PIE Tech
POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY
(Approved by AICTE and Affiliated to Anna University)
sky is the limit

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

REGULATION 2021

III YEAR / V SEM

CEC366 IMAGE PROCESSING

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COURSE OBJECTIVES:

- To become familiar with digital image fundamentals
- To get exposed to simple image enhancement techniques in Spatial and Frequency Domain
- To learn concepts of degradation function and restoration techniques.
- To study the image segmentation and representation techniques.
- To become familiar with image compression and recognition methods

UNIT I DIGITAL IMAGE FUNDAMENTALS

Steps in Digital Image Processing – Components – Elements of Visual Perception – Image Sensing and Acquisition – Image Sampling and Quantization – Relationships between pixels - Color image fundamentals - RGB, HSI models, Two-dimensional mathematical preliminaries, 2D transforms - DFT, DCT.

UNIT II IMAGE ENHANCEMENT

Spatial Domain: Gray level transformations – Histogram processing – Basics of Spatial Filtering– Smoothing and Sharpening Spatial Filtering, Frequency Domain: Introduction to Fourier Transform– Smoothing and Sharpening frequency domain filters – Ideal, Butterworth and Gaussian filters, Homomorphic filtering, Color image enhancement.

UNIT III IMAGE RESTORATION

Image Restoration - degradation model, Properties, Noise models – Mean Filters – Order Statistics – Adaptive filters – Band reject Filters – Band pass Filters – Notch Filters – Optimum Notch Filtering – Inverse Filtering – Wiener filtering

UNIT IV IMAGE SEGMENTATION

Edge detection, Edge linking via Hough transform – Thresholding - Region based segmentation – Region growing – Region splitting and merging – Morphological processing- erosion and dilation, Segmentation by morphological watersheds – basic concepts – Dam construction – Watershed segmentation algorithm.

UNIT V IMAGE COMPRESSION AND RECOGNITION

Need for data compression, Huffman, Run Length Encoding, Shift codes, Arithmetic coding, JPEG standard, MPEG. Boundary representation, Boundary description, Fourier Descriptor, Regional Descriptors – Topological feature, Texture - Patterns and Pattern classes - Recognition based on matching.

DIGITAL IMAGE PROCESSING.

UNIT-1

Elements of Digital Image Processing systems

(1) Image Acquisition:

- Camera Converts the image into a form which is, suitable for digital Computer.

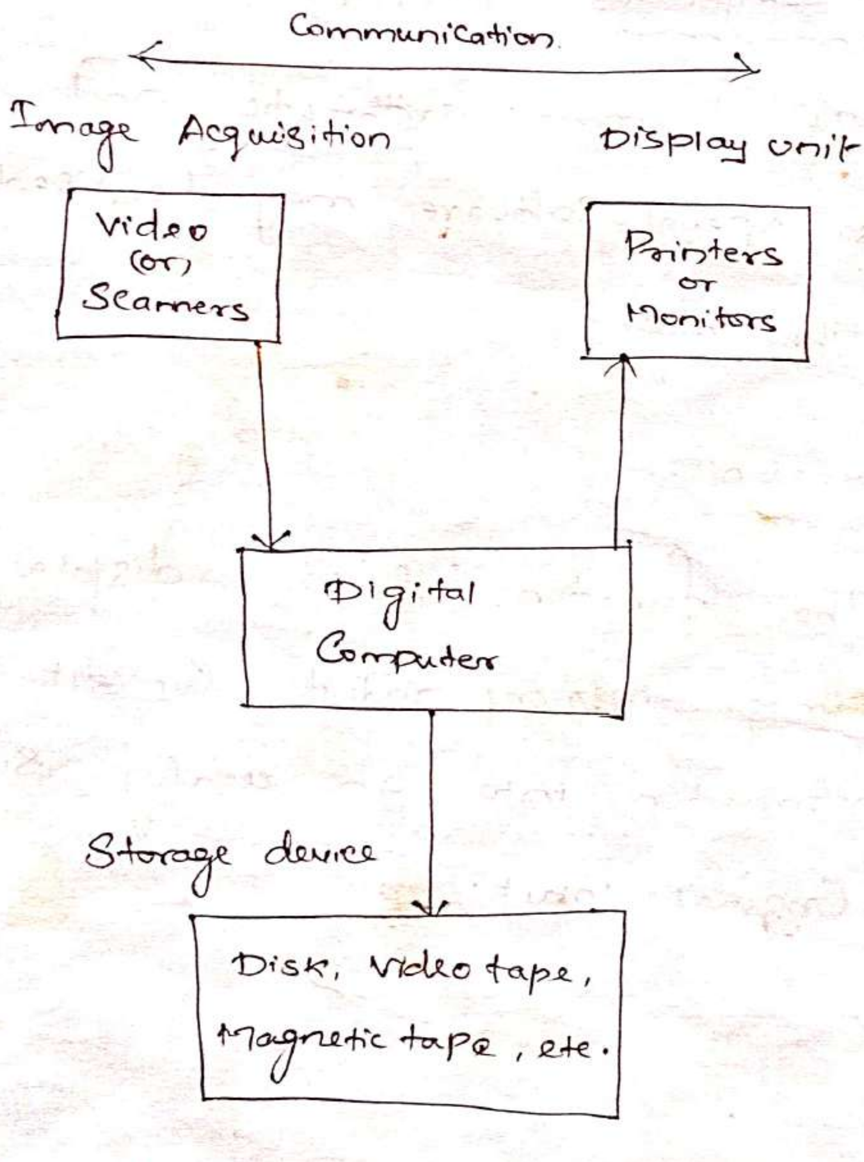


Fig: Basic Elements of digital Image Processing

(ii) Storage device:

* It is used to store entire digital Image (into a form which is, suitable for digital computer.)

* Its feature is that, the contents of the memory can be loaded or read at TV rates.

(iii) Digital computer: (Processor)

It performs arithmetic and logic operations. Special software may be used to process the image.

(iv) Display unit:

The function of the display is to read an image memory and to convert the stored digital information into an analog signal for getting original input.

Image:

An Image may be defined as a two - dimensional function, $f(x,y)$. where x and y are Spatial (Plane) coordinates, and the amplitude of f at any pair of coordinates (x,y) is called intensity or gray level of the image at that point.

Digital Image:

- When x, y and intensity values of f are all finite, discrete quantities, we call the image as digital Image.

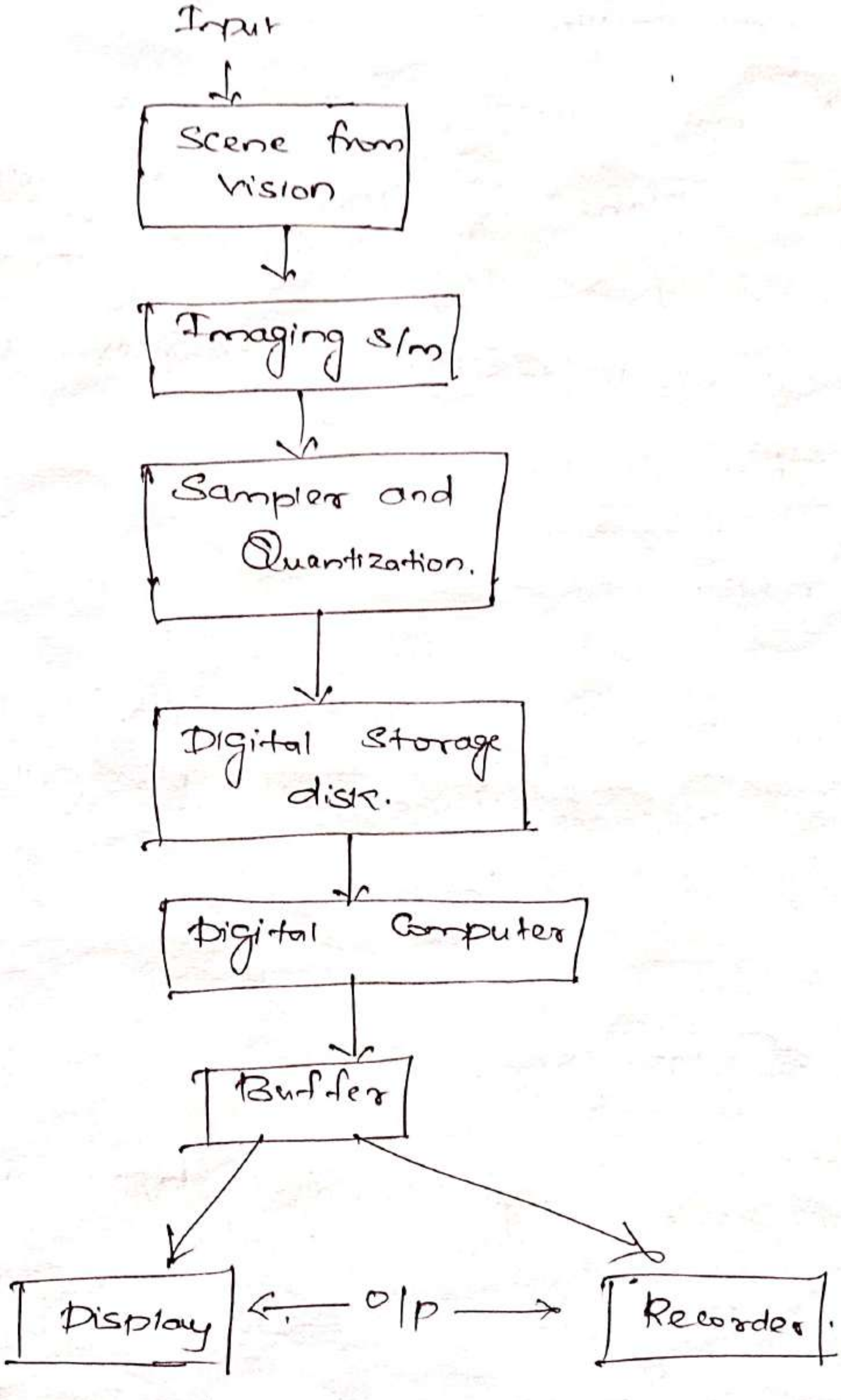
Digital Image Processing:

- Processing digital images by means of digital Computer.

Pixel- Each and every element in a Matrix form of Image.

Resolution- No. of Pixels accomodated in a unit area

Simple digital Image Processing System:



Stages of Image Processing:

(i) Image Representation and modeling

(ii) Image Enhancement

(iii) Image Restoration.

(iv) Image Analysis.

(v) Image reconstruction

(vi) Image data Compression.

Applications:

(1) Image - Remote Sensing via Satellites and Spacecraft

(2) Radar, Sonar

(3) Robotics.

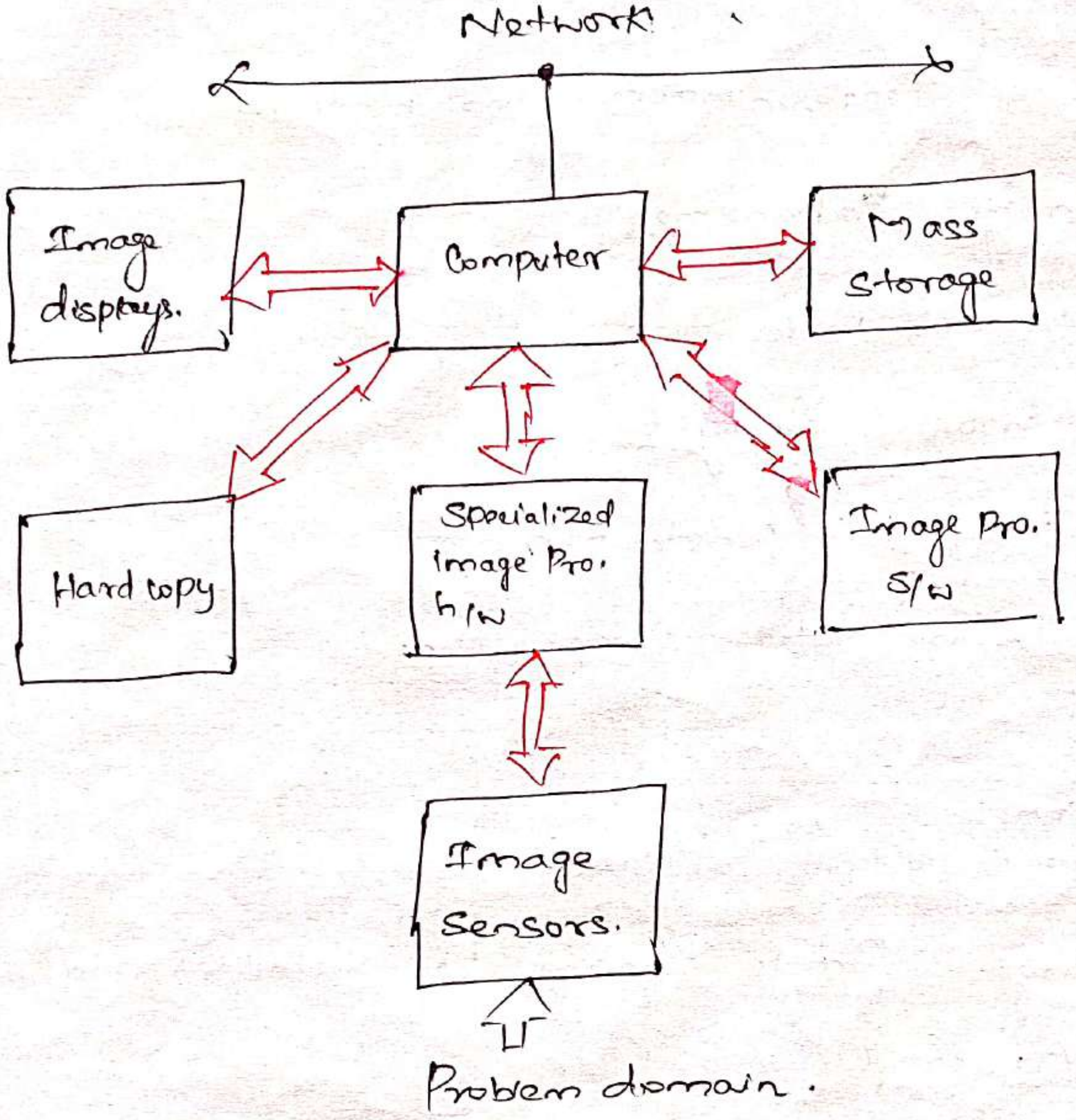
(4) Military Applns - Missile guidance and detection.

(5) Industrial automation, automatic inspection s/m, Non destructive testing.

(6) Biomedical - ECG, EEG, EMG analysis.

(7) Scientific, astronomy etc.

Components of DIP



Elements of Visual Perception:

Structure of human eye:

- The human eye is sphere shaped, having diameter of approx. 20 mm.
- 3 Major Parts:
  - (1) Cornea and Scelera
  - (2) Choroid.
  - (3) Retina.

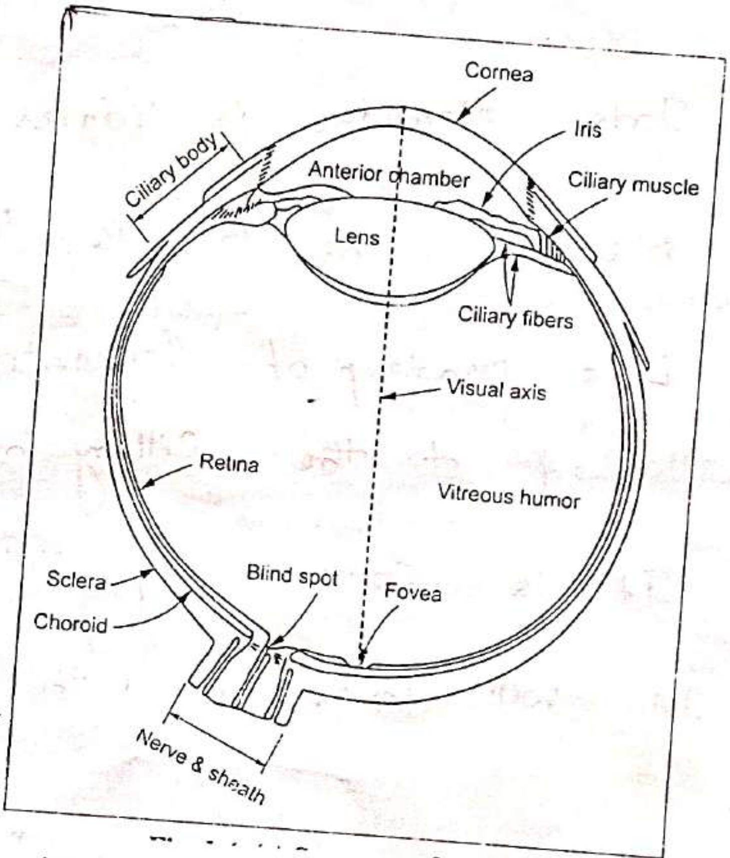


Fig: Structure of an eye.

Cornea:

- It is a transparent tissue.
- It covers the anterior surface of the eye.
- Remaining part is covered by Scelera.

Choroid:

- It lies below the Scelera.
- Contains no. of blood vessels and act as source of nutrition of the eye.

- The Choroid Coat is heavily pigmented to reduce the extraneous light entering the eye and back scattering.

- Divided into $\left\langle \begin{array}{l} \text{Iris} \\ \text{Cillary body.} \end{array} \right.$

- Iris - diameter is varies from 2 - 8 mm.

- Next to the Iris is lens.

- Lens madeup of Concentric Layers and it is attached to the Cillary body.

- It is Colored by yellow pigmentation.

- It will increase with increasing age. (Cataracts).

- It Contains 70 % water and 6 % fat and more Proteins.

- The Excessive amount of UV absorption will damage an eye.

Retina:

- This is the inner most part of an eye.
- When the eye is properly focused, light from an object outside the eye is imaged on retina.
- Pattern vision is afforded by the distribution of discrete light receptors over the surface of retina.
- Two types of receptors.
  (1) Cones
  (2) Rods.

Cones:

- Cones in each number between 6-7 million.
- It is located at the central portion of retina.
- Central portion of Retina - called fovea.
- Each cone is connected with own nerve end.

- Cone vision is known as Photopic vision (or) bright light vision. (Responsible for color vision)

Rods.

- Distributed over the retinal surface and it is ranged from 75-150 millions.

- Several rods are interconnected to a single nerve end. So it will reduce discernible by receptors.

- Not responsible for color vision.

- Rods vision is known as Scotopic vision (or) dim light vision.

Image Formation in the Eye:

Human eye has a small opening in the front, which allows light to inside. That light is passed through a transparent window called 'Cornea' and then

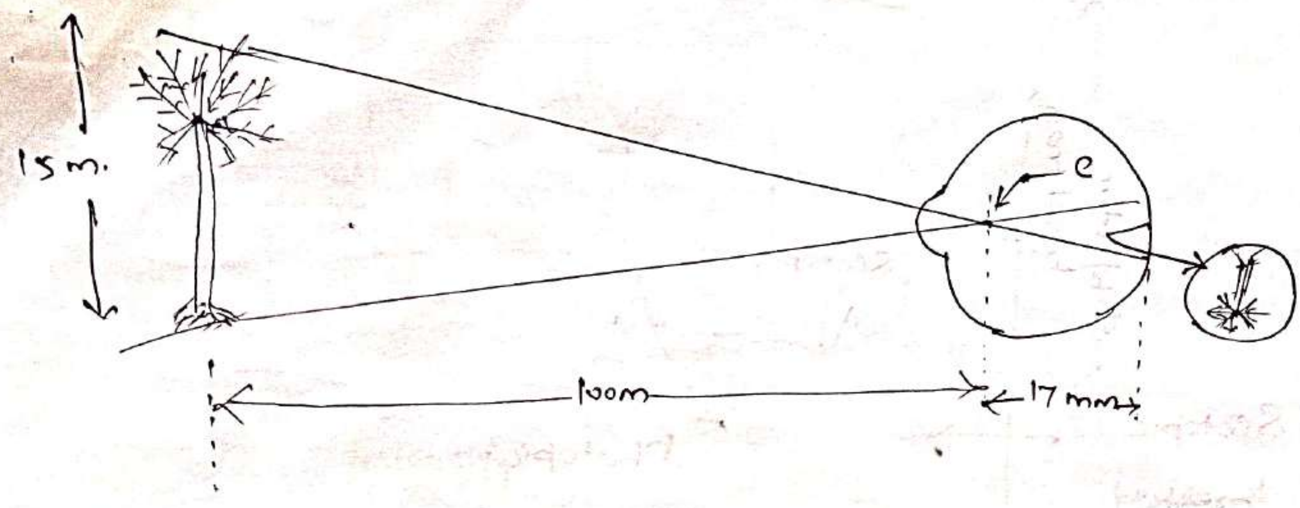


Fig: Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.

To calculate the size of retinal image

$h = 15$, $d = 100$.

$$\frac{15}{100} = \frac{x}{17}.$$

$$\boxed{x = 2.55\,mm}$$

Brightness Adaptation and Discrimination:

- The digital images are displayed as a discrete set of brightness points.

- Visual s/m does not operate on large range simultaneously. But, it achieves this large variation by changing its overall sensitivity – "brightness adaption".

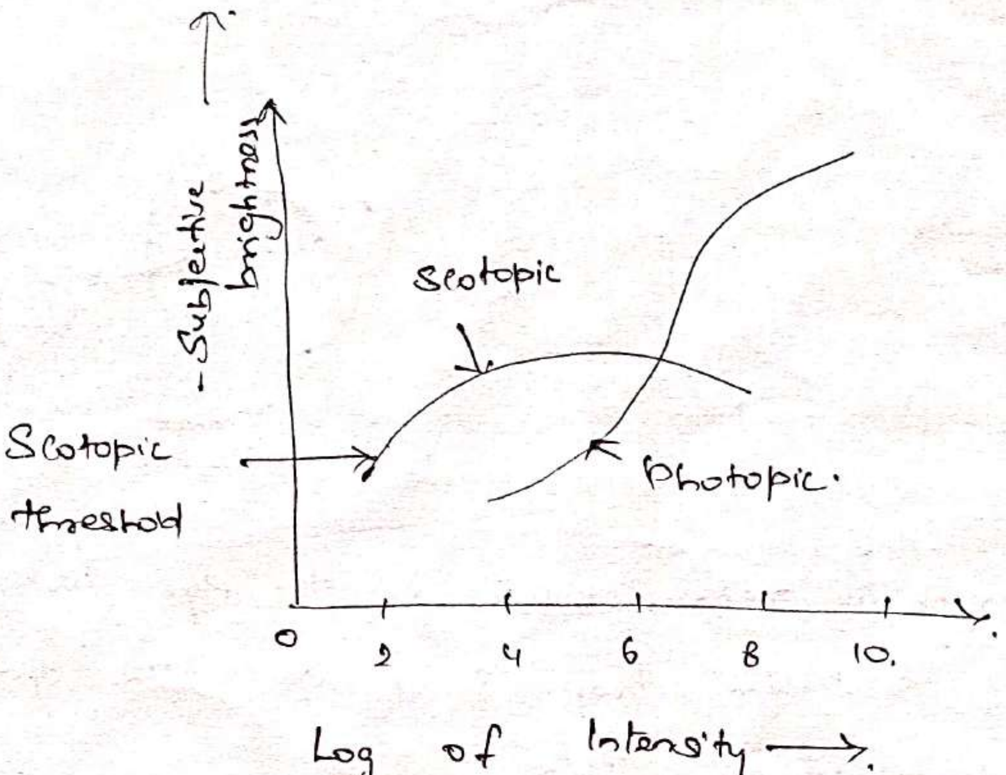


Fig: Subjective brightness Vs log intensity.

$$\boxed{L_{min} \leq l \leq L_{max}}$$

l – Gray level.

Brightness discrimination:

- The ability of the eye to discriminate b/w changes in light intensity at any specific adaptation level is most considerable one.

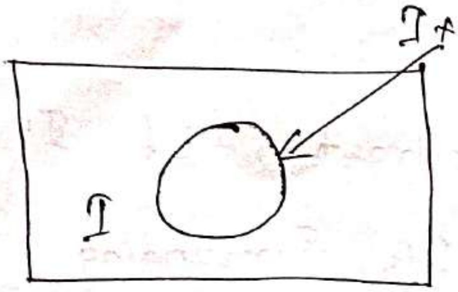


Fig: Experimental Setup about brightness discrimination.

I - Intensity.

ΔI - Increased Intensity. (Illumination)

$$\boxed{\text{Weber ratio} = \frac{\Delta I_c}{I}}$$

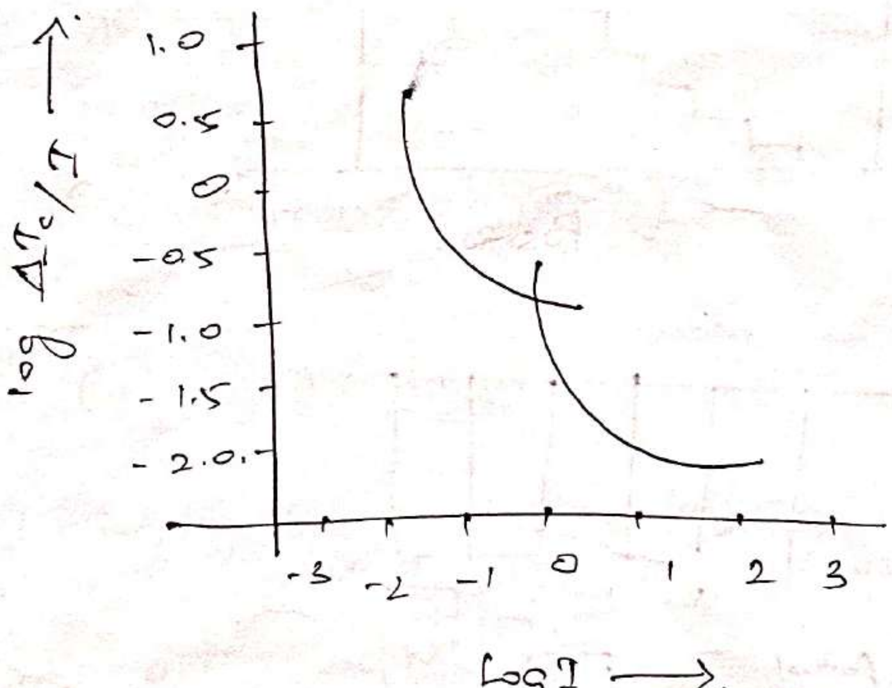
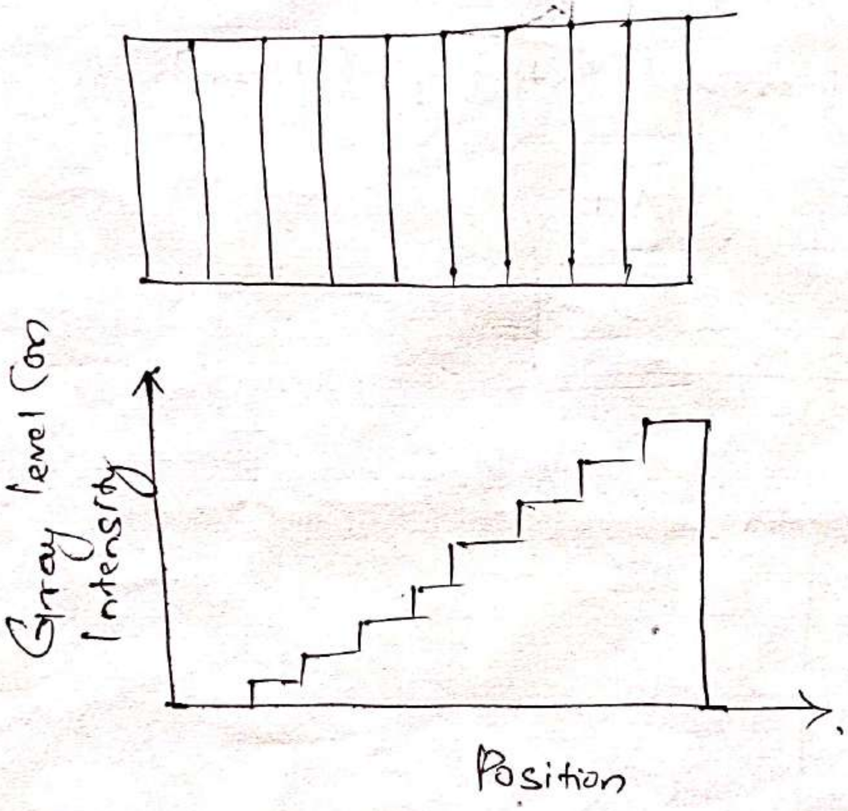
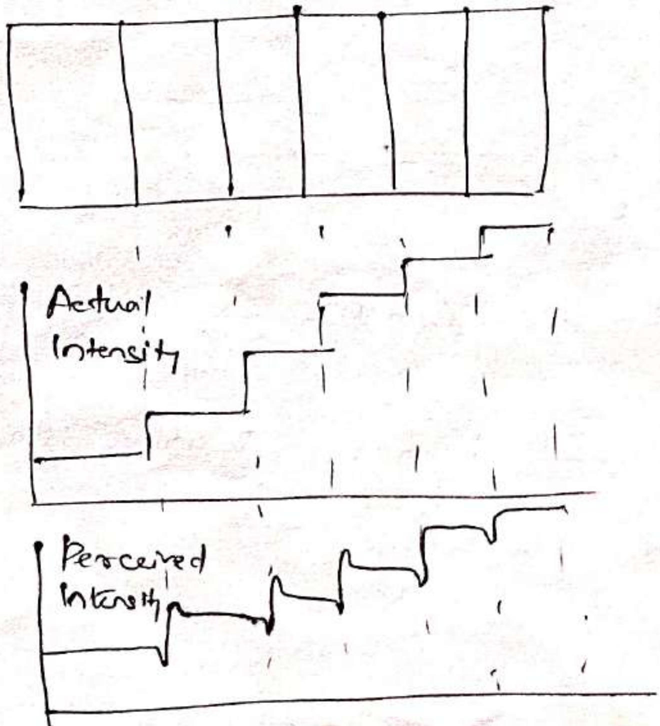


Fig: Typical Weber ratio as a function of Intensity.

Match band Effect:

The Spatial Intraction of luminances from an object & Its Surrounding Creates a Phenomenon Called "mach band effect".





(ii) Brightness is not a monotonic function of luminance. It under goes overshoot and undershoot.

(iii). In this figure Intensity of the stripes is constant.

(iv) ∴ We perceive a brightness pattern is strongly scalloped near the boundaries.

Image Sampling and Quantization:

- Digitizing the spatial coordinates is called Sampling
- Quantization - Digitizing the amplitude values.

Input (Analog) = $f(s,t)$

o/p (Sampling, Quantization) = $f(x,y)$,

$x = 0, 1, \ldots\ldots M-1$

$y = 0, 1, \ldots\ldots N-1$.

N, M - No. of Samples.

N and M are usually the integer powers of 2.

$M = 2^n$

$N = 2^k$.

$$\boxed{G = 2^m}$$

m - No of bits used to represent a gray level in the image.

$\therefore$ digital Image $b = m \times N \times M$.

If $M = N \Rightarrow b = N^2 m$.

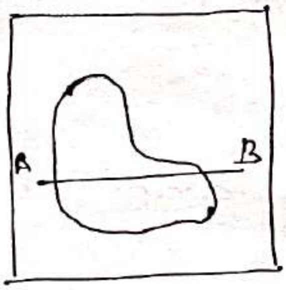


Fig: Continuous Image.

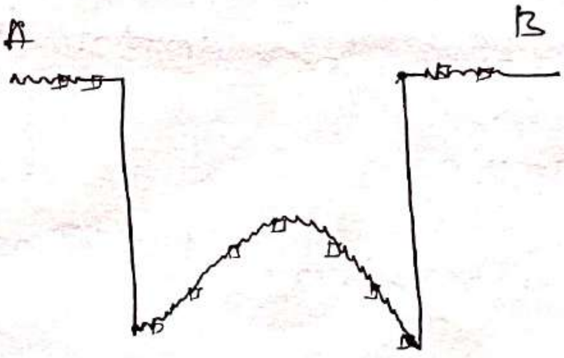


Fig: Sampling

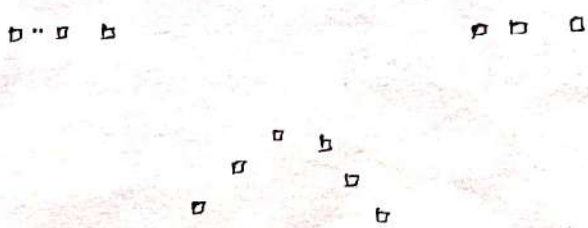


Fig: Quantization.

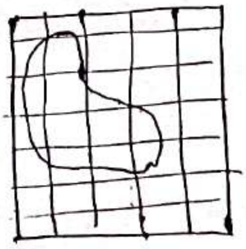
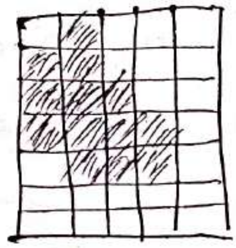


Fig: I/p. Image.

$\Rightarrow$



Result of Samp. Quantization

Hue:

- Hue represents dominant Colour as perceived by us. It is an attribute associated with the dominant wave length.

Saturation:

- Amount of white light mixed with hue.

Gray level

- It refers to a scalar measure of Intensity that ranges from black, to grays. and finally to white.

Brightness:

- Refers to Intensity
- Achromatic notion of Intensity.

Color Models

- 3D coordinates S/m.

Primary Color - RGB

Secondary Color - CMY.

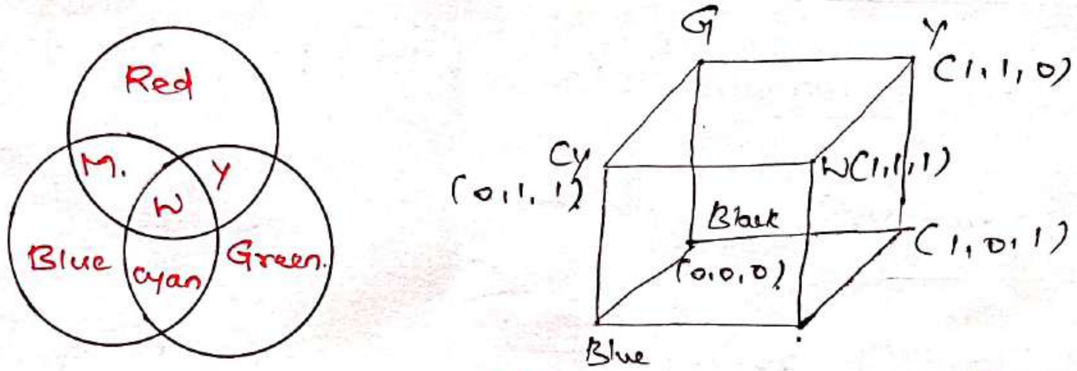


Fig: RGB Colour Model.

Additive Colour Model.

Magenta = Red + Blue

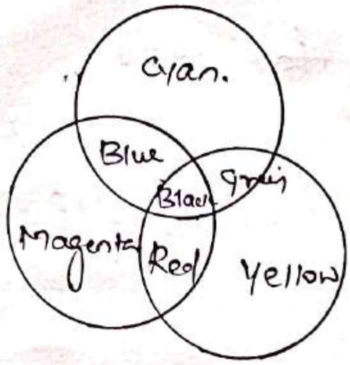
Yellow = Red + Green.

Cyan = Blue + Green.

C = I - R : White - Red

M = White - Green

Y = White - Blue.



CMY Colour Model.

Discrete Cosine Transform (DCT)

DCT is one of the popular x^m used for image compression.

1 dimensional DCT is given by

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left(\frac{(2x+1)u\pi}{2N}\right).$$

$u = 0, 1, 2, \ldots N-1.$

The inverse DCT can be given by

$$f(x) = \sum_{u=0}^{N-1} \alpha(u) C(u) \cos\left(\frac{(2x+1)u\pi}{2N}\right).$$

$x = 0, 1, 2 \ldots N-1.$

$\alpha(u)$ is given by

$$\alpha(u) = \sqrt{\frac{1}{N}} \quad \text{for } u = 0$$

$$\alpha(u) = \sqrt{\frac{2}{N}} \quad \text{for } u = 1, 2, 3 \ldots N-1$$

2-D DCT is given by

$$C(u,v) = \alpha(u)\,\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cdot \left[\cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)\right]$$

$u = 0, 1, 2 \ldots N-1$

$v = 0, 1, 2 \ldots N-1.$

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} \alpha(u)\,\alpha(v)\, C(u,v). \left[\cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)\right]$$

$\therefore x = 0, 1, 2 \ldots N-1$

$y = 0, 1, 2, \ldots N-1.$

Absorption of light in human eye :

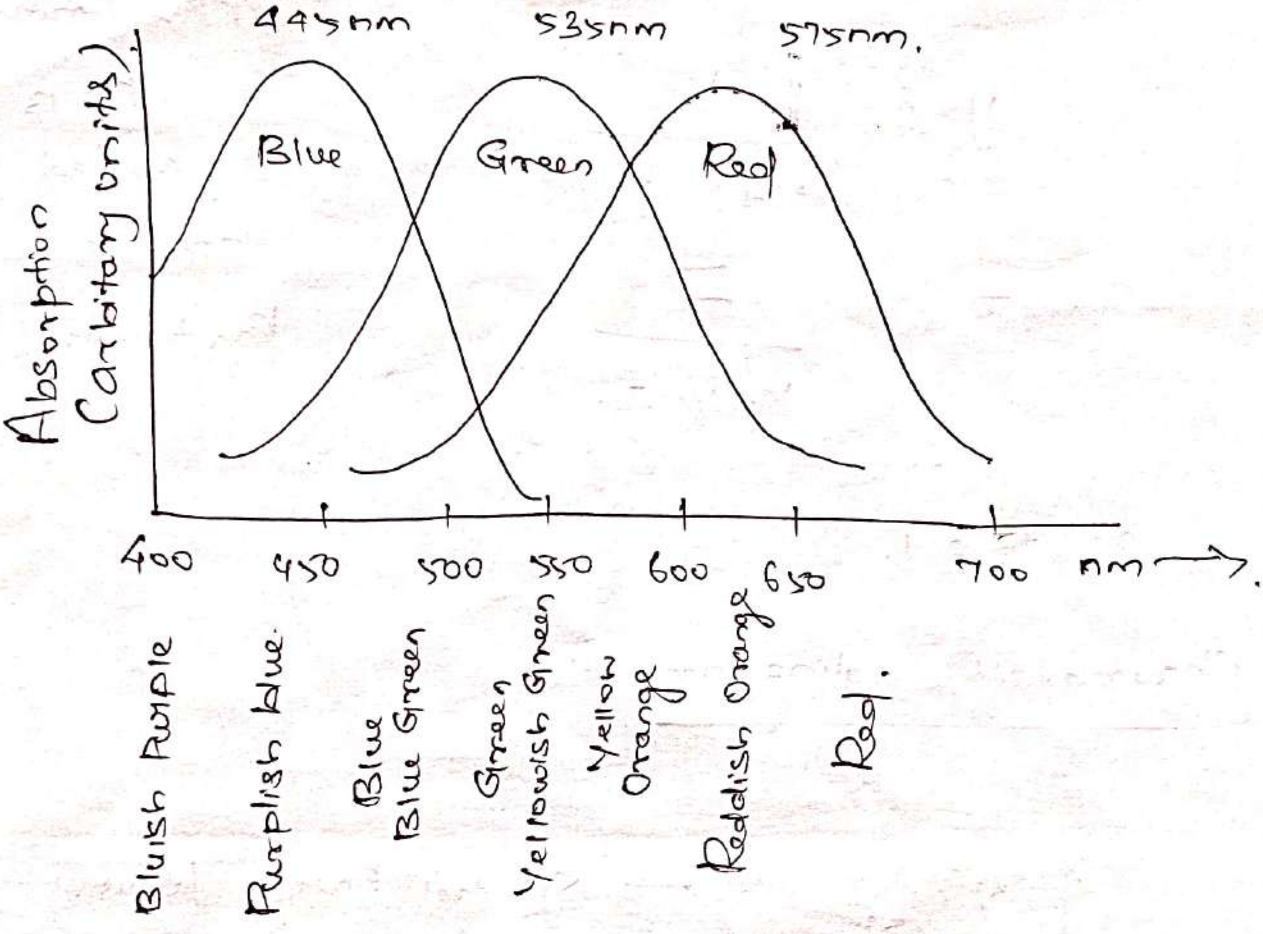


Fig: Absorption of light by the red, Green, Blue Cones in the human eye as a function of wave length.

- Hue and Saturation taken together are Called Chromaticity.
- A color is specified by its trichromatic Co-efficients, defined as

$$x = \frac{X}{X+Y+Z}.$$

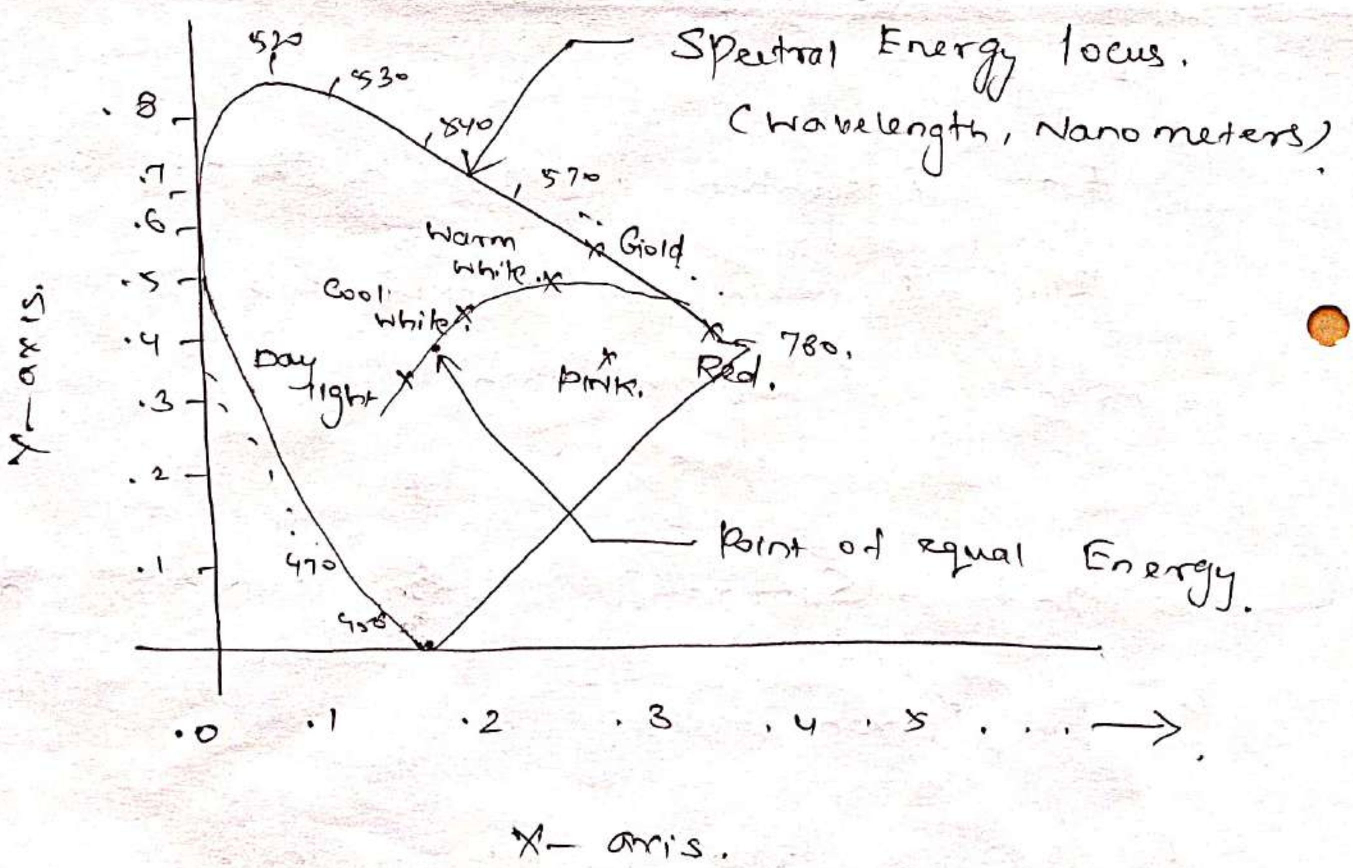
$$y = \frac{Y}{X+Y+Z}$$

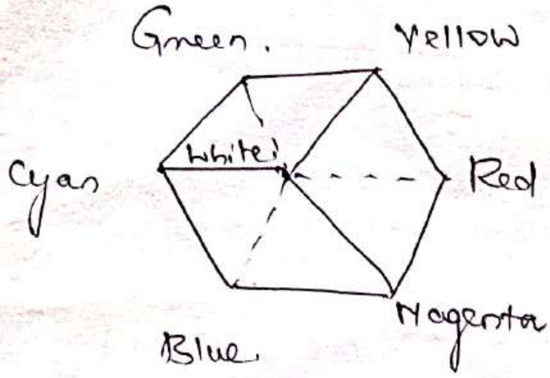
$$z = \frac{Z}{X+Y+Z}.$$

X, Y, Z - Amount of RGB needed to form a Particular color.

$x+y+z = 1.$

Chromaticity diagram:





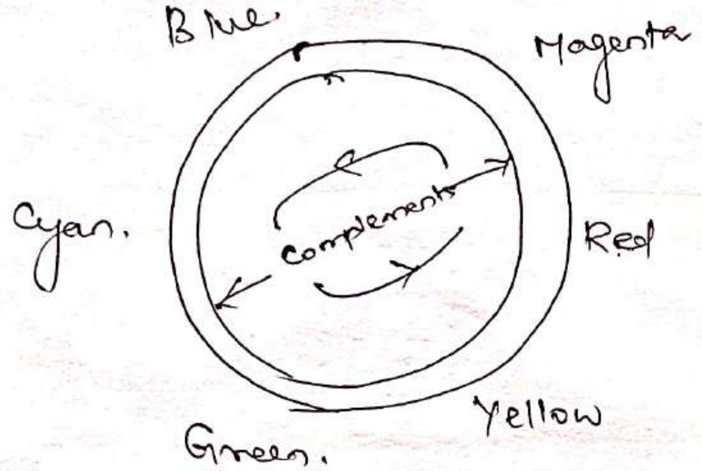


Fig: Complements on the Color circle.

Discrete Cosine Transform (DCT):

For N = 4

$$C[u] = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left(\frac{(2x+1)u\pi}{2N}\right).$$

$0 \le u \le N-1$

$$\alpha(u) = \sqrt{\frac{1}{N}} \quad \text{for } u = 0$$

$$\alpha(u) = \sqrt{\frac{2}{N}} \quad \text{for } u = 1, 2, 3 \ldots N-1.$$

N = 4.

$$C[u] = \alpha(u) \sum_{x=0}^{3} f(x) \left[\cos\left(\frac{(2x+1)u\pi}{2N}\right)\right].$$

If u = 0.

$$C[0] = \sqrt{\frac{1}{4}} \sum_{x=0}^{3} f(x) \cos\left(\frac{(2x+1)(0)\pi}{8}\right)$$

$$= \frac{1}{2} \sum_{x=0}^{3} f(x) \cos(0) = \frac{1}{2} \sum_{x=0}^{3} f(x)(1).$$

$$= \frac{1}{2} \sum_{x=0}^{3} f(x)$$

$$= \frac{1}{2} \times \{x(0) + x(1) + x(2) + x(3)\}$$

$$C(0) = \frac{1}{2}x(0) + \frac{1}{2}x(1) + \frac{1}{2}x(2) + \frac{1}{2}(x(3)) \quad \text{--- (1)}$$

Substituting $u = 1$

$$C(u) = \sqrt{\frac{2}{4}} \sum_{x=0}^{3} f(x) \cos\left[\frac{(2x+1)\pi \times 1}{8}\right]$$

$$= \sqrt{\frac{1}{2}} \sum_{x=0}^{3} f(x) \cos\left[\frac{(2x+1)\pi}{8}\right]$$

$$= 0.707 \times \left\{ x(0)\underbrace{\cos\left(\frac{\pi}{8}\right)}_{\Rightarrow 0.9239} + x(1)\underbrace{\cos\left(\frac{3\pi}{8}\right)}_{\Rightarrow 0.3827} + x(2)\cos\left(\frac{5\pi}{8}\right) + x(3)\cos\left(\frac{7\pi}{8}\right)\right\}$$

$$C(1) = 0.6532\,x(0) + 0.2706\,x(1) - 0.2706\,x(2) - 0.6532\,(x(3))$$

$$\text{--- (2)}$$

Subs $u = 2$.

$$C(2) = 0.5\,x(0) - 0.5\,x(1) - 0.5\,x(2) + 0.5\,x(3)$$

Subs $k = 3$ $(u = 3)$.

$$C(3) = 0.2706\,x(0) - 0.6533\,x(1) + 0.6533\,x(2) - 0.2706\,x(3)$$

Collecting the Co-efficient of $x(0), x(1), x(2), x(3)$ from $C(0), C(1), C(2), C(3)$ we get

$$\begin{bmatrix} C[0] \\ C[1] \\ C[2] \\ C[3] \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.6532 & 0.2706 & -0.2706 & -0.6532 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.2706 & -0.6533 & 0.6533 & -0.2706 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

Properties of Cosine X^m:

1. The Cosine X^m is real and Orthogonal, that is

$$C = C^* \Rightarrow C^{-1} = C^T$$

2. Vector of N elements Can be Calculated in $O(N \log_2 N)$.

3. DCT has excellent Energy Compaction for highly Correlated data.

KARHUNEN - LOEVE TRANSFORM (KL Transform)

(Hotelling x^m)

Developed by

Kari Karhunen

Michel Loeve

Drawbacks:

1). KL x^m is i/p dependent and the basic function has to be Calulated for each Signal model on which it operates.

2) No specific Mathematical Structure that leads to fast implementations.

3) It Requires $O(m^2)$ multiply / add operations.

But DFT and DCT Require $O(\log_2^m)$ multiplications.

Applications:

- Clustering Analysis.
- Image Compression.

Steps to Solve a Pbm:

<u>Step 1:</u> Formation of vectors from given matrix.

<u>Step 2:</u> Determination of Covariance Matrix.

$$Cov(x) = E[xx^T] - \bar{x}\bar{x}^T$$

$$\bar{x} = \frac{1}{M} \sum_{K=0}^{M-1} x_K.$$

M - No. of Vectors in x.

<u>Step 3:</u> Determination of eigen Values of Covariance Matrix.

$$|Cov(x) - \lambda I| = 0.$$

<u>Step 4:</u> Determination of eigen Vectors of the Covariance Matrix using ϕ_0, ϕ_1

$$(Cov(x) - \lambda_0 I)\,\phi_0 = 0.$$

$$(Cov(x) - \lambda_1 I)\,\phi_1 = 0.$$

<u>Step 5</u> Normalisation of eigen Vectors.

$$\frac{\phi_0}{\|\phi_0\|} = \frac{1}{\sqrt{\phi_{00}^2 + \phi_{01}^2}} \begin{bmatrix} \phi_{00} \\ \phi_{01} \end{bmatrix}$$

Step 6: KL x^m matrix from the eigen vector of Covariance Matrix.

$$TT^T = TT^{-1} = I.$$

Step 7 KL x^m of the i/p Matrix.

$$\boxed{Y = T[x]}$$

$$Y_0 = T[x_0]$$

$$Y_1 = T[x_1]$$

Step 8: Reconstruction of i/p values from the transformed Co-efficients.

$$X = T^T Y.$$

---

① Perform KL Transform for the following Matrix.

$$X = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

Sol

Step 1

$$x_0 = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \quad x_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Step 2

$$\text{Cov}(x) = E[xx^T] - \bar{x}\bar{x}^T$$

$$\bar{x} = \frac{1}{M} \sum_{k=0}^{M-1} x_k.$$

$$\bar{x} = \frac{1}{2}\sum_{k=0}^{1} x_k = \frac{1}{2}\{x_0 + x_1\}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 4 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}\right\}$$

$$\bar{x} = \frac{1}{2}\left\{\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right\} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boxed{\bar{x}\bar{x}^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$$

$$\therefore E[xx^T] = \frac{1}{M}\sum_{k=0}^{M-1} x_k x_k^T$$

$$= \frac{1}{2}\sum_{k=0}^{1} x_k x_k^T$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 4 \\ -1 \end{bmatrix}\begin{bmatrix} 4 & -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}\begin{bmatrix} -2 & 3 \end{bmatrix}\right\}$$

$$\therefore E[xx^T] = \frac{1}{2}\left\{\begin{bmatrix} 16 & -4 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 10 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\boxed{E[xx^T] = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}}$$

$$\therefore \quad Cov(x) = E[x x^T] - \bar{x}\bar{x}^T$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\boxed{Cov(x) = \begin{bmatrix} 1 & -2 \\ -2 & 0 \end{bmatrix}}$$

Step 3:

$$\left| [Cov(x) - \lambda I] \right| = 0.$$

$$\det\left(\begin{vmatrix} 1 & -2 \\ -2 & 0 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right) = 0.$$

$$\det\left(\begin{vmatrix} 1 & -2 \\ -2 & 0 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} \right) = 0.$$

$$\det\left(\begin{vmatrix} 1-\lambda & -2 \\ -2 & -\lambda \end{vmatrix} \right) = 0.$$

$$(1-\lambda)(-\lambda) \cancel{*} - (4) = 0.$$

$$-\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - \lambda - 4 = 0.$$

$$\lambda = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm 4.1213}{2}.$$

$$\lambda_0 = \frac{1 + 4.1213}{2} = 2.5615$$

$$\lambda_1 = \frac{1 - 4.1231}{2} = -1.5615.$$

Step 4

$$(\mathrm{cov}(x) - \lambda_0 I)\,\phi_0 = 0$$

$$(\mathrm{cov}(x) - \lambda_0 I)\phi_0 = \left[\begin{bmatrix} 1 & -2 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 2.5615 & 0 \\ 0 & 2.5615 \end{bmatrix}\right]\begin{bmatrix} \phi_{00} \\ \phi_{01} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1.5615 & -2 \\ -2 & -2.5615 \end{bmatrix}\begin{bmatrix} \phi_{00} \\ \phi_{01} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-1.5615\,\phi_{00} - 2\,\phi_{01} = 0$$

(Using Row detection method, we have found ϕ_{01} to be a free Variable. So, we choose the value of ϕ_{01} as 1.)

$$-1.5615 - 2\phi_{01} = 0$$

$$\phi_{00} = \frac{2}{-1.5615} = -1.2808$$

The eigen Vector $\phi_0 = \begin{bmatrix} -1.2808 \\ 1 \end{bmatrix}$

III'ly $\quad \lambda_1 = -1.5615$

$$\phi_1 = \begin{bmatrix} 0.7808 \\ 1 \end{bmatrix}$$

<u>Step. 5</u> Normalisation

$$\frac{\phi_0}{\|\phi_0\|} = \frac{1}{\sqrt{\phi_{00}^2 + \phi_{01}^2}} \begin{bmatrix} \phi_{00} \\ \phi_{01} \end{bmatrix}$$

$$\frac{\phi_0}{\|\phi_0\|} = \frac{1}{\sqrt{(-1.2808)^2 + 1^2}} \begin{bmatrix} -1.2808 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.7882 \\ 0.6154 \end{bmatrix}$$

III'ly

for $\quad \frac{\phi_1}{\|\phi_1\|} = \frac{1}{\sqrt{(0.7808)^2 + 1^2}} \begin{bmatrix} 0.7808 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 0.6154 \\ 0.7882 \end{bmatrix}$$

Step 6: KL Transformation

$$T = \begin{bmatrix} -0.7882 & 0.6154 \\ 0.6154 & 0.7882 \end{bmatrix}$$

$$TT^T = TT^{-1} = I$$

$$TT^T = \begin{bmatrix} -0.7882 & 0.6154 \\ 0.6154 & 0.7882 \end{bmatrix} \begin{bmatrix} -0.7882 & 0.6154 \\ 0.6154 & 0.7882 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9999 & 0 \\ 0 & 0.9999 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step 7: KLT for i/p Matrix.

$$Y = T[x].$$

$$Y_0 = T[x_0] = \begin{bmatrix} -0.7882 & 0.6154 \\ 0.6154 & 0.7882 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3.7682 \\ 1.6734 \end{bmatrix}$$

$$Y_1 = T[x_1] = [\,..\,] \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 3.4226 \\ 1.1338 \end{bmatrix}$$

Final Matrix for X^{mation}

$$Y = \begin{bmatrix} -3.7682 & 3.4226 \\ 1.6734 & 1.1338 \end{bmatrix}$$

<u>Step: 8</u> Reconstruction.

$$x_0 = T^T Y_0 = \begin{bmatrix} -0.7882 & 0.6154 \\ 0.6154 & 0.7882 \end{bmatrix} \begin{bmatrix} -3.7682 \\ 1.6734 \end{bmatrix}$$

$$= \begin{bmatrix} 3.9998 \\ -1 \end{bmatrix}$$

$$\text{III}^{ly} \quad x_1 = \begin{bmatrix} -1.9999 \\ 2.9999 \end{bmatrix}$$

$$\therefore \boxed{X = \begin{bmatrix} x_0 & x_1 \end{bmatrix} = \begin{bmatrix} +4 & -2 \\ -1 & 3 \end{bmatrix}}$$

Noise Modelling:

Error or uncertainty of an image arising from such sources as sensor noise, film grain irregularities and atmospheric light fluctuations. These all such effects are called as "Noise".

Freq. Properties. → Fourier spectrum of noise is Constant → White Noise.

Spatial Properties → Noise is independent of Spatial Coordinates.

Gaussian Noise:

The PDF of a Gaussian random Variable 'z' is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

z - Intensity

$\bar{z}$ - Mean (Average) Value of 'z'

σ = Its Std deviation.

σ^2 = Variance of 'z'.

(2) Rayleigh Noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance is given by

$$\bar{z} = a + \sqrt{\frac{\pi b}{4}}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

(3) Erlang (Gamma) Noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$$\bar{z} = \frac{b}{a} \; ; \quad \sigma^2 = \frac{b}{a^2}$$

(4) Exponential Noise:

$$p(z) = \begin{cases} a e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

where $a > 0$

$$\bar{z} = \frac{1}{a} \; ; \quad \sigma^2 = \frac{1}{a^2}$$

(5). Uniform Noise:

$$P(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise.} \end{cases}$$

$$\bar{z} = \frac{a+b}{2} \quad \text{and } \sigma^2 = \frac{(b-a)^2}{12}$$

(6) Impulse ('Salt and Pepper') Noise

$$P(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise.} \end{cases}$$

→ If $b > a$, intensity 'b' will appear as a light dot in the image otherwise 'a' will appear like a dark dot.

→ If either P_a (or) P_b is Zero the impulse noise is Called 'Uni~~form~~ Polar'.

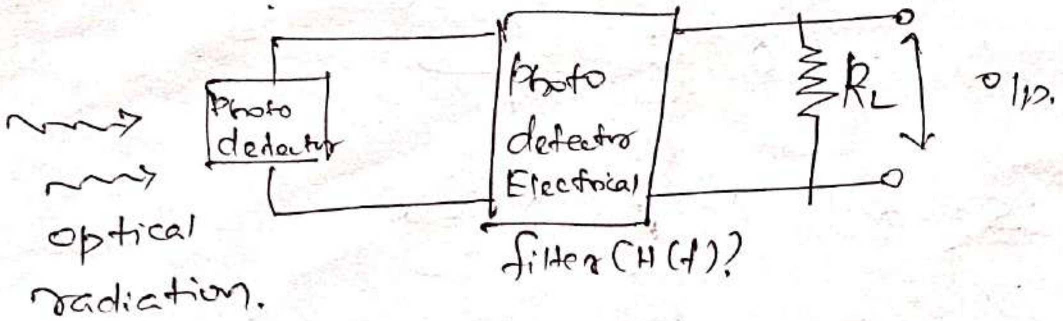
(7) Periodic Noise:

- Due to Electrical or Electromechanical interference during image acquisition.

(8) Photo - Detector Noise :

(1) Photo-detector Noise

(2) Film grain Noise.



Film grain Noise.

→ Silver halide grains → development of Photographic film.

UNIT-II

IMAGE ENHANCEMENT:

Directional Smoothing:

A directional averaging filter is used to protect the edges from blurring while smoothing.

Spatial averages $V(m,n:\theta)$ are calculated in various directions. Its equation is given below.

$$V(m,n:\theta) = \frac{1}{N} \sum_{(k,l)} \sum_{\in W_\theta} Y(m-k, n-l).$$

The direction θ^* at which $|y(m,n) - V(m,n:\theta^*)|$ is minimum is noted.

$$\therefore \boxed{V(m,n) = Y(m,n:\theta^*)}$$

The above equation can give the desired result.

Geometric Mean filter:

In this filter, the restored image is given as.

$$\hat{f}(x,y) = \left[\prod_{(u,v) \in S_{xy}} g(u,v) \right]^{\frac{1}{mn}}.$$

* Here, each restored Pixel is given by the Product of Pixels in the sub images window with Power of $\frac{1}{mn}$.

* In this filter there is a loss of less image details.

Harmonic Mean filter:

In this filter the restored image is given by,

$$\hat{f}(x,y) = \frac{mn}{\sum_{(u,v) \in S_{xy}} \frac{1}{g(u,v)}}$$

The filter is Suitable to remove Salt noise and not Suitable for removing Pepper Noise.

Histogram:

- represents the relative frequency of occurrence of the various gray levels in the image.

Histogram Equalization:

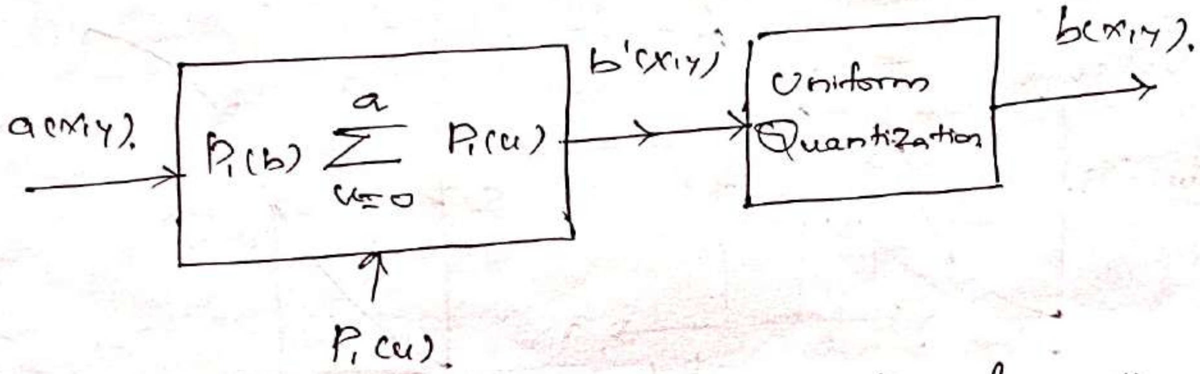


Fig: Histogram equalization transformation.

Consider the Intensity Variable as 'r'.

∴ r is in the range of $[0, L-1]$.

$r = 0 \Rightarrow$ black.

$r = L-1 \Rightarrow$ white.

∴ Transformation is given by

$$\boxed{s = T(r)} \quad 0 \le r \le L-1.$$

The equation satisfied the following conditions.

(i) $T(r)$ is monotonically increasing function in the interval $0 \le r \le L-1$ and

(ii) $0 \le T(r) \le L-1$ for $0 \le r \le L-1$

(iii) $r = T^{-1}(s)$ (Inverse xmation)

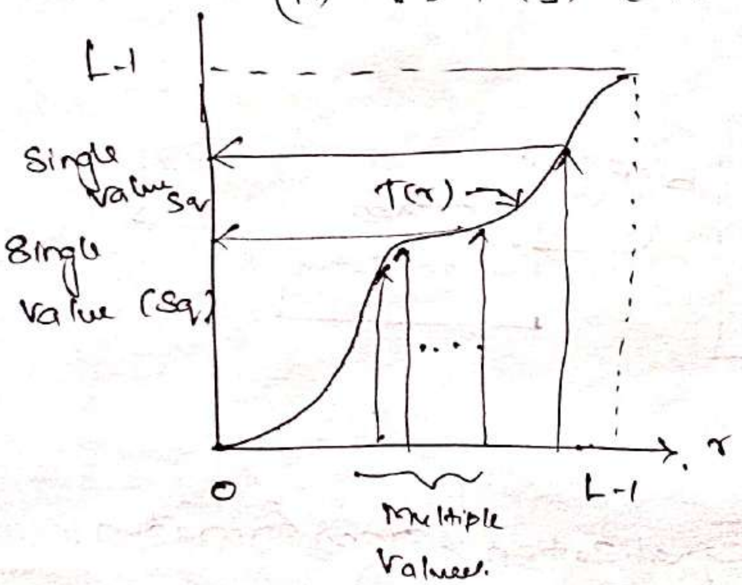


Fig: Monotonically Increasing function.

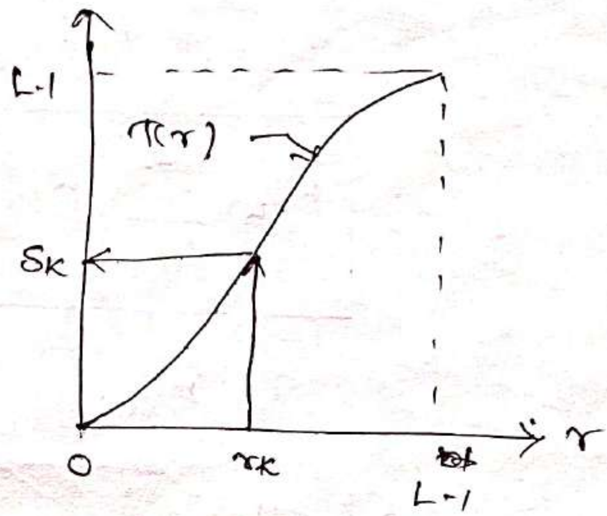


Fig: strictly monotonically increasing function.

*. Here the gray levels can be characterized by their probability density functions $P_r(r)$ and $P_s(s)$ respectively.

*. If $P_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies the condition(i) ∴ the pdf of xed gray level is

$$P_s(s) = \left[P_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)}$$

$$P_s(s) = \left[P_r(r) \frac{dr}{ds} \right]_{r = T^{-1}(s)} \quad \text{--- (1).}$$

Consider the Transformation.

$$S = T(r) = (L-1) \int_0^r P_r(w)\, dw \qquad 0 \le r \le 1. \quad \text{--- (2)}$$

where w is the dummy variable of integration.

* When Considering Cumulative Distribution function (CDF) from equ (2) derivative of S w.r.to 'r' is

$$\frac{ds}{dr} = P_r(r)\ (L-1). \quad \text{--- (3)}$$

Subs (3) in (1)

$$P_s(s) = \left[\cancel{P_r(r)} \frac{1}{\cancel{P_r(r)}} \right]_{r = T^{-1}(s)} \frac{1}{(L-1)}.$$

$$= [1]_{r = T^{-1}(s)} \cdot \frac{1}{(L-1)}$$

$$\boxed{P_s(s) = \frac{1}{L-1} \qquad 0 \le S \le L-1.}$$

Histogram Speification:

- Histogram equalization does not used in image Enhancement Appln. becoz it is Capable of generating only one result.

- Sometimes the ability to specify Particular histogram shapes capable of highlighting Certain desired gray level ranges in an image is desirable.

- Let $P_r(r)$ and $P_d(d)$ be the Original and desired p.d.f respectively.

- The histogram equalization of Origional image is

$$S = T(r) = (L-1)\int_0^r P_r(w)\,dw \quad (2)$$

The histogram equ. of desired image is

$$V = G(d) = (L-1)\int_0^d P_d(w)\,dw. \quad (3)$$

The inverse Process is $d = G^{-1}(v)$.

Here $P_s(s)$ and $P_v(v)$ are the identical uniform densities, bcoz the final result is independent of density inside the integral.

$$\boxed{d = G^{-1}(s)}$$

The Procedure is

(1) To equalize the levels of the Original image using transformation equ (2)

(2) Specify the desired density function and get the x^{ion} function $G(d)$ using equ (3)

(3) Apply the Inverse x^{ion} function, $d = G^{-1}(s)$ to the levels got in step (1).

This Procedure gives a Processed Value of Original image with the new gray levels characterized by the specified density ($P_d(d)$).

Spatial Averaging:

*. Image avg. is used to reduce the noise in an image.

Consider a noisy image represented as g(r,c) formed by the addition of noise n(r,c) to an original image f(r,c).

$$\boxed{g(r,c) = f(r,c) + n(r,c).}$$

Original Image + Noise = Noisy Image

We can use some following spatial filters to avoid noises

(1) Linear filters.

(2) Non Linear filters.

Linear spatial filters:

- x^{er} function and Impulse or Point Spread function of a linear system are inverse fourier x^{ms} of each other.

Three types of Linear filters.

(1) Low Pass filter,

(2) High Pass filter

(3) Bandpass filter.

LowPass filter:

- Eliminates high frequency Components.

High Pass filter:

- Eliminates Lowe freq. Components.

BandPass filters:

- It removes Selected frequency regions between low and high frequencies.

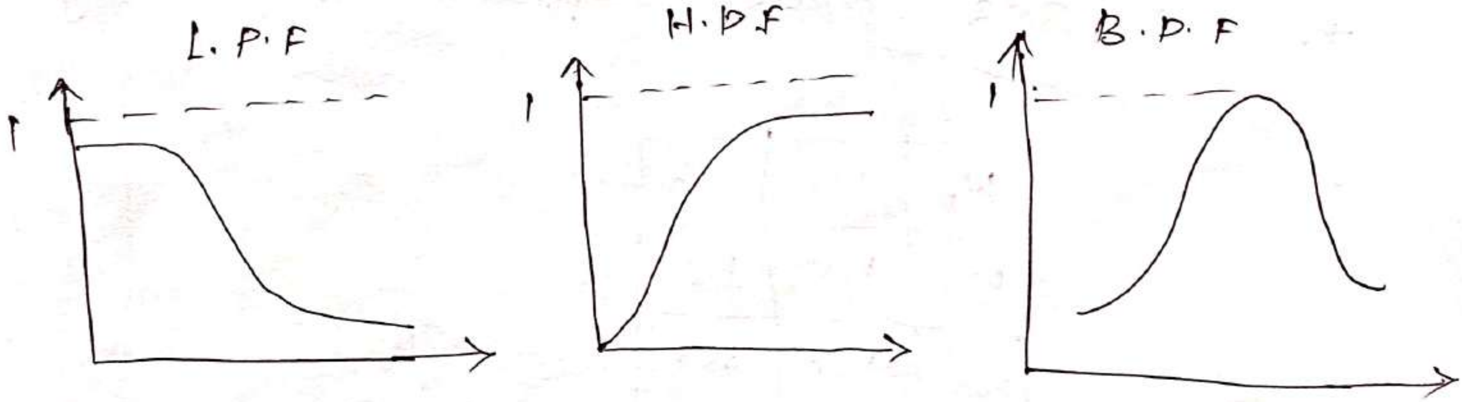


Fig: Frequency domain filters.

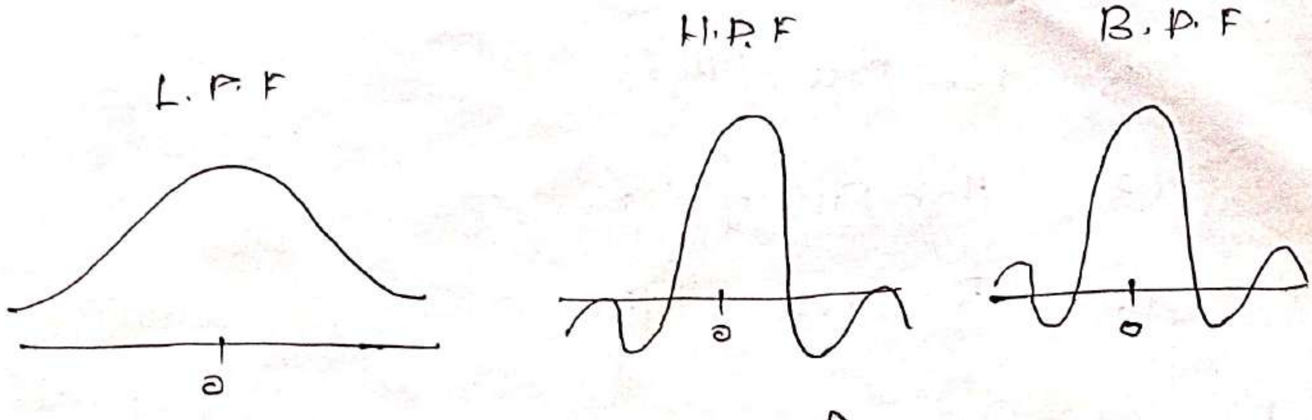


Fig. Spatial domain filter

* The Linear mask is the Sum of the Products between the mask Co-efficients and the intensities of the Pixels under the mask at a Specific location in the image.

$$R = w_1 z_1 + w_2 z_2 + \dots + w_9 z_9$$

*. 3x3 mask coefficient is given by

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Non Linear Spatial filter:

* This filter is based directly on the values of the pixels in the neighborhood and they did not use co-efficients.

* Using Non Linear filter Noise reduction can be achiered effectively.

* The basic function of the Non Linear filter is to compute the median gray level value in the neighborhood in which the filter is located.

Contra harmonic Mean filter:

The restored image function is given as

$$\boxed{\hat{f}(x,y) = \frac{\sum_{(u,v) \in S_{xy}} g(u,v)^{Q+1}}{\sum_{(u,v) \in S_{xy}} g(u,v)^{Q}}}$$

Q = Order of the filter.

Homomorphic filtering:

W.K.T 'illumination' and 'Reflectance' Components of the image light intensity function $f(x,y)$ are denoted by $i(x,y)$ and $r(x,y)$ respectively.

$\therefore$ $f(x,y)$ is given as

$$f(x,y) = i(x,y) \cdot r(x,y) \quad -(1)$$

Where

$$0 < i(x,y) < \infty \text{ and}$$
$$0 < r(x,y) < 1$$

Equ (1) Cannot be used directly, in Order to operate Separately on the freq. Components of illumination and reflectance. Bcoz in freq. domain the fourier x^m of Product of 2 function is not separable.

(i.e) $F\{f(x,y)\} \neq F\{i(x,y)\} \cdot F\{r(x,y)\}$.

but We Can Write

$$Z(x,y) = \ln f(x,y)$$
$$= \ln i(x,y) + \ln r(x,y).$$

Then we can write

$$F\{z(x,y)\} = F\{\ln f(x,y)\}.$$

$$= F\{\ln i(x,y)\} + F\{\ln r(x,y)\}.$$

Otherwise

$$Z(u,v) = I(u,v) + R(u,v).$$

W.K.T

- In freq. domain the convolution of an image $f(x,y)$ and linear position invariant operator $h(x,y)$ is given in F.T as

$$G(u,v) = H(u,v) \cdot F(u,v).$$

$$H(u,v) = \text{Homomorphic filter function.}$$

- Result in terms of F.T is given by.

$$S(u,v) = H(u,v)\, Z(u,v)$$

$$S(u,v) = H(u,v)\, I(u,v) + H(u,v)\, R(u,v).$$

In Spatial domain

$$s(x,y) = F^{-1}\{S(u,v)\}.$$

$$= F^{-1}\{H(u,v)\, I(u,v)\} + F^{-1}\{H(u,v)\, R(u,v)\}.$$

Let $i'(x,y) = F^{-1}\{H(u,v)\,I(u,v)\}$

$$r'(x,y) = F^{-1}\{H(u,v)\,R(u,v)\}$$

$$\therefore \; s(x,y) = i'(x,y) + r'(x,y)$$

→ $Z(x,y)$ is formed by taking the logarithm of Origional image $f(x,y)$.

→ The inverse operation yields the desired enhancement image $g(x,y)$.

→ So, take the exponential function

$$g(x,y) = \exp[s(x,y)]$$

$$= \underbrace{\exp[i'(x,y)]}\,.\,\underbrace{\exp[r'(x,y)]}$$

$$g(x,y) = i_0(x,y)\,.\,r_0(x,y)$$

Where the i_0, r_0 are the illumination and Reflectance components of the o/p image.

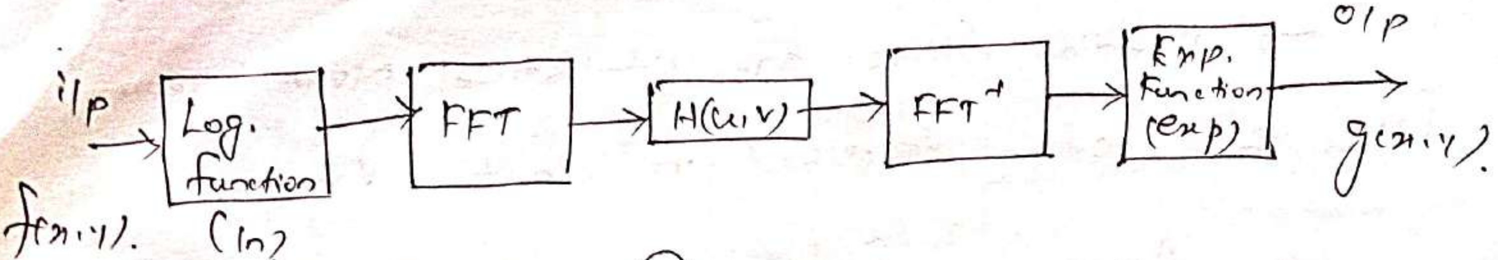


Fig: Homomorphic filtering approach for image Enhancement

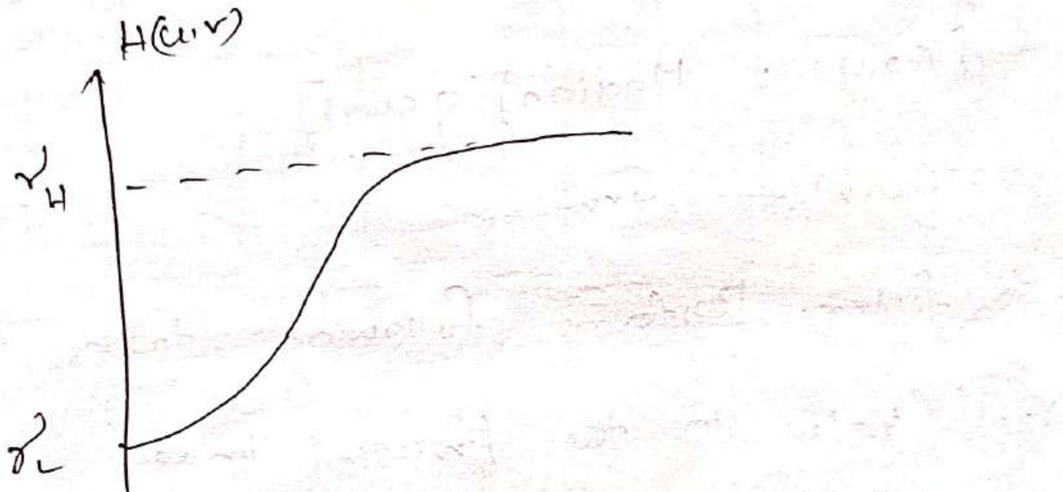


Fig: Cross Section of Circulatory Symmetric filter function

$D(u,v)$ - Distance from the Origin.

$\gamma_L < 1$ and $\gamma_H > 1$.

↓ ↓

Parameters.

Median Filtering:

→ Median filters are Statistical Non-Linear filters that often described in spatial domain.

→ A median filter smoothens the image by utilising the median of neighbourhood.

The restored function is given as

$$\hat{f}(x,y) = \text{Median}[g(u,v)]$$
$$(u,v) \leftarrow S_{xy}$$

Median filter Performs following tasks to find each Pixel Value in the Processed image.

1. All pixels in the Neighbourhood of the pixel in the Origional image which are identified by the Mask are Sorted in the ascending (or) descending Order
2. The Median of Sorted Value is Computed and is Chosen as the Pixel Value for the Processed image.

Prewitt operators:

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Sobel operators:

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Smoothing: of Spatial filtering:

- Removal of Blur & Noise

Linear filters:

Types:

1. Box filter
2. Weighted average filter

Box filter:

A spatial Avg. filter in which all Co.efficients are equal is called as box filter.

Eg:

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sum of all Co.eff = $1+1+1+1+1+1+1+1+1 = 9$.

∴ The mask Co.effs for box filter is

$$\boxed{R = \frac{1}{9} \sum_{i=1}^{9} z_i.}$$

Weighted Avg. filter:

- Pixels are multiplied by different Co-efficients.

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Sum of all Coeff = 1+2+1+2+4+2+1+2+1 = 16.

. The general implementation for filtering an M×N image with a weighted averaging filter of size m×n is given by

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)(f(x+s, y+t))}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

↓

The denominator → sum of the mask coeffs

Appln:

1. Noise Reduction.
2. Smoothing of false contours (or) outlines.

Smoothing by Non linear filters:

Median filter:

- Replaces the value of a pixel by the median of the gray levels in the Neighborhood of that Pixel

$$\boxed{\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{median}} \{g(s,t)\}}$$

Max & Min filters:

$$\boxed{\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\max} \{g(s,t)\}.}$$

Sharpening Spatial filters:

- Highlight the fine details in an image (or) to Enhance details that have been blurred either in ~~er~~ error or as a Natural Effect of Particular method for image acquisition.

First Order derivative:

Requirements:

1. Must be zero in areas of Constant Intensity
2. Must be non-zero at the Onset of an Intensity step or ramp.
3. Must be Non-Zero along ramps.

$$\boxed{\frac{\partial f}{\partial x} = f(x+1) - f(x)}$$

Second Order derivative:

Req:

1. Must be zero in Constant area
2. Must be non-zero at the Onset and end of an intensity step or ramp.
3. Must be zero along ramps of Constant slope.

$$\boxed{\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)}$$

Using the Second derivative for Image Sharpening: (Laplacian):

↓

Independent of direction.

Laplacian of 2D function $f(x,y)$ is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Partial second order derivative in x-direction is

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x,y) \quad \text{—} (1)$$

And "" in y-direction,

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x,y) \quad \text{—} (2)$$

(1) + (2)

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x,y)]$$

The eqn can be rep. by any one of following masks

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

Laplacian for Image Sharpening

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right].$$

$f(x,y)$ - i/p Image.

$g(x,y)$ - Sharped Image..

Using First Order derivative : (The Gradient)

$$\nabla f = \text{grad}(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Magnitude (Length) of Vector ∇f denoted as

$$M(x,y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$\boxed{M(x,y) = |g_x| + |g_y|}$$

Roberts Cross Gradient Operators.

-1	0
0	1

0	-1
1	0

Prewitt Operators:

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Sobel operators:

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Fourier Transform:

- To Enhance an Image.

- Multiply filter by Transfer function and take the Inverse x^m.

1-D F.T

$$\mathcal{F}\{f(x)\} = F(u) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi ux}\, dx$$

Inverse form,

$$\mathcal{F}^{-1}\{F(u)\} = f(x) = \int_{-\infty}^{\infty} F(u)\, e^{j2\pi ux}\, du.$$

2-D. F.T

$$\mathcal{F}\{f(x,y)\} = F(u,v) = \int_{-\infty}^{\infty}\int f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

Inverse

$$\mathcal{F}^{-1}\{F(u,v)\} = f(x,y) = \int_{-\infty}^{\infty}\int F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv.$$

Smoothing by Freq. domain filters:

Basic Model of filtering in freq domain is

$$G(u,v) = \underbrace{H(u,v)}_{\text{Filter function.}} \underbrace{F(u,v)}_{\text{F.T of image to be smoothed.}}$$

- 3 types of Low Pass filter.

① Ideal

② Butterworth.

③ Gaussian.

Ideal Low Pass filters:

- It Eliminates all H.F components of F.T.

∴ Transfer function of Ideal Low Pass Filter is

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 - Specified Non-Negative quantity.

* D(u,v) is the distance from point (u,v) to the center of freq. Rectangle.

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$$

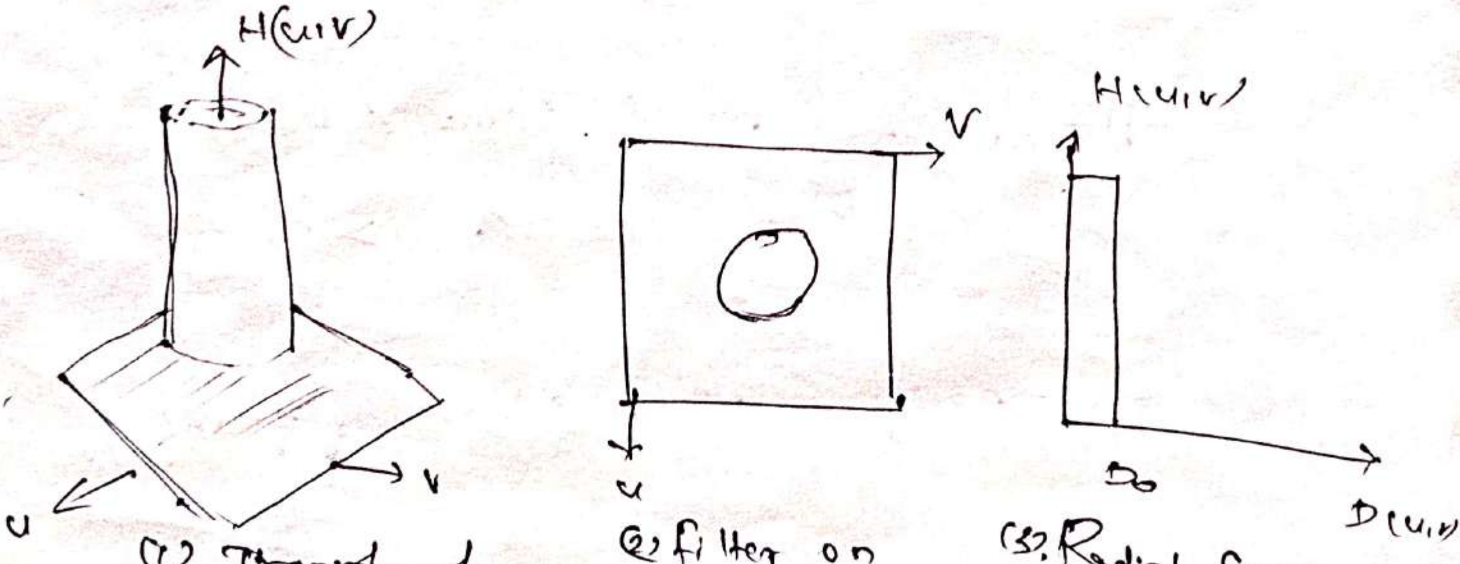
$$\therefore \quad D(u,v) = (u^2 + v^2)^{1/2}$$

* The Transition between $H(u,v)=1$ & $H(u,v)=0$ is Called ~~the~~ "Cut of frequency"

* Total Image Power

$$P_T = \sum_{u=0}^{P-1} \sum_{v=0}^{Q-1} P(u,v)$$

$$\alpha = 100 \left[\sum_u \sum_v \frac{P(u,v)}{P_T} \right]$$



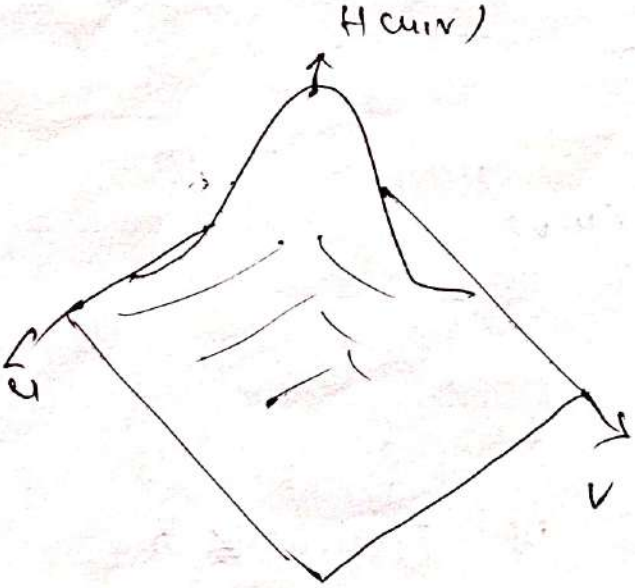
(1) Transfer function. (2) filter on Image. (3) Radial Cross Section.

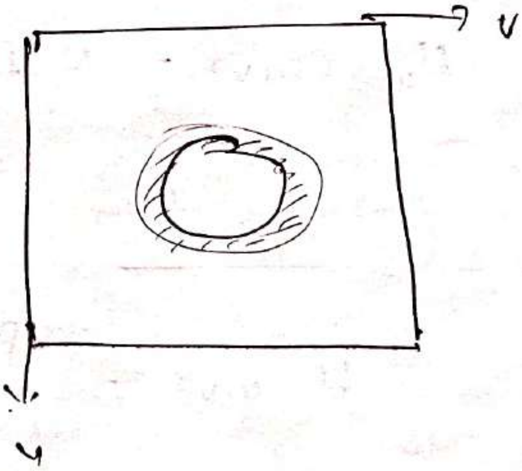
Butterworth Low Pass filters:

The x^{er} function of a Butterworth Low Pass filter (BLPF) of order n and with cut off freq at a distance D_0 from the Origin is defined as

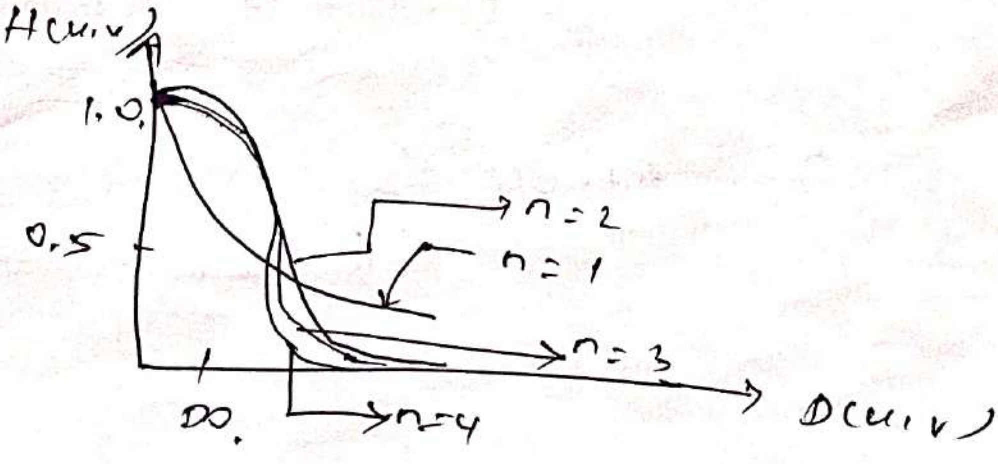
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

Most Appropriate value of $n = 2$.





&

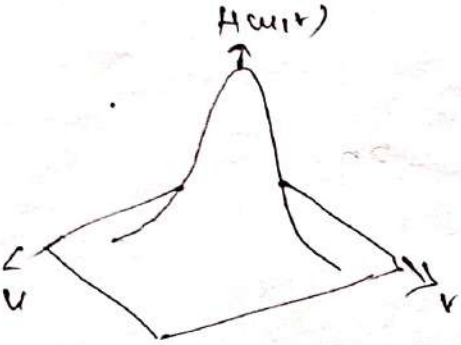


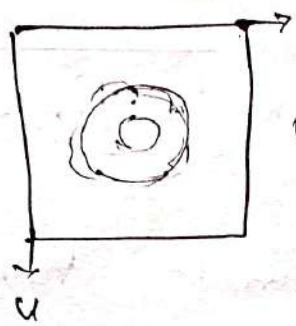
Gaussian L.P.F

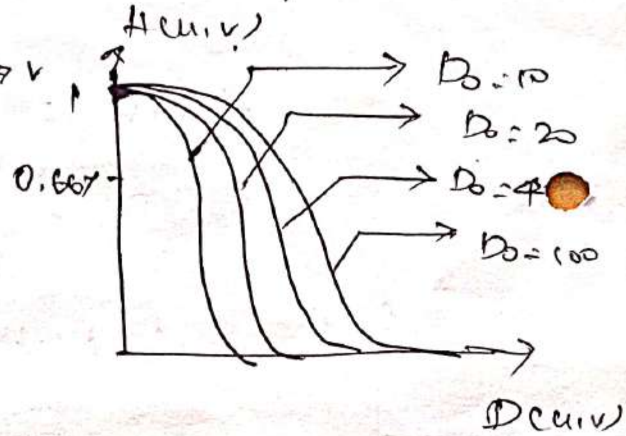
Transfer fun. of G.L.P.F is

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

$\sigma = D_0$ – Specifies cut off freq.







Sharping By freq. domain filters:

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

Ideal High Pass filter:

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

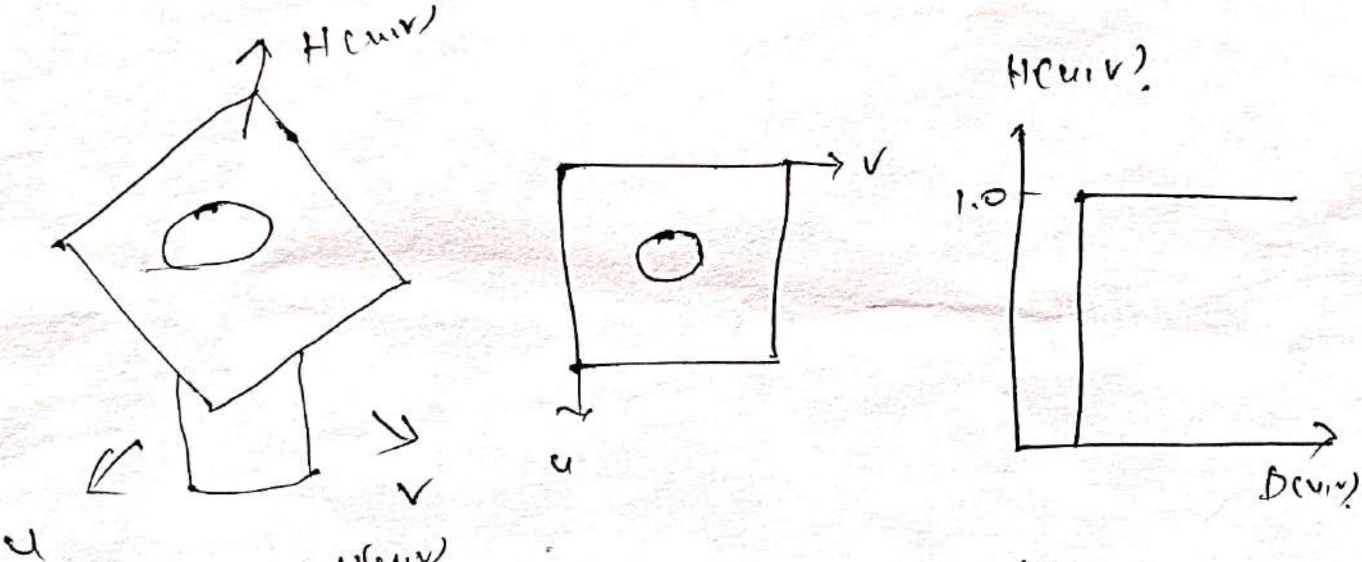
D_0 – cut-off freq.

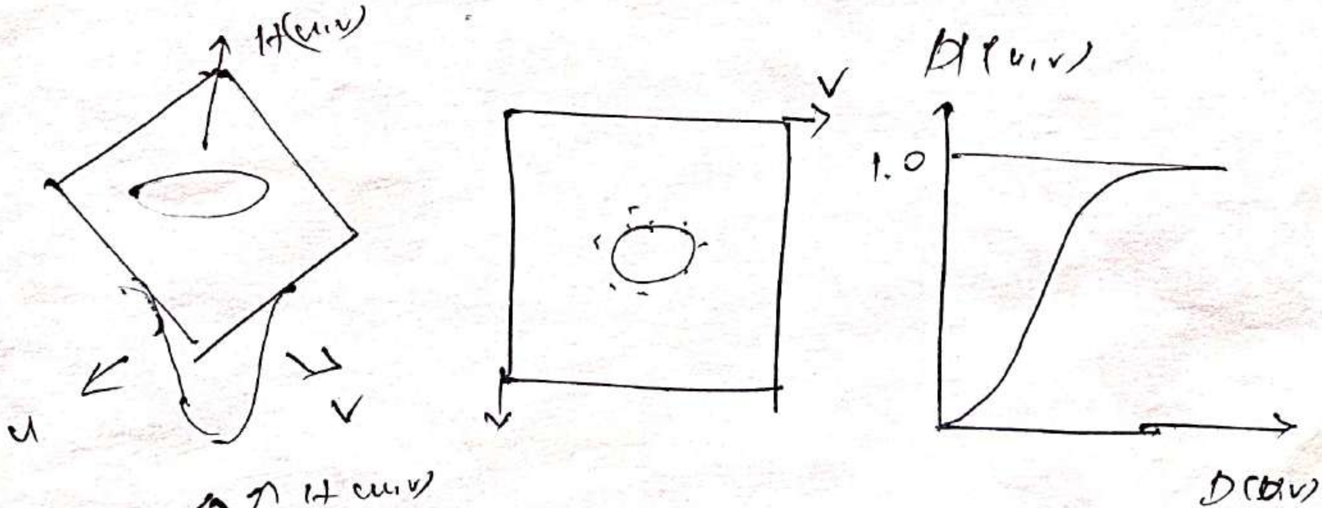
Butterworth HPF

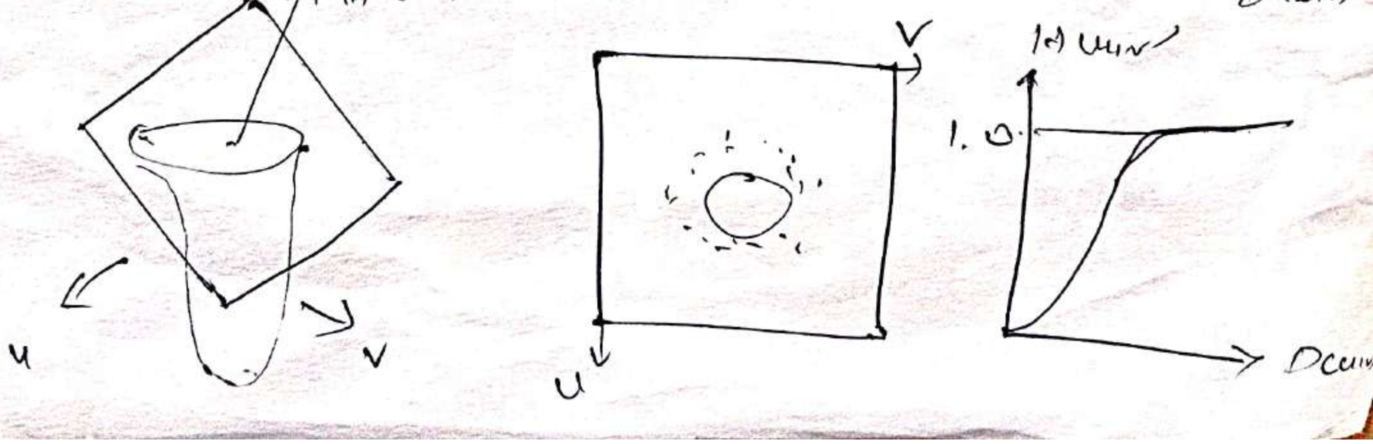
$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

Gaussian HPF

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2} \quad (\text{or}) \; 1 - e^{-D^2(u,v)/2\sigma^2}$$







UNIT-III

Image Restoration

- Image Restoration is the Process that reconstructs an image that has been degraded by using some Prior Knowledge of degradation Phenomenon.

- **Model of degradation:**

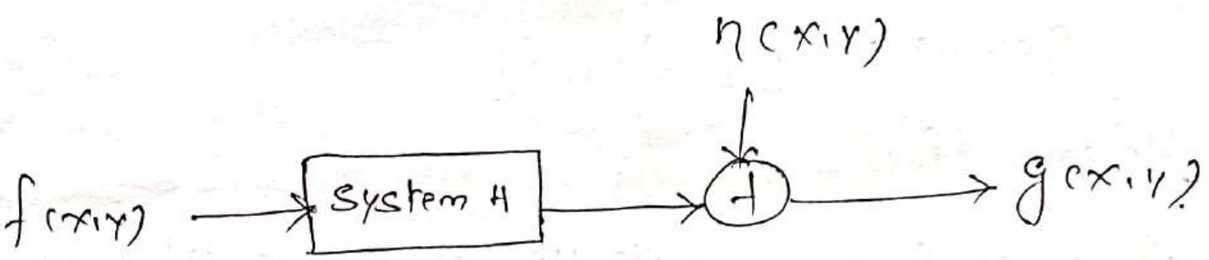


Fig: Model of Image degradation.

- The i/p image f(x,y) is applied to the system 'H'. The additive nois n(x,y) is also added to the S/m o/p to Produce a degraded image g(x,y).

The i/p, o/p relationship expressed as

$$\boxed{g(x,y) = H[f(x,y)] + n(x,y)} \quad - (1).$$

Properties of degradation Model

(1) Linearity

(2) Additive

(3) Homogeneity

(4) Position ~~into~~ invarient (or) Space varient.

Linearity

Assume $n(x,y) = 0$ in eqn (1)

$$g(x,y) = H[f(x,y)].$$

H is Linear if

$$H[K_1 f_1(x,y) + K_2 f_2(x,y)] = K_1 H[f_1(x,y)] + K_2 H[f_2(x,y)].$$

Additive

If $K_1 = K_2 = 1$

$$H[f_1(x,y) + f_2(x,y)] = H[f_1(x,y)] + H[f_2(x,y)].$$

Sum of 2 i/p's = Some of 2 response.

Homogenity

If $f_2(x,y) = 0.$

$$H[K_1 f_1(x,y)] = K_1 H[f(x,y)]$$

Space invarient

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta).$$

Inverse filtering:

Inverse filtering is the process of recovering the i/p of a S/m from its o/p.

Inverse filters are not physically reliable as they are unstable. They are sensitive to noise.

The unconstrained image restoration is given by,

$$\hat{f} = H^{-1} g \quad \text{— (1)}$$

Where H is a square Matrix

Substitute $H = WDW^{-1}$ in above equation.

$$\hat{f} = (WDW^{-1})^{-1} g = (WD^{-1}W^{-1}) g. \quad (2)$$

D - Density

W - Matrix

Multiply both sides by W^{-1} which gives

$$W^{-1}\hat{f} = D^{-1} W^{-1} g.$$

W.K.T

$$H(u,v) = \frac{G(u,v)}{\hat{F}(u,v)}$$

It can be represented as

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}.$$

for $u,v = 0,1,2 \ldots N-1$.

In this equ, $H(u,v)$ is considered as a filter function that multiplies $F(u,v)$ to produce the transform of the degraded image $g(x,y)$.

Divide $G(u,v)$ by $H(u,v)$ which gives an Inverse filtering operation.

The restored image is obtained by

$$\hat{f}(x,y) = F^{-1}\{\hat{F}(u,v)\}$$

$$= F^{-1}\{G(u,v)/H(u,v)\}$$

$$= g(x,y)/h(x,y).$$

for $y = 0,1 \ldots N-1$

$x = 0,1, \ldots N-1$.

* The computational difficulties may arise in the restoration process, if $H(u,v)$ becomes very small in any region of interest in the uv plane.

* So, the small values of $H(u,v)$ can be neglected in the computation of $F(u,v)$ without affecting the restored image.

* In some cases difficulty arises in the presence of Noise.

* For eg: Consider the degradation model with noise as given by

$$\boxed{G(u,v) = H(u,v) \cdot F(u,v) + N(u,v)}$$

Divide by $H(u,v)$ we get

$$\boxed{\frac{G(u,v)}{H(u,v)} = \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}}$$

*. This eqn clearly indicates that if $H(u,v)$ is small, the term $N(u,v)/H(u,v)$ dominates the restoration results.

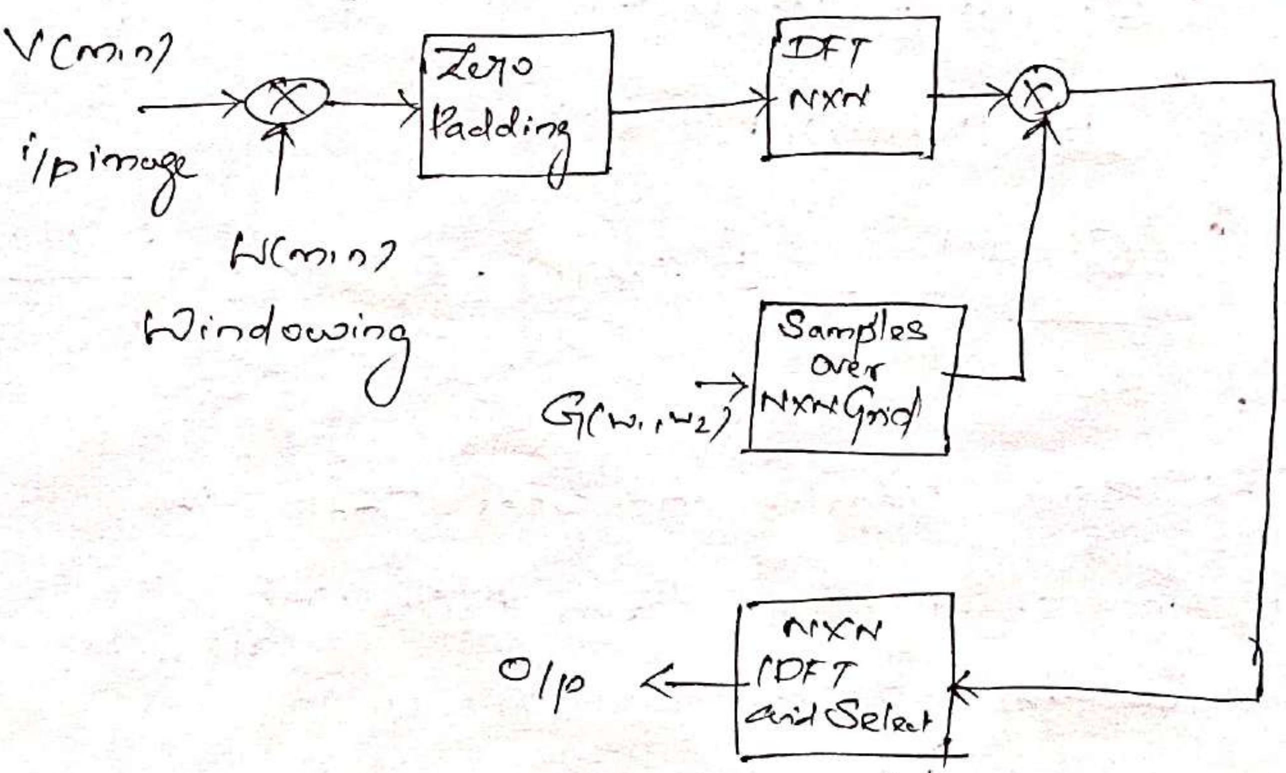
LEAST MEAN SQUARE FILTER (OR) WIENER FILTER (OR) UNCONSTRAINT RESTORATION.

→ Wiener filtering is a Method of restoring images in the presence of blur and Noise.

→ Let $u(m,n)$ and $v(m,n)$ be arbitrary zero mean random sequences.

→ To obtain an estimate $u(m,n)$ from $v(m,n)$ such that the mean square error is minimized.

$$\sigma_e^2 = E\left\{\left[u(m,n) - v(m,n)\right]^2\right\}$$



*. Let R_f and R_n be the Correlation matrixes of the image $f(x,y)$ and the noise term $n(x,y)$ defined respectively by

$$\left.\begin{aligned} R_f &= E\{f f^{T}\} \text{ and} \\ R_n &= E\{n n^{T}\} \end{aligned}\right\} - (1)$$

Where $E\{.\}$ denotes the expected Value.

*. The ij^{th} element of R_f is given by $E\{f_i, f_j\}$ Which is Correlated between the i^{th} and j^{th} element of f.

* Similarly ij^{th} element of R_n gives the Correlation between i^{th} and j^{th} element of n. The elements f and n are real so that

$$E\{f_i, f_j\} = E\{f_j f_i\} \text{ and}$$

$$E\{n_i n_j\} = E\{n_j n_i\}$$

and it follows that R_f and R_n are real and Symmetric Matrixes.

* R_f and R_n Can be approximately to block Circulant Matrices and therefore, It Can be diagnalised by the Matrix W.

* The diagnalised Matrices are denoted as A and B. then We Can Write

$$\left.\begin{aligned} R_f &= WAW^{-1} \\ R_n &= WBW^{-1} \end{aligned}\right\} \quad - (2)$$

* Where the elements A and B are the transforms of the Correlation elements in R_f and R_n respectively.

* The F.T of R_f and R_n is Called Power Spectrum of $f_e(x,y)$ & $n_e(x,y)$ are denoted as $S_f(u,v)$ and $S_n(u,v)$ defining

$$Q^T Q = R_f^{-1} R_n \quad - (3)$$

* The Block Circulant Matrix H Can be diagnolised using the equn

$$D = WHW^{-1} \quad - (4)$$

The Constrained Restoration function is given by

$$\hat{f} = (H^T H + \gamma Q^T Q)^{-1} H^T g. \quad - (5)$$

Substitute $Q^T Q = R_f^{-1} R_n$ in equ (5)

$$\hat{f} = (H^T H + \gamma R_f^{-1} R_n)^{-1} H^T g.$$

W.K.T

$$\left.\begin{aligned} H^T &= W D^* W^{-1} \\ H &= W D W^{-1} \end{aligned}\right\} - (6)$$

Subs equ (6), (2) in equ (5)

$$\hat{f} = (W D^* W^{-1} D + \gamma W A^{-1} B W^{-1})^{-1} W D^* W^{-1} g.$$

Multiplying both sides by W^{-1} and Performing Some Matrix Manipulation reduces the overall terms.

$$\boxed{W^{-1} \hat{f} = (D^* D + \gamma A^{-1} B)^{-1} D^* W^{-1} g.} \quad - (7)$$

$W^{-1} \hat{f}$ is the F.T of $\hat{f}$ and is denoted as $\hat{F}(u,v)$.

$$\therefore W^{-1} g = G(u,v).$$

$\therefore$ equ (7) becomes

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)^2| + \gamma[S_n(u,v)/S_f(u,v)]} \right] G(u,v)$$

$$\boxed{\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\right]\left[\frac{|H(u,v)|^2}{|H(u,v)^2| + \gamma[S_n(u,v)/S_f(u,v)]}\right] G(u,v)} \quad \text{--- (8)}$$

Where $u,v = 0,1,2 \ldots N-1$. $\quad \because |H(u,v)|^2 = H^*(u,v)H(u,v)$

$M = N$.

When $\gamma = 1 \Rightarrow$ Wiener filter; γ = variable $\Rightarrow$ Parametric Wiener filter.

If $S_n(u,v) = 0$ (noise) $\Rightarrow$ ideal Inverse filter.

When $S_n(u,v)$ and $S_f(u,v)$ are Unknown the equ (8) Can be approximated as

$$\boxed{\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\right]\left[\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)}$$

Where K - Constant

Geometric Transformation

Geometric transformation is usually applied to modify the spatial relationship between the Pixels and it is used to restore the image. It is also known as "Rubber Sheet transformations".

Because, this process is Similar to Printing an image in a rubber Sheet and Stretching the Sheet according to Predefined Set of rules.

Two basic operations in a geometric transformation are given below

1. Spatial x^{mation} - It describes rearrangement Procedures for Pixels.
2. Gray level Interpolation: It deals with assiging Gray levels to Pixels in the Spatially transformed Image.

Spatial Transformations:

$f(x,y)$ is an image with Coordinate (x,y)

$g(\hat{x},\hat{y})$ is a distorted image: An image is distorted (image) by the transformations given below.

$$\left.\begin{aligned} \hat{x} &= r(x,y) \\ \hat{y} &= s(x,y) \end{aligned}\right\} \text{Spatial transformations.}$$

Let

$$r(x,y) = x/4$$

$$s(x,y) = y/4.$$

* That means, the Size of the image is reduced by 4 times in the Spatial directions.

* When these transformations are Applied in the reverse direction, the Original image Can be obtained from $g(\hat{x},\hat{y})$.

*. Practically, it is not Possible to formulate analytically a single Set of functions $r(x,y)$ and $s(x,y)$ which describe the geometric distortion Process over the Entire image Plane.

∴ To overcome this problem, a subset of pixels whose locations in the distorted image and corrected image are known precisely. These are known as tie points.

$$r(x,y) = k_1 x + k_2 y + k_3 xy + k_4 \quad — (1)$$

$$s(x,y) = k_5 x + k_6 y + k_7 xy + k_8 \quad — (2)$$

and

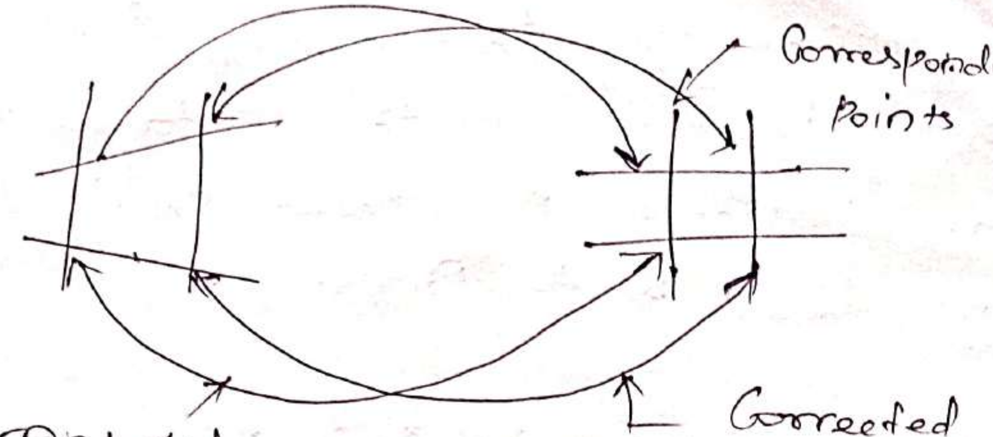
$$\hat{x} = k_1 x + k_2 y + k_3 xy + k_4 \quad — (3)$$

$$\hat{y} = k_5 x + k_6 y + k_7 xy + k_8 \quad — (4)$$

Here, totally 8 tie points are available in quadrilateral and these tie points are used to solve 8 unknown co-efficients $k_1, k_2 \ldots k_8$.

If we want to know $f(0,0)$ in the corrected image, then put $(x,y) = (0,0)$ in equ (3) and (4)

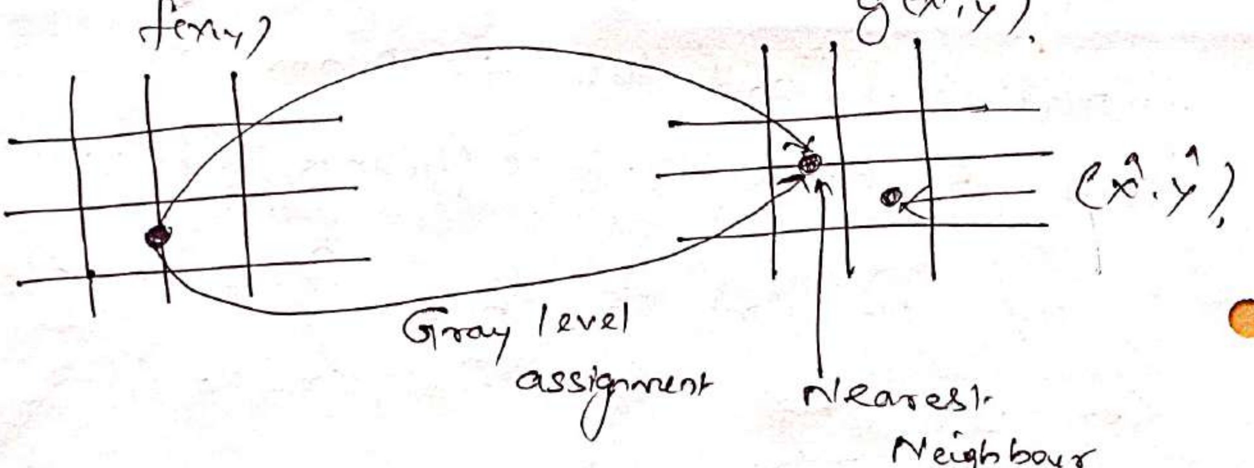
The pixel $(\hat{x}, \hat{y})$ in the distorted image is equivalent to the point $(0,0)$ in the corrected image. The same procedure is repeated for various values.



Gray level Interpolation:

A Procedure in which the nearest neighbour Pixels grey levels is used to assign a Pixel whose Co-ordinates is not confined to existing grid locations.

— fixed



The figure shows that mapping of integer Coordinate (x,y) into Coordinate $(\hat{x},\hat{y})$. The nearest integer is Selected and gray level of this nearest neighbour Pixel is assigned to the Pixel (x,y).

CONSTRAINED LEAST MEAN SQUARE FILTERING:

→ The least Square approach is a statistical approach.

→ In this the restoration Produce to be developed requires Knowledge about Noise mean and Variance.

→ $\hat{f} = (H^T H + \gamma Q^T Q)^{-1} H^T g$, In this equn Q only decides the restoration quality

→ By using the definition of Convolution we Can express the vector matrix is given by

$$\boxed{g = Hf + \eta}$$

$g(x,y) = M \times N$.

First we Can form - ~~reg~~ N element.

→ The Smoothness Criterion function is given by

$$\boxed{C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y)\right]^2}$$

Subject to the Constraint

$$\|g - H\hat{f}\|^2 = \|\eta\|^2$$

The C Matrix can be diagonalized by the Matrix. W. and it is defined as

$$E = W^{-1} C W$$

Then the smoothness criterion function takes the form minimize $\{f^T C^T C f\}$

Let $Q = C$

$$\|Qf\|^2 = (Qf)^T (Qf) = f^T Q^T Q f$$

Then the eqn is minimize $\|Qf\|^2$

In the basic eqn for Constrained restoration Substitute $Q = C$, we get

$$\hat{f} = (H^T H + \gamma C^T C) H^T g. \quad \text{--- } (1)$$

W.K.T

$$\left.\begin{aligned} H &= WDW^{-1} \\ H^T &= WD^* W^{-1} \\ C &= W^{-1} E W \end{aligned}\right\} \quad \text{--- } (2)$$

$\therefore \hat{f} = (H^T H + \gamma C^T C) H^T g$ can be Changed into

$$\therefore \hat{f} = (WD^*DW^{-1} + \gamma WE^*EW^{-1})\, WD^*W^{-1}g.$$

Multiply both sides by W^{-1}

$$W^{-1}\hat{f} = (D^*D + \gamma E^*E)\, D^*W^{-1}g.$$

Elements inside the Parentheses are diagonal,

So

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma\, |P(u,v)|^2}\right] G(u,v) \quad (3)$$

→ The value γ is to be adjusted to satisfy the constraint $\|g - H\hat{f}\|^2 = \|n\|^2$

→ $P(u,v)$ is the F.T of the Function $p(x,y)$

$$\therefore p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

→ An iterative Procedure to estimate the Parameter γ is as follows

The residual vector γ is defined as

$$\boxed{r = g - H\hat{f}} \quad — \quad \text{(A)}$$

Subs $\hat{f}$ value in above equn

$$\gamma = g - H(H^T H + \gamma C^T C)^{-1} H^T g.$$

→ This equn gives that γ is a function of γ.

$$\therefore \phi(\gamma) = \cancel{\gamma}\, r^T r = \|r\|^2.$$

→ Now to adjust γ such that $\boxed{\|\gamma\|^2 = \|n\|^2 \pm a}$

where 'a' is a accuracy factor. — (4)

*. If $\|\gamma\|^2 = \|n\|^2$ the constraint $\|g - H\hat{f}\|^2$ will be satisfied.

The simple iterative procedure steps can be given as

(1) Specify an initial value of γ.

(2) Compute $\hat{f}$ and $\|\gamma\|^2$ and

(3) Stop if equ (4) is satified. otherwise return to step (2) after ↑ing γ if $\|\gamma\|^2 < \|n\|^2 - a$ ~~and~~ or by ↓ing γ if $\|\gamma\|^2 < \|n\|^2 + a$.

Newton - Raphson Algorithm

This is the another Procedure method used to improve the speed of Convergence.

In order to use this algorithm, we need the quantities $\|r\|^2$ and $\|\eta\|^2$.

To Compute $\|r\|^2$ we note from equ (A)

$$R(u,v) = G(u,v) - H(u,v)\hat{F}(u,v).$$

Then

$$\|r\|^2 = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} r^2(x,y)$$

by $F.T^{-1}$.

To find $\|\eta\|^2$ we can use Sample avg. Method

$$\sigma_\eta^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\eta(x,y) - m_\eta\right]^2$$

Where $m_\eta = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \eta(x,y)$ is the Simple mean.

Finally $\boxed{\|\eta\|^2 = MN\left[\sigma_\eta^2 + m_\eta^2\right]}$

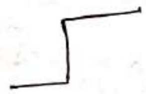
UNIT - IV

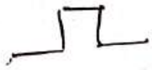
Image Segmentation:

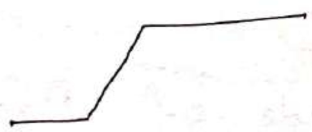
Edge detection:

- Used for meaningful discontinuities in gray level.
- First and Second Order digital derivatives are implemented to detect the Edges in an image.

Types of Edges:

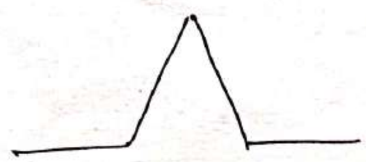
(1). Step Edge. → 

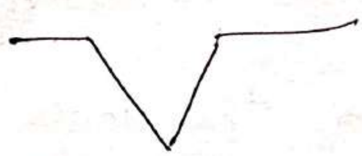
(2) Line Edge. → 

(3) Ramp Edge → 

(4) Roof Edge.

(i) Convex roof Edge (ii) Concave





Purpose of Edge detection:

(1) To Identify areas of an image where a large change in intensity occurs.

The image Gradient and its properties:

The tool of choice for finding edge strength and direction at location (x,y) of an image f is the gradient denoted by ∇f, and defined as the vector

$$\nabla f = \text{grad}(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \quad \text{—(1)}$$

The magnitude of vector ∇f, denoted as $M(x,y)$, where

$$M(x,y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2} \quad \text{— (2)}$$

The direction of gradient vector is given by the angle:

$$\boxed{\alpha(x,y) = \tan^{-1}\left[\frac{g_y}{g_x}\right]} \quad \text{— (3)}$$

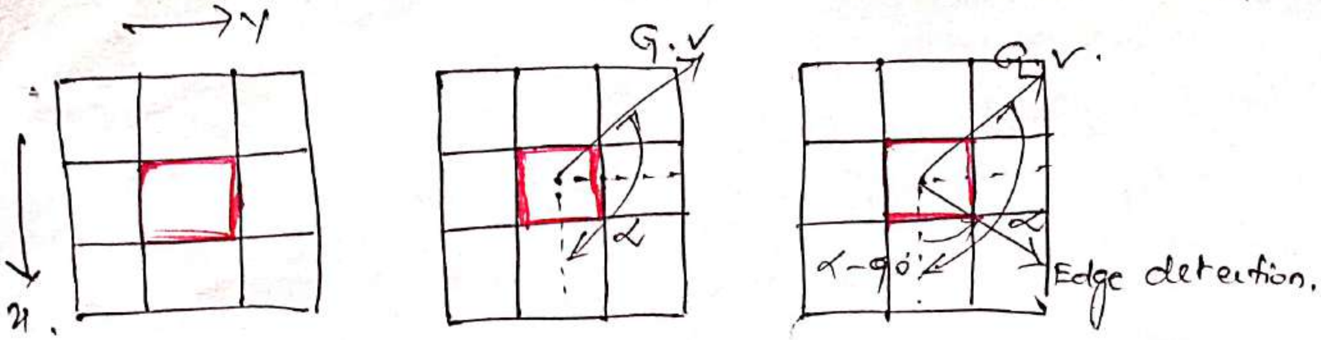


fig: Edge is ⊥

Roberts mask =>

-1	0
0	1

0	-1
1	0

Prewitt mask =>

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Edge linking.

Practically the set of pixels detected by the gradient operators will not form a complete boundary due to noise, Non-uniform illumination. Following techniques are used for Edge linking and boundary detection.

(1) Local Processing
(2) Regional Processing
(3) Global Processing using Hough X^m.

Local Processing.

→ Edge pixels are determined by using the gradient operators.

→ The boundary is not completely defined by the Edge pixels. Small gaps are there.

→ To fill those gaps, we consider the characteristics of pixels in a small neighbourhood.

→ The neighbourhood pixels similar to the boundary pixels are linked.

→ 2 Properties are used to check the similarity of the neighbourhood pixels with respect to the edge pixels.

(1) The strength of the gradient operator response to produce the edge pixel.

The direction of gradient is

$$|\nabla f(x,y)| - |\nabla f(x_1,y_1)| < T$$

(x,y) is edge pixel.

(x_1,y_1) - Neighbourhood pixel.

If the neighbourhood pixel is similar to the edge pixel, then

T = Non-Negative threshold value.

(2) The neighbourhood pixel with respect to the edge pixel has an angle similar to the edge pixel

if

$$|\alpha(x,y) - \alpha(x_1,y_1)| < \theta$$

$$\alpha(x,y) = \tan^{-1}\frac{G_y}{G_x}$$

θ = Angle of threshold.

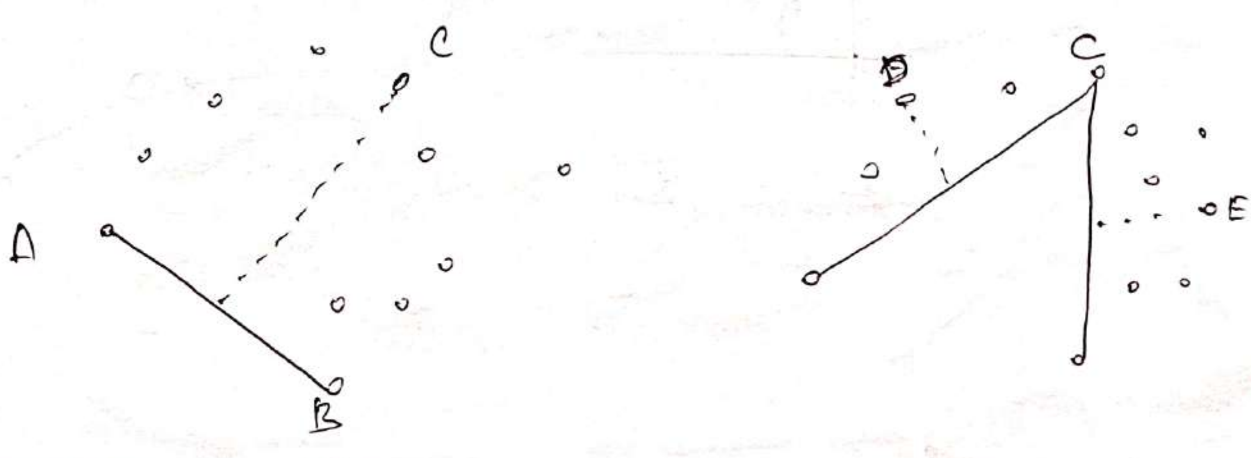
If the above eqn's are Satisfied then the neighbourhood Pixel is linked to the Edge Pixel. This Procedure is repeated for every edge Pixel location.

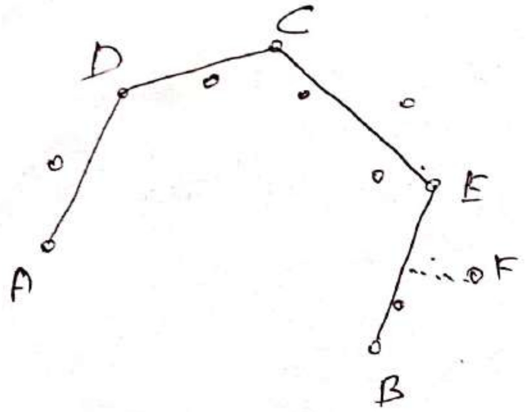
Regional Processing:

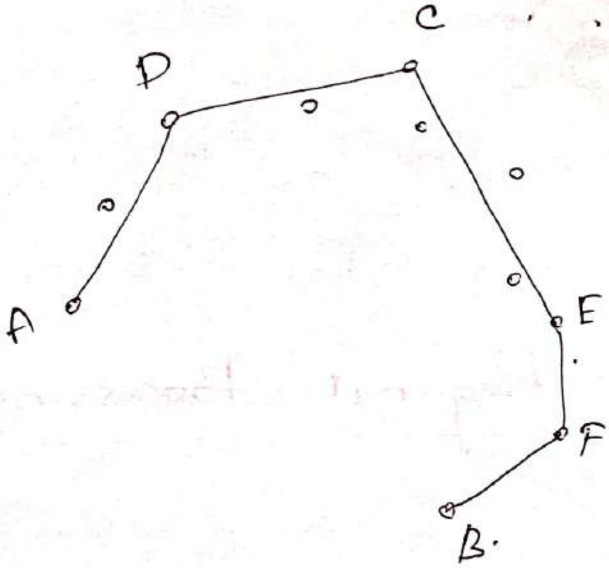
often, the location of regions of Interest in an image are Known or Can be determined.

This implies that Knowledge is available regarding the regional membership of Pixel in the Corresponding edge image.

Polygonal approximations are Particularly attractive because they Capture the essential Shape features of a region while Keeping the representation of boundary relatively simple.



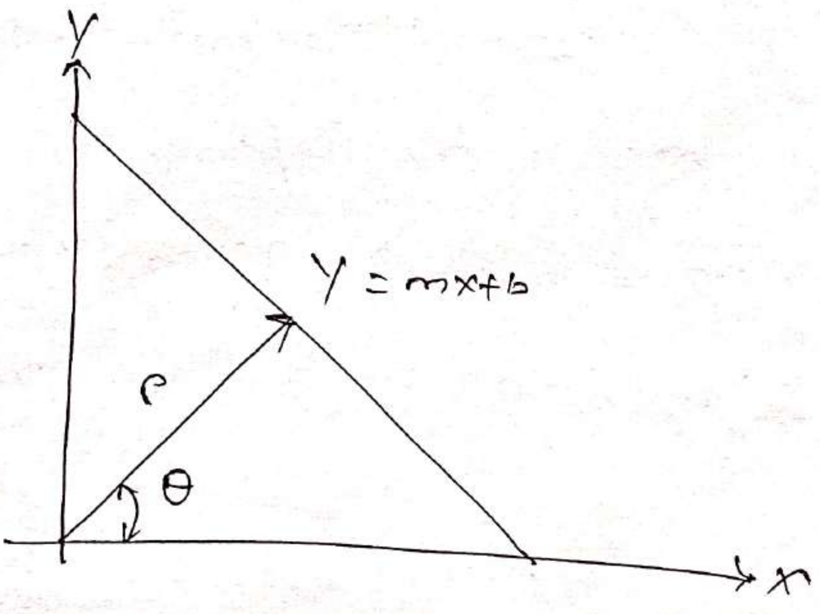




Hough x^m

The straight line $y = mx + b$ can be expressed in polar coordinate as

$$\rho = x\cos(\theta) + y\sin(\theta)$$



Straight Line

* Where ρ, θ defines a vector from the origin to the nearest point on the straight line.

$y = mx + b$.

* This vector will be perpendicular from the origin to the nearest point to the line.

* Any line in the x, y plane corresponds to a point in the 2D space defined by the parameter ρ and θ.

* Thus the hough x^m of a straight line in the x, y plane is a single point in the ρ, θ space.

* Every straight line that passes through a particular point x_i, y_i in the x, y plane plots to a point in the ρ, θ space and these points should satisfy with x_1, y_1 as constants.

Suppose we have a set of edge points x_i, y_i that lie along a straight line having parameters ρ_0, θ_0. Each edge point plots to a sinusoidal curve in the ρ, θ space, but those curves must intersect at a point ρ_0, θ_0 since this is a line they all have common.

Thresholding:

- This is an important technique used for image segmentation.
- We can consider the histogram of an image ($f(x,y)$) which consists of light objects on a dark background.

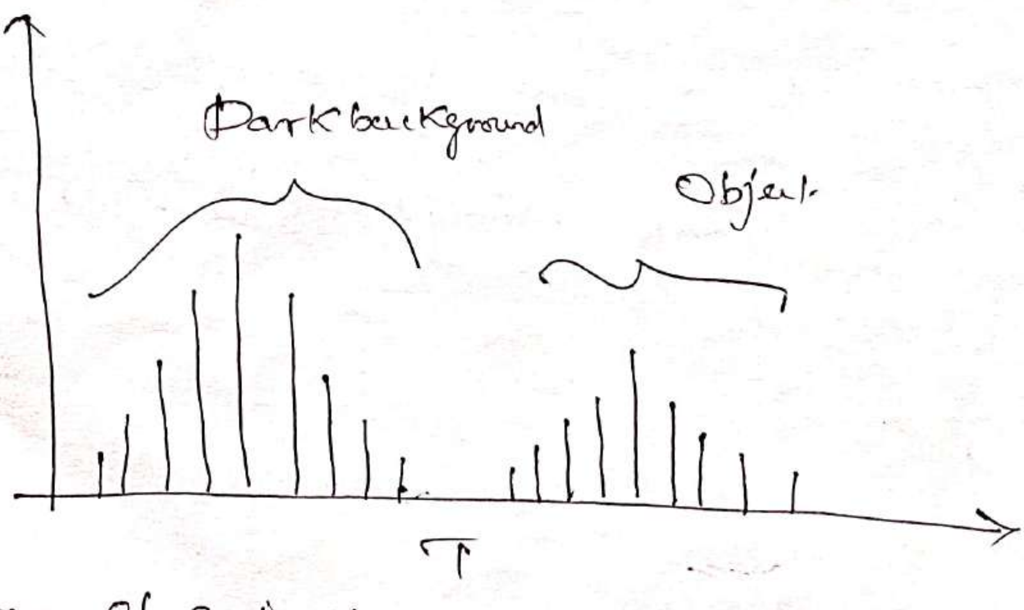


Fig: Histogram of an image.

This histogram consists of two dominant regions, one for the dark background and another for the object.

'T' is used to represent the threshold value. It separates object and background region.

- If $f(x,y) > T$ - Point is known as Object Point.

  Otherwise - Background Point.

- We can see the histogram which correspond to the 2 different light objects on a dark background.

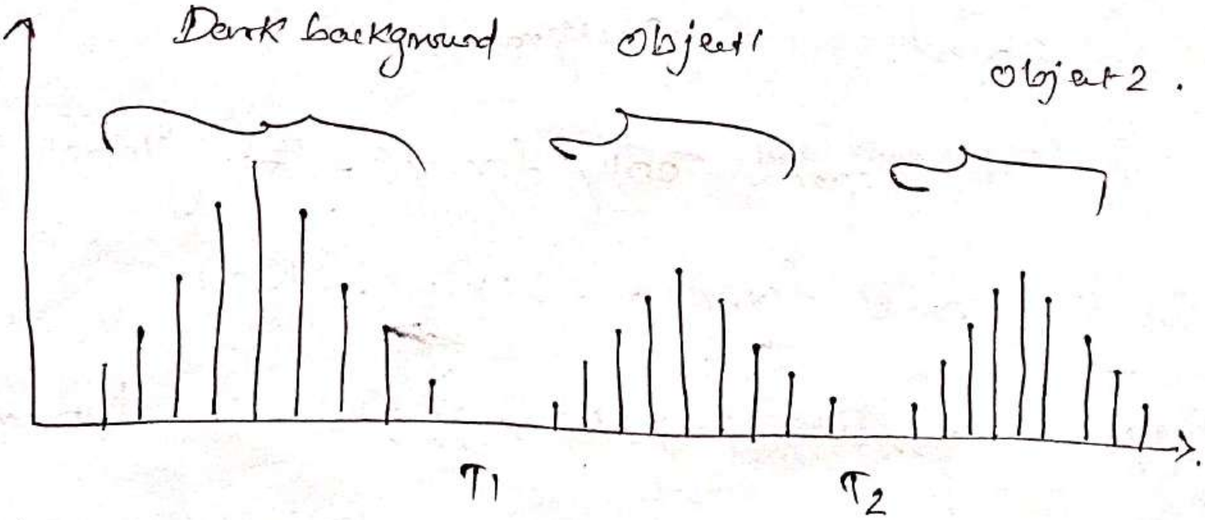


Fig: Histogram for two objects on a dark background (Multilevel Thresholding)

If $T_1 < f(x,y) \leq T_2$ then the Point (x,y). Correspond to object 1

If $f(x,y) > T_2$ - Object 2.

$f(x,y) \leq T_1$ - Background.

Then the associated Parameters of T are given below

$f(x,y)$ - gray level at any Point (x,y)

$p(x,y)$ - local Property at any Point.

$\therefore$ The threshold image $g(x,y)$ is given as

$$g(x,y) = \begin{cases} 1 & \text{for } f(x,y) > T \\ 0 & \text{for } f(x,y) \leq T \end{cases}$$

1 - Object, 0 - Background.

If T depends only $f(x,y)$ - The global thresholding.

If T depends $f(x,y)$ and $p(x,y)$ - local

If T depends (x,y); $p(x,y)$; $f(x,y)$ - dynamic

Optimal Thresholding:

If an image has 2 main brightness regions, then

$P(z)$ = P.D.F of the gray level in an image.

Overall $P(z)$ = Sum of densities.

= density of (light + Dark region)

$$P(z) = P_1 \times P_1(z) + P_2\, P_2(z)$$

P_1, P_2 - Probabilities of 2 gray levels.

$$P(z) = P_1 \cdot \frac{1}{\sqrt{2}\,\pi\,\sigma_1} e^{-\left[\frac{-(z-m_1)^2}{2\sigma_1^2}\right]} + P_2 \frac{1}{\sqrt{2}\,\pi\,\sigma_2} e^{-\left[\frac{-(z-m_2)^2}{2\sigma_2^2}\right]}$$

∴ Probability of error to classify object point as background point as $\int_{-\infty}^{T} P_2(z)\, dz$.

∴ " " " " background point as object point as

$$\int_{T}^{\infty} P_1(z)\, dz.$$

Over all Prob. error = $P_2 \cdot \int_{-\infty}^{T} P_2(z)\,dz + P_1 \int_{T}^{\infty} P_1(z)\,dz$.

If $P_1 = P_2$ then the n^{th} optimal threshold is the mean average. It is given as

$$\boxed{T = \frac{m_1 + m_2}{2}}$$

T - optimal threshold.

Region Based Segmentation:-

The Process of dividing an image into smaller regions is based upon some Predetermined rule that how to group the Pixels into one logical region.

Eg: All Pixels within the region must be exactly the same graylevel.

Basic rules for Segmentation:

R = Entire image region

R is Subdivided into 'n' no. of Subregions.

$R_1, R_2, R_3 \cdots R_n$ = Subregions.

(i) $\bigcup_{i=1}^{n} R_i = R$. Segmentation Process is Complete.

(ii) R_i is a Connected region for $i = 1, 2, \ldots n$.

(iii) $R_i \cap R_k = \phi$ for all i and k, where $i \neq k$.

It shows that the region must be disjoint

(iv) $P(R_i)$ = True for $i = 1, 2, \ldots n$. It shows that all the Pixels in the region R_i have the same intensity.

(v) $P(R_i \cup R_k)$ = False for $i \neq k$. It shows that the Regions R_i and R_k are different in the Sense of the Predicate P.

Region Growing:

- It is a Procedure that Groups Pixels or Subregions into larger regions based on Predefined criteria for growth.

- The basic approach is to start with a set of "Seed" points and from these grow regions by appending to each seed those neighbouring pixels that have Predefined properties similar to the seed.

R_1	R_1	R_2	R_2	R_2
R_1	R_1	R_2	R_2	R_2
R_1	R_1	R_2	R_2	R_2
R_1	R_1	R_2	R_2	R_2
R_1	R_1	R_2	R_2	R_2

Region 1 Region 2.

Region Splitting and Merging:

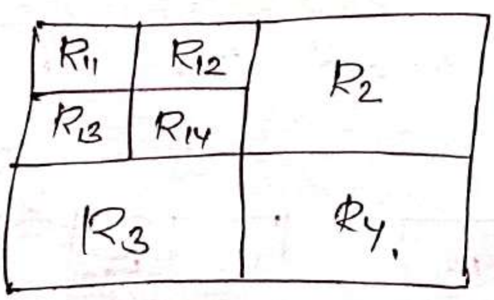
In this technique, an image is divided into various subimages of disjoint regions and then merge the connected regions together.

R: Entire region of Image.

→ The Predicate $P(R_i)$ is used to check the conditions. In any region $P(R_i)$ = True.

→ An Image is Subdivided into Various Sub images. If $P(R_i)$ = False, Then, divide the image into quadrants.

→ If $P(R_i)$ = False. then the further divide the quadrant into sub-quadrant.



Region R_1 is divided into quartrants (quadrants).
It is shown by using quad tree.

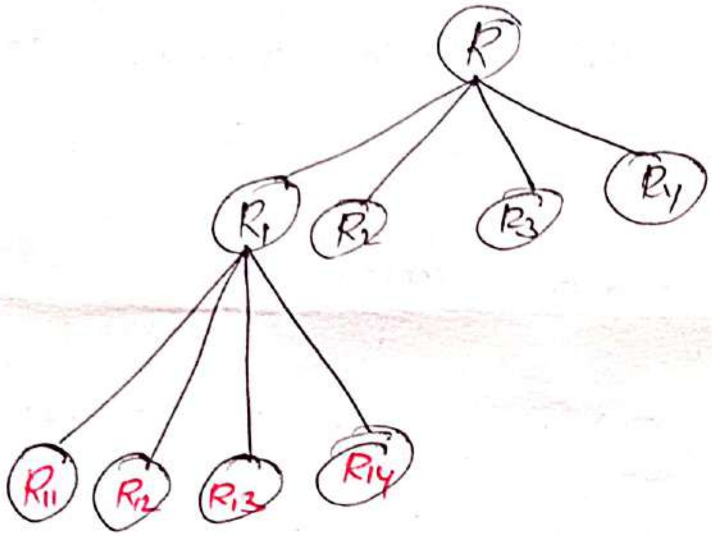
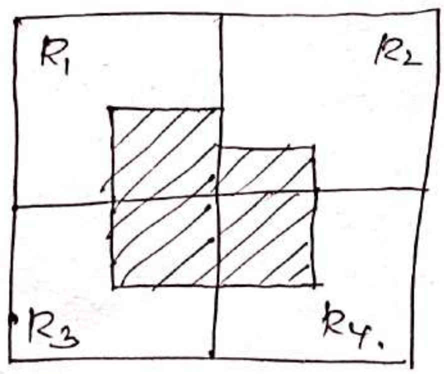
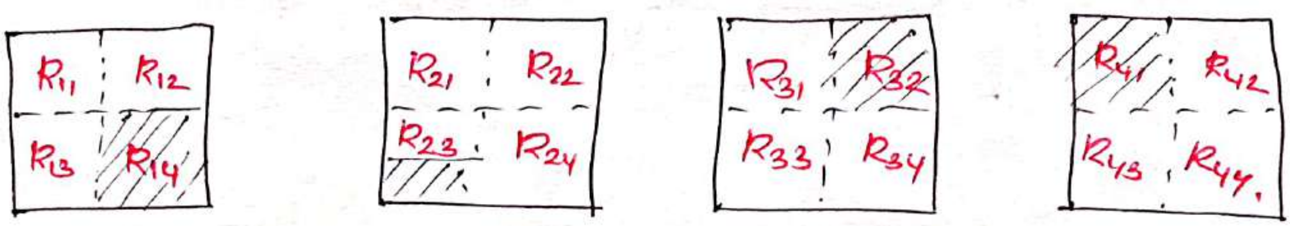


Fig: Quad tree representation.

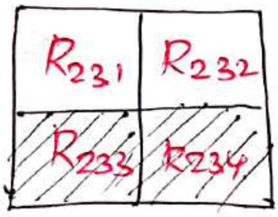
Eg: for split and Merge algorithm is shown below



Initally an image is divided into four regions $R_1, R_2 .. R_4$.



Here in the region R_2, R_{23} can be further subdivided shown below



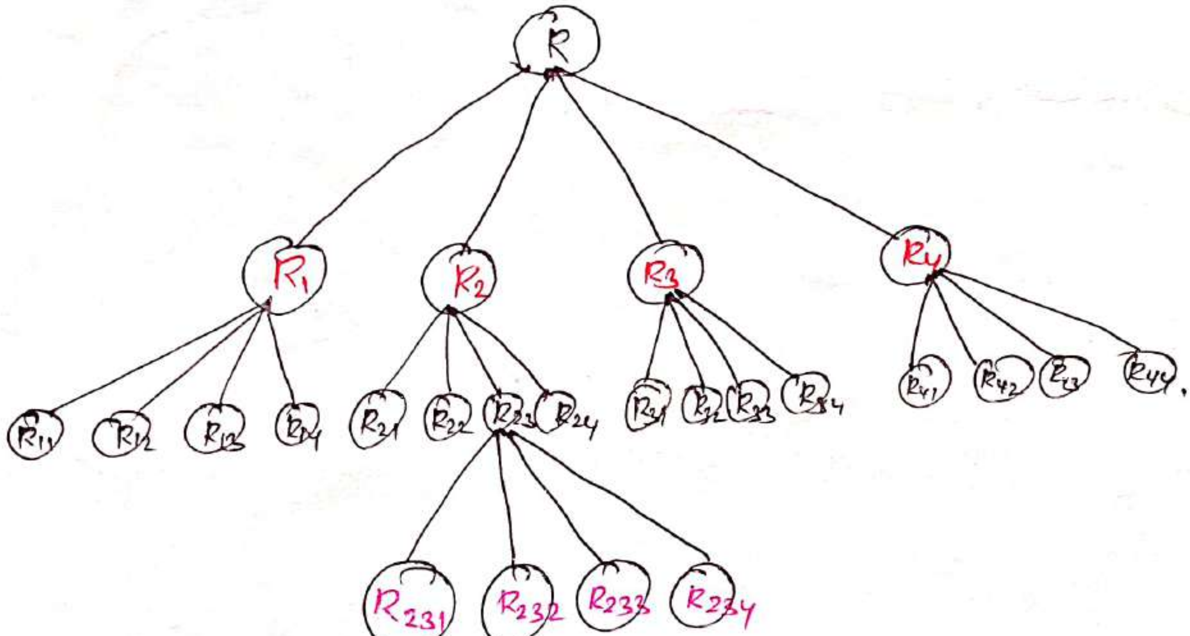


Fig: Quad tree representation.

The Process of splitting an image into small regions and then merging connected regions together is known as region Segmentation by splitting & Merging.

UNIT-V

Image Compression:

Data Compression:

- It refers to the process of reducing the amount of data required to represent a given quantity of information.

Compression Ratio:

$$\boxed{C_{Ratio} = \frac{n_1}{n_2}}$$

n_1, n_2 : No. of words / units used to represent the same information.

Data Redundancy:

$$\boxed{R_{data} = 1 - \frac{1}{C_{Ratio}}}$$

Categories

Information Preserving

Lossy Compression:

Information Preserving:

- Used in image archiving Eg. medical records. It Provides image without any loss in Information.

Lossy Compression:

It Provides higher level of data reduction. But does not give Perfect reproduction.

Appln

1. Televideo conferencing
2. Remote Sensing
3. Medical Imaging
4. FAX (Facsimile Transmission)
5. Military Applns.
6. Space App
7. Broad Cast TV.

Huffman Coding:

- It is framed by Huffman in 1952.
- It gives smallest possible no. of code symbols per source symbol.

Steps:

Step: 1

Create a series of source reduction by ordering the probabilities of symbols under consideration and combine lowest probability symbol that replaces them in next source reduction.

Source		Source Reduction			
Symbol	Probability	I	II	III	IV
b_2	0.4	0.4	0.4	0.4	0.6
b_1	0.3	0.3	0.3	0.3	0.4
b_3	0.1	0.1	0.2	0.3	
b_4	0.1	0.1	0.1		
b_5	0.06	0.1			
b_6	0.04				

First source reduction is formed by combining.

0.06 & 0.04

0.06+0.04 = 0.1.

Step: 2. Code Assignment Procedure:

Source			Source Reduction.							
Symbol	Prob.	Code	I		II		III		IV	
b_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
b_1	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
b_3	0 1	011	0.1	011	0.2	010	0.3	01		
b_4	0 1	0100	0.1	0100	0.1	011				
b_6	0.06	01010	0.1	0101						
b_5	0.04	01011								

In column IV

Assign 0 to 0.6

1 to 0.4

In Column II

0.4 — 1

0.3 — 00

0.2 — 01 0

0.1 — 01 1 } Appending 0 and 1

L_{avg} = Probability x No. of codes.

$= (0.4 \times 1) + (0.3 \times 2) + (0.1 \times 3) + (0.1 \times 4) +$
$(0.06 \times 5) + (0.04 \times 5)$

$L_{avg} = 2.2$ bit / symbol.

$$\text{Efficiency} = \frac{\text{Entropy}}{\text{Efficiency}}$$

$$= \frac{2.14}{2.2}$$

$$\boxed{\text{Efficiency} = 0.972727}$$

Arithmetic Coding:

* The Entire Sequence of Source Symbols is assigned to a Single arithmetic code Word.

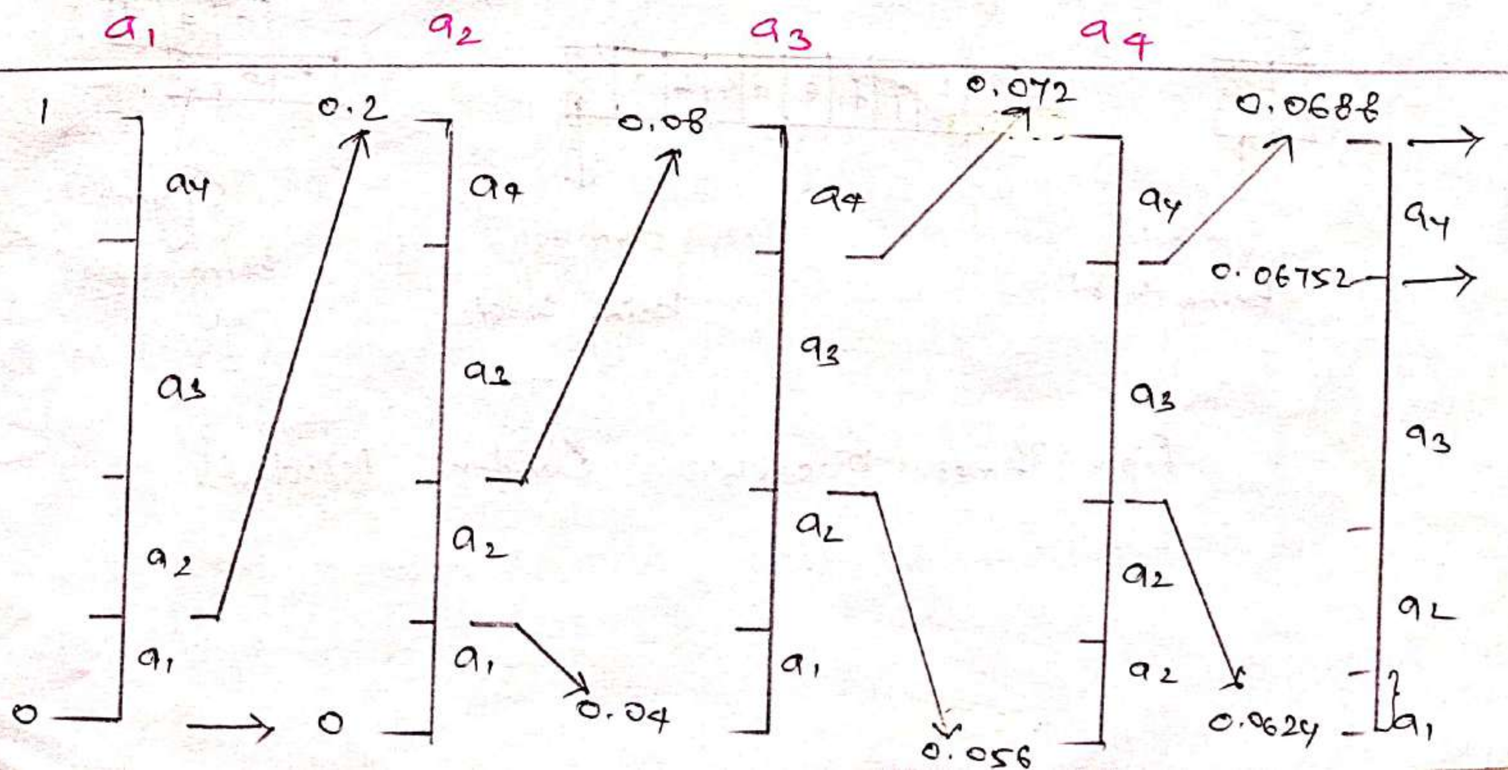
* The Code Word itself defines an interval of real no.'s between 0 and 1.

* Initially, message is assumed to occupy the interval [0,1].

* For Eg. We can consider the code sequence b_1, b_2, b_3, b_4. Using arithmetic code.

Source Symbol	Probability	Initial Sub-Interval
a_1	0.2	[0, 0.2]
a_2	0.2	[0.2, 0.4]
a_3	0.4	[0.4, 0.8]
a_4	0.2	[0.8, 1.0]

Encoding Sequence: →



I Sub Interval $(0, 0.2) = \frac{0.2}{5} = 0.04$

II $\rightarrow \frac{0.08 - 0.04}{5} = 0.008$

III $\rightarrow \frac{0.072 - 0.056}{5} = 0.0032$

III $- \frac{0.0688 - 0.0624}{5} = 0.00128$

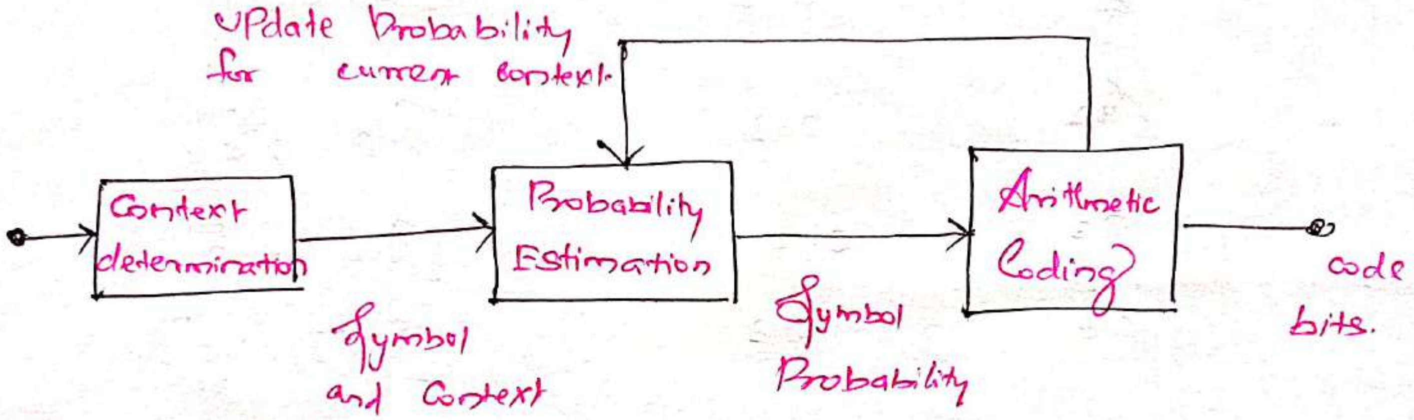


Fig: An Adaptive based arithmetic coding approach.

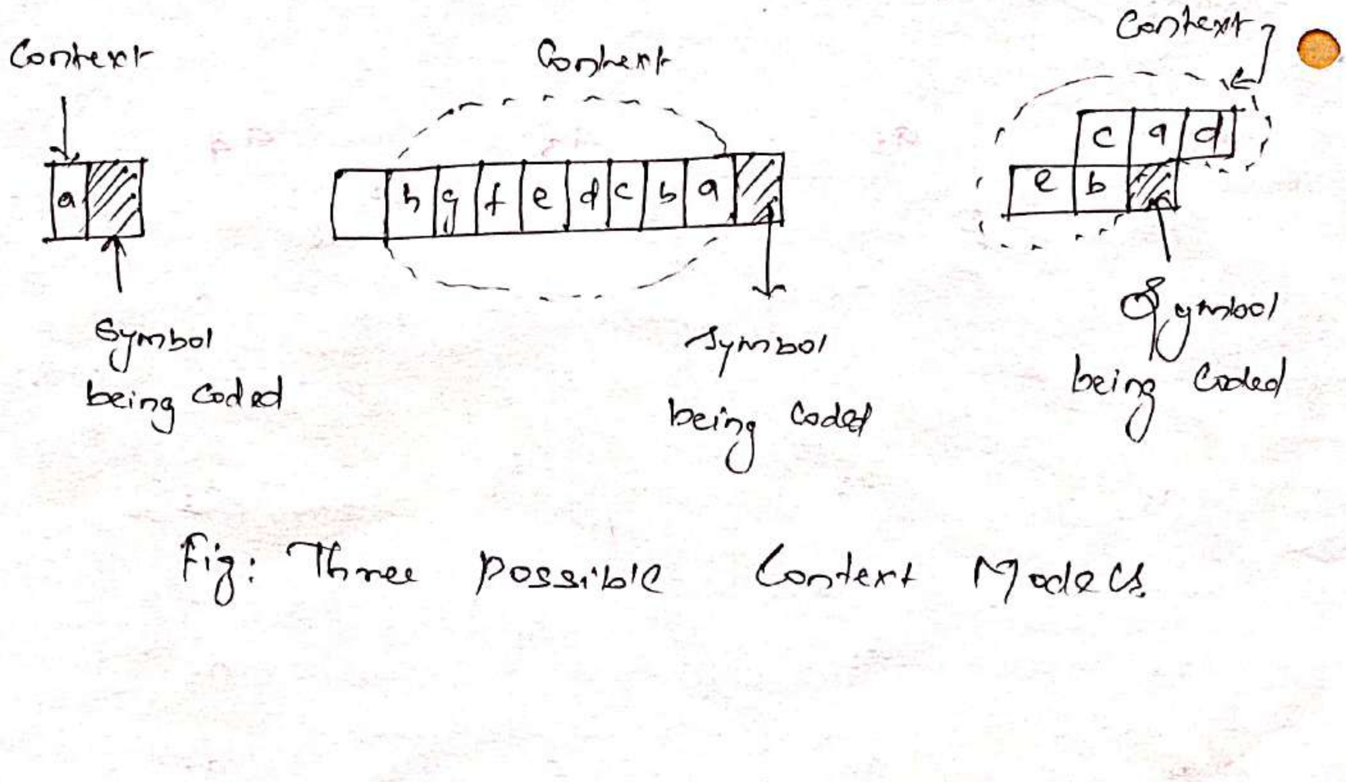


Fig: Three Possible Context Models.

Runlength coding:

- Runlength is coding is effective when long sequences of the same symbol occur.

- Runlength coding exploits the spatial redundancy by coding the no. of symbols in a run.

- The term run is used to indicate the repetition of a symbol, while the term run-length is used to represent the no. of repeated symbols.

Types:

+ 1- Dimensional Run length coding
+ 2- Dimensional Runlength coding

1-D Runlength coding:

- In 1-D. R.L.C Each scan line is encoded independently.

- Each scan line can be considered as a sequence of alternating, independent white runs and black runs.

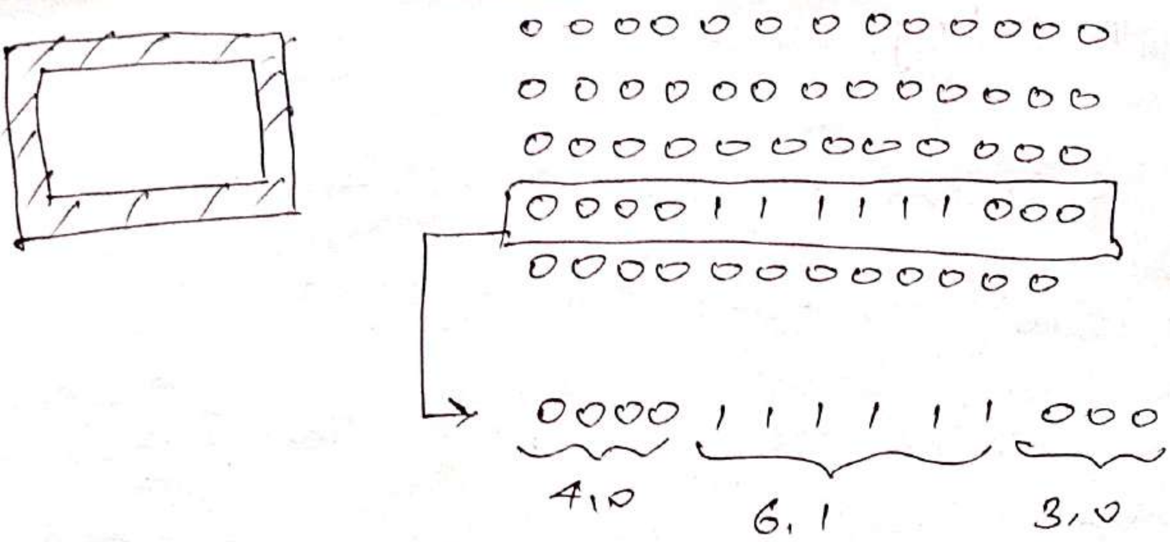


fig: Illustration of 1-D Runlength Coding.

2-D Runlength coding:

The 1-D R.L.C utilizes the Correlation between Pixels within a Scanline. In Order to utilise Correlation b/w Pixels in neighbouring Scanlines to achieve higher coding Efficiency, 2-D R.L.C was developed.

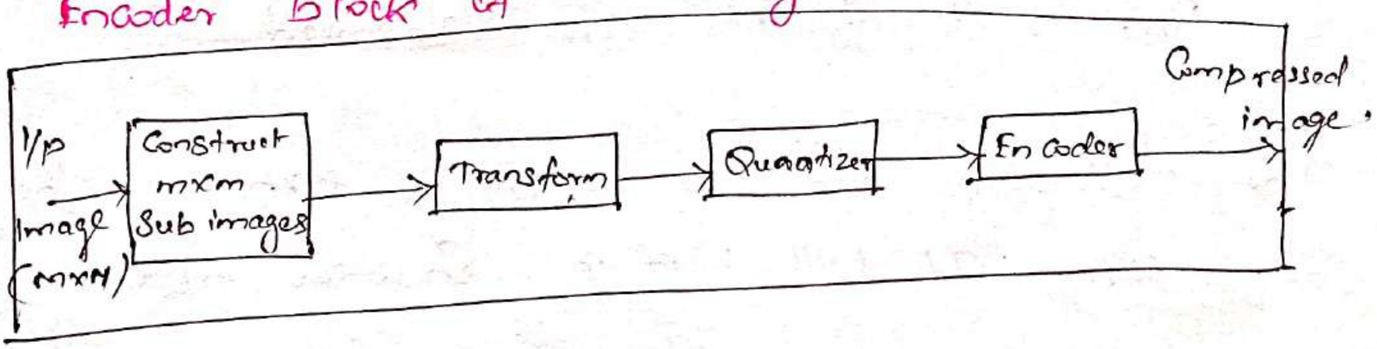
Transform Coding:

In this technique, an image is x^med using One of the image transform.

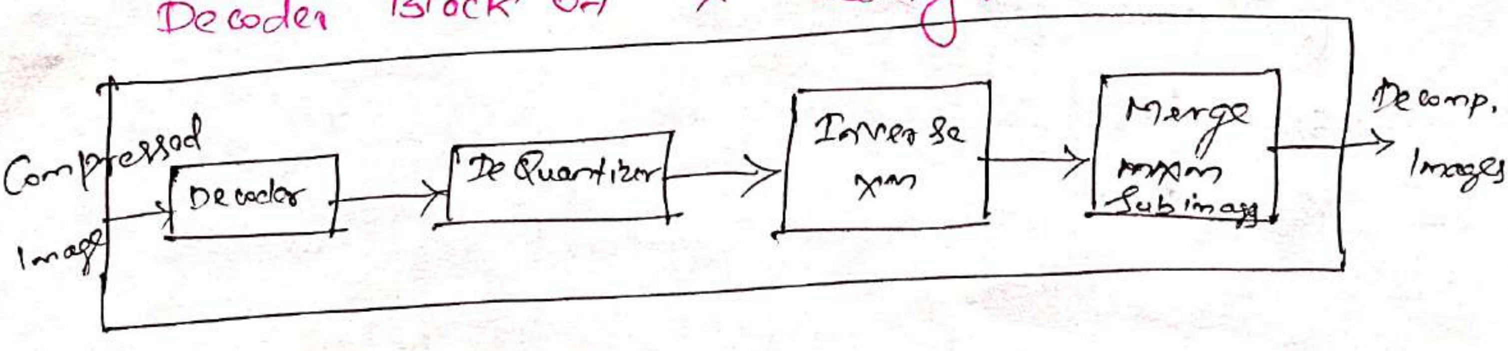
Here, the Concept is to Produce a new Set of Co-efficients that are de Correlated.

Then, the Set of Co-efficients in which more information Present in the image are Converted into a Small no. of Co efficients

Encoder block of x^n Coding



Decoder Block of x^n Coding.



I Sub Interval $(0, 0.2) = \frac{0.2}{5} = 0.04.$

II $\Rightarrow \frac{0.08 - 0.04}{5} = 0.008$

III

The Quantization block is used to eliminate the Co-efficients which carry least Information. These Omitted Co-efficients have the small impact on the Quality of the reconstructed Subimages.

Finally, Co-efficients are decoded.

The following x^{ms} are used to extract the Code. Hence It will used in Encoder block

(1) KLT

(2) DCT

(3) W-H-T

(4) DFT.

- At the Encoder, the i/p image is Partitioned into a Set of non-overlapping image blocks.

- The closest code word in the Code book is then found for each image block.

- Here, the closest Code Word for a given block is the one in the Code book that has the minimum Squared Euclidean distance from the i/p block.

- Next, the Corresponding index for each Searched Closest code word is xmitted to the decoder.

- When Encoder receives the index of the Code word, then the index is replaced with the associated Code word.

Before going into Vector Quantization, we should have knowledge about Vector Quantization of Two techniques.

1. Mapping technique (Grouping the Values)
2. Desiging a Code book (Mechanism of Mapping (or) Entries of Code words)

Vector Quantization: (VQ)

VQ is a block - coding technique that Quantises blocks of data instead of single sample.

In General a VQ Scheme Can be divided into two Parts.

1. Encoding Procedure.
2. Decoding Procedure.

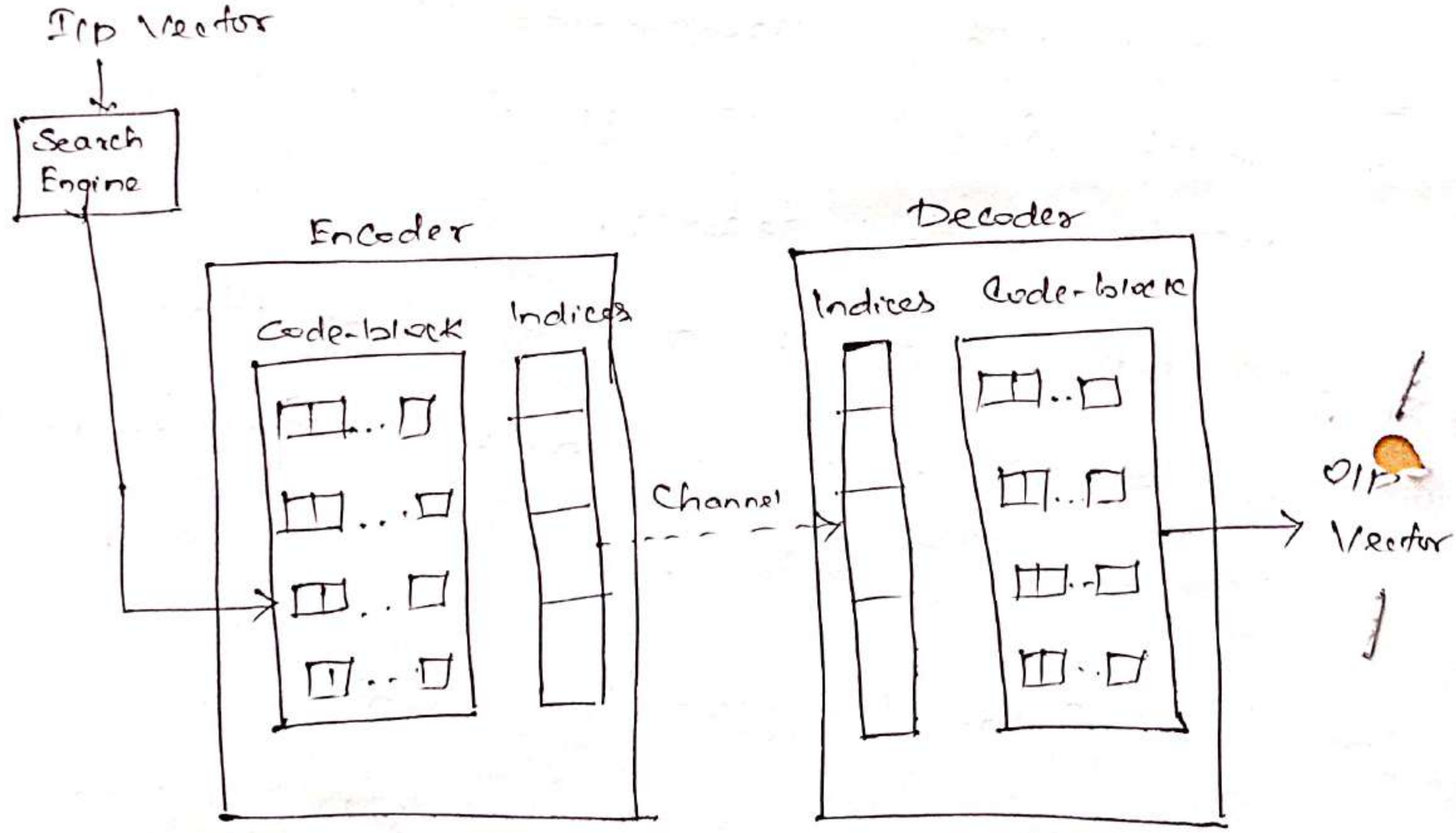


fig: Vector Quantization Scheme.

JPEG Compatible S/m's must Support the baseline S/m. It is a Sequential baseline S/m. Here i/p and o/p data length is limited to 8 bits.

Sequential JPEG Compression:

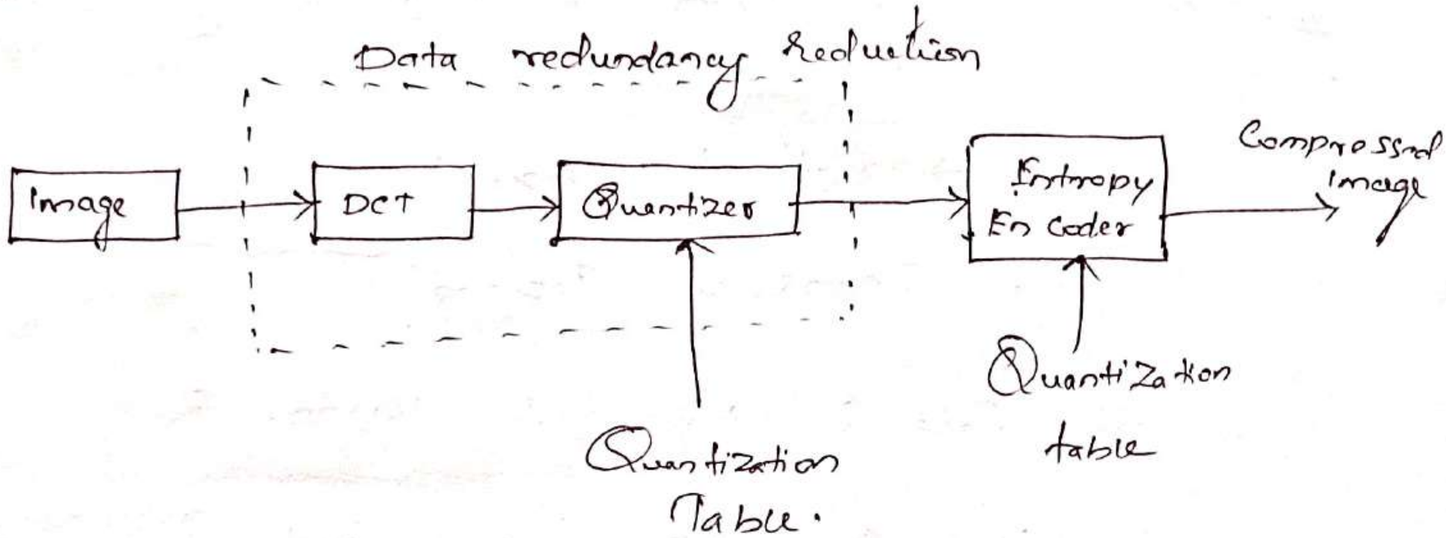


Fig: 1 JPEG- Encoder block

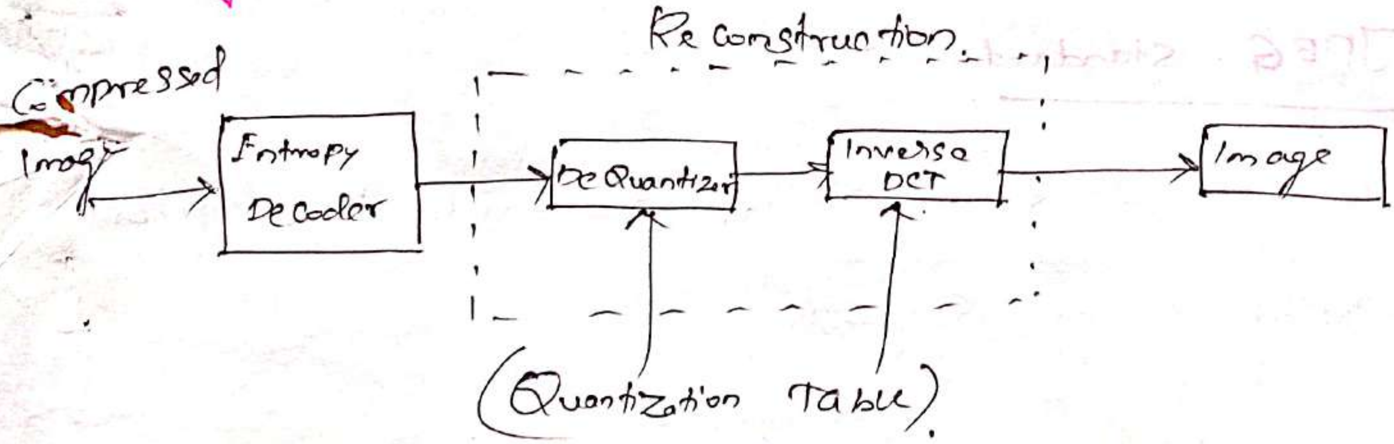


Fig: 2 : JPEG- Decoder Block.

The No. of Code Words (or) Code Vectors (N) depends upon two Parameters.

1. Rate (R)

2. Dimensions (L)

$\therefore$ No: of code Vectors (N) = $2^{R.L}$

R → Rate (bit/pixel)

L - Dimensions (Grouping)

When The Rate ↑ - No of Code Vectors ↑

No. of code Vector ↑ - Size of code book ↑.

JPEG - Standard:

It is a Popular Standard. It defines following coding Systems.

(i) Lossy base line Coding S/m (based on DCT)

(ii) Extended Coding S/m (using high Compression, high Precision)

(iii) Lossless independent Coding S/m (Used for reversible Compression)