be equal to the axial current. In a guide consisting inner conductor, the axial current is nothing but the conduction current in the inner inductor. But in a hollow waveguide like rectangular waveguide there is no inner conductor present. In this case the axial current must be equal to the displacement current. By the property, the displacement current needs the component of the electric field E in axial direction. But such axial component of E is not present in TEM waves, hence it cannot exist in rectangular or circular waveguide.

Transmission Of TM waves inside Circular Waveguide

To find field configuration or Components inside Circular waveguide, Consider a circular waveguide with inner radius 'r' and assume the wave is propagating in z-direction.

The maxwell's equation to be satisfied by the electric and magnetic field at the boundary are,

$$\nabla \times E = -j\omega\mu H \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon E \rightarrow (2)$$

Equation (1) can be written as $\quad r \quad \phi \quad z$

$$\frac{1}{r}\cdot\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_r \rightarrow (3)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi \rightarrow (4)$$

$$\frac{\partial E_\phi}{\partial r} - \frac{1}{r}\cdot\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z \rightarrow (5)$$

Equation (2) can be written as

$$\frac{1}{r}\cdot\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\mu E_r \rightarrow (6)$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\mu E_\phi \rightarrow (7)$$

$$\frac{\partial H_\phi}{\partial r} - \frac{1}{r}\cdot\frac{\partial H_r}{\partial \phi} = j\omega\mu E_z \rightarrow (8)$$

Manipulating equs. (3) → (8), the relation between field components inside the guide can be obtained as,

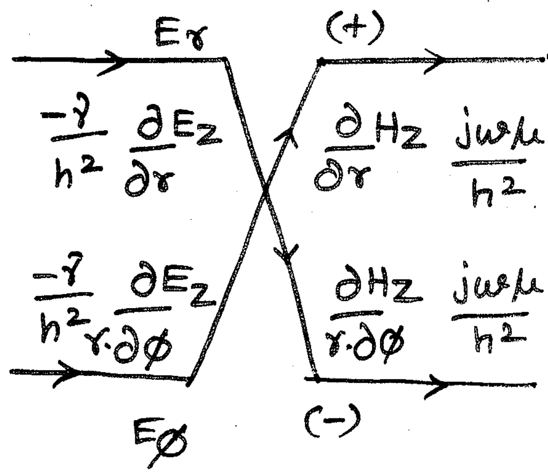
$$E_r = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial r} - \frac{j\omega\mu}{r \cdot h^2}\frac{\partial H_z}{\partial \phi} \rightarrow (9)$$

$$E_\phi = \frac{-\gamma}{r \cdot h^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial r} \rightarrow (10)$$

$$H_r = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial r} + \frac{j\omega\varepsilon}{r \cdot h^2}\frac{\partial E_z}{\partial \phi} \rightarrow (11)$$

$$H_\phi = \frac{-\gamma}{r \cdot h^2}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial r} \rightarrow (12)$$

where, $h^2 = \gamma^2 + \omega^2\mu\varepsilon \rightarrow (13)$



For TM waves, $H_z = 0$ and

For Circular WG. $\gamma = \gamma_{nm}$

∴ equs (9) → (12) becomes

$$E_r = -\frac{\gamma_{nm}}{h^2}\frac{\partial E_z}{\partial r} \rightarrow 14$$

$$E_\phi = \frac{-\gamma_{nm}}{r \cdot h^2}\frac{\partial E_z}{\partial \phi} \rightarrow 15$$

$$H_r = \frac{j\omega\varepsilon}{r \cdot h^2}\frac{\partial E_z}{\partial \phi} \rightarrow 16$$

$$H_\phi = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial r} \rightarrow 17$$

From equs. 14 → 17 the existing field components inside circular waveguide are E_r, E_ϕ, E_z, H_r and H_ϕ.

To find the field components of TM wave inside the circular waveguide, we can assume a value for one

one of the field components and from this value, we can get the value of other field components.

Let us assume a sine wave is propagating in z-direction. Let the value of E_z component be,

$$E_z = E_0 . J_n(hr) \cos(n\phi). \rightarrow (18)$$

$$H_z = H_0 \cdot J_n(h'r) \cos(n\phi)$$

Differentiate equ. (18) w.r.to r and ϕ

$$\frac{\partial E_z}{\partial r} = E_0 \cdot \frac{\partial}{\partial r} J_n(hr) \cdot h \cdot \cos n\phi$$

$$\frac{\partial E_z}{\partial r} = E_0 \cdot h \cdot J_n'(hr) \cdot \cos n\phi \rightarrow (19)$$

$$\frac{\partial E_z}{\partial \phi} = -E_0 \cdot n \cdot J_n(hr) \sin n\phi \rightarrow (20)$$

sub equs. (19) & (20) in (14) → (17)

$$E_r = -\frac{\gamma_{nm}}{h^{\cancel{2}}} . \cancel{h} . E_0 \cdot J_n'(hr) \cdot \cos(n\phi)$$

$$E_r = -\frac{\gamma_{nm}}{h} \cdot E_0 \cdot J_n'(hr) \cdot \cos(n\phi) \rightarrow (21)$$

$$E_\phi = \frac{\gamma_{nm}}{r \cdot h^2} \cdot n \cdot E_0 \cdot J_n(hr) \cdot \sin(n\phi) \rightarrow (22)$$

$$H_r = \frac{-j\omega\varepsilon}{r \cdot h^2} \cdot n \cdot E_0 \cdot J_n(hr) \cdot \sin(n\phi) \rightarrow (23)$$

$$H_\phi = \frac{-j\omega\varepsilon}{h^{\cancel{2}}} \cdot E_0 \cdot \cancel{h} \cdot J_n'(hr) \cdot \cos(n\phi)$$

$$H_\phi = \frac{-j\omega\varepsilon}{h} \cdot E_0 \cdot J_n'(hr) \cdot \cos(n\phi) \rightarrow (24)$$

The field components of TM wave inside the circular waveguide can be represented in terms of time and propagation variations. so, equs. 21 → 24 becomes

$$E_r = \frac{-\gamma_{nm}}{h} \cdot E_0 \cdot J_n'(hr) \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t \rightarrow (25)$$

$$E_\phi = \frac{\gamma_{nm}}{r \cdot h^2} \cdot n \cdot E_0 \cdot J_n(hr) \sin(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t \rightarrow (26)$$

$$H_r = \frac{-\cancel{j}\omega\varepsilon}{r \cdot h^2} \cdot n \cdot E_0 \cdot J_n(hr) \cdot \sin(n\phi)\, e^{-\gamma_{nm} z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$H_r = \frac{-\omega\varepsilon}{r h^2} \cdot n \cdot E_0 \cdot J_n(hr) \cdot \sin(n\phi)\, e^{-\gamma_{nm} \cdot z} \cdot \cos\omega t \rightarrow (27)$$

$$H_\phi = \frac{-\cancel{j}\omega\varepsilon}{h} \cdot E_0 \cdot J_n'(hr) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} \cdot z} \cdot \frac{\cos\omega t}{\cancel{j}}$$

$$H_\phi = \frac{-\omega\varepsilon}{h} \cdot E_0 \cdot J_n'(hr) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \cos\omega t \rightarrow (28)$$

The field components of TM waves inside the circular waveguide is summarized as,

$$\boxed{\begin{aligned}
E_r &= -\frac{\gamma_{nm}}{h} \cdot E_0 \cdot J_n'(hr) \cdot \cos(n\phi)\, e^{-\gamma_{nm} z} \cdot \sin\omega t \rightarrow 20\\
E_\phi &= \frac{\gamma_{nm}}{r \cdot h^2} \cdot n \cdot E_0 \cdot J_n(hr) \sin(n\phi)\, e^{-\gamma_{nm} z} \cdot \sin\omega t \rightarrow\\
E_z &= E_0 \cdot J_n(hr) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} \cdot z} \cdot \sin\omega t \rightarrow\\
H_r &= \frac{-\omega\varepsilon}{r h^2} \cdot n \cdot E_0 \cdot J_n(hr) \cdot \sin(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \cos\omega t \rightarrow\\
H_\phi &= \frac{-\omega\varepsilon}{h} \cdot E_0 \cdot J_n'(hr) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \cos\omega t \rightarrow\\
H_z &= 0
\end{aligned}}$$

Transmission of TE Waves inside Circular Waveguide

To find field configuration or components inside circular waveguide consider a circular waveguide with inner radius 'r' and assume the wave is propagating in z-direction.

The maxwell's equation to be satisfied by the electric and magnetic field at the boundary are,

$$\nabla \times E = -j\omega\mu H \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon E \rightarrow (2)$$

Equation (1) can be written as

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_r \rightarrow (3)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi \rightarrow (4)$$

$$\frac{\partial E_\phi}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z \rightarrow (5)$$

Equation (2) can be written as

$$\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\mu E_r \rightarrow (6)$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\mu E_\phi \rightarrow (7)$$

$$\frac{\partial H_\phi}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = j\omega\mu E_z \rightarrow (8)$$

Manipulating equs. (3) → (8), the relation between field components inside the guide can obtained as,

$$E_r = -\frac{\gamma}{h'^2}\frac{\partial E_z}{\partial r} - \frac{j\omega\mu}{r.h'^2}\cdot\frac{\partial H_z}{\partial \phi} \rightarrow (9)$$

$$E_\phi = \frac{-\gamma}{r.h'^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h'^2}\cdot\frac{\partial H_z}{\partial r} \rightarrow (10)$$

$$H_r = \frac{-\gamma}{h'^2}\frac{\partial H_z}{\partial r} + \frac{j\omega\varepsilon}{r.h'^2}\frac{\partial E_z}{\partial \phi} \rightarrow (11)$$

$$H_\phi = \frac{-\gamma}{r.h'^2}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\varepsilon}{h'^2}\cdot\frac{\partial E_z}{\partial r} \rightarrow (12)$$

where, $h'^2 = \gamma^2 + \omega^2\mu\varepsilon \rightarrow (13)$

For TE waves, $E_z = 0$ and for circular WG. $\gamma = \gamma_{nm}$

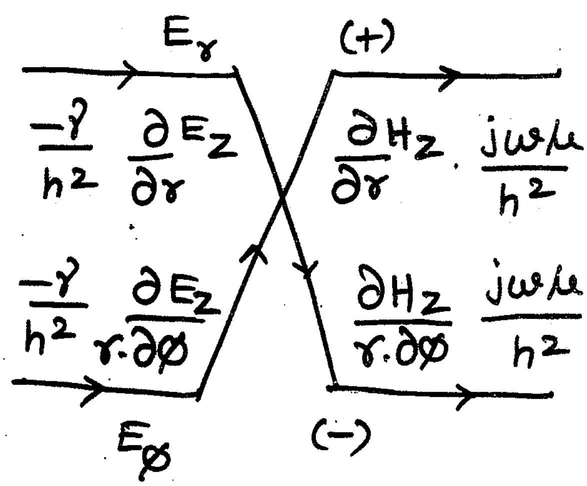
∴ equs. (9) → (12) becomes

$$E_r = \frac{-j\omega\mu}{r.h'^2}\cdot\frac{\partial H_z}{\partial \phi} \rightarrow 14$$

$$E_\phi = \frac{j\omega\mu}{h'^2}\cdot\frac{\partial H_z}{\partial r} \rightarrow 15$$

$$H_r = \frac{-\gamma}{h'^2}\frac{\partial H_z}{\partial r} \rightarrow 16$$

$$H_\phi = \frac{-\gamma}{r.h'^2}\frac{\partial H_z}{\partial \phi} \rightarrow 17$$



From equs. 14 → 17 the existing field components inside circular waveguide are,

E_r, E_ϕ, H_r, H_ϕ and H_z

To find the field components of TE wave inside the circular waveguide, we can assume a value for one of the field components and from this value, we can get the value of other field components.

Let us assume a sine wave is propagating in z-direction. Let the value of H_z component be,

$$H_z = H_0 \cdot J_n(h'r)\cos(n\phi) \rightarrow (18)$$

Differentiate equ (18) w.r.to r and ϕ

$$\frac{\partial H_z}{\partial r} = H_0 \cdot \frac{\partial}{\partial r} J_n(h'r) \cdot h' \cdot \cos(n\phi)$$

$$\frac{\partial H_z}{\partial r} = H_0 \cdot h' \cdot J_n'(h'r)\cos(n\phi) \rightarrow (19)$$

$$\frac{\partial H_z}{\partial \phi} = -H_0 \cdot J_n(h'r)\sin(n\phi) \cdot n$$

$$\frac{\partial H_z}{\partial \phi} = -H_0 \cdot n \cdot J_n(h'r)\sin(n\phi) \rightarrow (20)$$

sub. equs. (19) & (20) in (14) → (17)

$$E_r = \frac{j\omega\mu}{r \cdot h'^2} \cdot H_0 \cdot n \cdot J_n(h'r)\sin(n\phi) \rightarrow (21)$$

$$E_\phi = \frac{j\omega\mu}{h'^{\cancel{2}}} \cdot H_0 \cdot \cancel{h'} \cdot J_n'(h'r)\cos(n\phi) \rightarrow (22)$$

$$H_r = \frac{-\gamma_{nm}}{h'^{\cancel{2}}} \cdot H_0 \cdot \cancel{h'} \cdot J_n'(h'r)\cos(n\phi)$$

$$H_r = \frac{-\gamma_{nm}}{h'} \cdot H_0 \cdot J_n(h'r) \cdot \cos(n\phi) \rightarrow (23)$$

$$H_\phi = \frac{\gamma_{nm}}{r \cdot h'^2} \cdot H_0 \cdot n \cdot J_n(h'r)\sin(n\phi) \rightarrow (24)$$

The field components of TE wave inside the circular waveguide can be represented interms of time and propagation variations, so, equs. 21 → 24 becomes

$$E_r = \frac{j\omega\mu}{r \cdot h'^2} \cdot H_0 \cdot n \cdot J_n(h'r) \cdot \sin(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \frac{\cos\omega t}{j}$$

$$E_r = \frac{\omega\mu}{r \cdot h'^2} H_0 \cdot n \cdot J_n(h'r) \sin(n\phi)\, e^{-\gamma_{nm} z} \cdot \cos\omega t \rightarrow (25)$$

$$E_\phi = \frac{j\omega\mu}{h'} \cdot H_0 \cdot J_n'(h'r) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \frac{\cos\omega t}{j}$$

$$E_\phi = \frac{\omega\mu}{h'} \cdot H_0 \cdot J_n'(h'r) \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \cos\omega t \rightarrow (26)$$

$$H_r = \frac{-\gamma_{nm}}{h'} \cdot H_0 \cdot J_n(h'r) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t \rightarrow (27)$$

$$H_\phi = \frac{\gamma_{nm}}{r \cdot h'^2} \cdot H_0 \cdot n \cdot J_n(h'r) \cdot \sin(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t \rightarrow (28)$$

The field components of TE waves inside the circular waveguide is summarized as,

$$E_r = \frac{\omega\mu}{r \cdot h'^2} \cdot H_0 \cdot n \cdot J_n(h'r) \cdot \sin(n\phi)\, e^{-\gamma_{nm} z} \cdot \cos\omega t$$

$$E_\phi = \frac{\omega\mu}{h'} \cdot H_0 \cdot J_n'(h'r) \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \cos\omega t$$

$$E_z = 0$$

$$H_r = \frac{-\gamma_{nm}}{h'} \cdot H_0 \cdot J_n(h'r) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t$$

$$H_\phi = \frac{\gamma_{nm}}{r \cdot h'^2} \cdot H_0 \cdot n \cdot J_n(h'r) \cdot \sin(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t$$

$$H_z = H_0 \cdot J_n(h'r) \cdot \cos(n\phi) \cdot e^{-\gamma_{nm} z} \cdot \sin\omega t$$

Characteristics (or) Properties of TM waves of Circular Waveguide

1. Propagation constant, γ_{nm} :-

W.K.T $h^2 = \gamma_{nm}^2 + \omega^2\mu\varepsilon \rightarrow (1)$

$$\gamma_{nm}^2 = h^2 - \omega^2\mu\varepsilon$$

$$\gamma_{nm}^2 = \frac{(ha)^2}{a^2} - \omega^2\mu\varepsilon$$

$$\gamma_{nm} = \sqrt{\frac{(ha)^2}{a^2} - \omega^2\mu\varepsilon} \rightarrow (2)$$

2. cut-off frequency, f_c :-

At $f = f_c$, $\gamma_{nm} = 0$ and $\omega = \omega_c$

$\therefore$ equ (2) becomes

$$0 = \sqrt{\frac{(ha)^2}{a^2} - \omega_c^2\mu\varepsilon}$$

Squaring on both sides

$$0 = \frac{(ha)^2}{a^2} - \omega_c^2\mu\varepsilon$$

$$\omega_c^2\mu\varepsilon = \frac{(ha)^2}{a^2} \rightarrow (3)$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon} \cdot \frac{(ha)^2}{a^2}$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \cdot \frac{(ha)}{a}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \cdot \frac{(ha)}{a} \rightarrow (4)$$

for free space,

$$f_c = \frac{c}{2\pi} \frac{(ha)}{a} \rightarrow (5)$$

3. Phase constant, β_{nm} :

At HF, $\alpha_{nm} = 0$, $\gamma_{nm} = j\beta_{nm}$

so, equ (2) becomes

$$j\beta_{nm} = \sqrt{\frac{(ha)^2}{a^2} - \omega^2\mu\varepsilon}$$

$$\beta_{nm} = \frac{1}{j}\sqrt{\left[\frac{(ha)^2}{a^2} - \omega^2\mu\varepsilon\right]}$$

$$= \sqrt{\frac{1}{j^2}\left[\frac{(ha)^2}{a^2} - \omega^2\mu\varepsilon\right]}$$

$$\beta_{nm} = \sqrt{\left[\omega^2\mu\varepsilon - \frac{(ha)^2}{a^2}\right]} \rightarrow (6)$$

β_{nm} in terms of cut-off frequency

from equ. (3) $\frac{(ha)^2}{a^2} = \omega_c^2\mu\varepsilon$

$$\therefore \beta_{nm} = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c^2 \cancel{\mu\varepsilon}}{\omega^2\cancel{\mu\varepsilon}}\right)}$$

$$\beta_{nm} = \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(f_c/f\right)^2} \rightarrow (7)$$

β_{nm} in terms of λ_c.

$$\beta_{nm} = \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(\lambda/\lambda_c\right)^2} \rightarrow (8)$$

4. Guide Wavelength, λ_g :-

$$\lambda_g = \frac{2\pi}{\beta_{nm}} \rightarrow (9)$$

5. cut-off Wavelength, λ_c :-

$$\lambda_c = \frac{c}{f_c} \rightarrow (10)$$

6. Phase Velocity, v_p :-

$$v_p = \frac{\omega}{\beta_{nm}} \rightarrow (11)$$

7. Group velocity, v_g :-

$$v_p \times v_g = c^2 \rightarrow (12)$$

$$v_g = \frac{c^2}{v_p} \rightarrow (13)$$

8. Intrinsic Impedance, η :-

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow (14)$$

For air medium,

$$\eta = 377\,\Omega$$

9. Angle of Incidence, θ :-

$$\theta = \cos^{-1}\left(\frac{f_c}{f}\right) \rightarrow (15)$$

10. Characteristic Impedance, Z_{TM} :-

$$Z_{TM} = \frac{\beta_{nm}}{\omega\varepsilon} \rightarrow (16)$$

(or) $$Z_{TM} = \eta \cdot \sqrt{1-(f_c/f)^2} \rightarrow (17)$$

Order of the Bessel function n	$(ha)_{n1}$	$(ha)_{n2}$	$(ha)_{n3}$
0	✓ 2.405	5.52	8.65
1	3.83	7.06	10.17
2	5.13	8.41	11.62

Characteristics (or) Properties Of TE Waves of Circular Waveguide

1. Propagation constant, γ_{nm} :

W.K.T $\quad h^2 = \gamma_{nm}^2 + \omega^2\mu\varepsilon \rightarrow (1)$

$$\gamma_{nm}^2 = h^2 - \omega^2\mu\varepsilon$$

$$\gamma_{nm}^2 = \frac{(h'a)^2}{a^2} - \omega^2\mu\varepsilon$$

$$\gamma_{nm} = \sqrt{\frac{(h'a)^2}{a^2} - \omega^2\mu\varepsilon} \rightarrow (2)$$

2. cut-off frequency, f_c :-

At $f = f_c$, $\gamma_{nm} = 0$ and $\omega = \omega_c$

$\therefore$ equ (2) becomes

$$0 = \sqrt{\frac{(h'a)^2}{a^2} - \omega_c^2\mu\varepsilon}$$

squaring on both sides

$$0 = \frac{(h'a)^2}{a^2} - \omega_c^2\mu\varepsilon$$

$$\omega_c^2\mu\varepsilon = \frac{(h'a)^2}{a^2} \rightarrow (3)$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon} \cdot \frac{(h'a)^2}{a^2}$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \cdot \frac{(h'a)}{a}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \cdot \frac{(h'a)}{a} \rightarrow ④$$

for free space

$$f_c = \frac{c}{2\pi} \cdot \frac{(h'a)}{a} \rightarrow ⑤$$

3. Phase constant, β_{nm} :

At HF, $\alpha_{nm} = 0$, $\gamma_{nm} = j\beta_{nm}$

so, equ. ② becomes

$$j\beta_{nm} = \sqrt{\frac{(h'a)^2}{a^2} - \omega^2\mu\varepsilon}$$

$$\beta_{nm} = \frac{1}{j}\sqrt{\left[\frac{(h'a)^2}{a^2} - \omega^2\mu\varepsilon\right]}$$

$$= \sqrt{\frac{1}{j^2}\left[\frac{(h'a)^2}{a^2} - \omega^2\mu\varepsilon\right]}$$

$$\beta_{nm} = \sqrt{\left[\omega^2\mu\varepsilon - \frac{(h'a)^2}{a^2}\right]} \rightarrow ⑥$$

β_{nm} in terms of cut-off frequency

from equ. ③ $\frac{(h'a)^2}{a^2} = \omega_c^2\mu\varepsilon$

$$\therefore \beta_{nm} = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c^2\cancel{\mu\varepsilon}}{\omega^2\cancel{\mu\varepsilon}}\right)}$$

$$\beta_{nm} = \omega\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2} \rightarrow (7)$$

β_{nm} in terms of λ_c

$$\beta_{nm} = \omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-(\lambda/\lambda_c)^2} \rightarrow (8)$$

4. Guide Wavelength, λ_g :-

$$\lambda_g = \frac{2\pi}{\beta_{nm}} \rightarrow (9)$$

5. cut-off Wavelength, λ_c :-

$$\lambda_c = \frac{c}{f_c} \rightarrow (10)$$

6. Phase velocity, v_p :-

$$v_p = \frac{\omega}{\beta_{nm}} \rightarrow (11)$$

7. Group velocity, v_g :-

$$v_g = \frac{c^2}{v_p} \rightarrow (12)$$

8. Intrinsic Impedance, η :-

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow (13)$$

For air medium,

$$\eta = 377\,\Omega$$

9. Angle of Incidence, θ :-

$$\theta = \cos^{-1}\left(f_c/f\right) \rightarrow (14)$$

10. characteristic Impedance, Z_{TE} :-

$$Z_{TE} = \frac{\omega\mu}{\beta_{nm}} \rightarrow (15)$$

(or)

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(f_c/f\right)^2}} \rightarrow (16)$$

Order of the Bessel function n	$(h'a)_{n1}$	$(h'a)_{n2}$	$(h'a)_{n3}$
0	3.83	7.01	10.17
1	1.841	5.33	8.53
2	3.05	6.73	9.97

EXCITATION OF RECTANGULAR WAVEGUIDES :

* In Order to launch a particular mode, a type of probe is chosen which will produce lines of E and H that are roughly parallel to the lines of E and H for that mode.

* Generally, a guide is closed at one end by a conducting wall. An antenna probe is inserted through the end or side of the guide.

* The end of the waveguide closed by a conducting wall acts a reflector. By properly adjusting the distance between the probe and the end, we can make transmitted wave inphase with reflected wave. so, that both the waves will propagate as a single wave.

* The excitation methods of rectangular waveguide for various modes is as shown in fig. below.

* In Figure (a), the probe is connected in parallel to y-axis and produces lines of 'E' in y-direction. Lines of 'H' lie in x-z plane. This is the correct field configuration for TE_{10} mode.

* In Figure (b), two probes are connected in parallel to y-axis and produces lines of E in

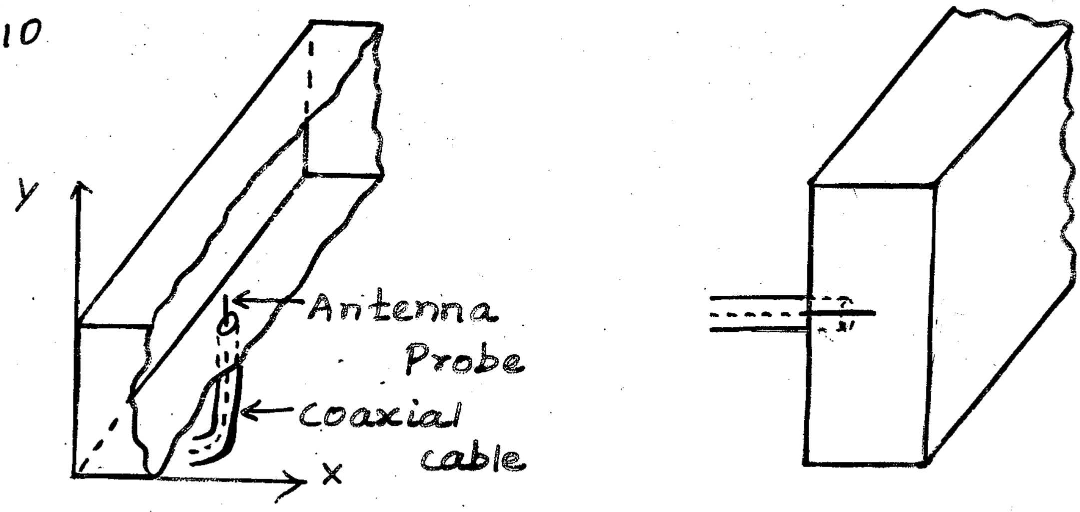
y-direction. The distance between the probes is $\frac{\lambda}{2}$. This is the correct field configuration for TE_{20} mode.

In Figure (c), the probe is connected at the terminated end. The probe is connected in parallel to z-axis and produces lines of E in z-direction. Lines of H lie in x-y plane. This is the correct field configuration for TM_{11} mode.

In Figure (d), two probes are connected in parallel to z-axis and produces lines of E in z-direction. The distance between the probes is $\lambda/2$. This is the correct field configuration for TM_{21} mode.

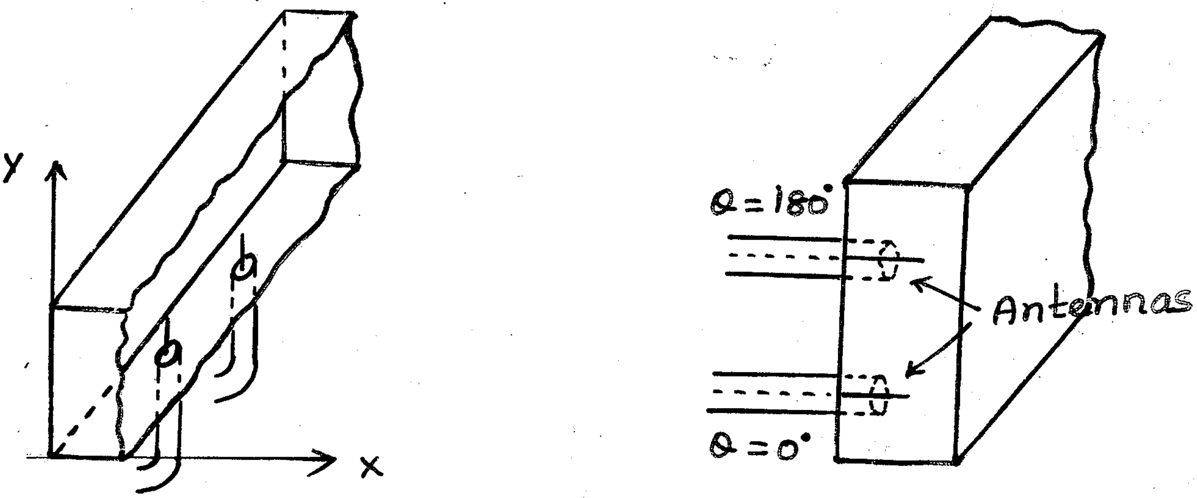
TE_{10}

a)



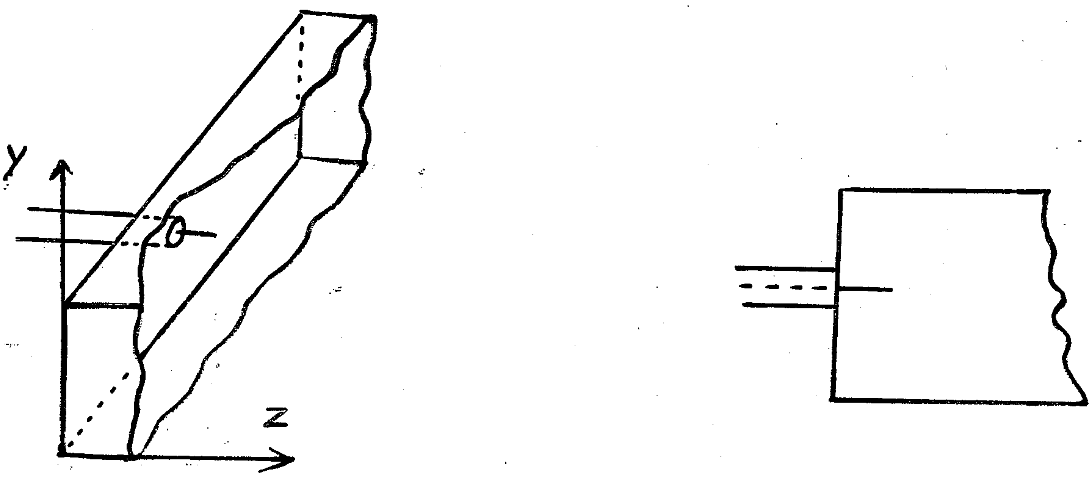
TE_{20}

b)



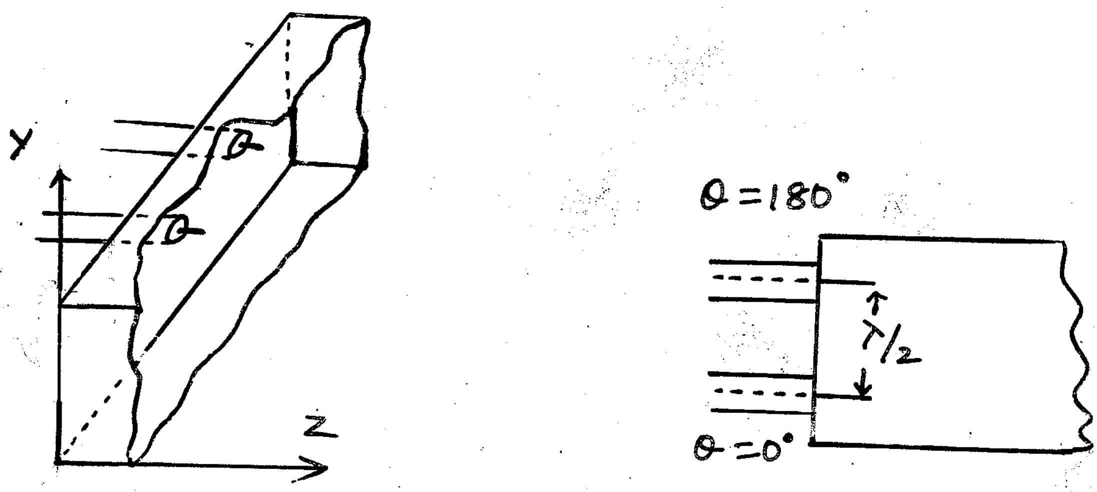
TM_{11}

c)

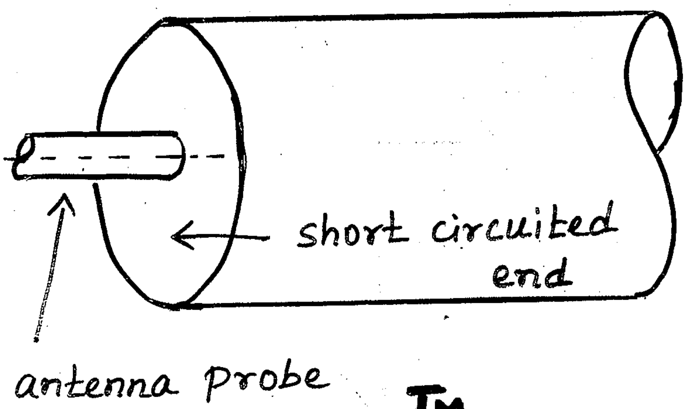


TM_{21}

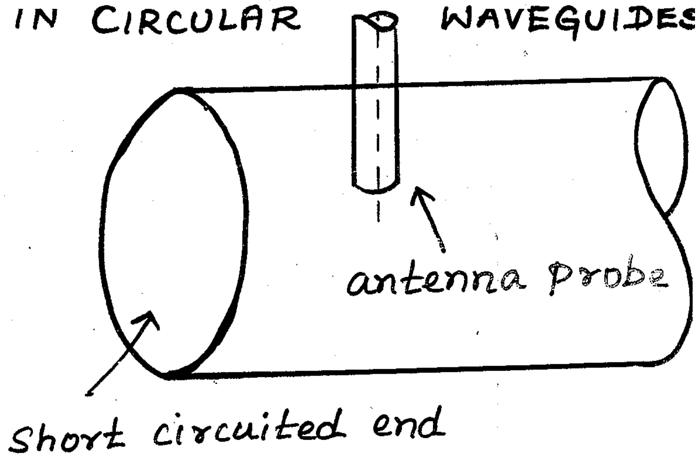
d)



EXCITATION OF MODES IN CIRCULAR WAVEGUIDES

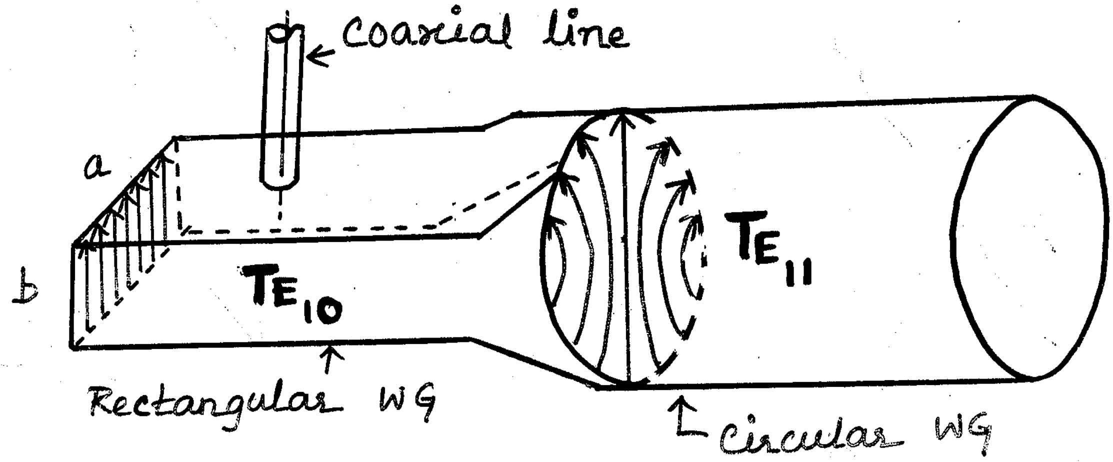


TM_{01}

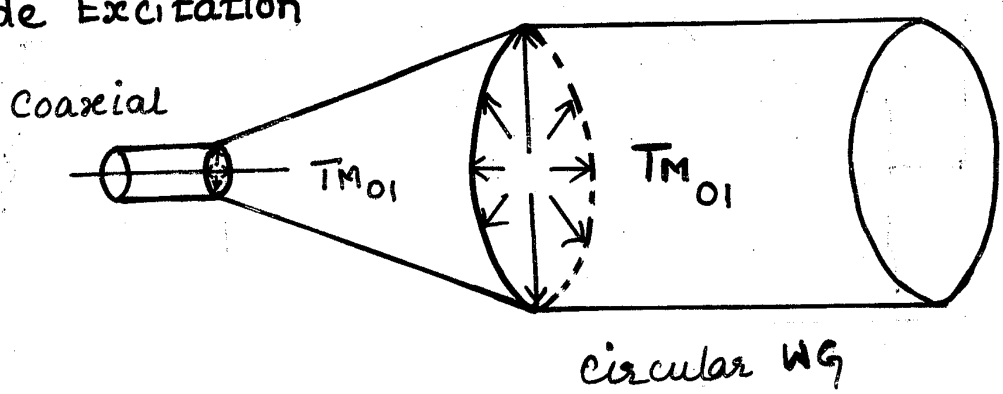


TE_{11}

TE_{11} mode Excitation



TM_{01} Mode Excitation



EXCITATIONS OF MODES IN CIRCULAR WAVEGUIDE :

The methods of excitation for various modes in circular waveguide is as shown below.

* In Figure (a) coaxial line probe excite the dominant mode TE_{10} in a rectangular waveguide which is converted to dominant mode TE_{11} in the circular waveguide through the transition length between them.

* In Figure (b) longitudinal coaxial line probe directly excites the symmetric mode TM_{01} in a circular waveguide.

* In Figure (c), TE_{01} mode is excited by means of two diametrically opposite placed longitudinal narrow slots parallel to the wall of the rectangular waveguide.

EC 8651 - Transmission lines & RF systems

Unit V

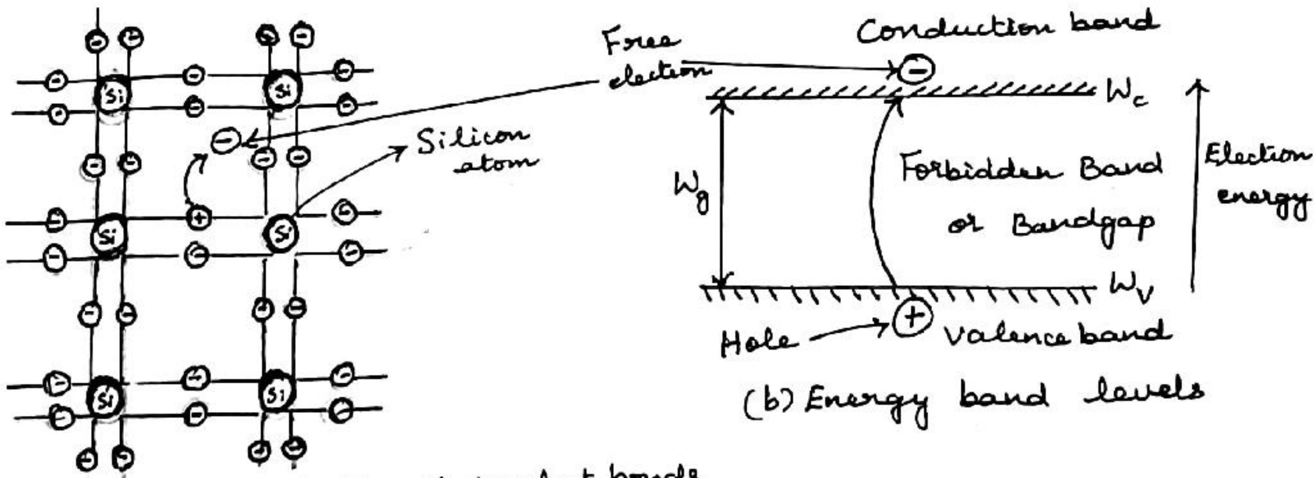
RF System Design concepts

Active RF components:

Semiconductor Physics:

The three most commonly used semiconductors are Germanium (Ge), Silicon (Si) and Gallium Arsenide (Ga As).

Bonding structure of Pure Silicon:



(a) Planar representation of covalent bonds

(b) Energy band levels

Each silicon atom shares its four valence elections with the four neighbouring atoms, forming four covalent bonds.

When temperature is equal to Zero degree Kelvin, all elections are bonded to the atoms and therefore the Semiconductor is not conductive. But when temperature increases, some of the elections obtain sufficient energy to break the covalent bond (called bandgap energy). These negatively charged free elections causes current conduction. The concentration of conduction elections is denoted as 'n'. The positively charged vacancy created due to break of covalent bond are called holes and their concentration is denoted as 'p'.

At $T > 0K$, recombinations of elections & holes may occur. In thermal equilibrium, the number of recombinations & generations of holes & elections are equal.

<u>Effective carrier concentrations:</u>

According to Fermi statistics,

$$n = N_c \exp\left(-\frac{W_c - W_F}{kT}\right) \;\&$$

$$p = N_v \exp\left(-\frac{W_F - W_v}{kT}\right)$$

where $N_{c,v} = 2\left(2 m_{n,p} \pi k T / h^2\right)^{3/2}$

Scanned with CamScanner

$N_c \rightarrow$ Conduction band

$N_v \rightarrow$ Valence band

$W_c \rightarrow$ Energy associated with conduction band

$W_v \rightarrow$ Energy associated with valence band

$W_F \rightarrow$ Fermi energy level

$m_{n,p} \rightarrow$ Effective mass of electrons / holes

$k \rightarrow$ Boltzmann's constant

$h \rightarrow$ Planck's constant

$T \rightarrow$ Absolute temperature in Kelvin.

Electrical conductivity of Intrinsic Semiconductor:

In an intrinsic semiconductor, the number of free electrons is equal to the number of holes.

(ie. $n = p = n_i$).

According to concentration law,

$$np = n_i^2 \qquad \text{——} \quad (1)$$

Substituting the expression of n & p in the above equation, we get

$$n_i^2 = N_c \cdot \exp\left(-\frac{W_c - W_F}{kT}\right) \cdot N_v \exp\left(-\frac{W_F - W_v}{kT}\right)$$

$$= N_c \cdot N_v \exp\left(\frac{-W_c + W_F - W_F + W_v}{2kT}\right)$$

$$= N_c \cdot N_v \exp\left(-\frac{W_c - W_v}{2kT}\right)$$

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or $n_i = \sqrt{N_c N_v} \exp\left(-\frac{W_c - W_v}{2kT}\right)$

Let $W_c - W_v = W_g$.

$$\therefore n_i = \sqrt{N_c N_v} \exp\left(-\frac{W_g}{2kT}\right) \quad \text{——— (2)}$$

The expression of electrical conductivity in a material due to applied electric field E is given as,

$$\sigma = qN\frac{V_d}{E} \quad \text{——— (3)}$$

where $V_d \rightarrow$ Drift velocity

$V_d \propto E$

$\Rightarrow V_d = \mu E$ [where μ is called carrier mobility]

$\therefore$ Equation (3) can be written as,

$$\sigma = qN\mu$$

or $\sigma = qn\mu_n + qp\mu_p$ [$\mu_n, \mu_p \rightarrow$ Mobilities of electrons & holes]

$$\sigma = qn_i(\mu_n + \mu_p) \quad [\because n = p = n_i] \quad \text{——— (4)}$$

Substituting equation (2) in (4), we get

$$\sigma = q\sqrt{N_c N_v} \exp\left(-\frac{W_g}{2kT}\right)(\mu_n + \mu_p)$$

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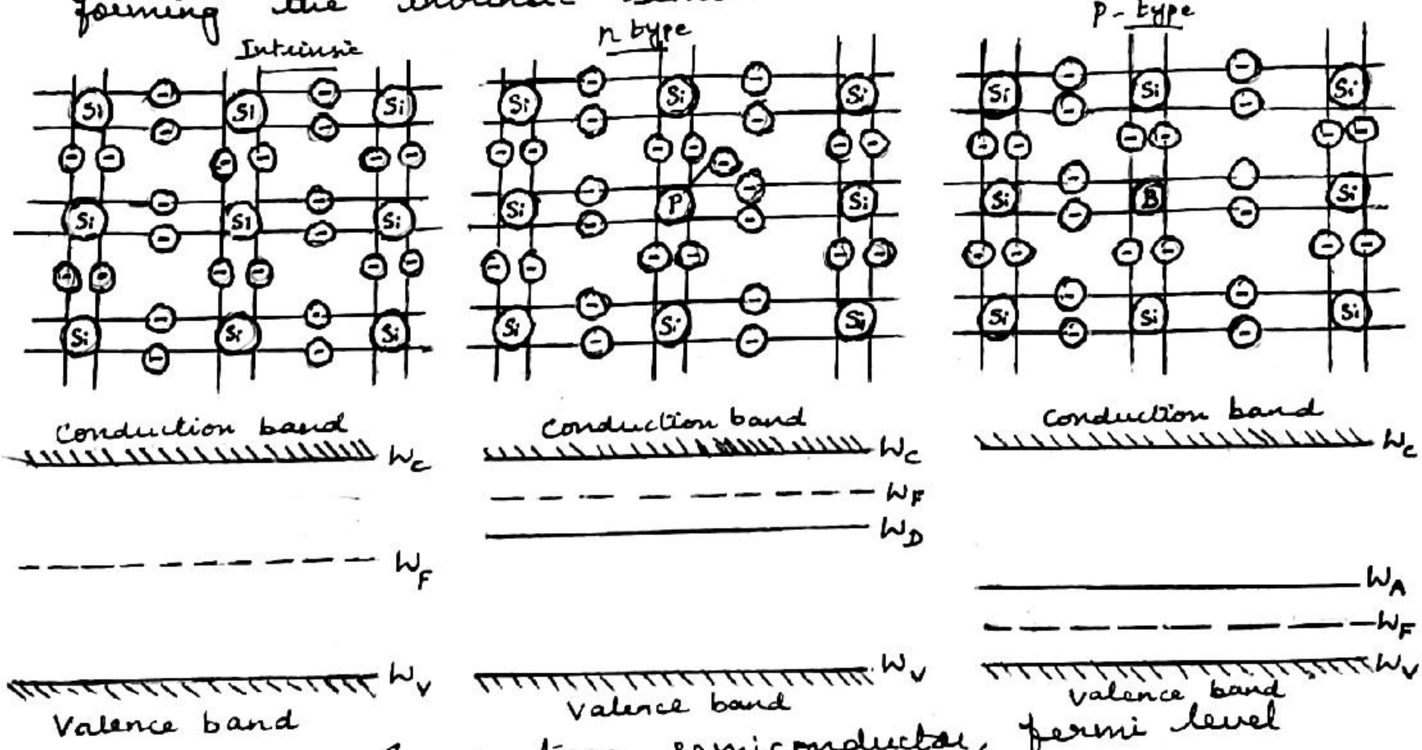
Doping:

The process of introducing impurity atoms to change the electrical properties of a Semiconductor is called Doping.

n-type Semiconductor:

This type of semiconductor is formed by adding pentavalent impurities (such as Phosphorous) with large number of valence elections than the atoms forming the intrinsic semiconductor lattice.



In n type semiconductor, fermi level is increased as more elections are located in the conduction band. The election concentiation is related to the hole concentiation as

$$n_n = N_D + p_n$$

$N_D \rightarrow$ Donor concentration

Where,
$$n_n = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2}$$

$$\& \quad p_n = \frac{-N_D + \sqrt{N_D^2 + 4n_i^2}}{2}$$

If $N_D >> n_i$

then $n_n \simeq N_D$

$$\& \quad p_n = \frac{-N_D + N_D\sqrt{1 + \frac{4n_i^2}{N_D^2}}}{2} = \frac{-N_D + N_D\left(1 + \frac{2n_i^2}{N_D^2}\right)}{2}$$

$$\therefore p_n \simeq \frac{n_i^2}{N_D}$$

<u>p-type</u> <u>Semiconductor</u>:

This type of semiconductor is formed by adding trivalent impurities (such as Boron) with fewer valence electrons than the atoms forming the intrinsic semiconductor lattice. The hole concentration in the p type semiconductor is given as,

$$p_p = N_A + n_p$$

where $N_A \rightarrow$ Acceptor electron concentration

$n_p \rightarrow$ electron concentration in p type semi-conductor.

$$p_p = \frac{N_A + \sqrt{N_A^2 + 4n_i^2}}{2}$$

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$$\& \; n_p = \frac{-N_A + \sqrt{N_A^2 + 4n_i^2}}{2}$$

For high doping level, $N_A >> n_i$

$$\therefore \; p_p \simeq N_A$$

$$\& \quad n_p \simeq \frac{-N_A + N_A\left(1 + 2n_i^2/N_A^2\right)}{2} \simeq \frac{n_i^2}{N_A}$$

~~The pn illustrations~~

Where,

$N_A \rightarrow$ Acceptor concentration

$n_i \rightarrow$ Intrinsic electron concentration

The pn Junction:

The physical contact of a p type & n type semiconductor is called pn junction.

The difference in the carrier concentration between the two types of semiconductor causes current flow across the interface and this current is called diffusion current. This current is composed of electrons & holes.

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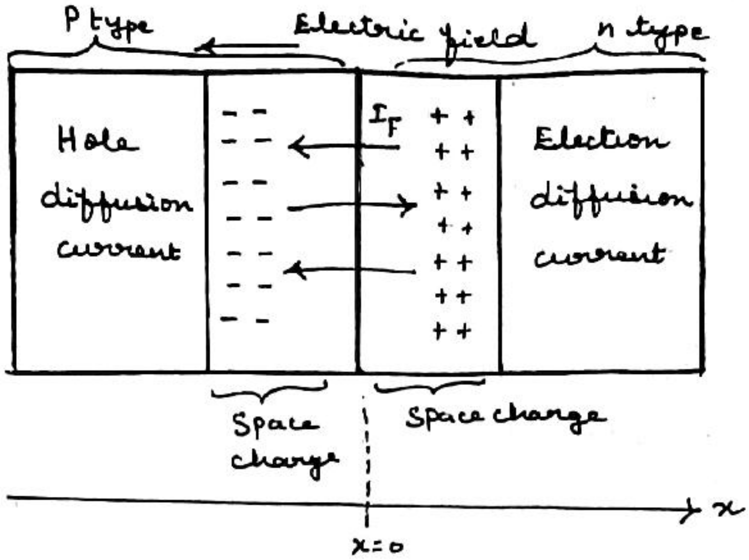

The diffusion current is composed of $I_{n\,diff}$ & $I_{p\,diff}$ components:

$$I_{diff} = I_{n\,diff} + I_{p\,diff}$$

$$= qA\left(D_n \frac{dn}{dx} + D_p \frac{dp}{dx}\right)$$

where

$A \rightarrow$ Semiconductor cross sectional area

$D_n \rightarrow$ Diffusion constants for elections

$D_p \rightarrow$ Diffusion constant for holes

$$D_{n,p} = \mu_{n,p} \frac{kT}{q} = \mu_{n,p} V_T$$

$V_T \rightarrow$ Thermal potential

$$V_T = \frac{kT}{q}$$

<u>Diffusion barrier voltage or built in potential:</u>

$$V_{diff} = V_T \ln\left(\frac{p_p}{p_n}\right) = V_T \ln\left(\frac{n_n}{n_p}\right)$$

If $N_D >> n_i$, then $n_n \simeq N_D$ & $n_p = \frac{n_i^2}{N_A}$

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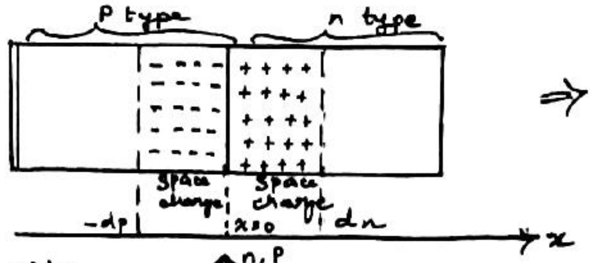
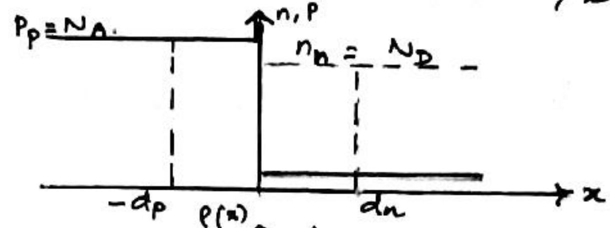
$$\therefore \; V_{diff} \simeq V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

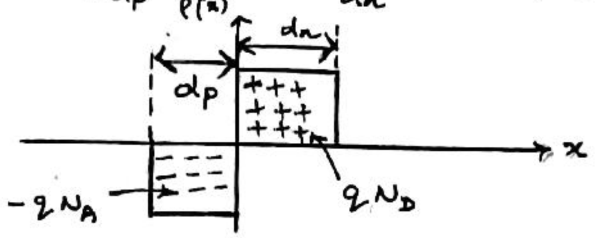
Pn junction in the absence of external applied voltage:



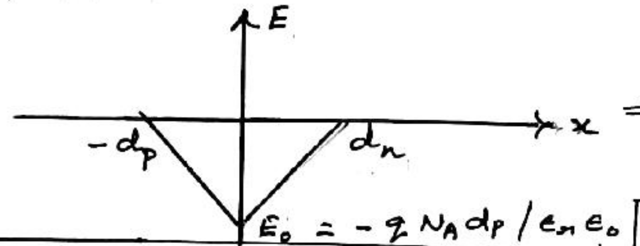
⇒ Pn junction with space charge extent



⇒ Acceptor & donor concentrations



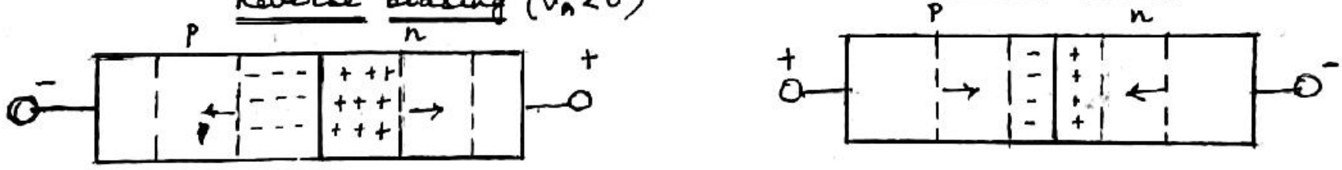
⇒ Polarity of charge density distribution



⇒ Electric field distribution

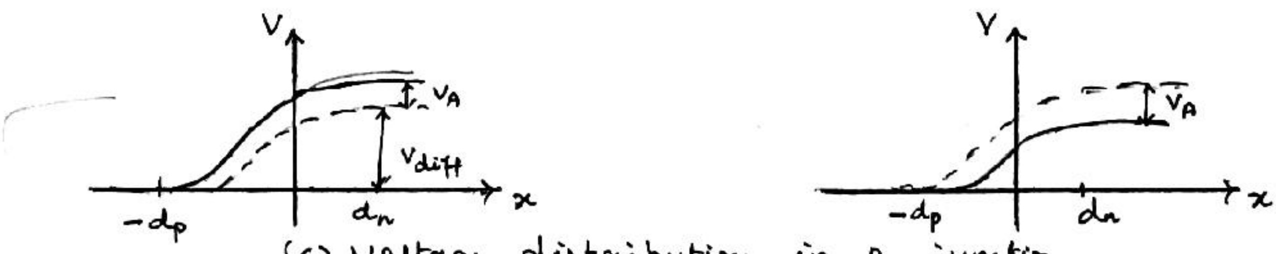
Pn junction in the presence of external applied voltage:

Reverse biasing ($V_A < 0$) Forward biasing ($V_A > 0$)



(a) Space charge distribution in pn junction

(b) Electric field distribution in pn junction



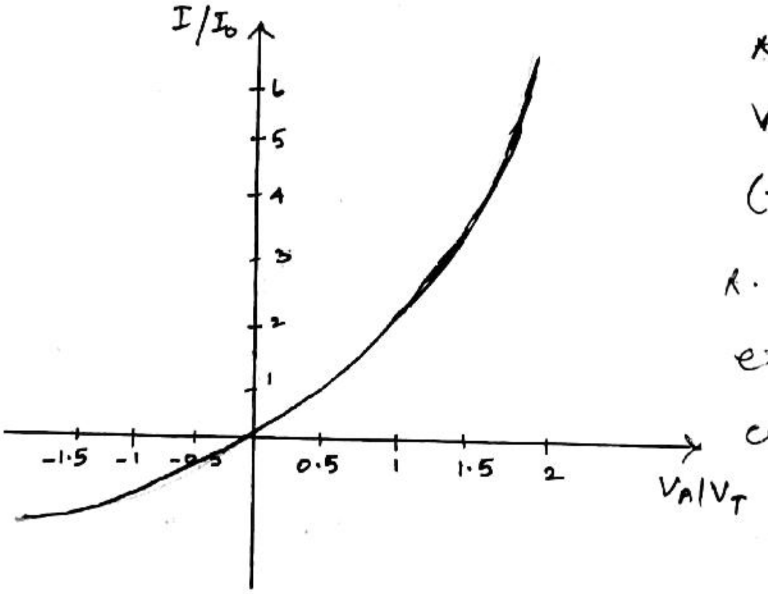
(c) Voltage distribution in pn junction

Forward bias	Reverse bias
1. Forward polarity decreases space charge domain	1. Reverse polarity increases the space charge domain
2. Increase in flow of current	2. Prevents the flow of current
3. Additional diffusion capacitance is encountered due to the presence of diffusion charges. $C_d = \frac{I_0 \tau_T}{V_T} e^{V_A/V_T}$	3. Leakage current occurs due to the movement of minority charge carriers. $I = I_0 \left(e^{V_A/V_T} - 1 \right)$ $I_0 \rightarrow$ Reverse saturation or leakage current.

Depletion layer Capacitance:

$$C_J = C_{J0} \left(1 - \frac{V_A}{V_{diff}} \right)^{-1/2}$$

I-V Charecteristics:



* For -ve voltage, a small voltage independent current ($-I_0$) will flows.
* For +ve voltages, an exponentially increasing current is observed.

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In general, the total capacitance C of a pn diode can be roughly divided into 3 regions.

(1) $V_A < 0$: Only the depletion capacitance is significant : $C = C_J$

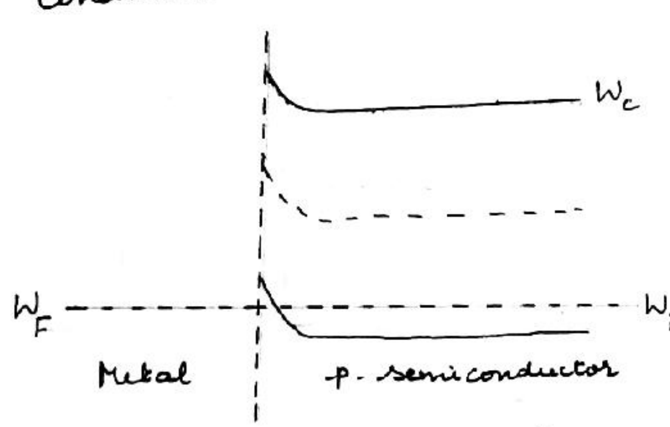
(2) $0 < V_A < V_{diff}$: depletion & diffusion capacitance combine : $C = C_J + C_d$

(3) $V_A > V_{diff}$: Only the diffusion capacitance is significant : $C = C_d$

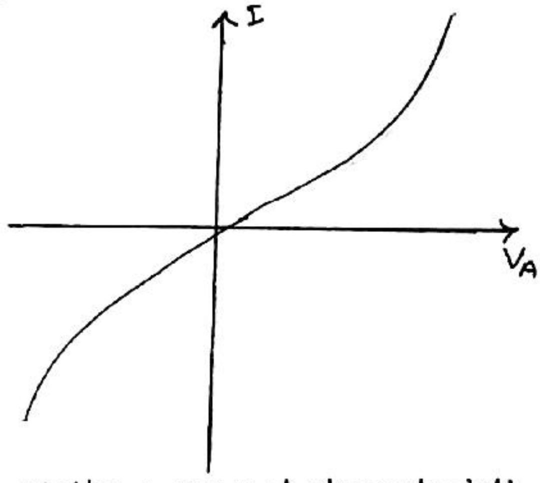
<u>Schottky</u> <u>contact</u>:

This refers to the contact between metallic electrode and semiconductor.

If a p semiconductor is in contact with a copper or alumunium electrode, there is a tendency for the electrons to diffuse into the metal, leading to increase in hole concentrations in the semi-conductor. This effect modifies the Valence and conduction band energy levels near the interface.



Energy band model

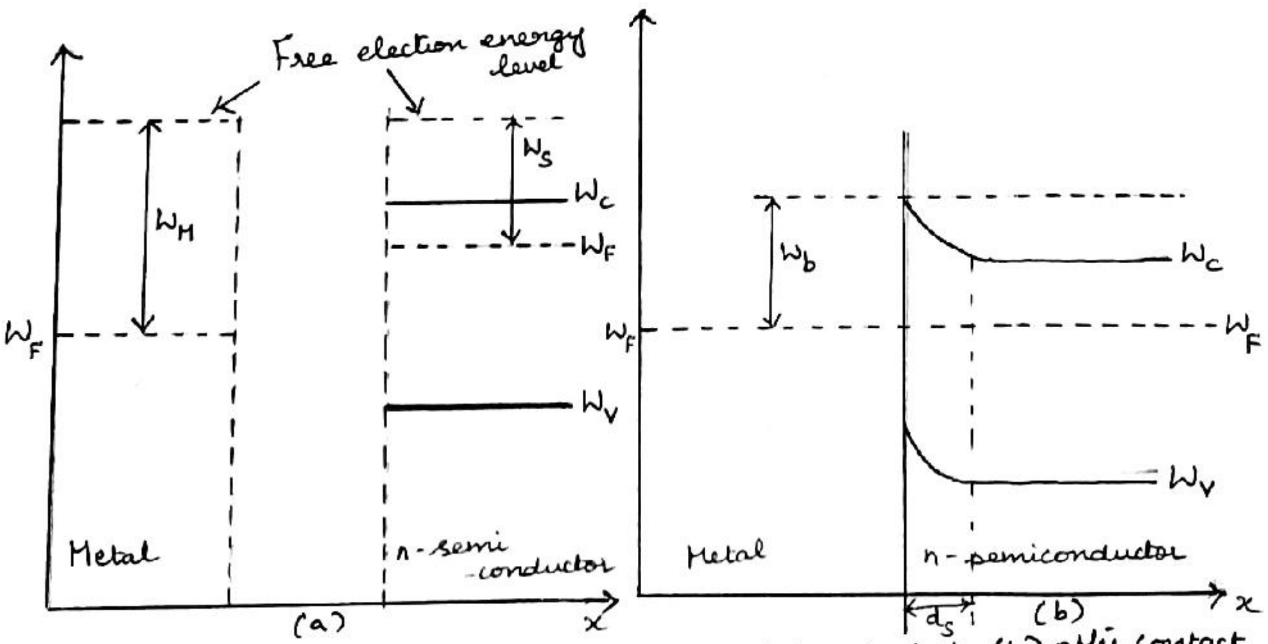


Voltage current characteristic

Metal electrode in contact with p-semiconductor

Because of the higher concentation of holes, the valence band bends toward the fermi level. The conduction band bends away from the fermi level. ∴ Irrespective of the polarity of the applied voltage, a low resistance contact is obtained.

When n type semiconductor is in contact with metal, electrons diffuse from the n semiconductor & leave behind positive space charge. The depletion zone grows until the electrostatic repulsion of the space charges prevents further electron diffusion.



Energy band diagram of Schottky contact (a) before contact (b) after contact

The junction capacitance of the Schottky contact is given as,

$$C_J = A\left[\frac{q\epsilon}{2(V_d - V_A)} N_D\right]^{1/2}$$

; V_d → Built in schottky barrier voltage

Bipolar Junction Transistor:

Bipolar Junction Transistor is a multi junction semiconductor device, where both the types of charge carriers take part in current carrying mechanism. two types of Bipolar Junction Transistors are n-p-n and p-n-p. The n-p-n Bipolar Junction Transistor is the complimentary structure of the p-n-p Bipolar Transistor.

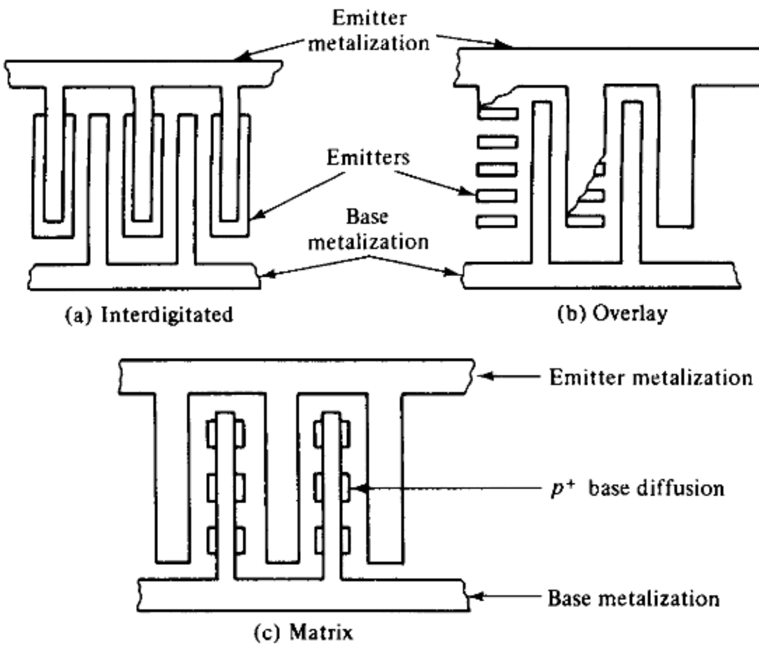
The principle of operation of microwave Bipolar Junction Transistor is similar to that of low frequency device bipolar transistor. All microwave Bipolar Transistor are planar in form and is of n-p-n type.

The majority of Bipolar Junction Transistors are fabricated from silicon because of low cost, more reliable integrative, offers higher gain and moderate noise figure when used as a microwave amplifier.

Microwave Bipolar Junction Transistors are capable of generating power upto a frequency of 22GHz.

Physical Structure:

The physical structure of microwave power transistor is as shown in figure below. the physical structure can be classified as a) inter-digitated b) Overlay c) Matrix type (also called as mesh or emitter grid)



Inter-digitated structure consists of large number of emitter strips alternating with base strips. Both of these are metallized. Overlay structure has a large number of segmented emitters overlaid through a number of wide metal strips. Matrix or mesh structure has emitter that forms the grid, the base filling the meshes of this grid with a p+ contact area in the middle of each mesh.

Inter-digitated structure is suitable for small signal applications in the L,S, and C bands whereas overlay and mesh structures are useful as power devices in the VHF and UHF regions.

Bipolar Transistor Configurations:

In general, there are two types of Bipolar transistors: n-p-n and p-n-p. A transistor can be connected as 3 different configurations: Common Base (CB), Common Emitter (CE) and Common Collector (CC).

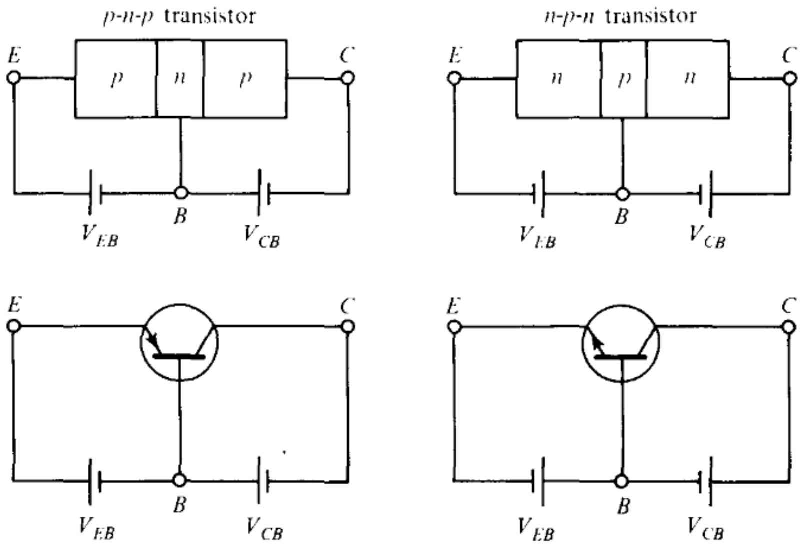
Common Base Configuration:

In common base configuration, the base terminal is common for both input circuit (Emitter) and output circuit(Collector). The common base configuration is also called as grounded base configuration.

common base configuration's input voltage V_{EB} and output current I_C can be expressed in terms of the output voltage V_{CB} and input current I_E as,

V_{EB} = some function (V_{CB}, I_E)

I_C = some function (V_{CB}, I_E)



(a) CB configuration (b) CB configuration

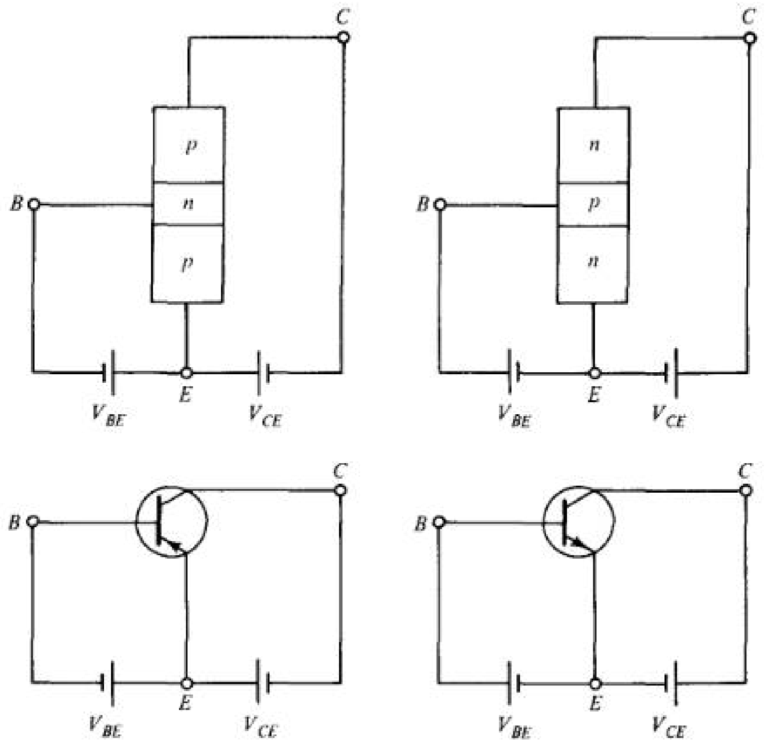
Common Emitter Configuration:

In common Emitter configuration, the emitter terminal is common for both input circuit (Base) and output circuit(Collector). The common emitter base configuration is also called as grounded emitter configuration.

common emitter configuration's input voltage V_{EB} and output current I_C can be expressed in terms of the output voltage V_{CB} and input current I_B as,

V_{EB} = some function (V_{CE}, I_B)

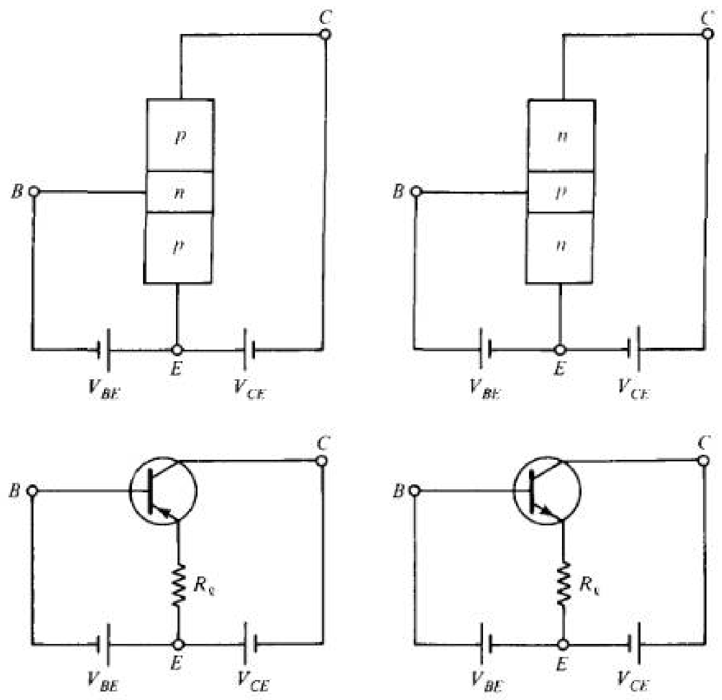
I_C = some function (V_{CE}, I_B)



Common Collector Configuration:

In common collector configuration, the collector terminal is common for both input circuit and output circuit. In a common collector configuration, the output voltage of the load is taken from the emitter terminal instead of the collector as in the common base and common emitter configuration.

The common collector configuration transistor can be used as a switch or pulse amplifier. The common collector amplifier has no voltage gain.



Principles of Operation:

The bipolar junction transistor is an active device which is commonly used as an amplifier or switch. A BJT can operate in four different modes depending on the voltage polarities across the two junctions.

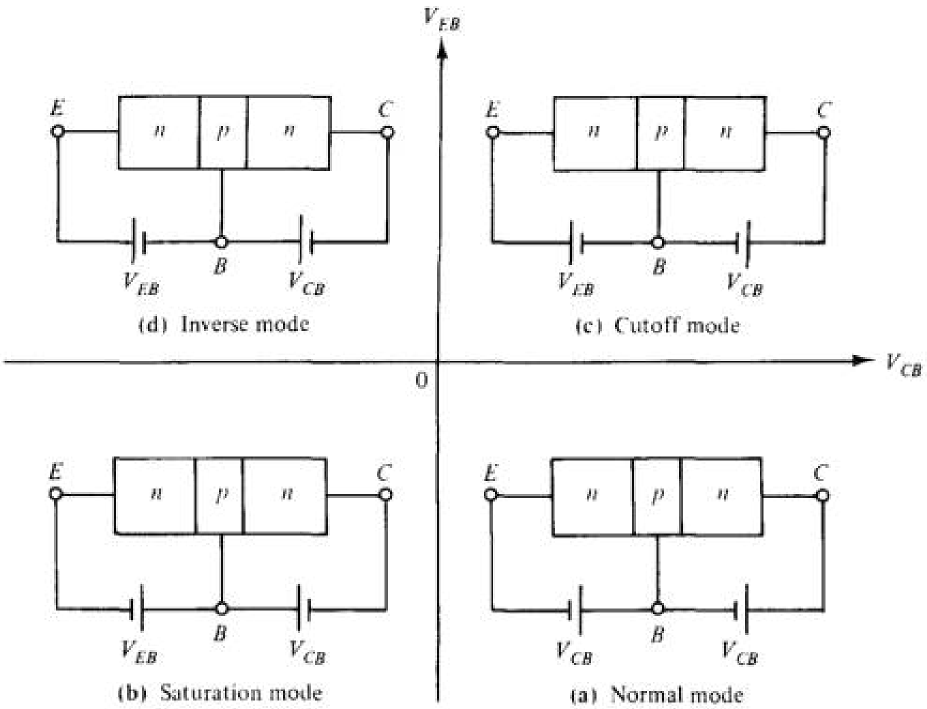
1. Normal Mode: In this mode, emitter junction of npn transistor is forward biased and collector junction is reverse biased . Generally at ON state a transistor remains in the normal mode.

2. Saturation Mode: When both the junctions are forward biased, the transistor is in its saturation mode with very low resistance and acts like a short circuit.

3. Cut-Off Mode: If both transistor junctions are reverse biased, the transistor is operated in cut-off mode and the transistor acts like an open circuit.

Thus saturation and cut-off modes are equivalent to the ON and OFF state of a switch.

4. Inverse Mode: A transistor is said to be in inverse mode when the emitter is reverse biased and collector is forward biased. In practice transistor is not commonly used in inverse mode.

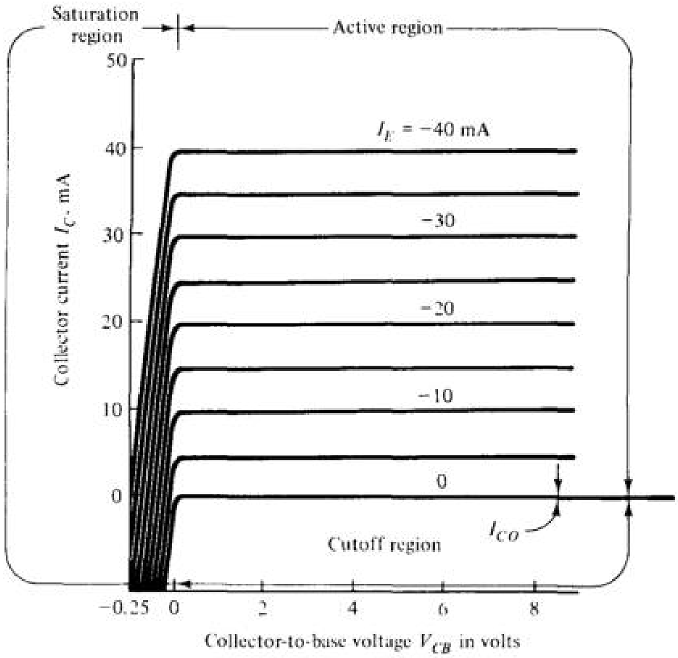


There are three regions for the 1-V characteristics of an *n-p-n* bipolar transistor:

1. Active Region: In this region the emitter junction is forward-biased and the collector junction is reverse-biased. The collector current I_C is essentially independent of collector voltage and depends only on the emitter current I_E. When the emitter current is zero, the collector current is equal to the reverse saturation current I_{CO}.

2. Saturation Region: In this region, as shown on the left side of figure, both emitter and collector junctions are forward-biased. The electron current flows from the *n* side across the collector junction to the p-type base. As a result, the collector current increases sharply.

3.Cutoff Region: In this region the emitter and collector junctions are both reverse-biased. Consequently, the emitter current is cut off to zero, as shown in the lower right side of figure.



Performance Parameter:

In high frequency operation, the performance of a microwave transistor depends on the cut-off frequency 'f_c' and maximum frequency of oscillation (f_{max}) rather than the two current gains α and β.

Now the cut-off frequency depends on the delay of the carrier results due to their movement from emitter to collector.

$$f_c = \frac{1}{2\pi\tau_{ec}} \qquad (1)$$

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c \qquad (2)$$

where, τ_e - Emitter base junction transit time

τ_c - Collector depletion layer charging time

τ_b - Base transit time

τ_d - Collector depletion layer transit time

Maximum frequency of operation is higher than f_c because although 'β' falls to unity at this frequency, power gain does not.

$$f_{max} = \sqrt{\frac{f_c}{8\pi r'_b C_C}} \qquad (3)$$

where, rb' - Base resistance

C_C - Collector Capacitance

RF FIELD EFFECT TRANSISTOR:

- Field effect transistor is a multi junction monopolar device, where only one carrier type either holes or electrons contribute to the current flow through the channel.
- Based on the contribution there are two types, n-Channel (electron) and p-channel (hole).
- FET is a voltage controlled device.
- RF field effect transistors has the capability of amplifying small signals up to the frequency range of X band with low noise figures.
- The RF field effect transistors has several advantages over the Bipolar junction transistor.
    1. Its Efficiency is higher than BJT.
    2. Its noise figure is low.
    3. Its operating frequency is up to X band
    4. Its input resistance is very high up to several mega ohms.

CONSTRUCTION:

FETs are classified according to how the gate is connected to the conducting channel. Specifically, there are four types. They are,

1. MISFET - Metal Insulator Semiconductor FET:

Here the gate is separated from the channel through an insulation layer. One of the most widely used type is MOSFET (Metal Oxide Semiconductor FET)

2. JFET - Junction FET:

This type relies on a revere biased pn-junction that isolates the gate from the channel.

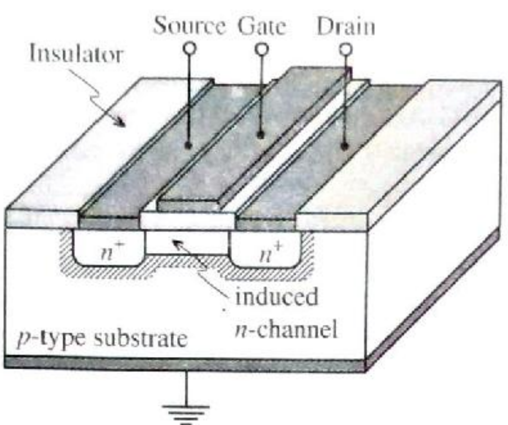
3.MESFET - Metal Semiconductor FET:

If the reverse biased pn-junction is replaced by a schottky contact, the channel can be controlled just as in the JFET case.

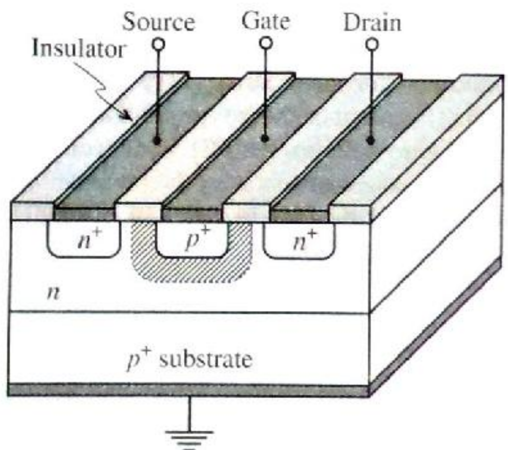
4. Hetro FET:

As the name implies, the transitions takes place between different layer of semiconductor materials. Examples: GaAlAs to GaAs or GaInAs to GaAlAs interfaces. High Electron Mobility Transistor(HEMT) belongs to this class.

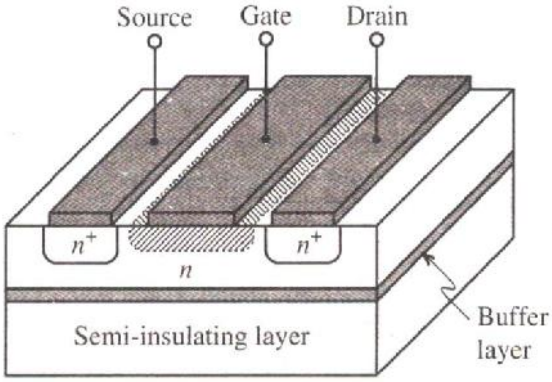
The construction of MISFET, JFET, and MESFET is as shown in the figure below.



(a) Metal insulator semiconductor FET (MISFET)



(b) Junction field effect transistor (JFET)



(c) Metal semiconductor FET (MESFET)

- In the above shown FETs, the current flows from source to drain and the gate controls the current flow. Due to the presence of a large capacitance formed by the gate electrode and the reverse biased pn-junction, MISFETs and JFETs have a relatively low cut-off frequency and are usually operated in low and medium frequency ranges of typically upto 1 GHz.
- GaAs MESFETs find applications upto 60-70 GHz and HEMT can operate beyond 100GHz.
- Electrically FETs can be classified into two types, 1) Enhancement and 2) Depletion type based on increase in carriers or depletion in carriers when the gate voltage is increased.

Functionality:

The functionality of MESFET for different drain-source voltages are shown in figure below. The transistor is operated in depletion mode. The schottky contact builds up channel space charge domain that affects the current flow from the source to drain. The space extent d_s can be controlled via the gate voltage.

$$d_s = \left(\frac{2\varepsilon V_d - V_{GS}}{qN_D} \right)^{\frac{1}{2}} \qquad (1)$$

where, d_s - Space extent or Space charge

N_D - Donor concentration

V_d - Barrier voltage 0.9v for GaAs-Au interface

q - Charge of an electron ($1.602\text{x}10^{-19}$)

V_{GS} - Gate source voltage

The resistance 'R' between source and drain is predicted by,

$$R = \frac{L}{\sigma(d - d_s)W} \qquad (2)$$

where, W - Gate Width

L - Gate Length

σ - Conductivity

d - Channel depth

d_s - Space charge

$$\sigma = q.\mu_n.N_D \qquad (3)$$

Where, $.\mu_n$ - Mobility of electron

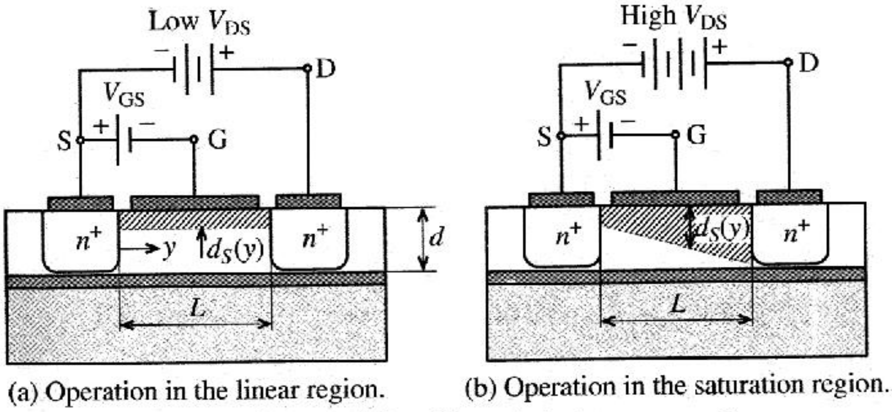
N_D - Donor Concentration

The drain current is given by,

$$I_D = \frac{V_{DS}}{R} = G_O \left[1 - \left(\frac{2\varepsilon}{qd^2} \frac{V_d - V_{GS}}{N_D} \right)^{\frac{1}{2}} \right] V_{DS} \qquad (4)$$

Where, Conductance G_O is,

$$G_O = \frac{\sigma q N_D W d}{L} \qquad (5)$$



(a) Operation in the linear region. (b) Operation in the saturation region.

Functionality of MESFET for different drain-source voltages.

The pinch-off voltage for the FET is independent of the gate-source voltage and is computed as,

$$V_P = \frac{qN_D d^2}{2\varepsilon} \qquad (6)$$

where, V_P - pinch-off voltage

q - Charge of an electron ($1.6\text{x}10^{-19}$)

ε - Permittivity

The threshold voltage for the FET is given as,

$$V_{TO} = V_d - V_P \qquad (7)$$

The Drain saturation current is

$$I_{DSat} = G_O \left[\frac{V_P}{3} - (V_d - V_{GS}) + \frac{2}{3\sqrt{V_P}} (V_d - V_{GS})^{\frac{3}{2}} \right] \qquad (8)$$

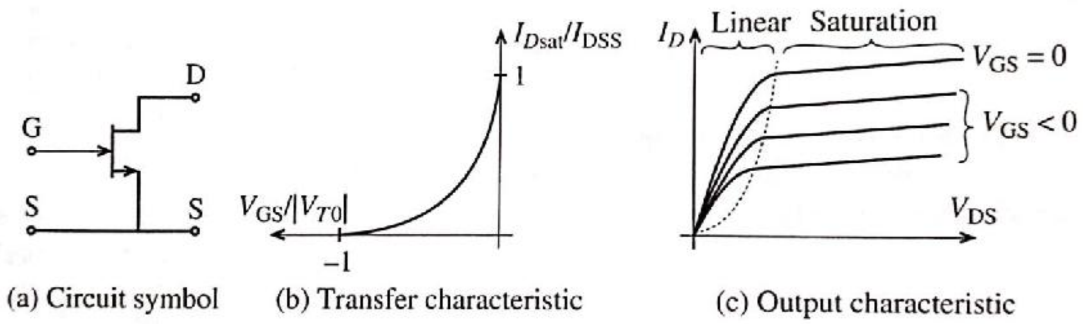
The maximum saturation current is obtained when $V_{GS} = 0$

$$I_{DSat} = G_O \left[\frac{V_P}{3} - (V_d) + \frac{2}{3\sqrt{V_P}} (V_d)^{\frac{3}{2}} \right] \qquad (9)$$

The saturation drain current is often approximated by the simple relation

$$I_{DSat} = I_{DSS} (1 - \frac{V_{GS}}{V_{TO}})^2 \qquad (10)$$

The transfer and output characteristics of an n-channel MESFET is as shown below.



Transfer and output characteristics of an *n*-channel MESFET

Problem:

1. A GaAs MESFET has the following parameters: $N_D = 10^{16} cm^{-3}$, d=0.75μm, W=10μm, L=2μm, ε_r=12.0, V_d=0.8v and μ_n=8500cm²/(Vs). Determine a) pinch-off voltage, b)Threshold Voltage, c) The maximum saturation current I_{DSS}.

Solution:

a) pinch-off voltage:

The pinch-off voltage for the FET is,

$$V_P = \frac{qN_Dd^2}{2\varepsilon} = \frac{1.6x10^{-19}\,x\,(10^{16}x10^6)\,x\,(0.75x10^{-6})^2}{2\,x\,8.854\,x10^{-12}\,x12} \textbf{ = 4.235V}$$

b)Threshold Voltage,

$V_{TO} = V_d - V_P$ = 0.8 - 4.235 = **-3.435v**

c) The maximum saturation current I_{DSS}

$$I_{DSat} = G_O\left[\frac{V_P}{3} - (V_d) + \frac{2}{3\sqrt{V_P}}(V_d)^{\frac{3}{2}}\right]$$

$$G_O = \frac{\sigma q N_D W d}{L} = \frac{q^2\mu_n N_D{}^2 W d}{L} = \frac{(1.6x10^{-19})^2\,x\,(8500\,x10^{-4})x\,(10^{16}x10^6)^2\,x(10x10^{-6})x(0.75x10^{-6})}{2x10^{-6}} \textbf{ =8.16}$$

$$I_{DSat} = 8.16\left[\frac{4.235}{3} - (0.8) + \frac{2}{3\sqrt{4.235}}(0.8)^{\frac{3}{2}}\right] \textbf{ =6.883A}$$

High Electron Mobility Transistor (HEMT)

High electron mobility transistor(HEMT) is a transistor that operates at higher frequencies, typically in the microwave range. They are used in applications that require high frequency, such as cell phones, RF applications, and some power applications.Essentially the device is a field-effect transistor that incorporates a junction between two materials with different band gaps (i.e. a heterojunction) as the channel instead of a doped region which is used in the standard MOSFET.

As a result of its structure, the HEMT may also be referred to as a heterojunction FET, HFET or modulation doped FET, MODFET on some occasions.

HEMTs are transistors that utilize the 2-dimensional electron gas(2DEG) created by a junction between two materials with different band gaps called a heterojunction. The two most commonly used materials to create the heterojunction are a highly doped n-type donor material, typically AlGaAs and an undoped material, typically GaAs.

HEMT's take advantage of 2DEG which is created at the AlGaAs/GaAsheterojunction. The 2DEG is confined at the heterojunction and free to move parallel to the channel. This results in a higher electron mobility which is good for large gain and high frequency characteristics.

HEMT structure & fabrication

The key element within a HEMT is the specialized PN junction that it uses. It is known as a hetero-junction and consists of a junction that uses different materials either side of the junction. The most common materials used aluminium gallium arsenide (AlGaAs) and gallium arsenide (GaAs).Gallium arsenide is generally used because it provides a high level of basic electron mobility which is crucial to the operation of the device. Silicon is not used because it has a much lower level of electron mobility.

In the manufacture of a HEMT, first an intrinsic layer of gallium arsenide is set down on the semi-insulating gallium arsenide layer. This is only about one micron thick.Next a very thin layer (between 30 and 60 Angstroms) of intrinsic aluminium gallium arsenide is set down on top of this. Its purpose is to ensure the separation of the hetero-junction interface from the doped aluminium gallium arsenide region. This is critical if the high electron mobility is to be achieved.

The doped layer of aluminium gallium arsenide about 500 Angstroms thick is set down above this. Precise control of the thickness of this layer is required and special techniques are required for the control of this.

There are two main structures that are used. These are the self-aligned ion implanted structure and the recess gate structure. In the case of the self-aligned ion implanted structure the gate, drain and source are set down and are generally metallic contacts, although source and drain contacts may sometimes be made from germanium. The gate is generally made from titanium, and it forms a minute reverse biased junction similar to that of the GaAsFET.

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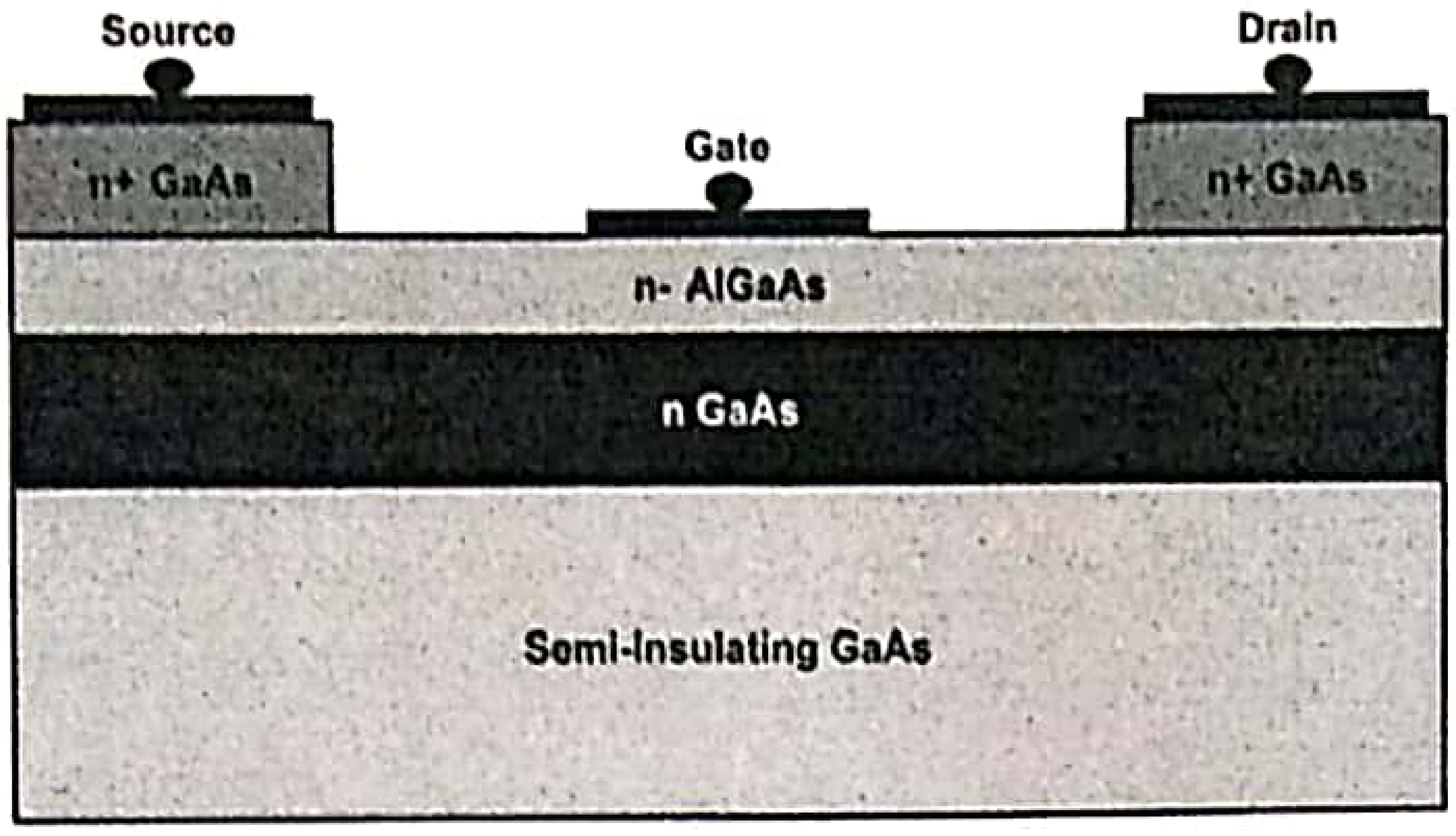
For the recess gate structure another layer of n-type gallium arsenide is set down to enable the drain and source contacts to be made. Areas are etched as shown in the diagram. The thickness under the gate is also very critical since the threshold voltage of the FET is determined by this. The size of the gate, and hence the channel is very small. Typically the gate is only 0.25 microns or less, enabling the device to have a very good high frequency performance.



HEMT operation

Electrons from the n-type region move through the crystal lattice and many remain close to the hetero-junction. These electrons form a layer that is only one electron thick forming what is known as a two dimensional electron gas. Within this region the electrons are able to move freely because there are no other donor electrons or other items with which electrons will collide and the mobility of the electrons in the gas is very high.

A bias applied to the gate formed as a Schottky barrier diode is used to modulate the number of electrons in the channel formed from the 2 D electron gas and in turn this controls the conductivity of the device. This can be compared to the more traditional types of FET where the width of the channel is changed by the gate bias.

Advantages of HEMT devices:

- ***High gain:*** HEMTs have a high gain at microwave frequencies because the charge carriers are almost exclusively the majority carriers and the minority carriers are not significantly involved.
- ***Low noise:*** HEMTs provide very low noise operation because the current variation in the devices is low when compared to other field effect devices.

Applications of HEMT

- Next generation wired/wireless communication
- Advanced radars
- Power electronics

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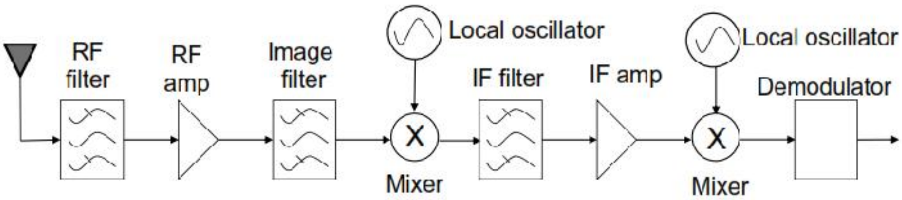
BASIC CONCEPTS OF RF DESIGN:

Radio-frequency (RF) engineering is about systems that operate at radio frequencies such as microwave frequencies. The RF portion of radio transmitters and receivers will be viewed as a subsystem of wireless systems. Thus, the relationships of the RF portion to other parts of the overall wireless system design will be pertinent. For example, radio receiver sensitivity depends on the RF design. RF generally includes other aspects, such as the device technologies and RF circuits (including active circuits and passive circuits). Real electronic components introduce noise and have other imperfections, such as nonlinearities. While the nature of noise, nonlinearities, and so on, is intimately related to the devices themselves, the results on the system can be studied and quantified at the systems level based on models of these effects.

The most popular wireless receiver architecture is known as the superheterodyne receiver. A block diagram of a superheterodyne radio receiver is as shown in figure below. In the figure shown below amplifiers, mixers, frequency synthesizers, and filters are the fundamental building blocks of the RF part of radios. Broadly speaking, an amplifier amplifies the power of a signal; a mixer is used to up-convert or down-convert a signal, by multiplying (also described as mixing) it with a periodic signal, such as would be produced by a frequency synthesizer. A frequency synthesizer may be as simple as an oscillator, or it may include an oscillator together with additional circuitry. A filter selects a band of frequencies to pass through, and attenuates signal components at other frequencies.



Superhetrodyne Receiver

The communications signals transmitted over wireless are at very high frequencies and so are often referred to as being “at RF.” To demodulate the signals and detect what was transmitted, the RF section of the receiver often needs to bring the signal down to around baseband. The superheterodyne receiver brings the signal from RF down to around baseband in two stages; first, it down-converts from RF to an intermediate frequency (IF), and second, it down-converts from IF to around baseband. Having two stages of down-conversion introduces some challenges.

Noise and distortion are the limiting factors in the RF circuit performance. Quantifying noise and distortion is necessary to quantify the performance of a transceiver.

1. NOISE:

Noise is always being picked up by a receiver from the rest of the universe when a desired signal is being fed into a system. Noises can be introduced into a circuit during a radio signal transformation. There is one kind of noise, which is called thermal energy, generated due to the temperature related motion of charged particles. Thermal energy is caused by atoms and electrons move in a random way resulting in random currents in a circuit itself. On the other hands, there are also many other man-made noises coming from outside the circuit system, such as microwave, cell phones, and even power chargers. In order to check out how much noise has been added to a source signal, a ratio of the signal to noise power is defined for a receiver. The sum of thermal noise power and circuit generated noise presented at the receiver front-end is defined as the noise floor. To detect a reliable signal, the minimum detectable signal level must typically be larger than its noise floor.

Thermal Noise:

Resistors are the most possible components that will cause noise in a circuit. Due to thermal energy, noise will be generated in resistors causing random currents in the circuit. The formula of thermal noise in spectral density from resistors can be expressed as follows:

$$N_{resistor} = 4kTBR \qquad (1)$$

where, k - Boltzman constant (1.38×10^{-23} J/K)

T - Kelvin temperature of resistor (300K)

B - Bandwidth

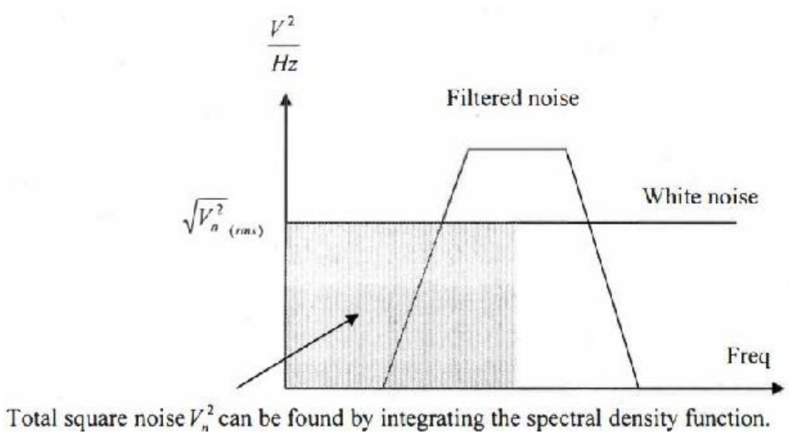
R - Value of Resistor

In additional, thermal noise is also white noise. This means that the thermal noise involves a constant power spectral density with respect to frequency. Therefore, to find out how much power is generated in a finite bandwidth in a resistor, the formula is presented as follows:

$$V_n^2 = 4kTR\Delta f \ (v^2/Hz) \qquad (2)$$

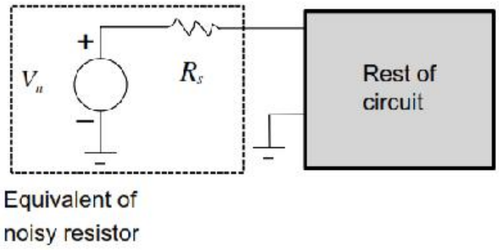
where, Δf - bandwidth

V_n^2 - noise voltage in rms value.



Noise power in spectral density respect to frequency.

Usually, the mean value of noise will be zero when noise is random. Therefore, in order to measure the dissipated noise power, it is needed to use mean square values. The below figure shows the spectral noise power density with respect to frequency. A model of resistor noise with a voltage source is as shown below.



Noise Factor and Noise Figure:

The Noise Figure (NF) describes how much noise is added to a signal by elements of a radio's receiver chain. There are many different ways to define NF, but the most common definition is,

$$NF = \frac{SNR_{in}}{SNR_{out}} \tag{1}$$

where SNR_{in} is the input SNR due to thermal noise and SNR_{out} is the device output SNR. The NF provides an indication of how the device degrades the SNR. The manufacturer of the device usually supplies the NF. In the case of passive components, the NF equals the loss of the passive components. Thus if a passive RF filter provides a 3 dB loss of signal, the NF is defined to be 3 dB.

Once the NF is determined, it is possible to provide an equivalent RF receiver NF, NF_{total}, that relates the noise back to the antenna. The Friis equation allows for the NF of all the devices in the RF chain to be combined.

$$NF_{total} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \frac{NF_4 - 1}{G_1 G_2 G_3} + \frac{NF_5 - 1}{G_1 G_2 G_3 G_4} + \ldots\ldots \tag{2}$$

Here NF_i represents the NF at the i^{th} stage and G_i represents the gain at the i^{th} stage. This equation assumes a linear scale, although NF is usually used with a dB scale. Given a component with a noisy input having noise power P_{i-1} dBm, gain G_i dB, and NF NF_i dB, the output noise power P_i dBm is given by

$$P_i \text{ dBm} = P_{i-1} \text{ dBm} + NF_i \text{ dB} + G_i \text{ dB} \tag{3}$$

Once the overall NF is determined, it is possible to determine the minimum input signal level discernible by the receiver or the sensitivity of the receiver to achieve a minimal SNR, SNR_{min}. Sensitivity, S dBm is calculated with the basic definition,

$$S \text{ dBm} = \text{Noise floor dBm} + SNR_{min} \text{ dB} \tag{4}$$

The NF is calculated from the source thermal noise, its magnification due to the total NF, and the bandwidth over which this noise exists.

Noise floor dBm = 10 $\log(kT_eB)$ + NF_{total} dB

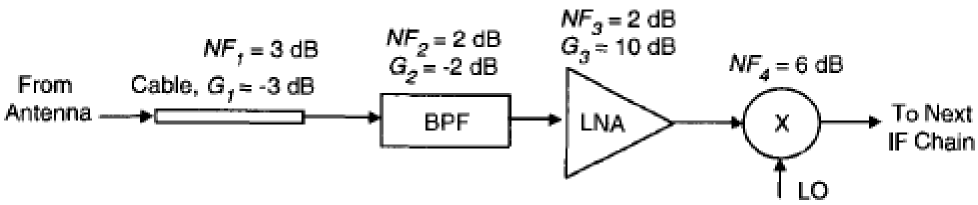
= 10 $\log(kT_e)$ dBm + 10 log (B) dB + NF_{total} dB (5)

Where, B is the end of the system bandwidth in Hz and T_e is taken as its usual value of room temperature, 290K. For room temperature, the sensitivity becomes

S = -174 dBm/Hz + NF dB + 10log(B) + SNR_{min} dB. (6)

Problem:

1. The block diagram of an RF stage of a receiver is as shown in figure. The transmission line is connected to the antenna, and the output of the mixer goes to the IF stage. Calculate the Noise factor and sensitivity of the receiver for a bandwidth of 1 MHz and minimal SNR of 12 dB.



Solution:

The linear values of the gain and individual NFs are

$NF_1 = 2$ $NF_2 = 1.585$ $NF_3 = 1.585$ $NF_4 = 4$

$G_1 = 0.5$ $G_2 = 0.631$ $G_3 = 10$

The total NF is given by

$$NF_{total} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \frac{NF_4 - 1}{G_1 G_2 G_3} + \frac{NF_5 - 1}{G_1 G_2 G_3 G_4} + \ldots\ldots$$

$$NF_{total} = 1 + (2 - 1) + \frac{1.585 - 1}{0.5} + \frac{1.585 - 1}{0.5x0.631} + \frac{4 - 1}{0.5x0.631x10} = 5.98 \text{ Or } 7.76dB$$

So for a bandwidth of 1 MHz and minimal SNR of 12 dB, the sensitivity is

S = -174 dBm/Hz + 60 dB + 7.76 dB + 12 dB

S = -94.24 dB

Flicker noise:

Flicker noise is a type of electronic noise with a 1/f power spectral density. It is therefore often referred to as 1/f noise or pink noise. device. It is basically due to variation in the conduction mechanism. There is no outstanding solution to decreasing it yet, but techniques do exist to minimize the effect. The power in spectral density of 1/f noise is inversely proportional to frequency.

2. Distortion Characterization:

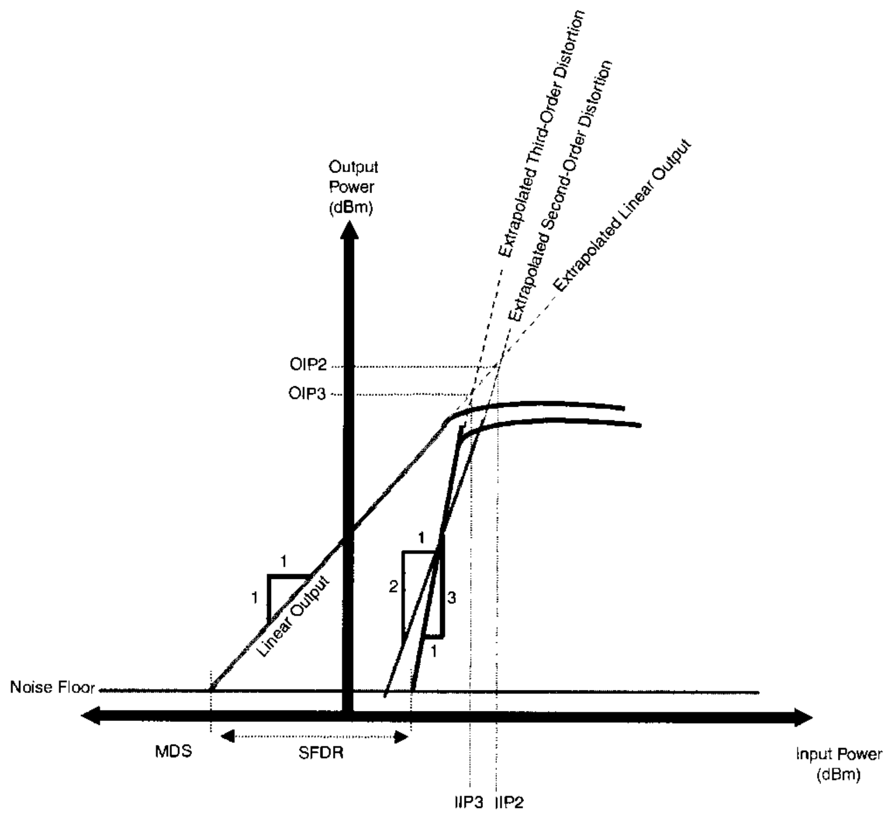
Distortion occurs in the RF chain because of the non-linearities in the system. The distortion takes the form of harmonics, i.e., sinusoidal terms that occur at multiples of the frequency of an input sinusoid. Distortion can take the form of cross modulation when a weak signal and a strong interferer enter a non-linearity and the amplitude of the interferer modifies the amplitude of the weak signal and vice versa.

In an ideal system, linear time-invariant (LTI) operations is expected which allows the outputs to be expressed as a linear combination of responses to inputs. For example, if there are two input signals, $x_1(t)$ and $x_2(t)$, the outputs of these signals can be: $x_1(t) \rightarrow y_1(t)$, $x_2(t) \rightarrow y_2(t)$

Therefore, a linear system has to be satisfied in the following condition.

$$a.x_1(t) + b.x_2(t) \rightarrow a.y_1(t) + b.y_2(t)$$

The relationship between increased input signal power and the output power of the desired signal and distortion is shown in Figure below. In a linear circuit, a linear relationship exists between input power and output power, and this linear relationship is represented as a line with a slope of one in below figure.



MDS = Minimum Detectable Signal (Output Noise Floor)
IIP3 = Third-Order Intercept Point
SFDR =1/3 IIP3 - MDS
OIP3 = Output Referred Third-Order Intercept Point
OIP2 = Output Referred Second-Order Intercept Point

Non-Linear Output Distortion Characterization Using Input Versus Output Power Characteristics.

However, in any real RF component, the transfer function is much more complicated. These complexities can be due to active or passive components in a RF circuit. It is common to have nonlinearity and time variance present in a system. Mathematically, any nonlinearity function can be written as a series expansion of power terms. Assume a nonlinear system $y(t) = \alpha_1.x(t) + \alpha_2.x_2(t) + \alpha_3.x_3(t)$ is memoryless and has an input signal $x(t) = A\cos(\omega t)$. where α_1, α_2, α_3 are functions of time. then, the result of the system is,

$$y(t) = \alpha_1.\ A\cos(\omega t) + \alpha_2.\ A^2\cos^2(\omega t) + \alpha_3.\ A^3\cos^3(\omega t).$$

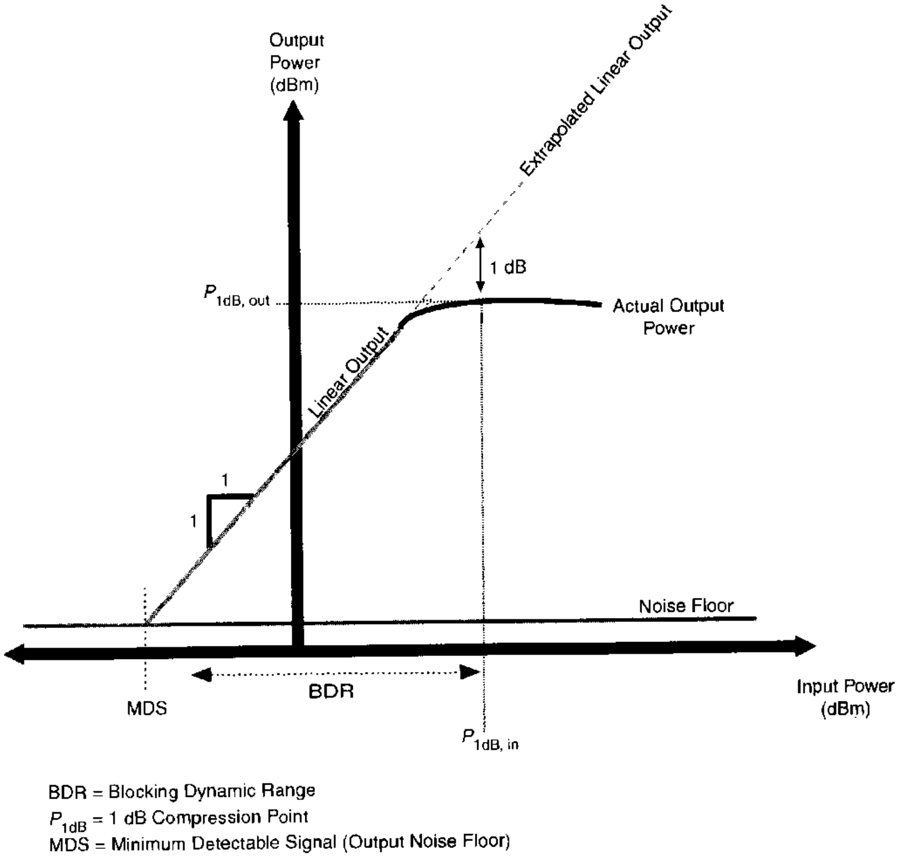
Typically a non-linear device output can be modeled as

$$v_o = a_0 + a_1 v_{in} + a_2 v^2_{in} + a_3 v^3_{in} + \ldots$$

where v_{in} is the input voltage and a_is are constant terms. The square term produces second-order products, and the cubic term represents third-order products.

For non-linear devices that exhibit a pure cubic characteristic, the third-order distortion power grows at three times the rate of the desired signal. Eventually, there is a practical limitation on how much the power of the desired signal can be raised with a corresponding linear increase in the output signal level. Eventually the device begins to saturate, and when the actual desired signal's output power level differs by 1 dB with the desired signal's ideal output value, the 1 dB compression, point P_1dB, is reached. This amplitude compression characteristic, shown in Figure below, tends to block the detection of lower level signals in the presence of stronger signals, and the blocking dynamic range (BDR), i.e., the difference between the minimum detectable signal (MDS) level and the input that produces a 1 dB compression, quantifies this effect:

$$BDR = P_1dB - MDS$$



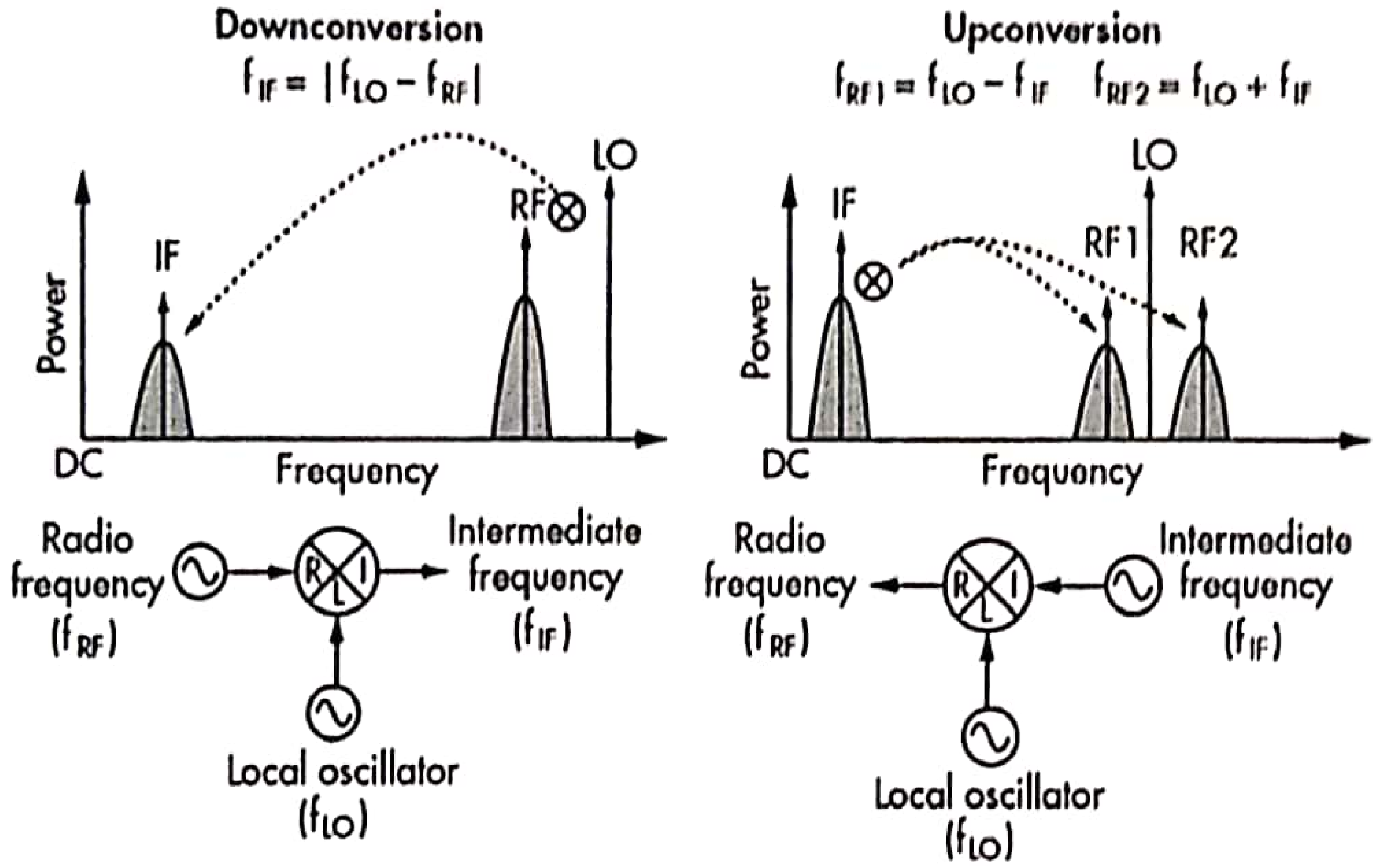
1 dB Compression Point

ADC and DAC Distortion:

The ADC and DAC introduce both noise and distortion. The main sources of noise are quantization noise, internal thermal noise, and sampling aperture jitter. Quantization noise occurs because of the limited number of states that can be represented by the ADC and DAC. The round-off or truncation is often modeled as an additive noise process onto a true signal representation. Thermal noise is a problem for all components. Aperture jitter (aperture uncertainty), the result of sampling at unevenly spaced intervals, produces a modulation on the phase of the signal that is typically modeled as background noise. This type of distortion is especially apparent for signals that have high frequency content. if the signal level exceeds the maximum range of the ADC, nonlinear distortion results.

Mixers

A mixer is a three-port component, which performs the task of frequency conversion. Mixers translate the frequency of an input signal to a different frequency. This functionality is vital for a wide range of applications, including military radar, satellite-communications (satcom), cellular base stations, and more. Mixers are used to perform both frequency upconversion and downconversion.



1.

These simple diagrams provide an illustration of frequency conversion. "Two of a mixer's three ports serve as inputs, while the other port serves as an output port. An ideal mixer produces an output that consists of the sum and difference frequencies of its two input signals. In other words:

$$f_{out} = f_{in1} \pm f_{in2}$$

The three ports of a mixer are known as the intermediate-frequency (IF), radio-frequency (RF), and local-oscillator (LO) ports. The LO port is usually an input port."

The RF and IF ports can be used interchangeably, depending on whether the mixer is being used to perform upconversion or downconversion. The LO signal is typically the strongest signal injected into a mixer. The required LO drive level is dependent on several factors, including the mixer's configuration and device technology.

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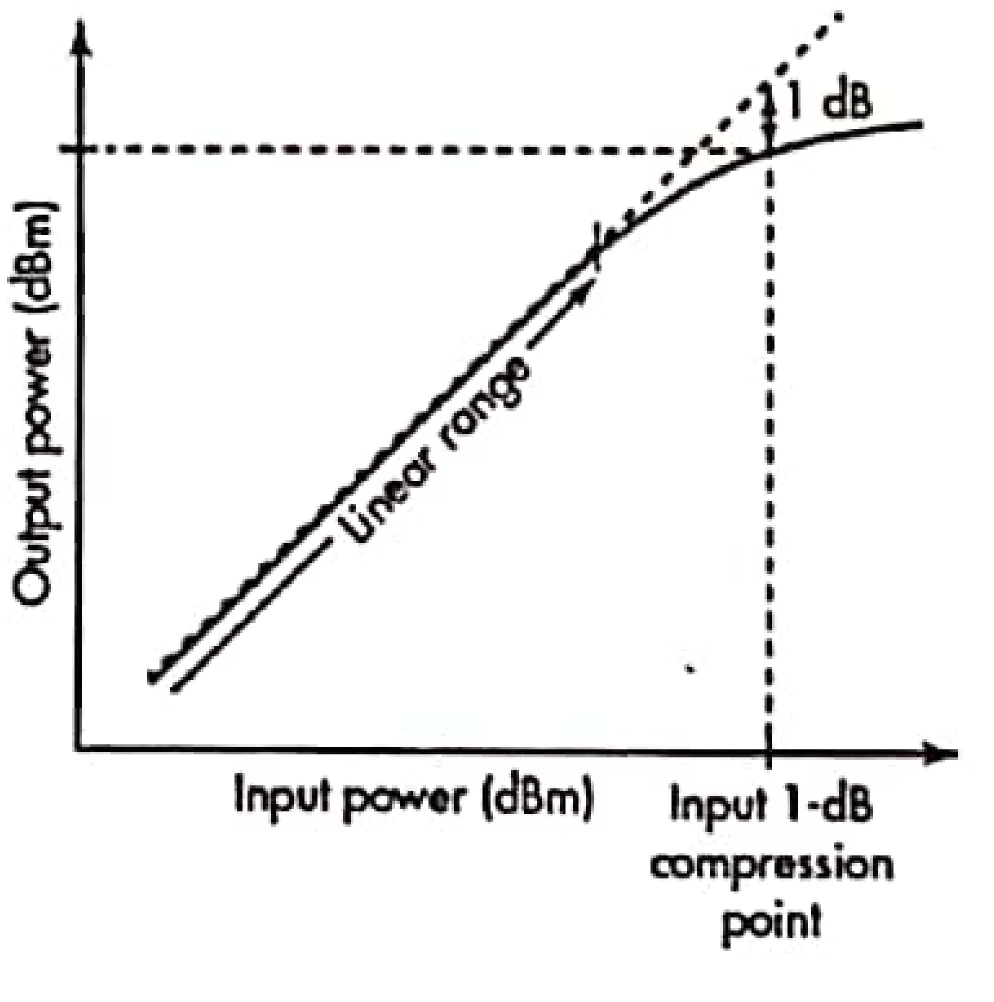
When a mixer is used to perform downconversion, an input signal enters the RF port and an LO signal enters the LO port. These two input signals produce an output signal at the IF port. The frequency of this output signal is equal to the difference of the RF input signal's frequency and the LO signal's frequency.

When a mixer is used to perform upconversion, an input signal enters the IF port and an LO signal enters the LO port. These two input signals produce an output signal at the RF port. The frequency of this output signal is equal to the sum of the IF input signal's frequency and the LO signal's frequency. Both downconversion and upconversion are shown graphically in *Fig. 1.* Upconversion is normally part of a transmitter, while downconversion is typically used in a receiver.

Mixer Performance Parameters

Conversion Loss: In passive mixers, conversion loss is defined as the difference in signal level between the amplitude of the input signal and the amplitude of the desired output signal. In a mixer used for downconversion, the conversion loss is the difference between the RF input signal's amplitude and the IF output signal's amplitude. In a mixer used for upconversion, the conversion loss is the difference between the IF input signal's amplitude and the RF output signal's amplitude.



The above fig. is a graphical representation of 1-dB compression point.

Isolation: "Isolation is a measurement of the amount of power that leaks from one port to another. Isolation is defined as the difference in signal level between the amplitude of an input signal and the amplitude of the leaked power from that input signal to another port." When isolation is high, the amount of power leaked from one port to a different port is small.

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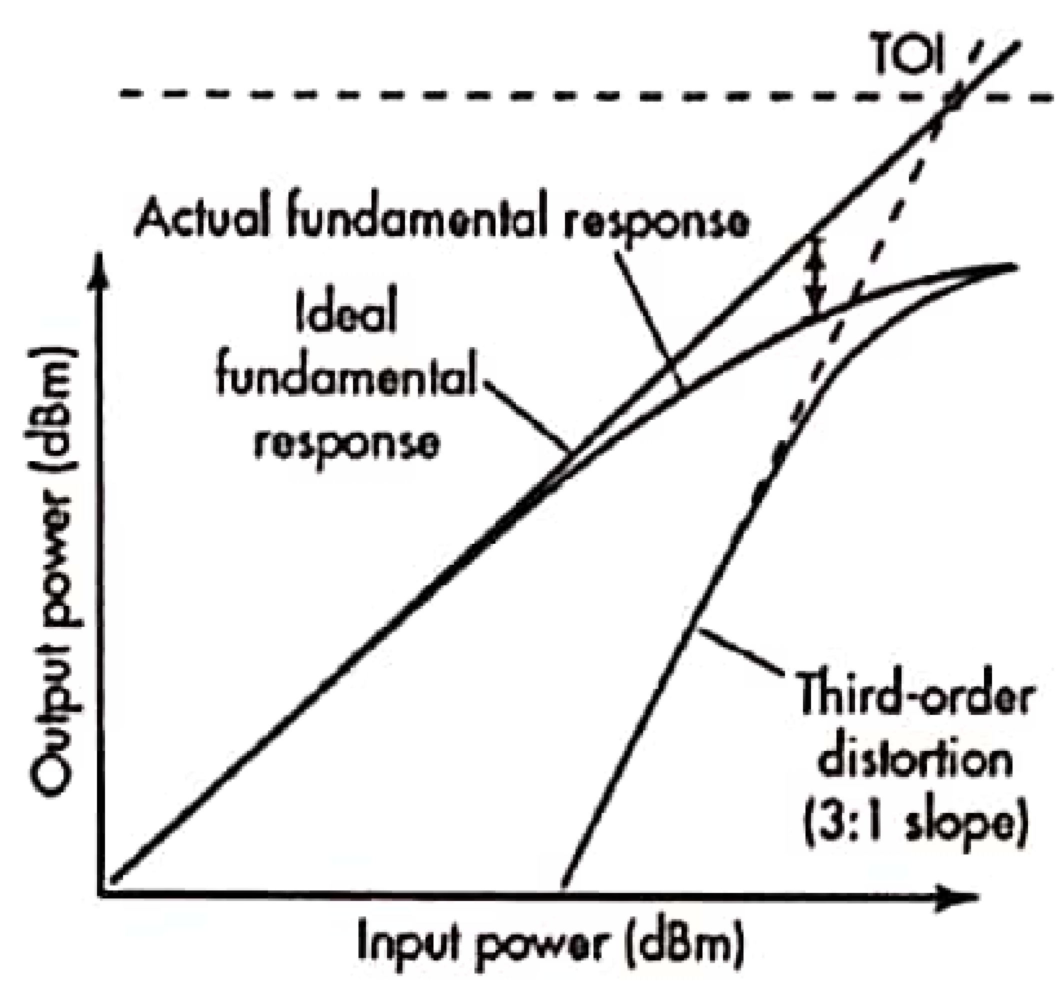
"Three types of isolation are commonly quoted in microwave mixers: LO-RF isolation, LO-IF isolation, and RF-IF isolation."

1-dB Compression Point: A mixer's conversion loss remains constant when the mixer is in linear operation. As the amplitude of the input signal increases, the amplitude of the output signal rises by the same amount. However, once the input signal's amplitude reaches a certain level, the amplitude of the output signal ceases to exactly follow the input signal. The mixer deviates from linear behavior and its conversion loss begins to increase.



Intermodulation Distortion: Two-tone third-order intermodulation distortion (IMD) occurs when two signals simultaneously enter the mixer's IF or RF input port. In practice, this could happen in a multi-carrier signal environment. These two signals interact with each other and with the LO signal, which creates distortion. In a receiver, two-tone third-order IMD is a serious problem because it can generate third-order distortion products that fall within the IF bandwidth.

If f_{RF1} and f_{RF2} represent two separate RF input signals and f_{LO} represents the LO signal, the third-order distortion products generated at the mixer's IF port are:

$\text{Interferer}_1 = 2f_{RF1} - f_{RF2} - f_{LO}$
$\text{Interferer}_2 = 2f_{RF2} - f_{RF1} - f_{LO}$

These third-order distortion products are extremely close to the desired IF output frequency. No amount of filtering can remove these unwanted distortion products. Thus, the signal-to-noise ratio of the received signal is degraded, highlighting the need to suppress these distortion products.

The third-order input intercept point (TOI or IP3) is a widely accepted figure of merit used to describe a mixer's capability to suppress third-order distortion products. TOI is used in predicting the nonlinear behavior of a mixer as the amplitude of its input signal increases, which

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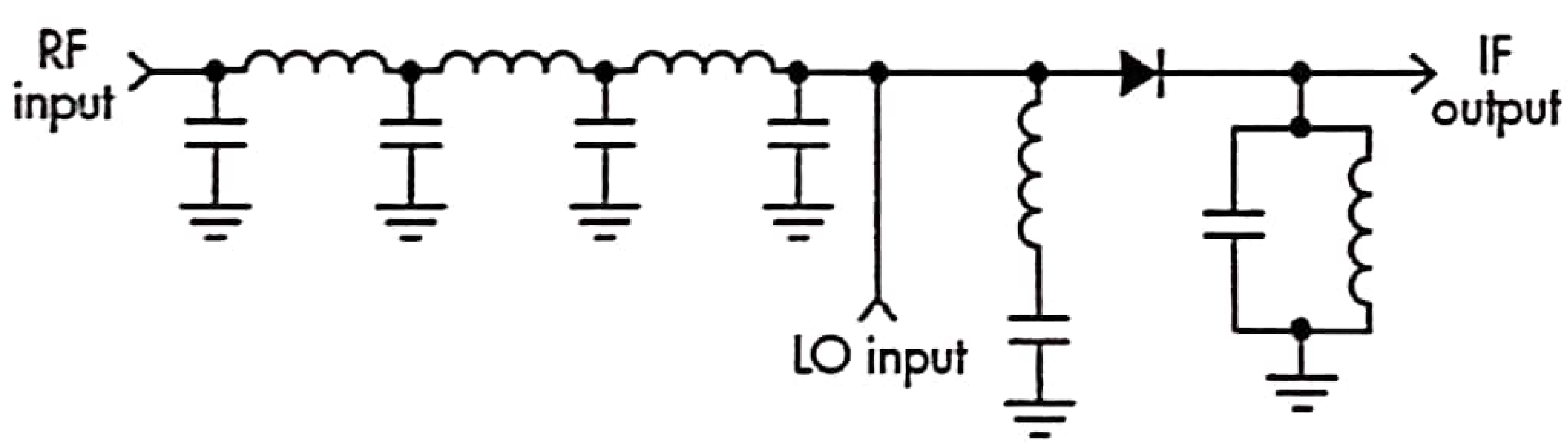
causes the third-order products to increase by a 3:1 ratio. For any 1-dB increase in the input signal's amplitude, the third-order products increase by 3 dB

Mixer Design Techniques

In theory, any nonlinear device can be used to create a mixer circuit. However, only a few devices satisfy the requirements needed to design mixers with acceptable performance. Devices that are commonly used to design modern mixers include Schottky diodes, gallium-arsenide (GaAs) field-effect transistors (FETs), and CMOS transistors. Various topologies can be used to design mixers. Mixers can be designed as either passive or active components.

Single diode mixer



Passive mixers primarily use Schottky diodes, although the FET resistive mixer has recently become another popular passive mixer. Active mixers use either FETs or bipolar devices. Schottky diodes, in comparison with FETs and bipolar devices, have the advantage of possessing an inherently wide bandwidth. This is a major reason why diodes are still widely used to design mixers.

Mixers can be designed with just a single diode, which is the simplest mixer topology. Balanced mixers, which consist of two, four, or even eight diodes in a balanced structure, build upon the single-diode mixer. The majority of mixers available today incorporate some form of mixer balancing.A single diode can be used to create a mixer . Here, the RF and LO signals combine at the anode of the diode. The LO signal needs to be large enough to switch the diode on and off, which causes the actual mixing process. The frequency components generated by single-diode mixers are:

$f_{IF} = nf_{LO} \pm mf_{RF}$ (m and n are all integers)

where:

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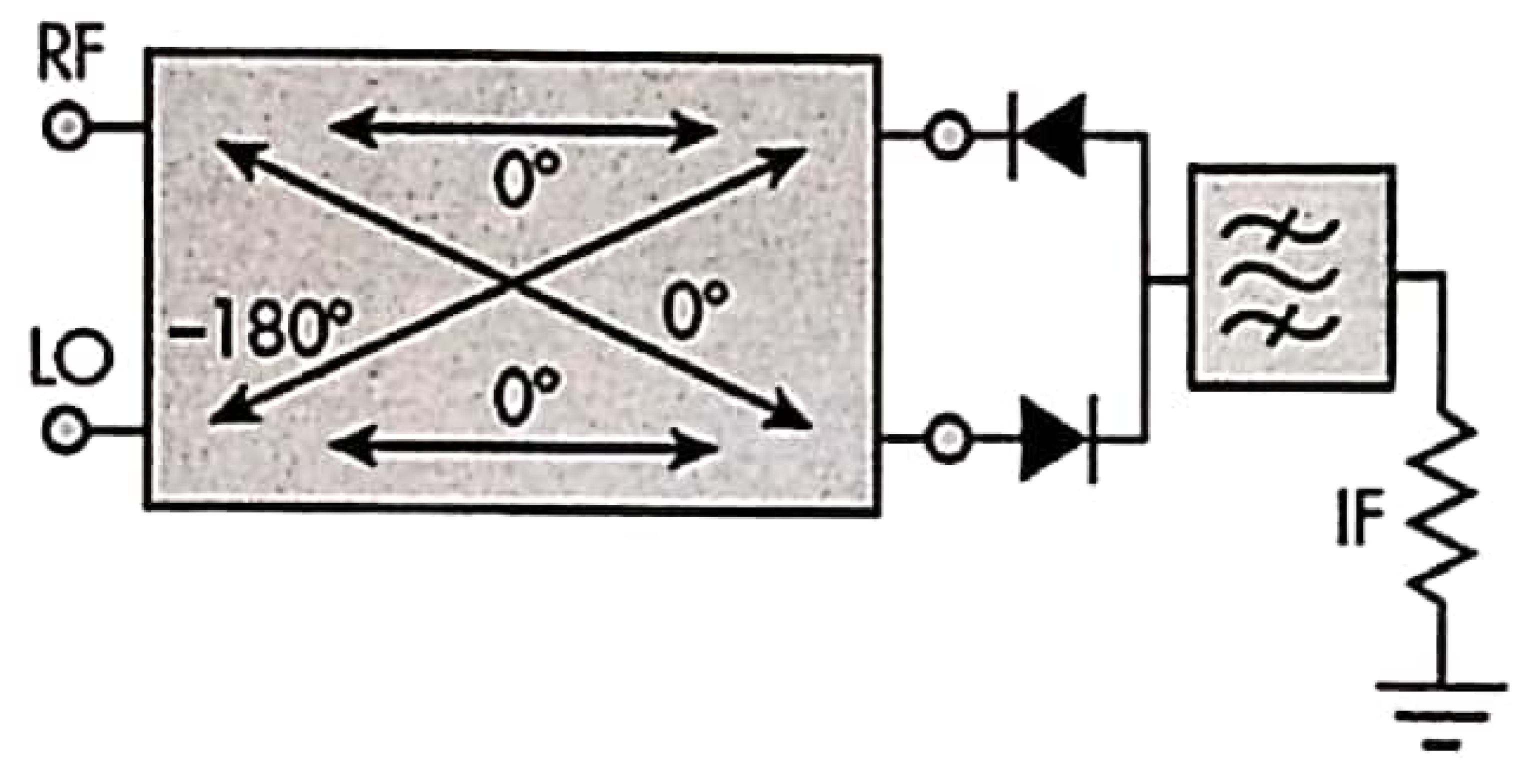
f_{LO} = the LO input signal frequency
f_{RF} = the RF input signal frequency
f_{IF} = the IF output signal frequency

Although only one output frequency is desired (when n = 1 and m = 1), additional unwanted harmonics are generated by the diode's current-voltage (I-V) characteristics and the transconductance modulation caused by the RF signal. Because the single-diode mixer has no inherent isolation between the RF and LO ports, external filters also are needed to achieve isolation between ports. This need for external filtering makes it difficult to achieve wideband mixers with just a single diode.

A single-balanced mixer consists of two diodes and a hybrid.



Balanced mixers overcome some of the limitations of single-diode mixers. They do require baluns or hybrids, which largely determine the bandwidth and overall performance of the mixer. Inherent isolation between ports is achieved by balanced mixers as well as increased cancellation of intermodulation products. Common-mode noise cancellation is another advantage gained by balanced mixers. However, balanced mixers do require a higher LO drive level.

Single-balanced mixers consist of two diodes along with a hybrid (*Fig. 5*). Although 90-deg. and 180-deg. hybrids can both be used to design single-balanced mixers, the majority of single-balanced mixers incorporate a 180-deg. hybrid. The 180-deg. hybrid's input ports are mutually isolated, enabling the LO port to be isolated from the RF port. This provides frequency-band independence and equal power division to the load. In comparison with single-diode mixers, single-balanced mixers also have 50% fewer intermodulation products.

Two single-balanced mixers can be combined to form a double-balanced mixer. Traditional double-balanced mixers are typically based on four Schottky diodes in a quad ring configuration. Baluns are placed at both the RF and LO ports, while the IF signal is tapped off from the RF balun. The IF signal can also be tapped off from the LO balun, but this would worsen the LO-IF isolation.

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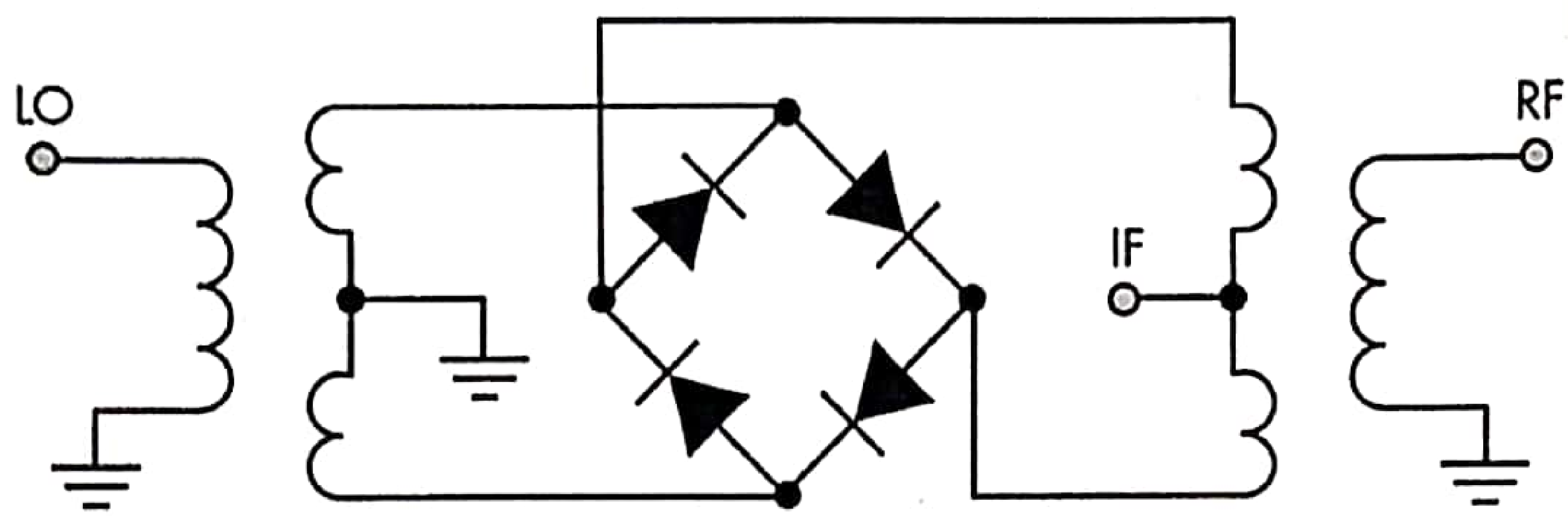
For this reason, it is usually preferred to tap off the IF signal from the RF balun instead of the LO balun. An example of a double-balanced mixer is shown in Fig. 6. This mixer has high LO-RF isolation and LO-IF isolation along with moderate RF-IF isolation. Double-balanced mixers also have the benefit of reducing intermodulation products by as much as 75% in comparison with single-diode mixers.

Double balanced mixer



An even more complex mixer circuit is the triple-balanced mixer. Triple-balanced mixers have separate baluns for the LO, RF, and IF ports, which enables them to achieve high LO-RF isolation, LO-IF isolation, and RF-IF isolation. Triple-balanced mixers also offer higher suppression of intermodulation products than double-balanced mixers. The downside of triple-balanced mixers is that they need a higher LO drive level. They also are greater in both size and complexity.

Applications

Mixer circuits can be used to shift the frequency of an input signal like as in a receiver. They can also be used as a product detector, modulator, frequency multiplier or phase detector.

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VCO

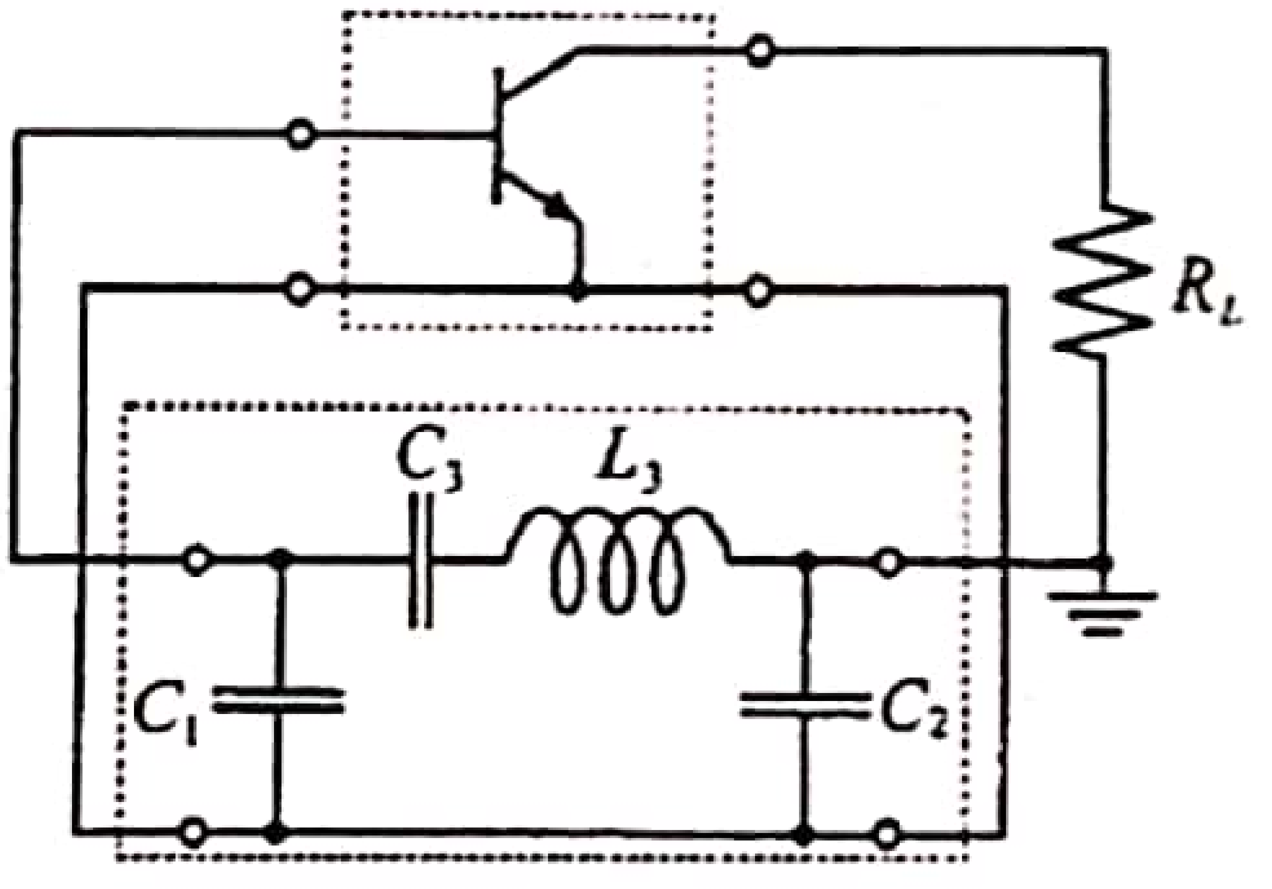
A voltage-controlled oscillator (VCO) is an electronic oscillator whose oscillation frequency is controlled by a voltage input. The applied input voltage determines the instantaneous oscillation frequency. Consequently, a VCO can be used for frequency modulation (FM) or phase modulation (PM) by applying a modulating signal to the control input. A VCO is also an integral part of a phase-locked loop.

Types of Voltage Controlled Oscillators

- Harmonic Oscillators: The output is a signal with sinusoidal waveform. Examples are crystal oscillators and tank oscillators
- Relaxation Oscillators: The output is a signal with saw tooth or triangular waveform and provides a wide range of operational frequencies. The output frequency depends on the time of charging and discharging of the capacitor.

Applications of VCO

- Tone Generators
- Function generators
- Phase-Locked Loops
- In synthesizers to generate variable tones for the production of electronic music
- In communication equipment these are used as frequency synthesizers
- Clock generators
- Frequency Shift Keying



(a) Pi-type feedback loop

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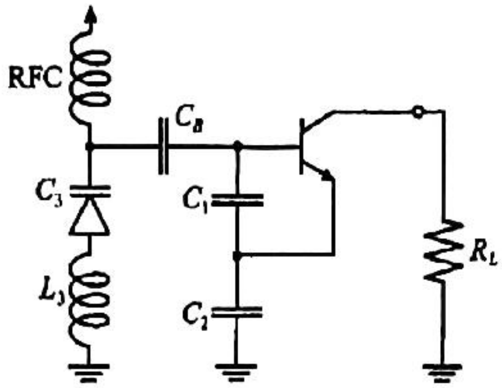
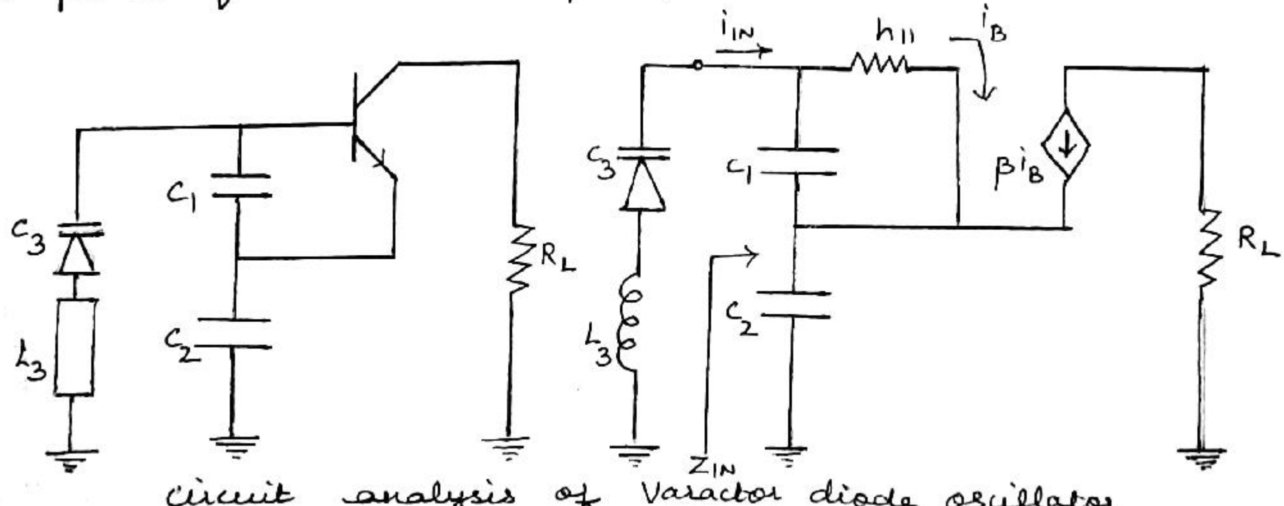


(b) Redrawn circuit with DC isolation

The feedback loop for the clapp oscillator shown in Fig. 1(a) can be modified by replacing C_3 with a Varactor diode as shown in Fig. 1.(b).

In Fig. 1.(c) the Varactor diode and a transmission line element, whose length is adjusted to be inductive, form the termination circuit connected to the input of the oscillator. If the Varactor diode & transmission line segment is disconnected, the input impedance Z_{IN} can be computed from two loop equations:



circuit analysis of Varactor diode oscillator

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$$V_{IN} - i_{IN} X_{c1} - i_{IN} X_{c2} + i_B X_{c1} - \beta i_B X_{c2} = 0 \quad \text{—— (1)}$$

$$h_{11} i_B + i_B X_{c1} - i_{IN} X_{c1} = 0 \quad \text{—— (2)}$$

From equation (2),

$$i_B (h_{11} + X_{c1}) = i_{IN} X_{c1}$$

$$\therefore \quad i_B = \frac{i_{IN} X_{c1}}{h_{11} + X_{c1}} \quad \text{—— (3)}$$

Sub. (3) in (1)

$$V_{IN} + i_B (X_{c1} - \beta X_{c2}) = i_{IN} X_{c1} + i_{IN} X_{c2}$$

$$V_{IN} + \frac{i_{IN} X_{c1}}{h_{11} + X_{c1}} (X_{c1} - \beta X_{c2}) = i_{IN} (X_{c1} + X_{c2})$$

$$V_{IN} = i_{IN} (X_{c1} + X_{c2}) - \frac{i_{IN} X_{c1}}{h_{11} + X_{c1}} (X_{c1} - \beta X_{c2})$$

$$= i_{IN} \left[(X_{c1} + X_{c2}) - \frac{X_{c1}}{h_{11} + X_{c1}} (X_{c1} - \beta X_{c2}) \right]$$

$$= i_{IN} \left[\frac{(h_{11} + X_{c1})(X_{c1} + X_{c2}) - X_{c1}(X_{c1} - \beta X_{c2})}{h_{11} + X_{c1}} \right]$$

$$V_{IN} = \frac{i_{IN}}{h_{11} + X_{c1}} \left[h_{11} (X_{c1} + X_{c2}) + X_{c1} X_{c2} (1+\beta) \right]$$

$$\therefore \quad Z_{IN} = \frac{V_{IN}}{i_{IN}} = \frac{1}{h_{11} + X_{c1}} \left[h_{11} (X_{c1} + X_{c2}) + X_{c1} X_{c2} (1+\beta) \right] \quad \text{—— (4)}$$

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Equation (4) can further be simplified by considering

$(1+\beta) \simeq \beta$ and $h_{11} >> X_{c1}$

$$\therefore Z_{IN} = \frac{1}{h_{11}} \left[h_{11} (X_{c1} + X_{c2}) + X_{c1} X_{c2} \beta \right]$$

$X_{c1} = \frac{1}{j\omega C_1}$

$\& \; X_{c2} = \frac{1}{j\omega C_2}$

$$= (X_{c1} + X_{c2}) + \frac{\beta}{h_{11}} X_{c1} X_{c2}$$

$$= \frac{1}{j\omega c_1} + \frac{1}{j\omega c_2} + \frac{\beta}{h_{11}} \cdot \frac{1}{j\omega c_1} \cdot \frac{1}{j\omega c_2}$$

$$\therefore Z_{IN} = \frac{1}{j\omega} \left(\frac{1}{c_1} + \frac{1}{c_2} \right) \; \& \; - \frac{\beta}{h_{11}} \left(\frac{1}{\omega^2 c_1 c_2} \right) \quad \text{--- } 5$$

From equation 5 & by considering $g_m = \frac{\beta}{h_{11}}$

$$R_{IN} = - \frac{g_m}{\omega^2 c_1 c_2}$$

$$\& \quad X_{IN} = \frac{1}{j\omega C_{IN}} \qquad \left[\text{where } C_{IN} = \frac{C_1 C_2}{C_1 + C_2} \right]$$

The resonant frequency can be obtained by considering the condition $X_1 + X_2 + X_3 = 0$.

$$\text{ie. } j \left(\omega_0 L_3 - \frac{1}{\omega_0 C_3} \right) - \frac{1}{j\omega_0} \left[\frac{1}{c_1} + \frac{1}{c_2} \right] = 0$$

By simplifying the above equation we get,

$$\boxed{f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{L_3} \left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} \right)}}$$

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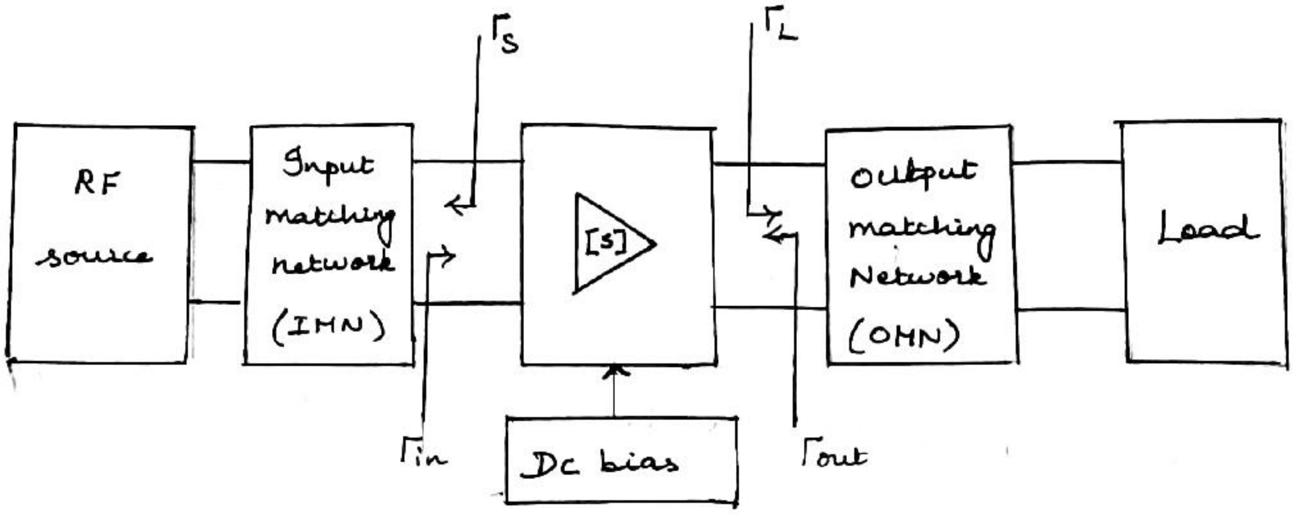
Amplifier

A generic single stage amplifier configuration embedded between input and output matching networks is shown in the following figure; The amplifier is characterized through its S-parameter matrix at a particular DC bias point.



Generic amplifier system

S-Parameter matrix:

S-parameters (Scattering parameters) are power wave descriptors that permit us to define the input-output relations of a network in terms of incident and reflected power waves.

a_n refers to incident normalized power wave

b_n refers to reflected normalized power wave

where index n refers to port number 1 or 2.

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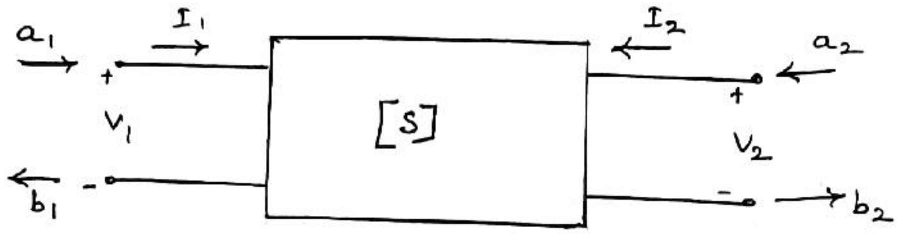
Consider a 2 port network. a_1 & a_2 represent incident power wave at port 1 and port 2 respectively. b_1 and b_2 represent reflected / transmitted power wave at port 1 and port 2 respectively.



Two port network

The S parameter matrix for the above network is given as;

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

where $$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{\text{reflected power wave at port 1}}{\text{incident power wave at port 1}}$$

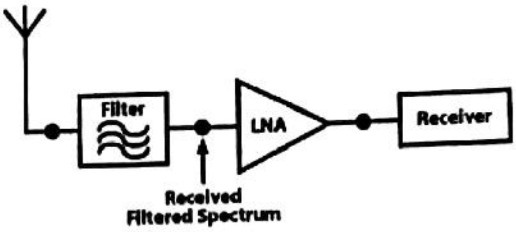
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{\text{transmitted power wave at port 2}}{\text{incident power wave at port 1}}$$

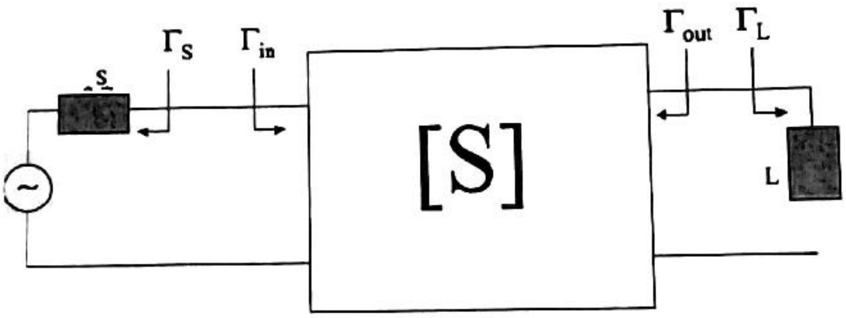
$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{\text{reflected power wave at port 2}}{\text{incident power wave at port 2}}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{\text{transmitted power wave at port 1}}{\text{incident power wave at port 2}}$$

Low noise Amplifier

A **low-noise amplifier (LNA)** is an electronic amplifier that amplifies a very low-power signal without significantly degrading its signal-to-noise ratio. An amplifier will increase the power of both the signal and the noise present at its input, but the amplifier will also introduce some additional noise. LNAs are designed to minimize that additional noise. Designers can minimize additional noise by choosing low-noise components, operating points, and circuit topologies. Minimizing additional noise must balance with other design goals such as power gain and impedance matching.





LNAs are found in radio communications systems, medical instruments and electronic test equipment. A typical LNA may supply a power gain of 100 (20 decibels (dB)) while decreasing the signal-to-noise ratio by less than a factor of two (a 3 dB noise figure (NF)). Although LNAs are primarily concerned with weak signals that are just above the noise floor, they must also consider the presence of larger signals that cause intermodulation distortion.

Low noise amplifiers are the building blocks of communication systems and instruments. The four important parameters in LNA design are: gain, noise figure, non-linearity and impedance matching.

Applications

LNAs are used in communications receivers such as in cellular telephones, GPS receivers, wireless LANs (WiFi), and satellite communications.

In a satellite communications system, the ground station receiving antenna uses an LNA because the received signal is weak since satellites have limited power and therefore use low-power transmitters. The satellites are also distant and suffer path loss: low earth orbit satellites might be 200 km (120 miles) away; a geosynchronous satellite is 35,786 miles (57,592 km) away. The

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LNA boosts the antenna signal to overcome feed line losses between the antenna and the receiver.

LNAs are becoming increasingly popular for enhancing the performance of software-defined radio (SDR) receiver systems. SDRs are typically designed to be general purpose and therefore the noise figure is not optimized for any one particular application. With a LNA and appropriate filter, the receive sensitivity and performance can be greatly enhanced at any particular frequency or range of frequencies.

RF Power amplifier

A **radio frequency power amplifier (RF power amplifier)** is a type of electronic amplifier that converts a low-power radio-frequency signal into a higher power signal. Typically, RF power amplifiers drive the antenna of a transmitter. Design goals often include gain, power output, bandwidth, power efficiency, linearity (low signal compression at rated output), input and output impedance matching, and heat dissipation.

Amplifier classes

Many modern RF amplifiers operate in different modes, called "classes", to help achieve different design goals. Some classes are class A, class AB, class B, class C, which are considered the linear amplifier classes. In these classes the active device is used as a controlled current source. The bias at the input determines the class of the amplifier.

A common trade-off in power amplifier design is the trade-off between efficiency and linearity. The previously named classes become more efficient, but less linear, in the order they are listed. Operating the active device as a switch results in higher efficiency, theoretically up to 100%, but lower linearity.Among the switch-mode classes are Class D, Class F and class E.[2] The Class D amplifier is not often used in RF applications because the finite switching speed of the active devices and possible charge storage in saturation could lead to a large I-V product, which deteriorates efficiency.

Applications

The basic applications of the RF power amplifier include driving to another high power source, driving a transmitting antenna and exciting microwave cavity resonators. Among these applications, driving transmitter antennas is most well known. The transmitter–receivers are used not only for voice and data communication but also for weather sensing (in the form of a radar)

RF power amplifiers using LDMOS (laterally diffused MOSFET) are the most widely used power semiconductor devices in wireless telecommunication networks, particularly mobile networks. LDMOS-based RF power amplifiers are widely used in digital mobile networks such as 2G, 3G, and 4G.

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Transducer power gain:

The transducer power gain (G_T) quantifies the gain of the amplifier placed between source and load.

$$G_T = \frac{\text{Power delivered to the load}}{\text{available power from the source}} = \frac{P_L}{P_A}$$

Where,

$$P_L = \frac{1}{2}|b_2|^2 \cdot (1 - |\Gamma_L|^2)$$

$$\& \quad P_A = \frac{1}{2}|b_S|^2 (1 - |\Gamma_S|^2)$$

$$b_2 = \frac{S_{21}\, a_1}{1 - S_{22}\Gamma_L}$$

$$b_S = \left[1 - \left(S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L}\right)\Gamma_S\right] a_1$$

where,

* Γ_L & Γ_S refers to reflection coefficient at load & source respectively.
* $b_S \rightarrow$ Source transmitted power

$$\therefore G_T = \frac{P_L}{P_A} = \frac{|b_2|^2}{|b_S|^2} (1 - |\Gamma_L|^2)(1 - |\Gamma_S|^2)$$

To find $\frac{b_2}{b_S}$:

$$\frac{b_2}{b_S} = \frac{\dfrac{S_{21}\, a_1}{1 - S_{22}\Gamma_L}}{\left[1 - \left(S_{11} + \dfrac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L}\right)\Gamma_S\right] a_1}$$

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$$= \frac{\dfrac{S_{21}}{1-S_{22}\Gamma_L}}{\left\{(1-\dfrac{S_{22}\Gamma_L) - (S_{11}(1-S_{22}\Gamma_L) + S_{21}S_{12}\Gamma_L)\Gamma_S}{1-S_{22}\Gamma_L}\right\}}$$

$$= \frac{S_{21}}{(1-S_{22}\Gamma_L) - S_{11}\Gamma_S(1-S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_S}$$

$$= \frac{S_{21}}{(1-S_{22}\Gamma_L)(1-S_{11}\Gamma_S) - S_{21}S_{12}\Gamma_L\Gamma_S}$$

$\therefore$ G_T is given as,

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_S|^2}$$

The input & output reflection coefficient is given as

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}$$

& $$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

Therefore, with respect to Γ_{in}, G_T can be expressed as,

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{\left|\left(1-\left(\Gamma_{in} - \dfrac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}\right)\Gamma_S\right)(1-S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_S\right|^2}$$

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$$= \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{\left|(1-S_{22}\Gamma_L) - (\Gamma_{IN}\Gamma_S(1-S_{22}\Gamma_L)) - S_{21}S_{12}\Gamma_L\Gamma_S + S_{21}S_{12}\Gamma_L\Gamma_S\right|^2}$$

$$\therefore G_T = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{|1-\Gamma_{in}\Gamma_S|^2 \, |1-S_{22}\Gamma_L|^2}$$

Similarly, in terms of output reflection coefficient, G_T can be expressed as,

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{|1-\Gamma_L\Gamma_{out}|^2 \, |1-S_{11}\Gamma_S|^2}$$

Unilateral power gain G_{TU} can be obtained by neglecting S_{12}. (ie. $S_{12}=0$).

$$\therefore G_{TU} = \frac{(1-|\Gamma_L|^2)\,|S_{21}|^2\,(1-|\Gamma_S|^2)}{|1-\Gamma_L S_{22}|^2 \, |1-S_{11}\Gamma_S|^2}$$

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Stability considerations:

Stability circles:

The main requirement of an amplifier circuit is to possess stable performance over the entire frequency range. If $|\Gamma| > 1$, then the return voltage increases in magnitude (positive feedback) causing instability. Conversely, if $|\Gamma| < 1$, the return voltage decreases in magnitude (negative feedback) causing stability.

Let us consider amplifier as a two port network characterized through its S-parameters, with external terminations described by Γ_L and Γ_S. Stability implies that the magnitude of the reflection coefficients are less than unity.

ie. $|\Gamma_L| < 1$, $|\Gamma_S| < 1$

$$|\Gamma_{in}| = \left| \frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L} \right| < 1$$

$$|\Gamma_{out}| = \left| \frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S} \right| < 1$$

Where $\Delta = S_{11}S_{22} - S_{12}S_{21}$

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Since the S parameters are fixed for a particular frequency, the only factors that have a parametric effect on stability are Γ_L and Γ_S.

The output stability circle equation is given as,

$$\left(\Gamma_L^R - C_{out}^R\right)^2 + \left(\Gamma_L^I - C_{out}^I\right)^2 = r_{out}^2$$

Where the circle radius is given by,

$$r_{out} = \frac{|S_{12}\, S_{21}|}{\left|\, |S_{22}|^2 - |\Delta|^2 \right|}$$

& the center of this circle is located at

$$C_{out} = \frac{(S_{22} - S_{11}^{*}\, \Delta)^{*}}{|S_{22}|^2 - |\Delta|^2}$$

Similarly, the input stability circle equation is given as,

$$\left(\Gamma_S^R - C_{in}^R\right)^2 + \left(\Gamma_S^I - C_{in}^I\right)^2 = r_{in}^2$$

Where,

$$r_{in} = \frac{|S_{12}\, S_{21}|}{\left|\, |S_{11}|^2 - |\Delta|^2 \right|}$$

& $$C_{in} = \frac{(S_{11} - S_{22}^{*}\, \Delta)^{*}}{|S_{11}|^2 - |\Delta|^2}$$

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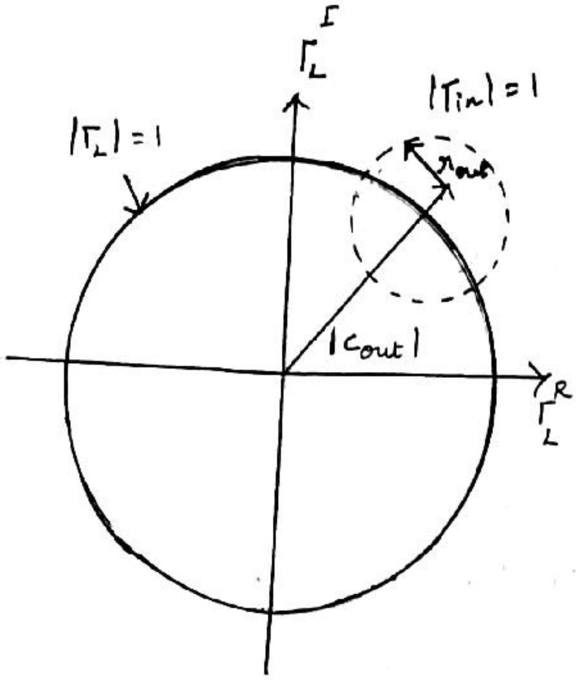
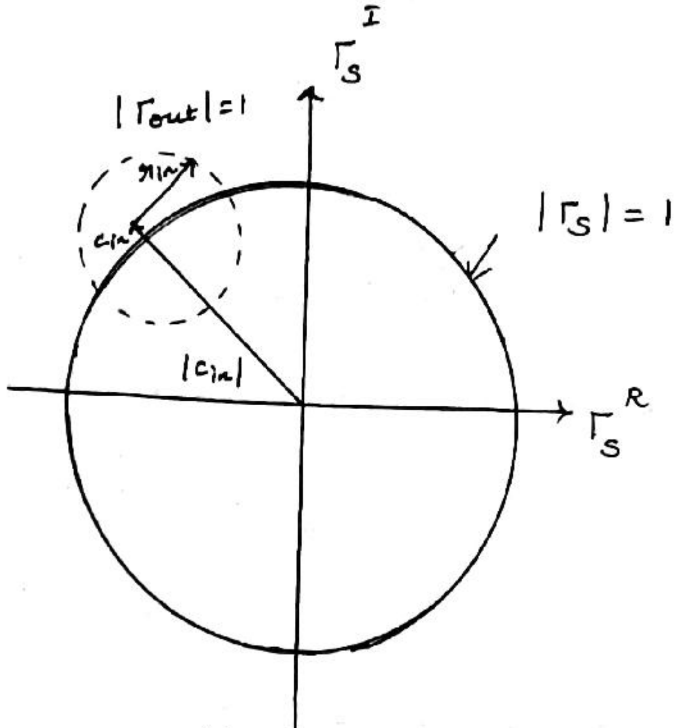
The output & input stability circle is shown in the following figure;



(a) Output stability circle



(b) Input stability circle

Fig. (1)

For output stability circle, the rule is that if $|S_{11}|<1$, the center $(\Gamma_L=0)$ must be stable; otherwise the center becomes unstable for $|S_{22}|>1$.

Output stability circles denoting stable & unstable region is shown in Fig. 2.

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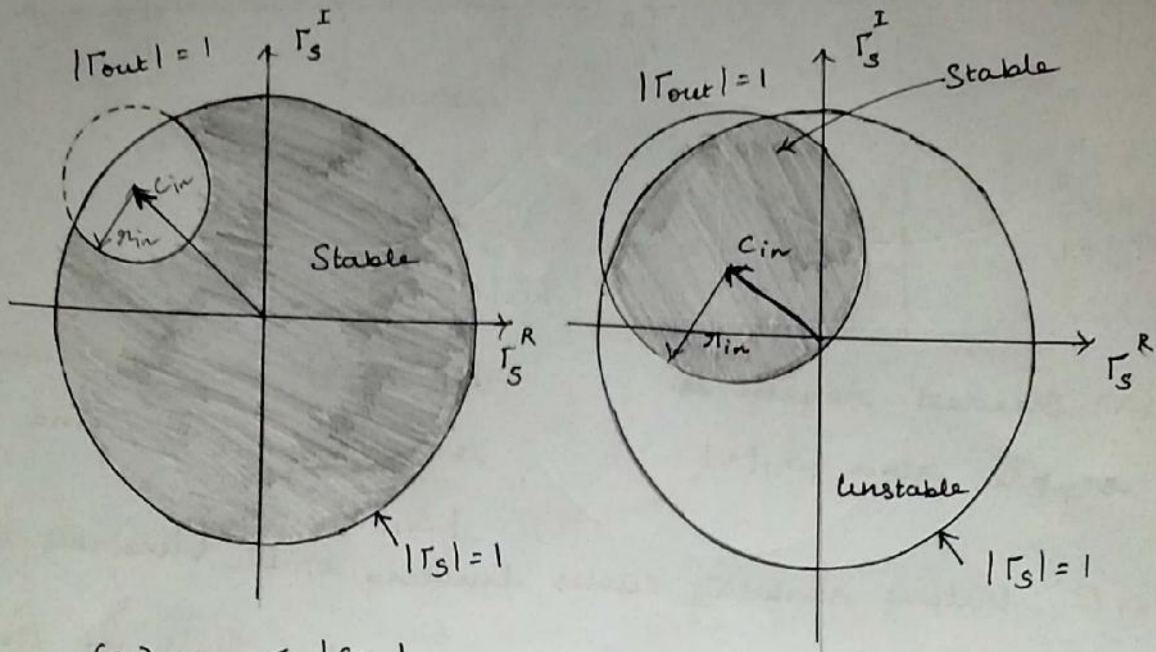
The following diagram depicts the input stability circles for $|S_{22}| < 1$ & the two possible stability domains depending on $r_{in} < |C_{in}|$ or $r_{in} > |C_{in}|$.



(a) $r_{in} < |C_{in}|$

(b) $r_{in} > |C_{in}|$

Different input stability regions for $|S_{22}| < 1$ depending on ratio between r_s and $|C_{in}|$.

Unconditional stability:

Unconditional stability refers to the situation where the amplifier remains stable for any passive source & load at the selected frequency & bias conditions. For $|S_{11}| < 1$ & $|S_{22}| < 1$, it is stated as

$$\big|\,|C_{in}| - r_{in}\big| > 1$$

$$\big|\,|C_{out}| - r_{out}\big| > 1$$

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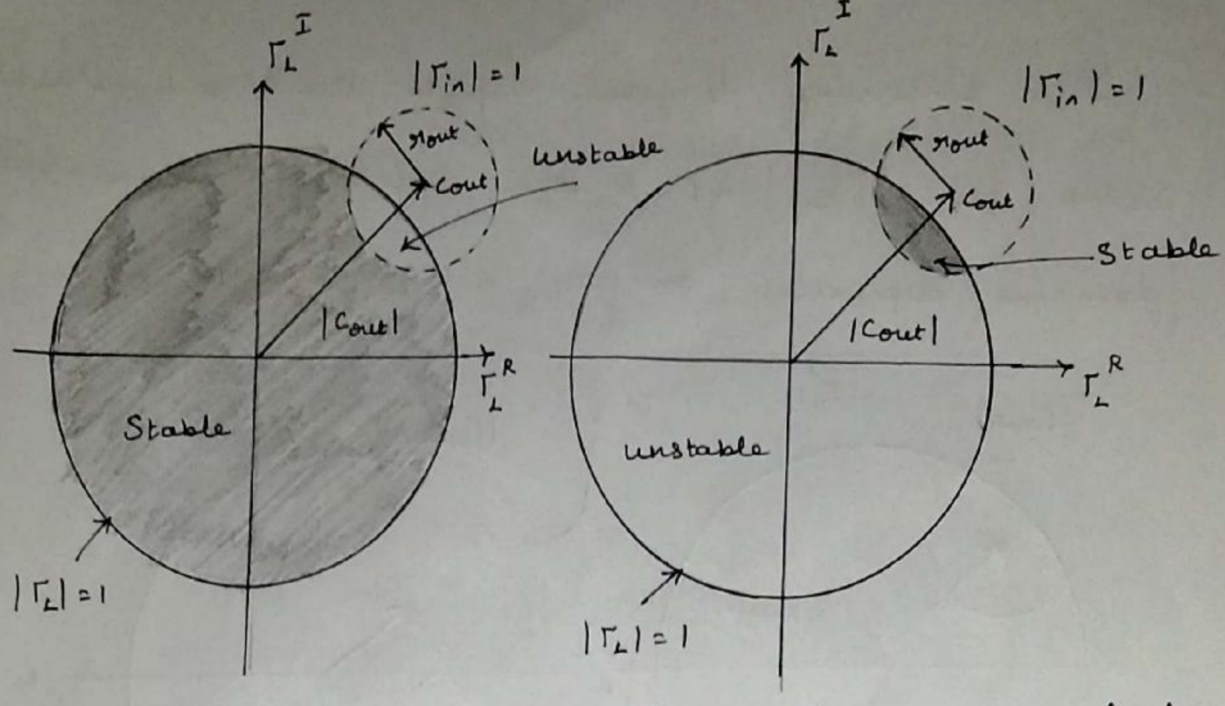
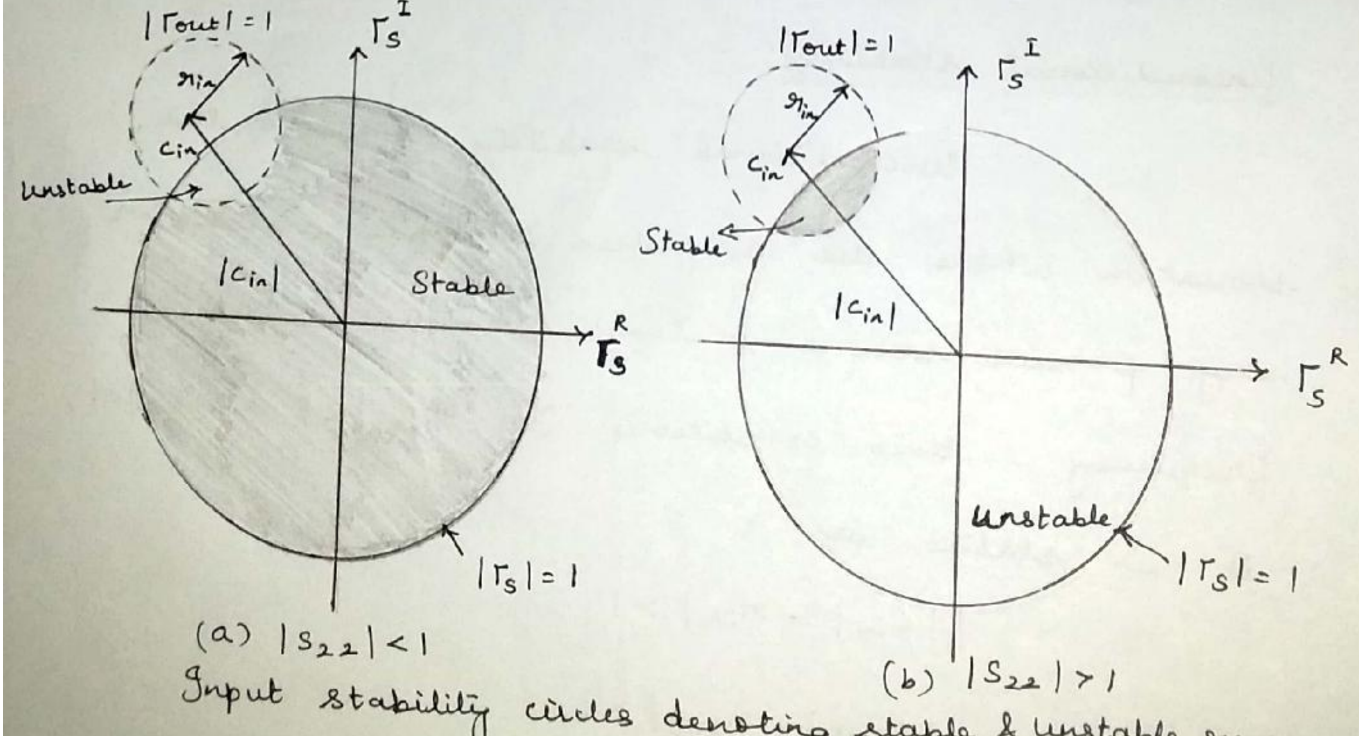


(a) Shaded region is stable, since $|S_{11}| < 1$

(b) Stable region excludes the origin, $\Gamma_L = 0$, sine $|S_{11}| > 1$.

Fig (2) Output stability circles denoting stable & unstable regions

For input stability circle, the rule is that if $|S_{22}| < 1$, the center ($\Gamma_S = 0$) must be stable; otherwise the center becomes unstable for $|S_{22}| > 1$.



(a) $|S_{22}| < 1$

(b) $|S_{22}| > 1$

Input stability circles denoting stable & unstable regions

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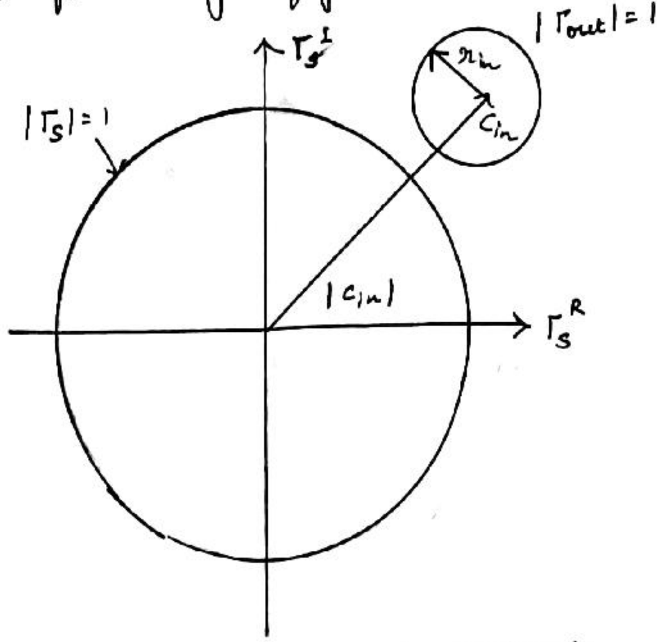
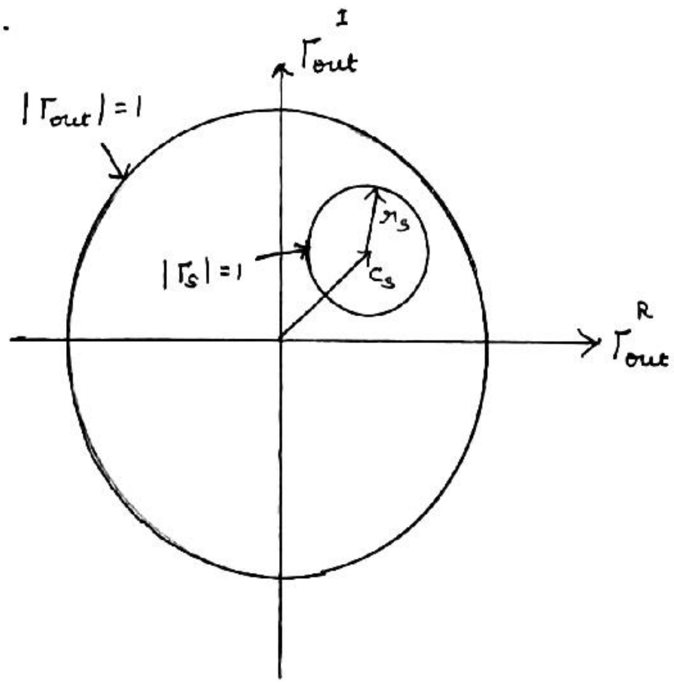
In other words, stability circles have to reside completely outside the $|\Gamma_S|=1$ and $|\Gamma_L|=1$ circles as shown in the following figure.



$|\Gamma_{out}|=1$ circle must reside outside

Alternatively, unconditional stability can also be viewed in terms of Γ_S behavior in Γ_{out} plane. Here, the $|\Gamma_S| \le 1$ domain must reside completely within the $|\Gamma_{out}|=1$ circle.



$|\Gamma_S|=1$ circle must reside inside