PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Approved by **AICTE** and Affiliated to **Anna University**)

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Department of Electronics and Communication Engineering

Regulation 2021

II Year – IV Semester

EC3492- DIGITAL SIGNAL PROCESSING

EC3492 **DIGITAL SIGNAL PROCESSING** **L T P C**

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COURSE OBJECTIVES:

- To learn discrete fourier transform, properties of DFT and its application to linear filtering
- To understand the characteristics of digital filters, design digital IIR and FIR filters and apply these filters to filter undesirable signals in various frequency bands
- To understand the effects of finite precision representation on digital filters
- To understand the fundamental concepts of multi rate signal processing and its applications
- To introduce the concepts of adaptive filters and its application to communication engineering

UNIT I DISCRETE FOURIER TRANSFORM 9

Sampling Theorem, concept of frequency in discrete-time signals, summary of analysis & synthesis equations for FT & DTFT, frequency domain sampling, Discrete Fourier transform (DFT) - deriving DFT from DTFT, properties of DFT - periodicity, symmetry, circular convolution. Linear filtering using DFT. Filtering long data sequences - overlap save and overlap add method. Fast computation of DFT - Radix-2 Decimation-in-time (DIT) Fast Fourier transform (FFT), Decimation-in-frequency (DIF) Fast Fourier transform (FFT). Linear filtering using FFT.

UNIT II INFINITE IMPULSE RESPONSE FILTERS 9

Characteristics of practical frequency selective filters. characteristics of commonly used analog filters - Butterworth filters, Chebyshev filters. Design of IIR filters from analog filters (LPF, HPF, BPF, BRF) - Approximation of derivatives, Impulse invariance method, Bilinear transformation. Frequency transformation in the analog domain. Structure of IIR filter - direct form I, direct form II, Cascade, parallel realizations.

UNIT III FINITE IMPULSE RESPONSE FILTERS 9

Design of FIR filters - symmetric and Anti-symmetric FIR filters - design of linear phase FIR filters using Fourier series method - FIR filter design using windows (Rectangular, Hamming and Hanning window), Frequency sampling method. FIR filter structures - linear phase structure, direct form realizations

UNIT IV FINITE WORD LENGTH EFFECTS 9

Fixed point and floating point number representation - ADC - quantization - truncation and rounding - quantization noise - input / output quantization - coefficient quantization error - product quantization error - overflow error - limit cycle oscillations due to product quantization and summation - scaling to prevent overflow.

UNIT V DSP APPLICATIONS 9

Multirate signal processing: Decimation, Interpolation, Sampling rate conversion by a rational factor – Adaptive Filters: Introduction, Applications of adaptive filtering to equalization-DSP Architecture-Fixed and Floating point architecture principles

45 PERIODS

PRACTICAL EXERCISES: **30 PERIODS**

MATLAB / EQUIVALENT SOFTWARE PACKAGE/ DSP PROCESSOR BASED IMPLEMENTATION

1. Generation of elementary Discrete-Time sequences
2. Linear and Circular convolutions
3. Auto correlation and Cross Correlation
4. Frequency Analysis using DFT
5. Design of FIR filters (LPF/HPF/BPF/BSF) and demonstrates the filtering operation
6. Design of Butterworth and Chebyshev IIR filters (LPF/HPF/BPF/BSF) and demonstrate the filtering operations
7. Study of architecture of Digital Signal Processor
8. Perform MAC operation using various addressing modes
9. Generation of various signals and random noise
10. Design and demonstration of FIR Filter for Low pass, High pass, Band pass and Band stop filtering
11. Design and demonstration of Butter worth and Chebyshev IIR Filters for Low pass, High pass, Band pass and Band stop filtering
12. Implement an Up-sampling and Down-sampling operation in DSP Processor

COURSE OUTCOMES:

At the end of the course students will be able to:

CO1: Apply DFT for the analysis of digital signals and systems

CO2: Design IIR and FIR filters

CO3: Characterize the effects of finite precision representation on digital filters

CO4: Design multirate filters

CO5: Apply adaptive filters appropriately in communication systems

TOTAL:75 PERIODS

TEXT BOOKS:

1. 1.John G. Proakis and Dimitris G.Manolakis, Digital Signal Processing – Principles, Algorithms and Applications, Fourth Edition, Pearson Education / Prentice Hall, 2007.
2. 2.A. V. Oppenheim, R.W. Schafer and J.R. Buck, —Discrete-Time Signal Processing‖, 8th Indian Reprint, Pearson, 2004.

REFERENCES

1. Emmanuel C. Ifeachor& Barrie. W. Jervis, "Digital Signal Processing", Second Edition, Pearson Education / Prentice Hall, 2002.

EC 3492 - Digital signal processing

Unit - 1 Discrete Fourier Transform

⟹ Sampling Theorem

⟹ concept of freq. domain sampling in DT signals

⟹ Summary of analysis & synthesis

⟹ Eqns for FT & DTFT

⟹ freq. domain sampling

⟹ DFT — deriving DFT from DTFT

⟹ properties of DFT —

⟹ periodicity, symmetry, circular convolution

⟹ Linear filtering using DFT

⟹ Filtering long data sequences.
— overlap save and overlap add method

⟹ Fast computation DFT

⟹ Radix - 2 Decimation in time (DIT)

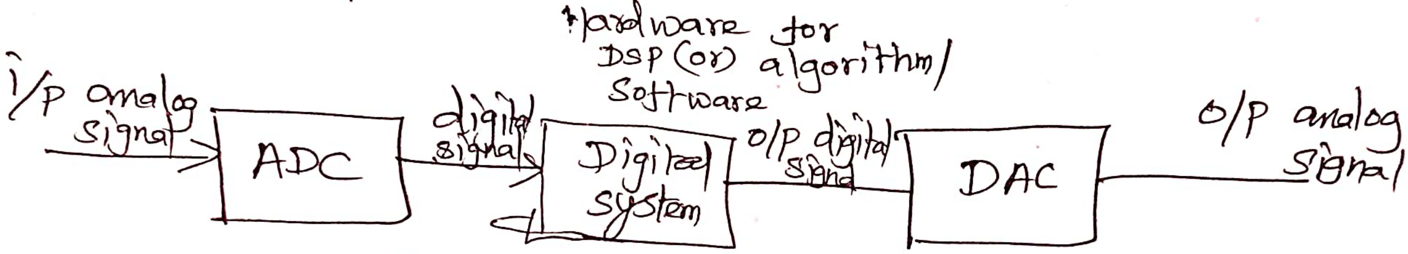
⟹ FFT

⟹ Decimation in freq. (DIF)

⟹ Fast fourier transform

⟹ Linear filtering using FFT.

DSP :- It refers to processing of signals by digital systems like PC and system designed using digital ICs, microprocessor and micro controller.



Basic Components of DSP system.

Advantages of DSP :-

* The digital hardware are compact, relaible less expensive and programmable.

* DSP systems are programmable, the performance of the system can be easily upgraded and modified.

* High speed operation..

* The digital signals can be permantly stored in magnetic media, so they are transportable and can be processed in real time or off line.

Importance of DSP :

* Biomedical Applications.
* Speech processing
* Audio & video equipments.
* power electronics.
* Image processing
* Geology
* Astronomy.

Deriving DFT from DTFT :-

* The DFT of $x(n)$ is obtained Sampling one perid of the discrete time FT $X(e^{jw})$ at finite no.. of freq. points

* The freq. domain sampling is conventionally performed at N equal spaced freq. points. in the perid of 0 to 2π.

* The sampling freq. points are denoted as w_k

$$w_k = \frac{2\pi K}{N} \qquad K = 0, 1, \ldots N-1$$

$$X(K) = X(e^{jw})\Big|_{w = \frac{2\pi K}{N}}$$

~~X(K)~~ * The DFT is defined along with no.. of samples and is called N-point

DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi Kn/N}$$

$K = 0, 1, 2 \ldots N-1$

IDFT

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j2\pi Kn/N}.$$

$n = 0, 1, 2, \ldots N-1$

Discrete Fourier transform

The formulas for DFT and IDFT are

$$\text{DFT}:- \quad X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$0 \leq k \leq N-1$$

$$\text{IDFT} \quad x(n) = \sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

$$0 \leq n \leq N-1$$

(1) Find the DFT of a sequence $x(n) = \{1,1,0,0\}$ and find IDFT of $Y(k) = \{1,0,1,0\}$

$N = L = 4.$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

$k = 0, 1, \cdots N-1$

$k = 0, 1, 2, 3$

$\underline{k=0}$

$$X(0) = x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0$$

$$= 2$$

$\underline{k=1}$

$$X(1) = x(0) + x(1)\, e^{-j\cancel{2}\pi/\cancel{4}2} + x(2)\, e^{-j\cancel{2\pi 2\times 1}/\cancel{4}} + x(3)\, e^{-j\cancel{6}3\pi/\cancel{4}2}$$

$$= 1 + x(1)\, e^{-j\pi/2} + x(2)\, e^{-\pi} + x(3)\, e^{-3j\pi/2}$$

$= 1 + (1)\, e^{-j\pi/2}$

$= 1 + \cos \pi/2 - j \sin \pi/2$

$= 1 - j$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$= x(0) + x(1)\, e^{-j\pi} + x(2)\, e^{-j2\pi} + x(3)\, e^{-j3\pi}$

$= 1 + 1\, e^{-j\pi} + 0 + 0$

$= 1 + \cos \pi - j \sin \pi$

$= 1 - 1 - 0$

$= 0$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j3\pi n/2}$$

$= x(0)\, e^{0} + x(1)\, e^{-j3\pi/2} + x(2)\, e^{-j3\pi} + x(3)\, e^{-j9\pi/2}$

$= 1 + 1\, e^{-j3\pi/2} + 0 + 0$

$= 1 + \cos 3\pi/2 - j \sin 3\pi/2$

$= 1 + j$

$X(K) = \{2, 1-j, 0, 1+j\}$

$$y(n) = \frac{1}{N} \sum_{K=0}^{N-1} y(k)\, e^{j 2\pi n k / N}$$

$n = 0, 1, 2, \ldots N-1$

$$Y(0) = \frac{1}{4} \sum_{K=0}^{3} Y(k) \qquad n = 0, 1, 2, 3$$

$$= \frac{1}{4}\left[y(0) + y(1) + y(2) + y(3) \right]$$

$$= \frac{1}{4}\left[1 + 0 + 1 + 0 \right]$$

$$= \frac{1}{4}[2]$$

$$= \frac{1}{2}$$

$$= 0.5$$

$$y(1) = \frac{1}{4} \sum_{K=0}^{3} y(k)\, e^{j 2\pi k/4}$$

$$= \frac{1}{4} \sum_{K=0}^{3} y(k)\, e^{j \pi k/2}$$

$$= \frac{1}{4}\left[y(0) e^{0} + y(1) e^{j\pi/2} + y(2) e^{j\pi} + y(3) e^{j3\pi/2} \right]$$

$$= \frac{1}{4}\left[1 + 0 + 1 e^{j\pi} + 0 \right]$$

$$= \frac{1}{4}\left[1 + \cos\pi + j \sin\pi \right]$$

$= \frac{1}{4}[1+0-1+0]$

$= 0$

$y(2) = \frac{1}{4}[y(0) + y(1)e^{j\pi} + y(2)e^{j2\pi} + y(3)e^{j3\pi}]$

$= \frac{1}{4}[1+0+1e^{j2\pi}+0]$

$= \frac{1}{4}[1+\cos 2\pi + j\sin 2\pi]$

$= \frac{1}{4}[1+1+0]$

$= \frac{2}{4}$

$= \frac{1}{2}$

$= 0.5$

$y(3) = \frac{1}{4}[y(0) + y(1)e^{j3\pi/2} + y(2)e^{j3\pi} + y(3)e^{j9\pi/2}]$

$= \frac{1}{4}[1+0+e^{j3\pi}+0]$

$= \frac{1}{4}[1+\cos 3\pi + j\sin 3\pi]$

$= \frac{1}{4}[1+0+(-1)+0]$

$= 0$

$y(n) = \{0.5, 0, 0.5, 0\}$.

DTFT :-

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$X(\omega)$ is FT of $x(n)$

freq. range for ω from $-\pi$ to π (or) 0 to 2π

$$X(\omega + 2\pi k) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\omega + 2\pi k)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \cdot e^{-j2\pi nk}$$

$$e^{-j2\pi nk} = \cos(2\pi kn) - j\sin(2\pi kn)$$

k & n are integers

$$\left.\begin{array}{l} \cos 2\pi kn = 1 \\ \sin 2\pi kn = 0 \end{array}\right\} \text{always}$$

$$\therefore e^{-j2\pi nk} = 1$$

$$= \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$$

$$= X(\omega)$$

The inverse Fourier transform is

$$\boxed{x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n} \cdot d\omega.}$$

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

This is the sufficient conditions for existence of FT.

Freq. domain sampling :-

The fourier transform of signal can be calculated as

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$x(n)$ – discrete time signal

Ω – freq. (i.e continuous fn 0 to 2π)

* This means $x(n)$ is discrete its spectrum $X(\Omega)$ is continuous.

* Such continuous fn can not be ~~evaluated~~ processed on digital processor.

* To overcome the problem of digital processing, the spectrum $X(\Omega)$ is sampled Uniformly.

* Let N samples are taken from 0 to 2π Spacing b/w successive samples will be $2\pi/N$

$$\therefore \Omega = \frac{2\pi}{N} k$$

$$X\left(\frac{2\pi}{N} k\right) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{2\pi k n}{N}}$$

K – index for the sample.

$K = 0, 1, 2 \ldots N-1$

Freq. domain sampling :-

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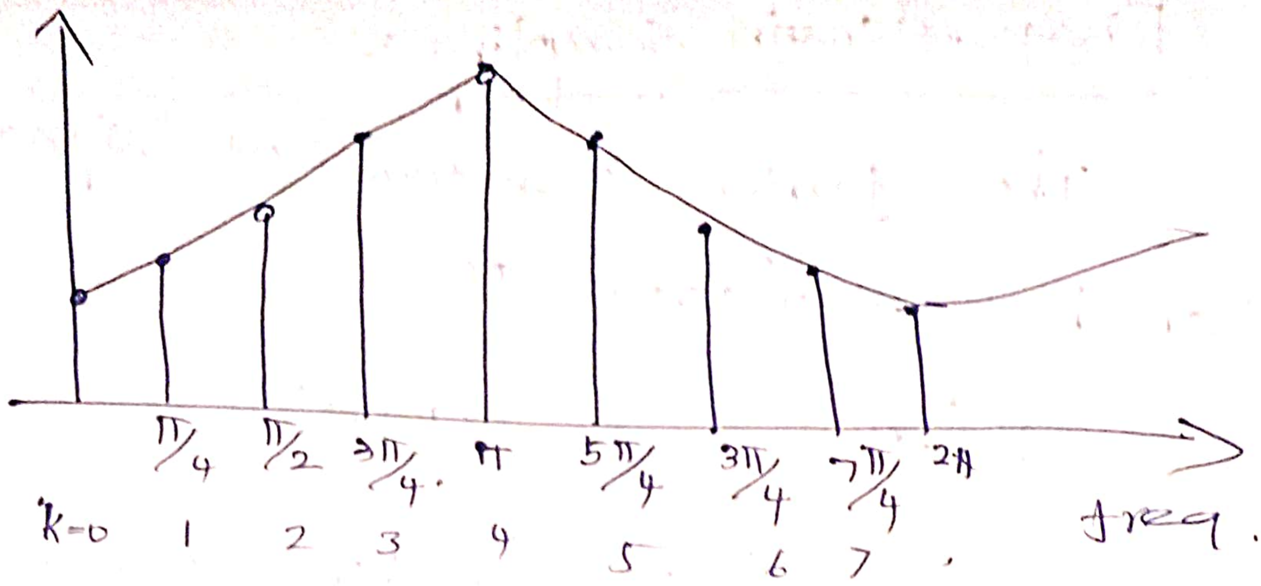
$k = 0, 1, 2 \ldots N-1$ — k - index for the sample.



π/4
π/2
3π/4
π
5π/4
3π/4
7π/4
2π
k=0
1
2
3
4
5
6
7
freq.

Properties of DFT

1. Linearity :-

$$DFT\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 X_1(K) + a_2 X_2(K)$$

Proof :-

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi kn/N}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(K)\, e^{-j2\pi kn/N}$$

LHS.

$$= DFT\{a_1 x_1(n) + a_2 x_2(n)\}$$

$$= \sum_{n=0}^{N-1} \{a_1 x_1(n) + a_2 x_2(n)\}\, e^{-j2\pi Kn/N}$$

$$= \sum_{n=0}^{N-1} a_1 x_1(n)\, e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} a_2 x_2(n)\, e^{-j2\pi kn/N}$$

$$= a_1 \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi kn/N} + a_2 \sum_{n=0}^{N-1} x_2(n)\, e^{-j2\pi kn/N}$$

$$= a_1 X_1(K) + a_2 X_2(K)$$

$$= R.H.S.$$

2. Periodicity :- $X(k+N) = X(k)$

LHS

$$X(k+N) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi n(k+N)/N}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi k n/N} \cdot e^{-j2\pi n N/N}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi k n/N} \cdot e^{-j2\pi n}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi k n/N} \qquad e^{-j2\pi n} = 1$$

$$= X(k)$$

Circular time shift :-

$$DFT\{x(n-m)_N\} = X(k)\, e^{-j2\pi k m/N}$$

$$LHS = DFT\{x(n-m)_N\}$$

$$= \sum_{n=0}^{N-1} x(n-m)_N\, e^{-j2\pi k n/N}$$

$$p = n-m \qquad n = p+m$$

$$= \sum_{p=0}^{N-1} x(p)\, e^{-j2\pi k(p+m)/N}$$

$$= \sum_{p=0}^{N-1} x(p)\, e^{-j2\pi k p/N} \cdot e^{-j2\pi k m/N}$$

$$= X(k)\, e^{-j2\pi k m/N} = RHS.$$

Time reversal :-

LHS $\quad DFT\{x(N-n)\} = X(N-k)$

$$DFT\{x(N-n)\} = \sum_{n=0}^{N-1} x(N-n)\, e^{-j2\pi kn/N}$$

$m = N-n$

$n = N-m$

$$= \sum_{n=0}^{N-1} x(m)\, e^{-j2\pi k(N-m)/N}$$

$$= \sum_{n=0}^{N-1} x(m)\, e^{-j2\pi k \cancel{N}/\cancel{N}} \cdot e^{+j2\pi km/N}$$

$$= \sum_{n=0}^{N-1} x(m)\, e^{j2\pi km/N} \cdot e^{-j2\pi m}$$

$e^{-j2\pi k} = 1$

$$= \sum_{n=0}^{N-1} x(m)\, e^{j2\pi km/N} \cdot e^{-j2\pi m N/N} \quad e^{-j2\pi m} = 1$$

$$= \sum_{n=0}^{N-1} x(m)\, e^{-j2\pi m(N-k)/N}$$

$$= \sum_{n=0}^{N-1} x(m)\, e^{-j2\pi m(N-k)/N}$$

$$= X(N-k).$$

Circular convolution :-

The circular convolution of two N-point sequence $x_1(n)$ and $x_2(n)$ is defined as.

$$x_1(n) * x_2(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)$$

$$DFT\{x_1(n) * x_2(n)\} = X_1(K)\, X_2(K)$$

Proof :-

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi nK/N} \qquad n = m$$

$$= \sum_{m=0}^{N-1} x_1(m)\, e^{-j2\pi mK/N} \qquad K = 0, 1, \ldots N-1$$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j2\pi nK/N} \qquad n = P$$

$$= \sum_{p=0}^{N-1} x_2(P)\, e^{-j2\pi PK/N} \qquad K = 0, 1, \ldots N-1$$

Consider the product $X_1(K)\, X_2(K)$.

The inverse DFT of the product is

$$DFT^{-1}\{X_1(K)\, X_2(K)\} = \frac{1}{N} \sum_{K=0}^{N-1} X_1(K)\, X_2(K)\, e^{j2\pi nK/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1}\left\{\left[\sum_{m=0}^{N-1} x_1(m)\, e^{-j2\pi mk/N}\right]\left[\sum_{p=0}^{N-1} x_2(p)\, e^{-j2\pi pk/N}\right]\right\} e^{j2\pi nk/N}$$

$$= \frac{1}{N}\sum_{m=0}^{N-1} x_1(m)\sum_{p=0}^{N-1} x_2(p)\sum_{k=0}^{N-1} e^{\frac{j2\pi k(n-m-p)}{N}}$$

Consider the summation

$$\sum_{k=0}^{N-1} e^{j2\pi k(n-m-p)/N}$$

$$n-m-p = qN$$

$$= \sum_{k=0}^{N-1} e^{j2\pi k(qN)/N} = \sum_{k=0}^{N-1}\left(e^{j2\pi q}\right)^k \qquad e^{j2\pi q} = 1$$

$$= \sum_{k=0}^{N-1} 1^k = N \quad \text{——— (1)}$$

Consider $\sum_{p=0}^{N-1} x_2(p)$

$$n-m-p = qN, \quad p = n-m-qN$$

$$\sum_{p=0}^{N-1} x_2(n-m-qN) = \sum_{m=0}^{N-1} x_2(n-m, \bmod N)$$

$$= \sum_{m=0}^{N-1} x_2((n-m))_N \quad \text{——— (2)}$$

Using eqn ① & ②

$$DFT^{-1}\{X_1(k) \cdot X_2(k)\} = \frac{1}{\cancel{N}} \sum_{m=0}^{N-1} x_1(m) \sum_{m=0}^{N-1} x_2((n-m))_N \cancel{N}$$

$$= \sum_{m=0}^{N-1} x_1(m)\, x_2((n-m))_N$$

$$= x_1(n) \circledast x_2(n)$$

$$X_1(k)\, X_2(k) = DFT\{x_1(n) \circledast x_2(n)\}$$

<u>Parseval's relation</u> :-

$$\sum_{n=0}^{N-1} x_1(n)\, x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k)\, X_2(k)$$

Let $x_1(n)$ and $x_2(n)$ be N-point sequence

Using DFT $\quad X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi nk/N}.$

By IDFT $\quad x_2(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_2(k)\, e^{j2\pi nk/N}$

Consider R.H.S.

$$\frac{1}{N} \sum_{k=0}^{N-1} X_1(k)\, X_2^*(k)$$

$$= \frac{1}{N}\left[\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nk/N}\right] X_2^*(k)\right]$$

$$= \sum_{n=0}^{N-1} x_1(n)\left[\frac{1}{N}\sum_{k=0}^{N-1} X_2^*(k)\, e^{-j2\pi nk/N}\right]$$

$$= \sum_{n=0}^{N-1} x_1(n)\left[\frac{1}{N}\sum_{k=0}^{N-1} X_2(P)\, e^{j2\pi nk/N}\right]^*$$

$$= \sum_{n=0}^{N-1} x_1(n)\, x_2^*(n)$$

$$= L.H.S.$$

① Compute 4-point DFT and and 8-point DFT of casnal 3 sample sequence Given by $x(n) = 1/3 \quad 0 \leq n \leq 2$

$\quad\quad 0 \quad$ else.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}.$$

<u>N=4</u>

$$= \sum_{n=0}^{3} x(n)\, e^{-j2\pi nk/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi nk/2}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi k/2} + x(2)\, e^{-j\pi k}$$

$$= \frac{1}{3} + \frac{1}{3}\left[\cos \pi k/2 - j \sin \pi k/2\right] + \frac{1}{3}\left[\cos \pi k - j \sin \pi k\right]$$

$$= \frac{1}{3}\left[1 + \cos \pi k/2 - j \sin k\pi/2 + \cos \pi k - j \sin \pi k\right]$$

$k = 0$

$$x(0) = \frac{1}{3}\left[1 + \cos 0 - j \sin 0 + \cos 0 - j \sin 0\right]$$

$$= \frac{1}{3}\left[1 + 1 - 0 + 1 - 0\right]$$

$$= \frac{3}{3} = 1 = 1\angle 0$$

Compute circular convolution of the following two sequences using DFT

$x_1(n) = \{\underset{\uparrow}{0}, 1, 0, 1\}$ $x_2(n) = \{\underset{\uparrow}{1}, 2, 1, 2\}$

$x_1(n) = \{0, 1, 0, 1\}$

$$X_1(k) = \sum_{n=0}^{3} x_1(n) e^{-j\cancel{4}\pi nk/\cancel{4}2}$$

$$= \sum_{n=0}^{3} x_1(n) e^{-j\pi nk/2}$$

$$= x_1(0) e^{0} + x_1(1) e^{-j\pi k/2} + x_1(2) e^{-j\pi k} + x_1(3) e^{-j3\pi k/2}$$

$$= 0 + e^{-j\pi k/2} + 0 + e^{-j3\pi k/2}$$

$K = 0 \Rightarrow X_1(0) = e^{0} + e^{0} = 2$

$K = 1 \Rightarrow X_1(1) = e^{-j\pi/2} + e^{-j3\pi/2}$

$$= \cos \pi/2 - j\sin \pi/2 + \cos 3\pi/2 - j\sin 3\pi/2$$

$$= 0 - j + 0 + j$$

$$= 0$$

$K = 2$

$$X_1(2) = e^{-j\pi} + e^{-j3\pi}$$

$= -1-1 = -2$

$K=3 \Rightarrow X_1(3) = e^{-j3\pi/2} + e^{-j9\pi/2}$

$= j - j = 0$

$\cancel{X} X_1(K) = \{2, 0, -2, 0\}$

$x_2(n) = \{1, 2, 1, 2\}$

$$X_2(K) = \sum_{n=0}^{3} x_2(n)\, e^{-j\cancel{2}\pi nk/\cancel{4}}$$

$$= \sum_{n=0}^{3} x_2(n)\, e^{-j\pi nk/2}$$

$$= x_2(0)e^0 + x_2(1)\, e^{-j\pi K/2} + x_2(2)e^{-j\pi K}$$
$$+ x_2(3)e^{-j3\pi K}$$

$$= 1 + 2e^{-j\pi K} + e^{-j\pi K} + 2e^{-j3\pi/2}$$

$K=0$

$X_2(0) = 1 + 2e^{-j\pi} + e^0 + 2e^0$

$= 1+2+1+2$

$= 6$

$\underline{K=1}$

$x_2(1) = 1 + 2(\cos\pi/2 - j\sin\pi/2)$.

$X_2(k) = \{6, 0, -2, 0\}$

$k=0 \quad X_1(0) \times X_2(0) = 2 \times 6 = 12$

$X_1(1) \times X_2(1) = 0 \times 0 = 0$

$X_1(2) \times X_2(2) = -2 \times -2 = 4$

$X_1(3) \times X_2(3) = 0 \times 0 = 0$

$X_3(k) = \{12, 0, 4, 0\}$

By circular convolution Theorem of DFT.

$DFT\{x_1(n) * x_2(n)\} = X_1(k) X_2(k)$

$\{x_1(n) * x_2(n)\} = DFT^{-1}\{X_1(k) X_2(k)\}$

$$x_3(n) = 1/4 \sum_{k=0}^{4-1} X_3(k) \, e^{j2\pi nk/4}$$

$$= 1/4 \sum_{k=0}^{3} X_3(k) \, e^{j\pi nk/2}$$

$= \{4, 2, 4, 2\}$

Linear filtering using DFT :-

Linear convolution using DFT :-

* Linear filtering operation is implemeted with help of linear convolution.

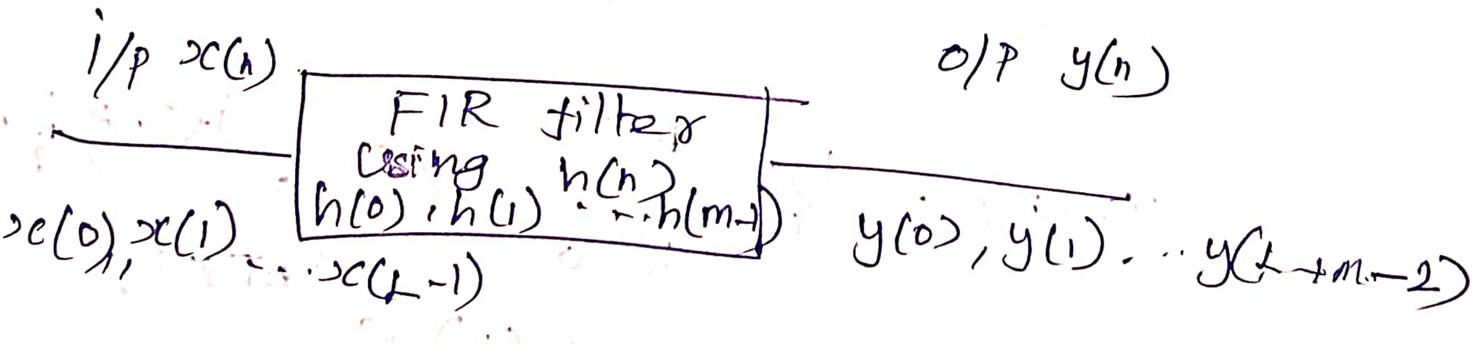
$$y(n) = h(n) * x(n)$$

Let unit sample response $h(n)$ of length M

i.e. $h(0), h(1) \ldots h(M-1)$

Let i/p to this LTI system be $x(n)$ of length L.

i.e. $x(0), x(1), \ldots x(L-1)$



$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

FT of above eqn

$$Y(\omega) = F\left\{ \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right.$$

$$F\{x_1(n) * x_2(n)\} = X_1(\omega) X_2(\omega)$$

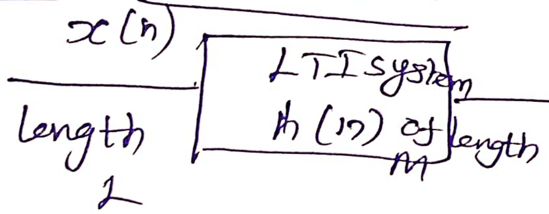
$$Y(\omega) = H(\omega) X(\omega)$$

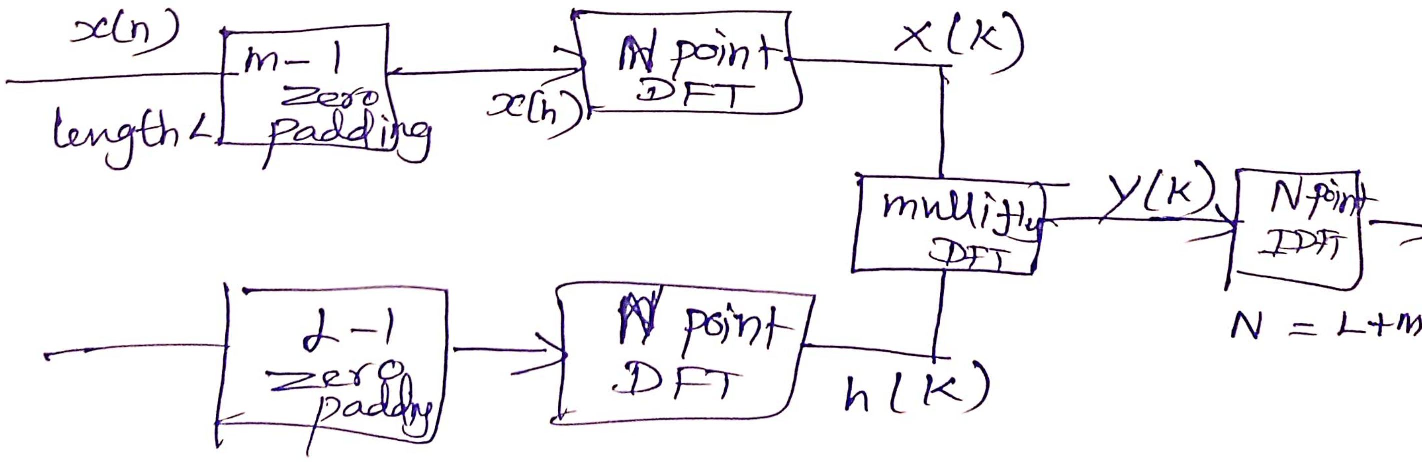
$$Y(K) = Y(\omega)\Big|_{\omega = \frac{2\pi K}{N}} \qquad K = 0, 1, 2 \ldots N-1$$

$$Y(K) = X(K) \cdot H(K)$$

$$y(n) = \text{IDFT}\{Y(K)\}$$
$$= \text{IDFT}\{X(K) \cdot H(K)\}$$

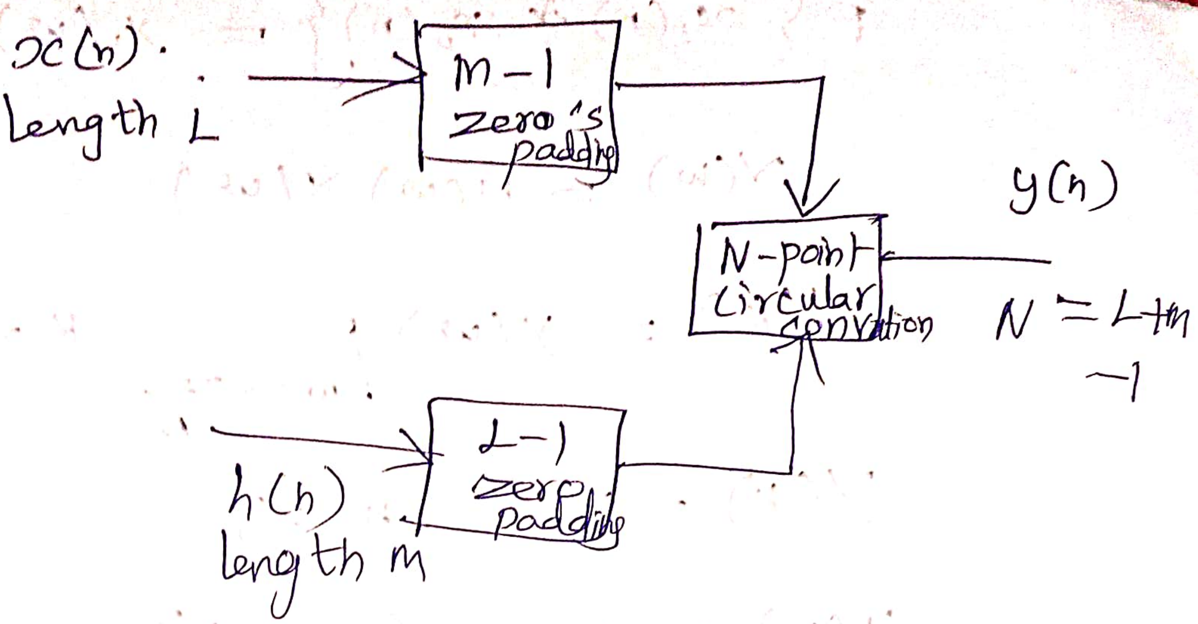
Linear convolution:





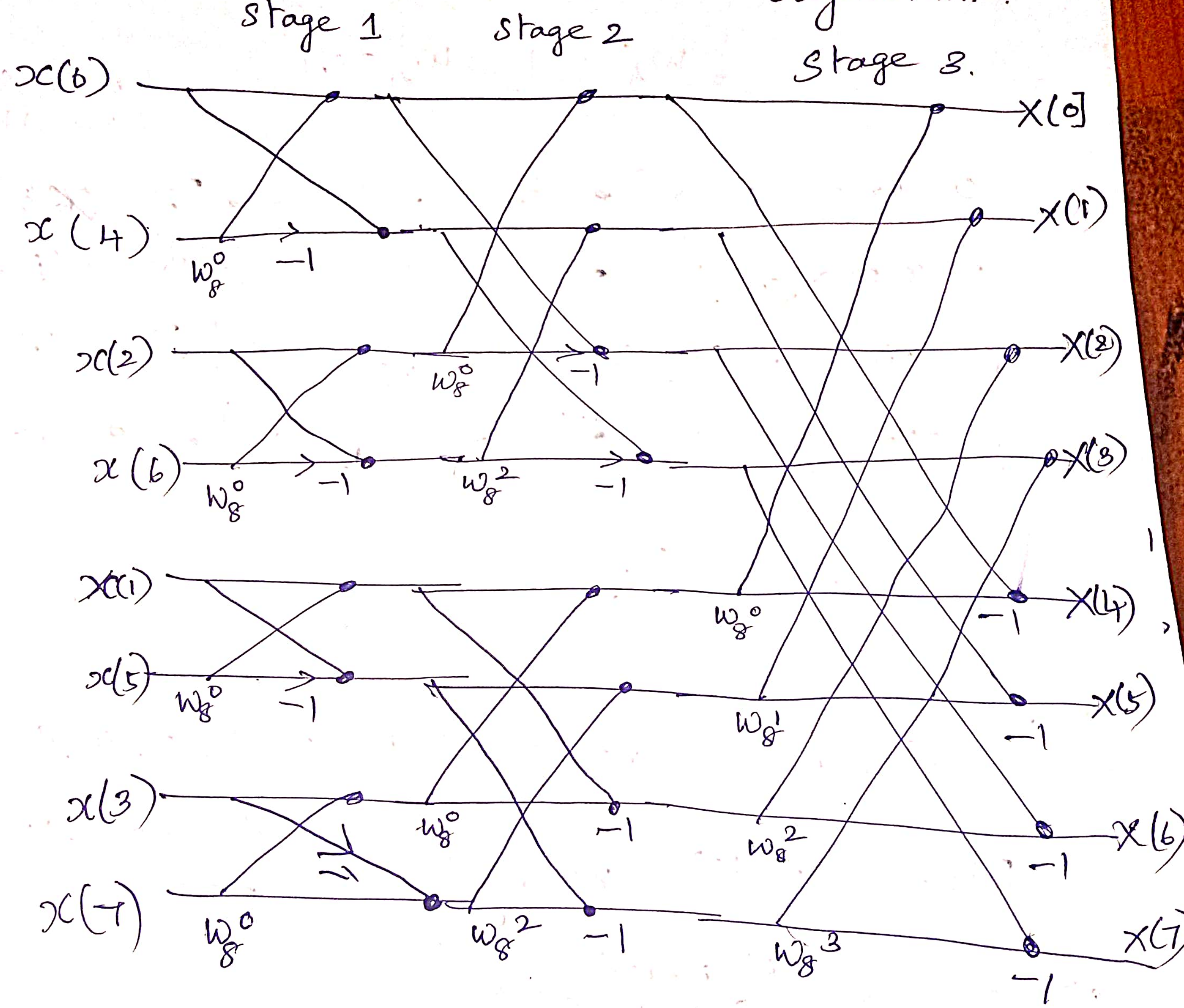
$$x_1(n) \; (N) \; x_2(n) \xleftrightarrow{\text{DFT}} X_1(K)\, X_2(K)$$

$$y(n) = x(n) \; (N) \; h(n)$$


x(n).
Length L
M-1
zero's
padding
y(n)
N-point
circular
convolution
N = L+M
-1
L-1
zero
padding
h(n)
length M

FFT :- (Fast fourier transform)

N=8 point decimation in time FFT algorithm.



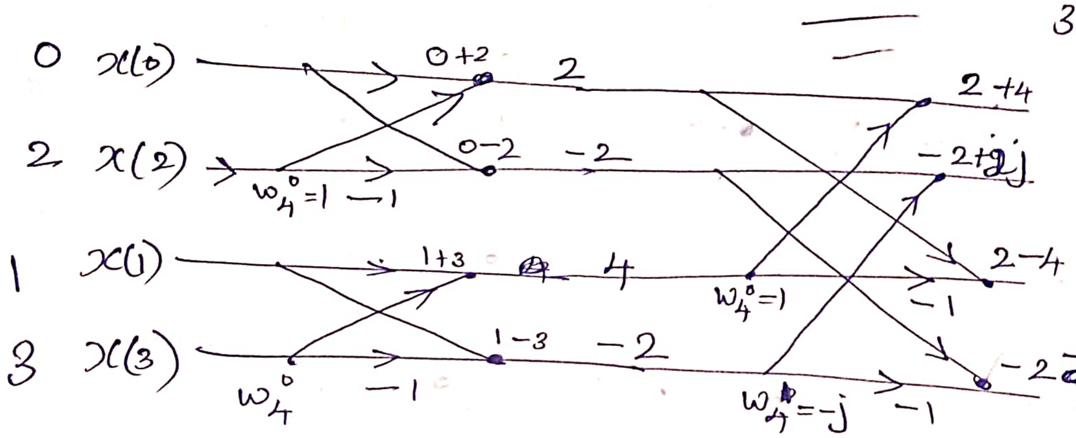
① $x(n) = \{0, 1, 2, 3\}$ find $X(k)$ using DIT - FFT algorithm.

$N = 4$

$W_4^0 = 1 \quad W_4^1 = -j$

bit reversal

0	→ 00	0 0 0
1	→ 01	1 0 2
2	→ 10	0 1 1
3	→ 11	1 1 3



$X(0) = 6$

$X(1) = -2+2j$

$X(2) = -2$

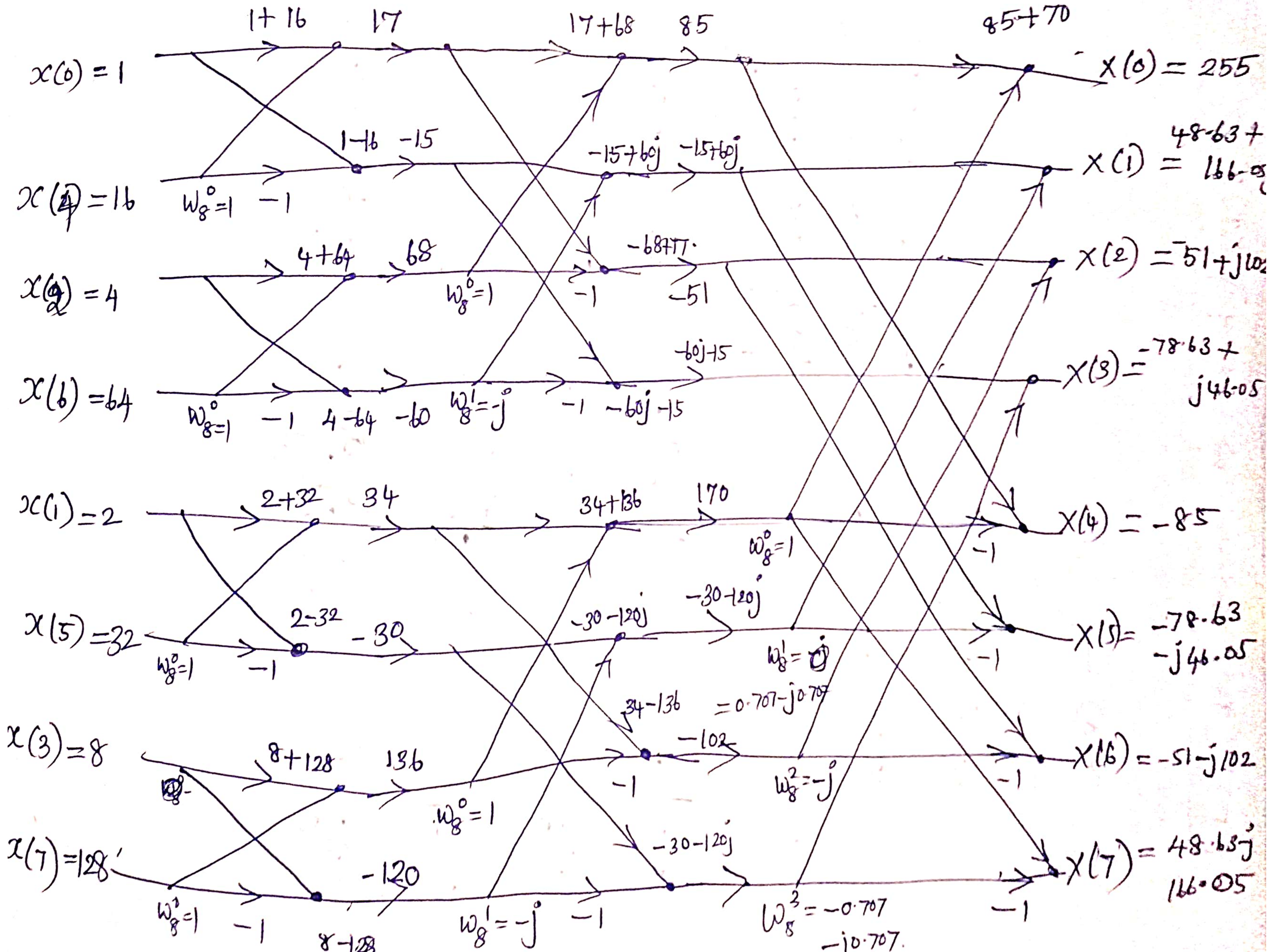
$X(3) = -2-2j$

$$X(k) = \{6, -2+2j, -2, -2-2j\}$$

② Given sequence $x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$

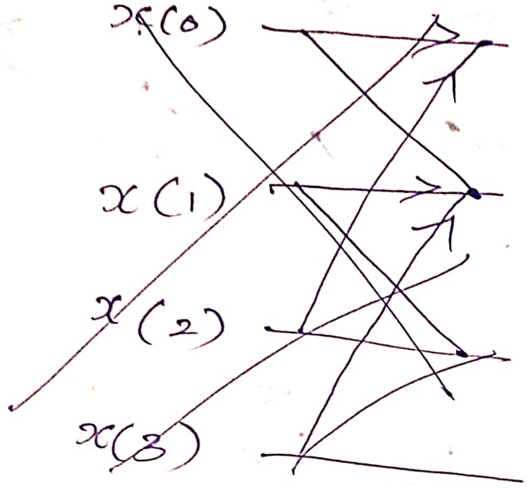
Find $X(K)$ using DIT - FFT

				Bit reversal			
x(0)	0	0	0	0	0	0	x(0)
x(1)	0	0	1	1	0	0	x(4)
x(2)	0	1	0	0	1	0	x(2)
x(3)	0	1	1	1	1	0	x(6)
x(4)	1	0	0	0	0	1	x(1)
x(5)	1	0	1	1	0	1	x(5)
x(6)	1	1	0	0	1	1	x(3)
x(7)	1	1	1	1	1	1	x(7)

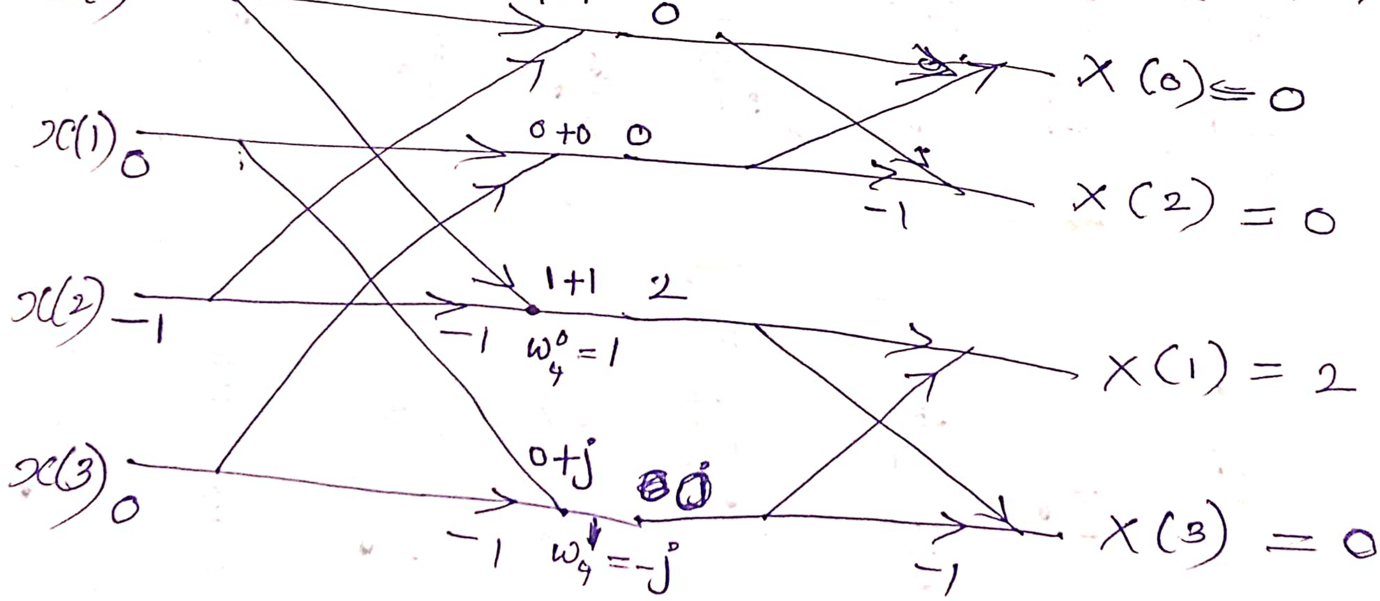


x(0) = 1
x(4) = 16
x(2) = 4
x(6) = 64
x(1) = 2
x(5) = 32
x(3) = 8
x(7) = 128
1+16
17
1-16
-15
4+64
68
4-64
-60
2+32
34
2-32
-30
8+128
136
-120
17+68
85
-15+60j
-51
-60j -15
34+136
170
-30-120j
34-136
-102
$W_8^0=1$
$W_8^1=-j$
$W_8^1=0.707-j0.707$
$W_8^2=-j$
$W_8^3=-0.707-j0.707$
-1
85+70
X(0) = 255
X(1) = 48.63 + 166.05
X(2) = -51+j102
X(3) = -78.63 + j46.05
X(4) = -85
X(5) = -78.63 -j46.05
X(6) = -51-j102
X(7) = 48.63j 166.05

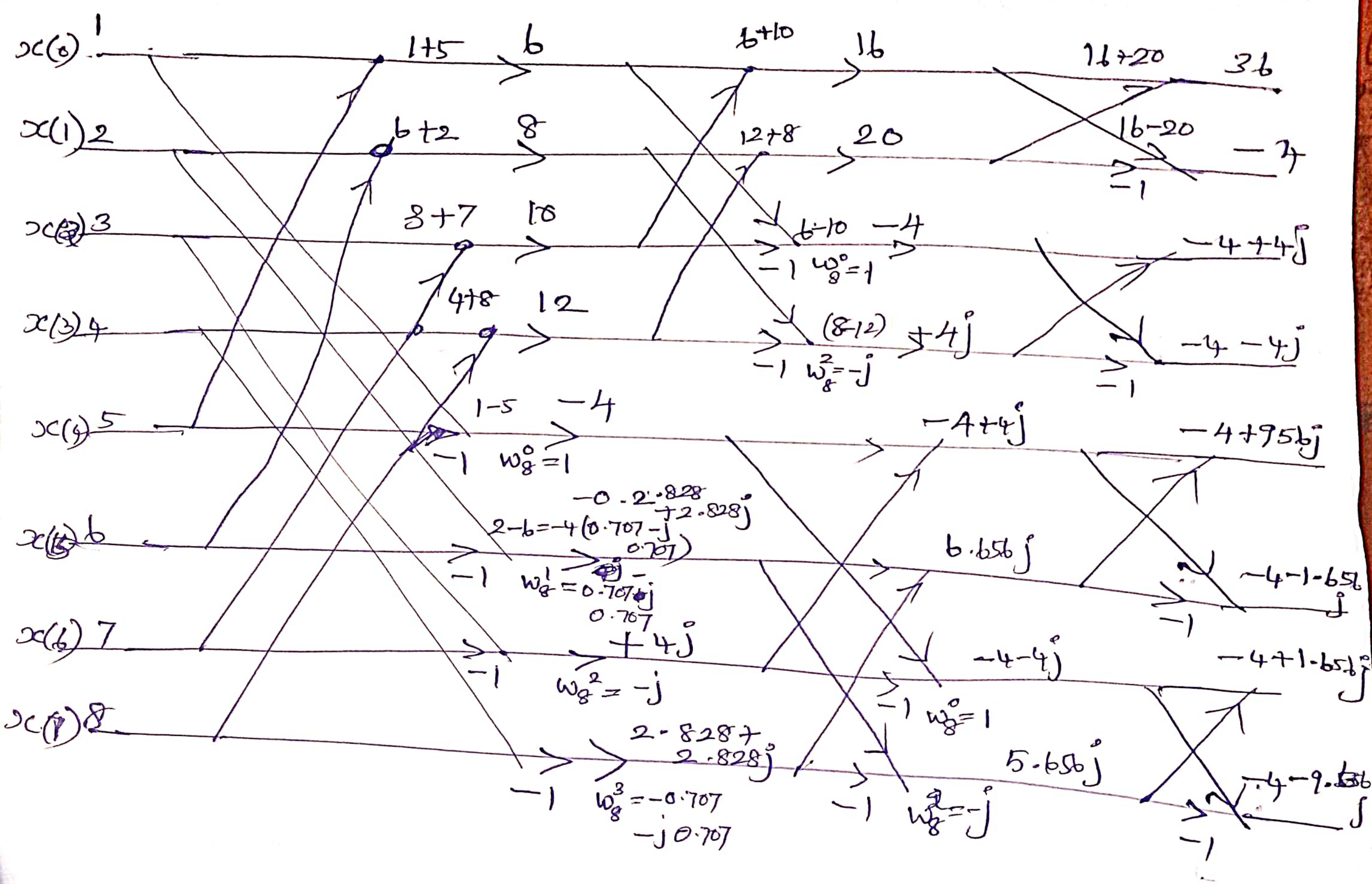
FFT (Decimation in freq) - DIF



compute the DFT of $x(n) = \cos\frac{n\pi}{2}$ where $N=4$
using DIF - FFT
$n = 0$ to 3
$x(n) = \{1, 0, -1, 0\}$



Given $x(n) = n+1$ for $0 \leq n \leq 7$ Find $X(K)$ using DIF – FFT



Unit - II infinite impulse response filters.

* characteristics of practical freq. selective filters.
* characteristeristics of commonly used analog filters.
* Butterworth filters
* Chebyschev' filter
* Design of IIR filters from analog filters (LPF, HPF, BPF, BRF).
* Approximation Derivatives.
* Impulse invariance method.
* Bilinear transformation
* Freq transformation in the analog domain.
* structure of IIR filter - direct form - direct form II
* cascade
* parallel realizations.

Filter :-

* Filters are used to reduce or remove the unwanted signals present at the o/p.
* Two types
  * Analog filter
  * Digital filter.

Analog filter	Digital filter
* Used for filtering analog sequence	Used for filtering Digital sequence.
* Designed with various components like R, L, C.	Designed with digital hardware like FF, counter, shift regs. Using softwares like C, C++, MATLAB etc.
* Less accuracy	More accuracy
* More sensitive to environmental changes	Less sensitive to environmental changes.
* Less flexible	More flexible.
* multiple operation is very difficult.	multiple operations can be performed.

*

Types of ~~digital~~ filter

* FIR - finite impulse response filter
* IIR - infinite " " "

Two types of Analog filter design

* Butterworth filter
* Chebyshev filter

Analog ~~LP~~ Butterworth LPF is

$$H(j\Omega) = \frac{1}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}}$$

$N = 1, 2, 3 \ldots$

N — order of the filter

Ω_c — cut off freq.

$\Omega < \Omega_c \Rightarrow |H(j\Omega)| = 1$

$\Omega > \Omega_c \Rightarrow |H(j\Omega)|$ decreases

$\Omega = \Omega_c$

$|H(j\Omega)| = -3\,dB$, the curve passes thro' 0.707 which corresponds to $-3\,dB$ pt.

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$= \frac{1}{1+\Omega^{2N}}$$

To ensure stability, put

$$\Omega = \frac{s}{j} \text{ (or) } s = j\Omega$$

$$|H(s)|^2 = \frac{1}{1+\left(\frac{s}{j}\right)^{2N}}$$

$$H(s) \cdot H(-s) = \frac{1}{1+(-js)^{2N}}$$

* To find poles equate Dr = 0

$$1+(-js)^{2N} = 0 \quad , \quad 1 - s^{2N} = 0$$

When N is $\Rightarrow$ odd, $s^{2N} = +1 = e^{j2\pi k}$

N is $\Rightarrow$ even, $s^{2N} = -1 = e^{j(2k-1)\pi}$

odd $\Rightarrow$ $S_k = e^{j2\pi k/N}$ $k = 1, 2, \ldots 2N$

even $\Rightarrow$ $S_k = e^{j(2k-1)\pi/2N}$ $k = 1, 2, \ldots 2N$

$N = 3$

$S_k = e^{j2\pi k/3}$ $k = 1, 2, \ldots 6$

$S_{\cancel{k}} = e^{j2\pi/3}$

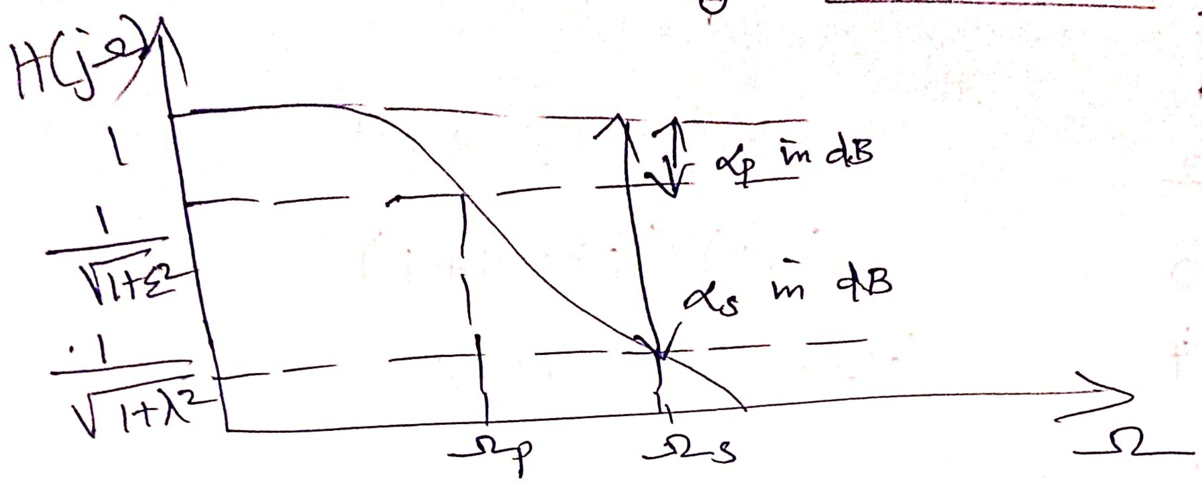
$= 0.5 + j0.866$

$S_2 = e^{j4\pi/3} = -0.5 + j0.866$

$S_3 = e^{j3\pi} = -1$

$S_4 = e$

Design of an Analog Butterworth filter



Response of LPF.

α_p – maximum pass band attenuation in dB

α_s – minimum stop " " in dB

Ω_p – pass band freq in radian/sec

Ω_s – stop band " in "

Ω_c – cutoff freq $= 2\pi f_c$

Steps to design an analog Butterworth LPF

(i) Find the order of the filter N

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

(ii) Roundoff the order to the next higher integer.

(iii) Find the transfer function H(S) for the value of N

$$H(s) = \frac{N(s)}{D(s)}$$

N	Denominator polynomial of H(S)
1	$s+1$
2	$s^2+\sqrt{2}\,s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.61803s+1)(s^2+1.61803s+1)$

(iv) Calculate the cut off freq:

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{1/2N}} \quad (\text{or}) \quad \frac{\Omega_s}{\left(10^{0.1\alpha_s}-1\right)^{1/2N}}$$

(v) Find the transfer function Ha(s) of an analog butterworth filter for Ω_c

$$H_a(s) = H(s)\Big|_{s \to s/\Omega_c}$$

problem ① Design an analog Butterworth filter that has -2 dB pass band attenuat at a freq 20 rad/sec and atleast -10 Stop band attenuation at 30 rad/sec.

Given $\alpha_p = 2\,dB$ $\quad \alpha_s = 10\,dB$

$\Omega_p = 20$ rad/sec $\quad \Omega_s = 30$ rad/sec.

(i) order of the filter

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\times10}-1}{10^{0.1\times2}-1}}}{\log\left(30/20\right)}$$

$$N \geq 3.37$$

(ii) Round off the order to the next higher integer

$$N = 4$$

(iii) The transfer fn for $N=4$ & $\Omega_c = 1$

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.84775+1)}$$

(iv) calculate the Ω_c

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}} = \frac{\Omega_s}{(10^{0.1\alpha_s}-1)^{1/2N}}$$

$$= \frac{20}{(10^{0.1\times2}-1)^{1/8}} = \frac{30}{(10^{0.1\times10}-1)^{1/8}}$$

$$= \frac{20}{0.935} = 21.39$$

(V) The transfer function can be obtained by sustituting

$$s \to s/\Omega_c \text{ in } H(s)$$

$$s \to \frac{s}{21.39}$$

$$H_a(s) = \frac{1}{\left(\left(\frac{s}{21.39}\right)^2 + 0.76537\left(\frac{s}{21.39}\right) + 1\right)\left[\left(\left(\frac{s}{21.39}\right)^2 + 1.8477\left(\frac{s}{21.39}\right) + 1\right]\right.}$$

$$H_a(s) = \frac{209835.53}{\left(s^2 + 16.3934s + 457.532\right)\left(s^2 + 39.5216s + 457.532\right)}$$

IIR filter design by bilinear Transformation

* $$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

(or)

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

* s-plane into z-plane mapping

$$s = \sigma + j\Omega$$

$$z = re^{j\omega}$$

$$s = \frac{2}{T}\left(\frac{re^{j\omega}-1}{re^{j\omega}+1}\right)$$

$$e^{j\omega} = \cos\omega + j\sin\omega.$$

$$s = \frac{2}{T}\left(\frac{r(\cos\omega + j\sin\omega)-1}{r(\cos\omega + j\sin\omega)+1}\right)$$

Seperating the real & imaginary parts

$$s = \frac{2}{T}\left(\frac{r^2-1}{1+r^2+2r\cos\omega} + j\,\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right)$$

$$\sigma = \frac{2}{T} \cdot \frac{r^2 - 1}{1 + r^2 + 2r\cos\omega}$$

$$\Omega = \frac{2}{T} \frac{2r\sin\omega}{1 + r^2 + 2r\cos\omega}$$

$r > 1$, $\sigma > 0 \Rightarrow$ R.H.S of s-plane maps outside the unit circle.

$r < 1$ $\sigma < 0 \Rightarrow$ LHS of s-plane maps inside the unit circle.

$r = 1$ $\sigma = 0 \Rightarrow j\Omega$ axis in s-plane maps on the unit circle.

<u>Frequency</u> <u>warping :-</u>

The imaginary axis $j\Omega$ is mapped on unit circle

$$\Omega = \frac{2}{T} \frac{2r\sin\omega}{1 + r^2 + 2r\cos\omega}$$

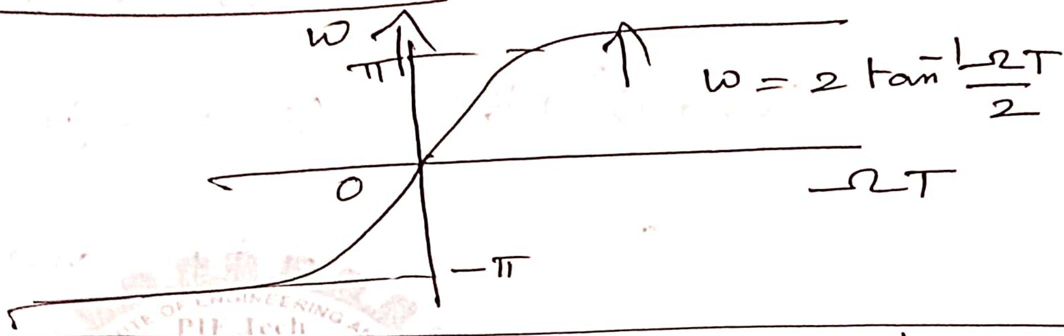
$r = 1$

$$\Omega = \frac{2}{T} \frac{2\sin\omega}{1 + 1 + 2\cos\omega}$$

$$= \frac{2}{T} \frac{\cancel{2}\sin\omega}{\cancel{2}(1 + \cos\omega)}$$

$$= \frac{2}{T} \; \frac{\cancel{2} \sin \frac{w}{2} \; \cancel{\cos \frac{w}{2}}}{\cancel{2} \cancel{\cos^2 \frac{w}{2}}}$$

$$\boxed{\Omega = \frac{2}{T} \tan \frac{w}{2}}$$



Impulse invariant	Bilinear transform
1. $\frac{1}{s - P_k} \rightarrow \frac{1}{1 - e^{P_k T} z^{-1}}$	$s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$
2. There is aliasing in freq. domain	There is no aliasing.
3 $\omega = \Omega T$	$\omega = 2 \tan^{-1} \frac{\Omega T}{2}$
4. freq. relationship is Linear	is non Linear
5. Maps only poles	Maps poles and zero's.
6. used for LPF	used for all types of filter.

(1) The system fn of the analog filter is given as $H_a(s) = \dfrac{s+0.1}{(s+0.1)^2+16}$

obtain the digital filter using bilinear transformation which is resonant $\omega_r = \pi/2$

$$(s+0.1)^2+16 = (s+0.1-j4)(s+0.1+j4)$$

$$s = -0.1+j4 \quad \text{and} \quad s = -0.1-j4$$

$$\sigma = -0.1 \qquad \Omega = \pm 4$$

$$s = -0.1 \pm 4j$$

$$T = \frac{2}{\Omega} \tan \frac{\omega}{2}$$

$$= \frac{2}{4} \tan \frac{\pi/2}{2}$$

$$= \frac{1}{2} \tan \pi/4$$

$$\boxed{T = 1/2} \quad \& \quad \boxed{\Omega = 4} \quad \boxed{\omega_r = \pi/2}$$

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$T = 1/2$ $\quad s = 4\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$

$$H(z) = \frac{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.4}{\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.1\right]^2 + 16}$$

$$= \frac{\frac{4(1-z^{-1}) + 0.4(1+z^{-1})}{1+z^{-1}}}{16\left(\frac{(1-z^{-1})^2}{(1+z^{-1})^2}\right) + 0.2 + 2\times 4 \times 0.1\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 16}$$

$$= \frac{\frac{4 - 4z^{-1} + 0.4 + 0.4z^{-1}}{1+z^{-1}}}{\frac{16(1-z^{-1})^2}{(1+z^{-1})^2} + 16.2 + 0.8\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{\frac{4.4 - 3.6z^{-1}}{\cancel{1+z^{-1}}}}{\frac{16(1-z^{-1})^2 + 16.2(1+z^{-1})^2 + 0.8\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{(1+z^{-1})^2}}$$

$$\frac{4.4 - 3.6\, z^{-1}(1+z^{-1})}{16(1-z^{-1})^2 + 16.2(1+z^{-1})^2 + 0.8(1-z^{-1})(1+z^{-1})}$$

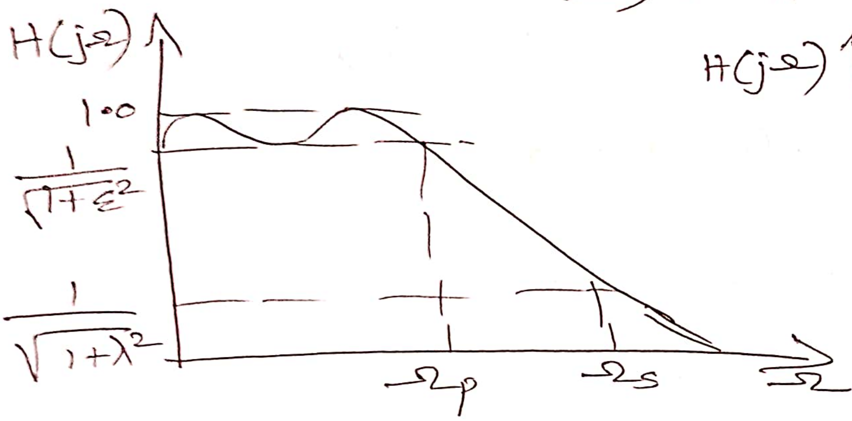
$$H(z) = \frac{0.128 \cancel{+} 0.006\, z^{-1} - 0.122\, z^{-2}}{1 + 0.0006\, z^{-1} + 0.975\, z^{-2}}$$

Chebyshev filters :-

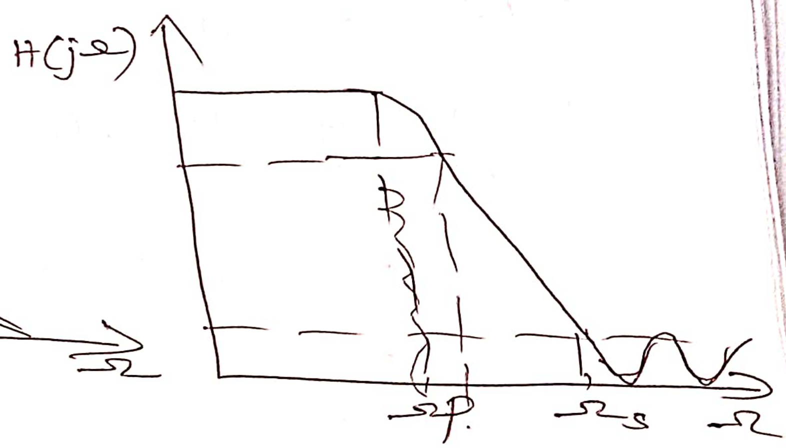
* Two types

① Type 1 chebyshev filters
$\Rightarrow$ all pole filters.

② Type 2 " "
$\Rightarrow$ both poles and zero's.



Type I



Type II

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)} \qquad N = 1, 2 \ldots$$

ε – parameter of filter related to the ripple in the pass band

$C_N(x)$ – N^{th} order chebyshev polynomial

$C_N(x) = \cos(N\cos^{-1}x)$ $|x| \leq 1$ pass band

$C_N(x) = \cosh(N\cosh^{-1}x)$ $|x| > 1$ stopband

Steps to design an analog chebyshev LPF :-

* Find the order of the filter N

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\Omega_s/\Omega_p}$$

* Round off it to next higher integer.

* Using the formula find the value of a & b

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2}+1}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_s}-1}$$

∴ For normalised chebyshev filter $\Omega_p = 1$ rad/sec.

$$a = \Omega_p \left\{ \frac{\mu^{1/N} - \mu^{-1/N}}{2} \right\}$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right]$$

* Calculate the poles of chebyshev filter which lies on the ellipse

$$s_k = a\cos\phi_k + jb\sin\phi_k \qquad k = 1, 2, \ldots N$$

$$\phi_k = \pi/2 + \left(\frac{2k-1}{2N}\right)\pi \qquad k = 1, 2 \ldots N$$

* Find the denominator polynomial of the transfer fn. using above poles

* The Nr of the T.F depends on value of N.

For N - odd substitute $s = 0$ in Dr polynomial. and find the value

The value is equal to Nr.

$\Rightarrow$ For N even substitute $s = 0$ in Dr polynomial & divide the result by $\sqrt{1+\varepsilon^2}$. This value equal to Nr.

(1) Design a chebyshev filter with max. pass band attenuation of 2.5 dB at $\Omega_p = 20$ rad/sec & the stop band attenuation of 30 dB and stop band freq $\Omega_s = 50$ rad/sec.

$\Omega_p = 20$ rad/sec $\quad \alpha_p = 2.5$ dB

$\Omega_s = 50$ " $\quad \alpha_s = 30$ dB

Step 1. Find the order of the filter

$$N = \frac{\cosh^{-1}[\lambda/\varepsilon]}{\cosh^{-1}\left[\frac{\Omega_p}{\Omega_s}\right]}$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = \sqrt{10^{0.1 \times 50} - 1} = 31.607$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = \sqrt{10^{0.1 \times 2.5} - 1} = 0.882$$

$$N = \frac{\cosh^{-1}\left(\frac{31.607}{0.882}\right)}{\cosh^{-1}\left[\frac{20}{50}\right]} = 2.726$$

Step 2:- Round off $N = 3$

Step 3:-

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.65$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 6.6$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 21.06$$

$$S_k = a\cos\phi_k + jb\sin\phi_k \qquad k = 1,2,3$$

$$\phi_k = \pi/2 + \left(\frac{2k-1}{2N}\right)\pi$$

$$\phi_1 = 120^\circ \quad \phi_2 = 180^\circ \quad \phi_3 = 240$$

$$S_1 = -3.3 + j\,18.23$$

$$S_2 = -6.6$$

$$S_3 = -3.3 - j\,18.23$$

$$D_0 \quad H(s) = (S + 0.66)(s^2 + 6.6s + 343.2)$$

$$N_r \text{ of } H(s) = 2265.27$$

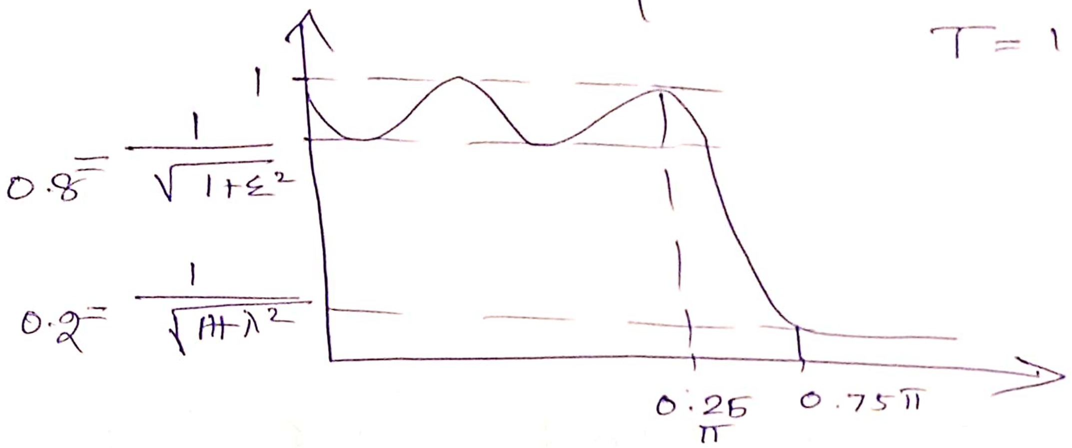
$$H(s) = \frac{2265.27}{(S+0.66)(s^2+6.6S+343.2)}$$

Design the chebyshev digital filter to satisfy the following constraints

$$0.8 \leq |H(\omega)| \leq 1 \quad \text{for} \quad 0 \leq \omega \leq 0.25\pi$$

$$|H(\omega)| \leq 0.2 \quad 0.75\pi \leq \omega \leq \pi$$

$T = 1$ sec.



$$\Omega_p = \frac{\omega_p}{T} = \frac{0.25\pi}{1} = 0.25$$

$$\Omega_s = \frac{\omega_s}{T} = \frac{0.75\pi}{1} = 0.75\pi$$

$$\alpha_p = 0.8$$

$$\alpha_s = 0.2$$

Step 1 :- Find the order of the filter

$$N \geq \frac{\cosh^{-1}\left[\sqrt{\lambda/\varepsilon}\right]}{\cosh^{-1}\left[\Omega_s/\Omega_p\right]}$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = \sqrt{10^{0.1 \times 0.2} - 1}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = \sqrt{10^{0.1 \times 0.8} - 1}$$

$N \geq 1.453$

Step 2:-

$$\boxed{N = 2}$$

Step 3:-

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 3$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.4534$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.906$$

Step 4:-

$$S_K = a \cos\phi_K + j b \sin\phi_K$$

$$\phi_K = \frac{\pi}{2} + \left(\frac{2K-1}{2N}\right)\pi$$

$$\phi_1 = \frac{\pi}{2} + \frac{1}{4}\pi = \frac{3\pi}{4} = 135^\circ$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = \frac{5\pi}{4} = 225^\circ$$

$$S_1 = 0.4534 \cos 135^\circ + j0.906 \sin 135^\circ$$

$$= -0.3206 + j0.6406$$

$$S_2 = 0.4534 \cos 225^\circ + j0.906 \sin 225^\circ$$

$$= -0.3206 - j0.6406.$$

Step 5 :-

$$H(s) = \frac{1}{(s-s_1)(s-s_2)}$$

$$= \frac{1}{(s+0.3206-j0.6406)(s+0.3206+j0.6406)}$$

Since N is even $= \frac{1}{s^2+0.641s+0.5131}$

put $s=0$ in denominator

$$\therefore K = \frac{0.5131}{\sqrt{1+\varepsilon^2}} = 0.4105$$

$$H(s) = \frac{0.4105}{s^2+0.641s+0.5131}.$$

Step 5 :- $H(s)$ into $H(z)$

$$H(s) = \frac{0.4105}{s^2+0.641s+0.5131}$$

Using partial fraction method

$$= \frac{A}{s+0.3206+j0.6406} + \frac{B}{s+0.3206-j0.640}$$

$$A = \frac{1}{s+0.3206+j0.6406}\Bigg|_{s=-0.3206+0.j6406}$$

$A = j0.32$

$B = A^* = -j0.32$

$$= \frac{j0.32}{s + 0.3206 + j0.6406} + \frac{(-j0.32)}{s + 0.3206 - j0.6406}$$

Applying impulse invariant method

$$\frac{1}{s - p_K} = \frac{1}{1 - e^{p_K T} z^{-1}}$$

$$H(z) = \frac{j0.32}{1 - e^{-0.3206 - j0.6406} z^{-1}} - \frac{j0.32}{1 - e^{-0.3206 + j0.6406} z^{-1}}$$

$$H(z) = \frac{0.277 z^{-1}}{1 - 1.163 z^{-1} - 0.527 z^{-2}}$$

IIR filter structure are

1. Direct form - I
2. Direct form - II
3. cascade form
4. parallel form
5. Lattice -ladder structure.

Direct form I :-

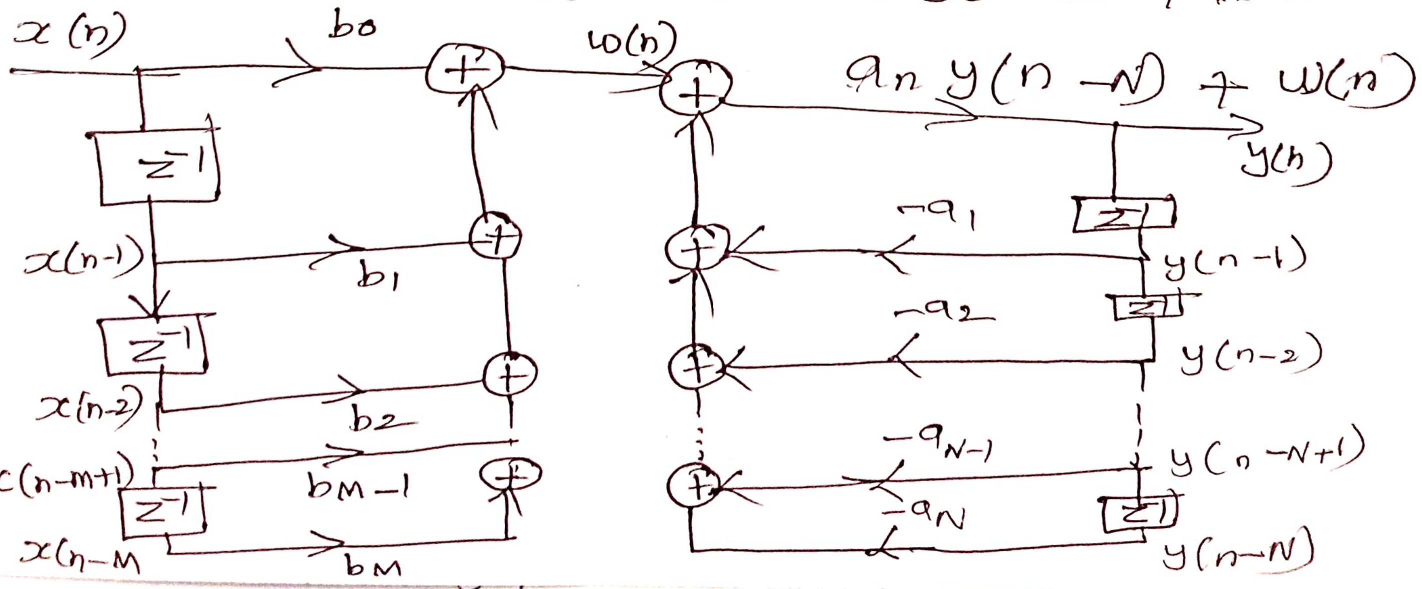
$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$$= -a_1 y(n-1) - a_2 y(n-2) - \cdots - a_{N-1} y(n-N+1) - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M)$$

Let

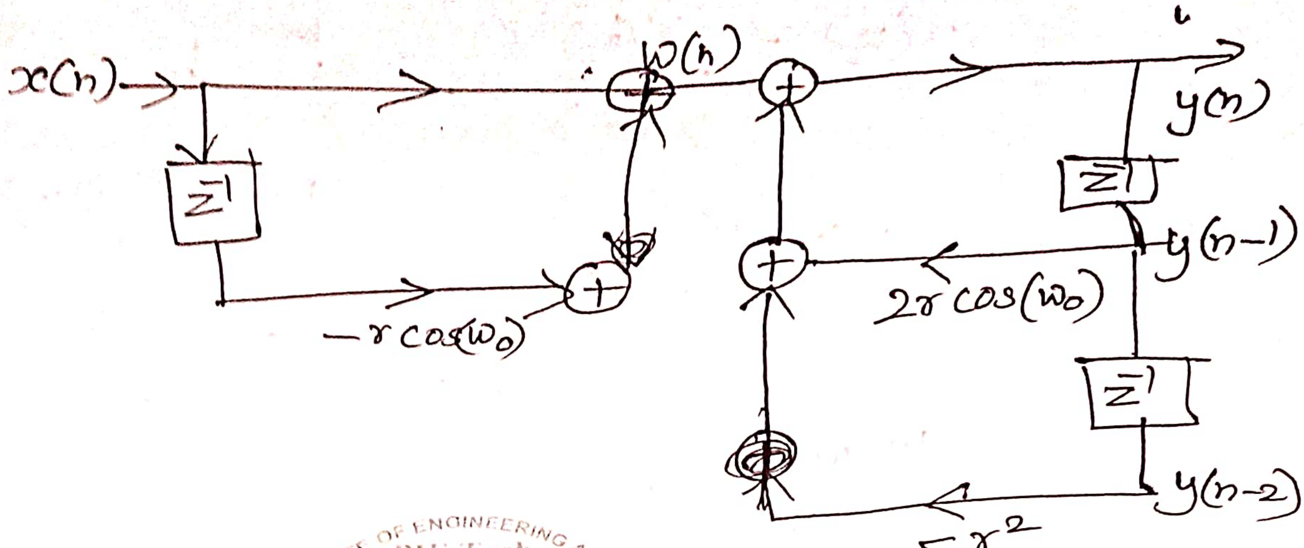
$$b_0 x(n) + b_1 x(n-1) + \cdots b_M x(n-M) = w(n)$$

$$\therefore y(n) = -a_1 y(n-1) - a_2 y(n-2) + \cdots a_N y(n-N) + w(n)$$



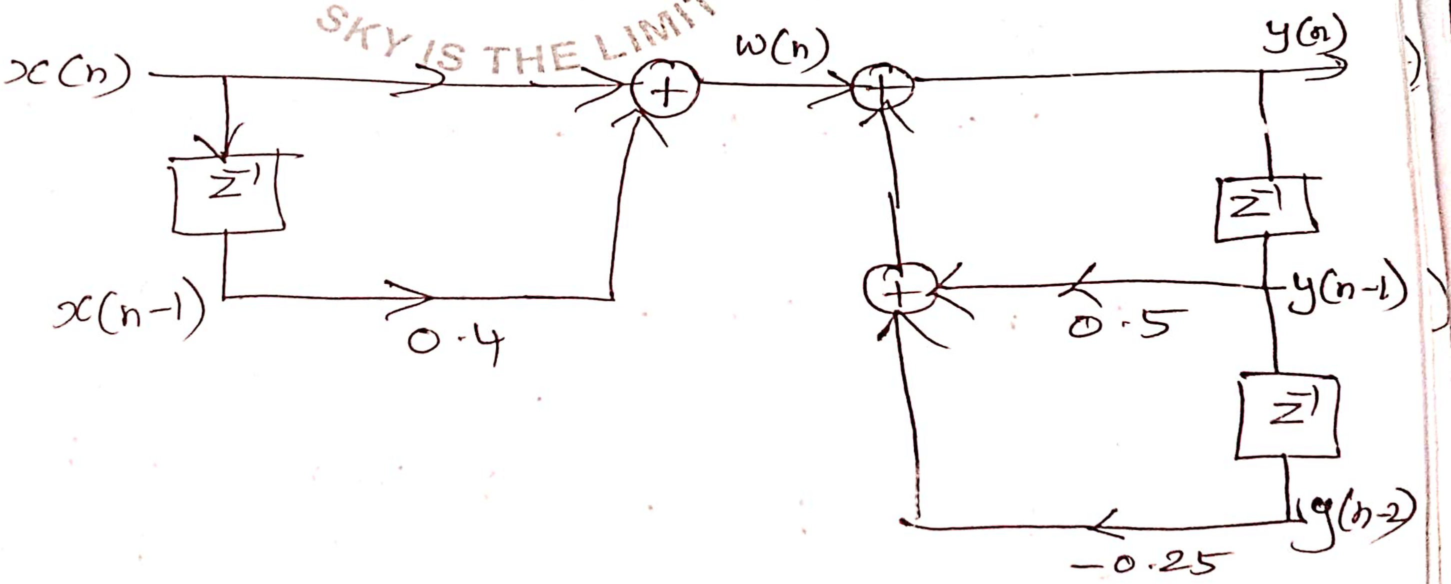
① Realize The 2nd order digital filter

$$y(n) = 2r\cos(w_0)\, y(n-1) - r^2 y(n-2) + x(n) - r\cos(w_0)\, x(n-1)$$



② obtain the direct form-II realization for the system described by difference eqn.

$$y(n) = 0.5\, y(n-1) - 0.25\, y(n-2) + x(n) + 0.4\, x(n-1)$$



Direct form - II :

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k\, x(n-k)}{1 + \sum_{k=0}^{N} a_k\, y(n-k)}$$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=0}^{N} a_k\, y(n-k)}$$

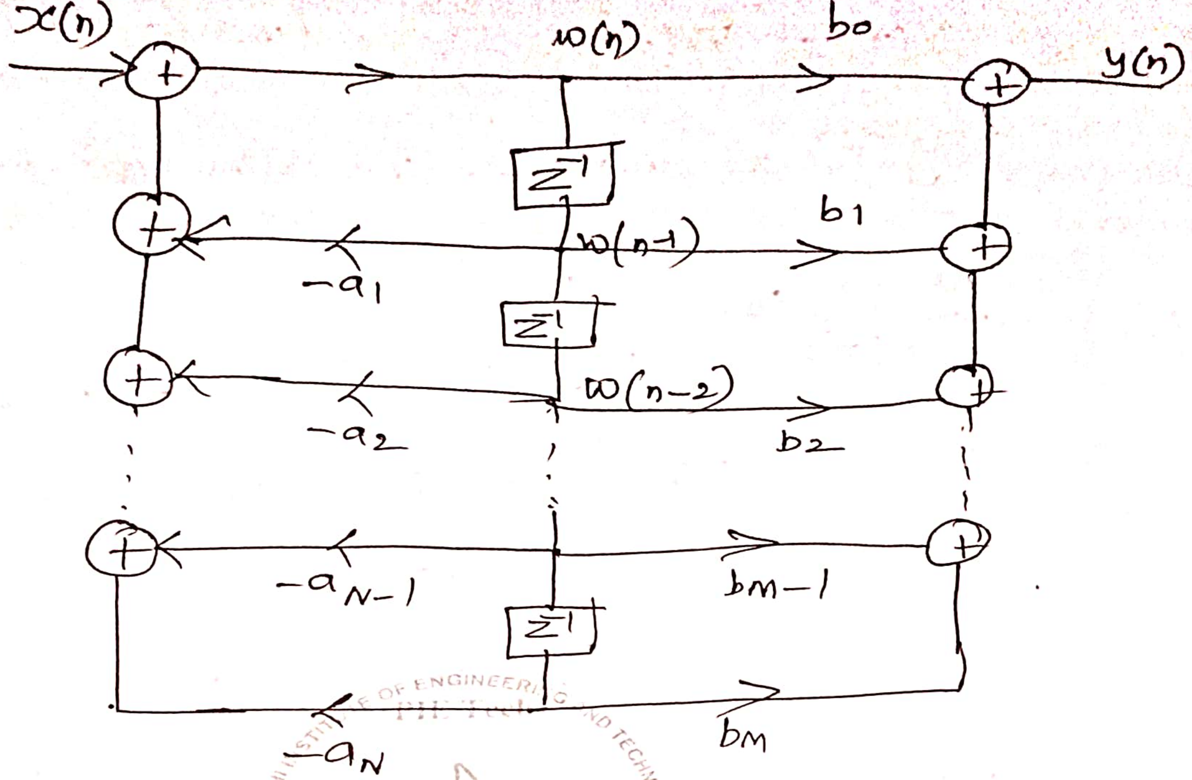
$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) + \cdots - a_N z^{-N} W(z)$$

$$\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k\, x(n-k)$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + \cdots b_M z^{-N} W(z)$$

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \cdots - a_N w(n-N)$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \cdots b_M w(n-N)$$



1. Realize the second order system

$$y(n) = 2r\cos(\omega_0)\, y(n-1) - r^2 y(n-2) + x(n) - r\cos(\omega_0)\, x(n-1)$$

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos\omega_0\, z^{-1}}{1 - 2\cos(\omega_0)\, z^{-1} - r^2 z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = 1 - r\cos(\omega_0)\, z^{-1}$$

$$W(z)/X(z) = \frac{1}{1 - 2\cos(\omega_0)\, z^{-1} - r^2 z^{-2}}$$

$$Y(z) = W(z) - r\cos(w_0) z^{-1} W(z)$$

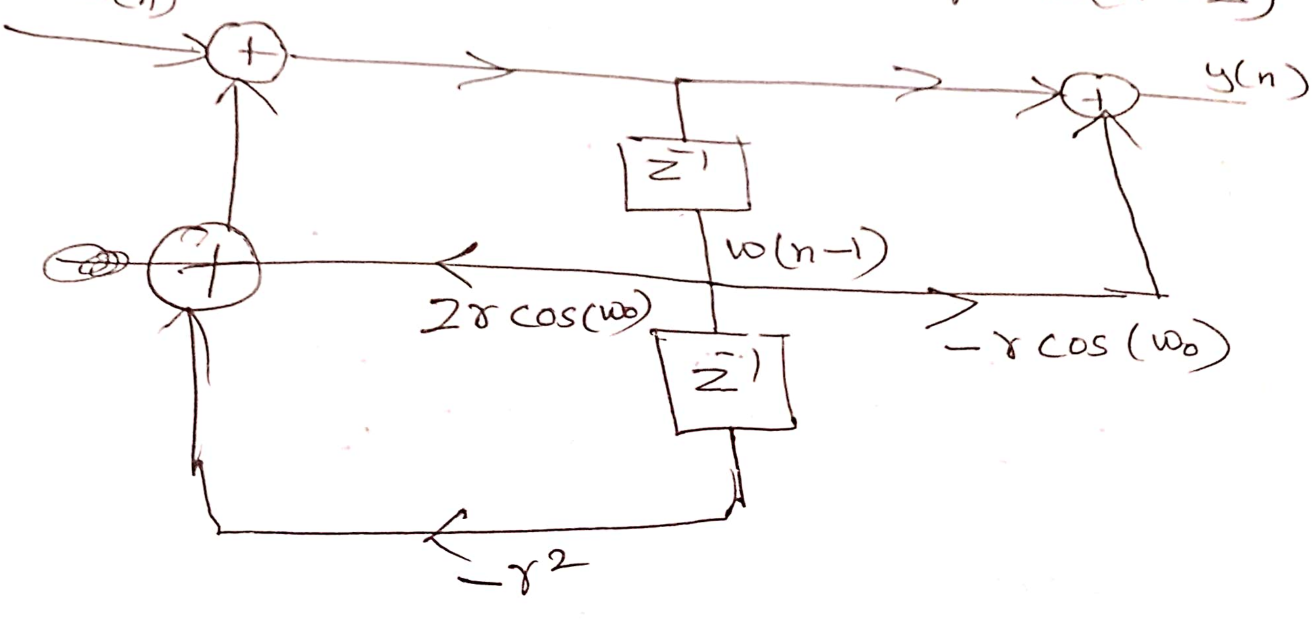
$$y(n) = w(n) - r\cos(w_0) w(n-1)$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - 2r\cos(w_0) z^{-1} + r^2 z^{-2}}$$

$$W(z) - 2r\cos(w_0) z^{-1} W(z) + r^2 z^{-2} W(z) = X(z)$$

$$W(z) = X(z) + 2r\cos(w_0) z^{-1} W(z) - r^2 z^{-2} W(z)$$

$$w(n) = x(n) + 2r\cos(w_0) w(n-1) - r^2 w(n-2)$$



$S + 0.3206 + j0.0400$ | ... +0.1

(2) Determine the direct form-II realization for the following system

$$y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$$

$$Y(z) = -0.1\,Y(z)z^{-1} + 0.72\,Y(z)z^{-2} + 0.7\,X(z) - 0.252\,X(z)z^{-2}$$

$$Y(z) + 0.1\,Y(z)z^{-1} - 0.72\,Y(z)z^{-2} = 0.7\,X(z) - 0.252\,X(z)z^{-2}$$

$$Y(z)\left[1 + 0.1z^{-1} - 0.72z^{-2}\right] = X(z)\left[0.7 - 0.252z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$\frac{Y(z)}{W(z)} = 0.7 - 0.252\, z^{-2}$$

$$Y(z) = 0.7\, W(z) - 0.252\, z^{-2} W(z)$$

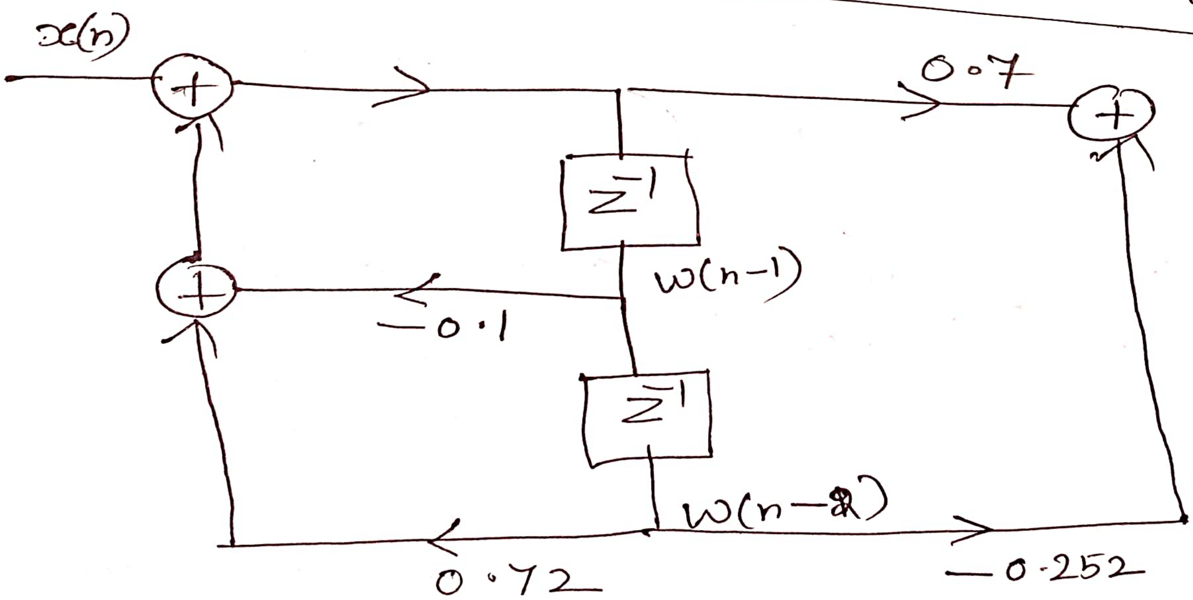
$$\boxed{y(n) = 0.7\, w(n) - 0.252\, w(n-2)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + 0.1\, z^{-1} - 0.72\, z^{-2}}$$

$$W(z) + 0.1\, z^{-1} W(z) - 0.72\, z^{-2} W(z) = X(z)$$

$$W(z) = X(z) - 0.1\, z^{-1} W(z) - 0.72\, z^{-2} W(z)$$

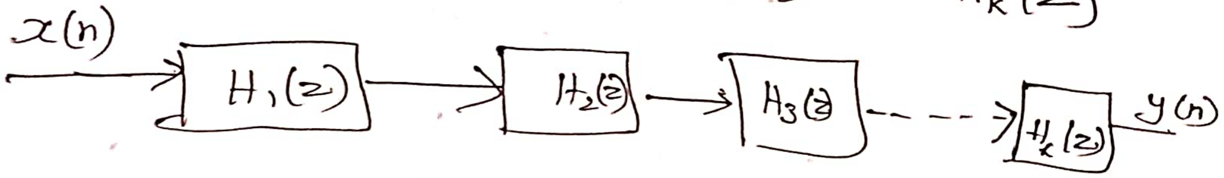
$$\boxed{w(n) = x(n) - 0.1\, w(n-1) + 0.72\, w(n-2)}$$



Cascade form :-

Let us consider IIR system with system function

$$H(z) = H_1(z)\, H_2(z) \cdots H_K(z)$$



Now realize each $H_K(z)$ in direct form $-II$ and cascade all structures.

For eg :- Let us take a system whose transfer fun.

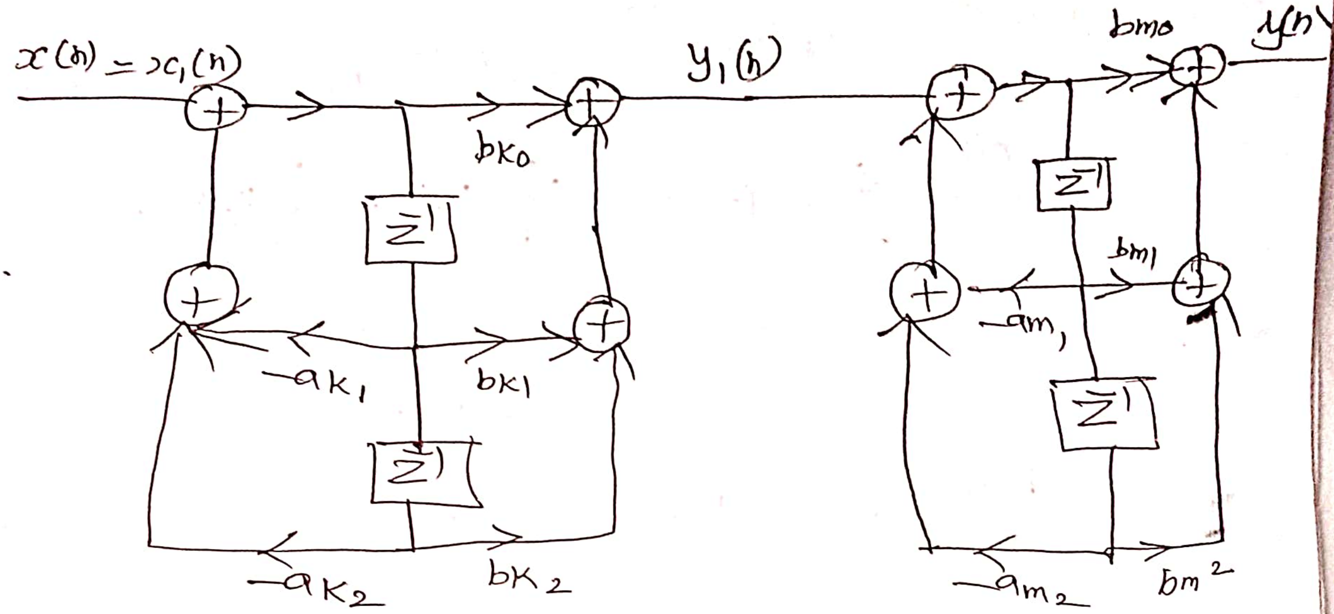
$$H(z) = \frac{(b_{K0} + b_{K1} z^{-1} + b_{K2} z^{-2})(b_{m0} + b_{m1} z^{-1} + b_{m2} z^{-2})}{(1 + a_{K1} z^{-1} + a_{K2} z^{-2})(1 + a_{m1} z^{-1} + a_{m2} z^{-2})}$$

$$= H_1(z)\, H_2(z)$$

$$H_1(z) = \frac{b_{K0} + b_{K1} z^{-1} + b_{K2} z^{-2}}{1 + a_{K1} z^{-1} + a_{K2} z^{-2}}$$

$$H_2(z) = \frac{b_{m0} + b_{m1} z^{-1} + b_{m2} z^{-2}}{1 + a_{m1} z^{-1} + a_{m2} z^{-2}}$$

Realizing $H_1(z)$ and $H_2(z)$ in direct form II and cascading, we obtain cascade form.



(1) Realize the system with difference equation $y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$ in cascade form.

$$Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} Y(z) z^{-2} + X(z) + \frac{1}{3} z^{-1} X(z)$$

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} Y(z) z^{-2} = X(z) \left[1 + \frac{1}{3} z^{-1}\right)$$

$$Y(z)\left[1-\tfrac{3}{4}z^{-1}+\tfrac{1}{8}z^{-2}\right] = X(z)\left[1+\tfrac{1}{3}z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$\times$ $\quad +$

$\frac{1}{8}$ $\quad -\frac{3}{4}$

$-\frac{1}{2} \times -\frac{1}{4}$ $\quad -\frac{3}{4}$

$$= H_1(z)\, H_2(z)$$

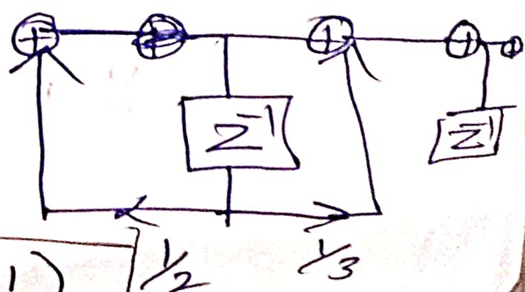
$$H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}, \quad H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$H_1(z)$ can be realized in direct form II

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\frac{Y(z)}{W(z)} = 1+\frac{1}{3}z^{-1}$$

$$Y(z) = W(z) + \frac{1}{3}z^{-1}$$

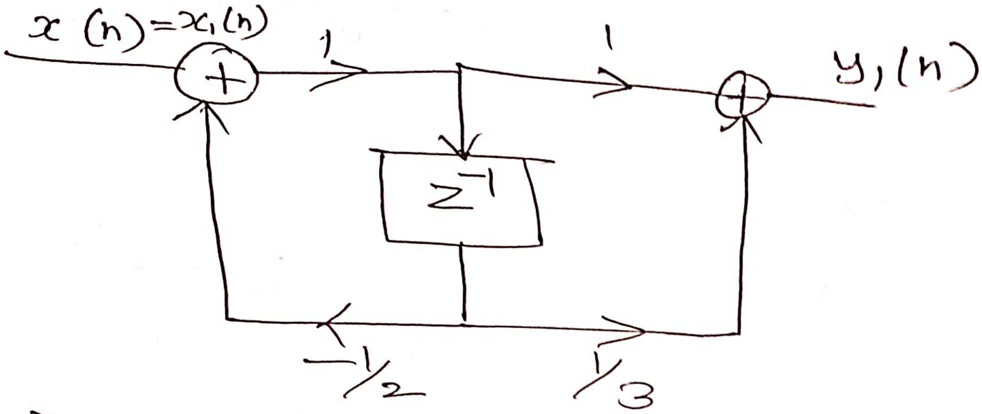


$$\boxed{y(n) = w(n) + \frac{1}{3}w(n-1)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

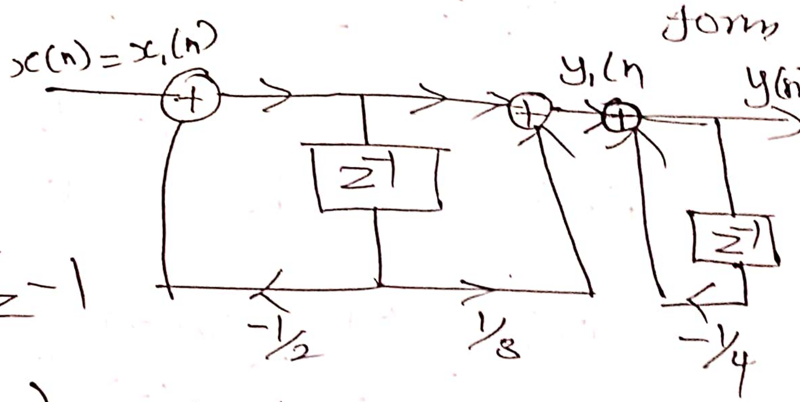
$$\cancel{X}(z) = W(z) - \frac{1}{2} z^{-1} W(z)$$

$$x(n) = w(n) - \frac{1}{2} w(n-1)$$



$$H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$ can be realized in direct-II form

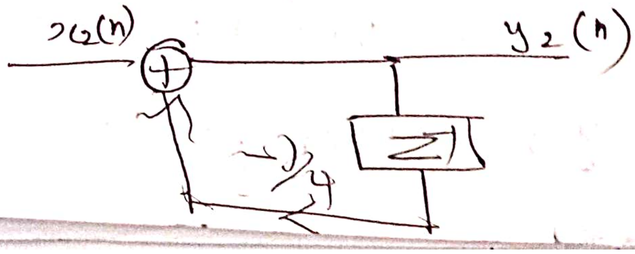
$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$



$$\frac{Y(z)}{W(z)} = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$Y(z) - \frac{1}{4} z^{-1} Y(z) = W(z)$$

$$y(n) - \frac{1}{4} y(n-1) = w(n)$$



Parallel form :-

$$H(z) = c + \sum_{k=1}^{N} \frac{C_k}{1 - P_k z^{-1}}$$

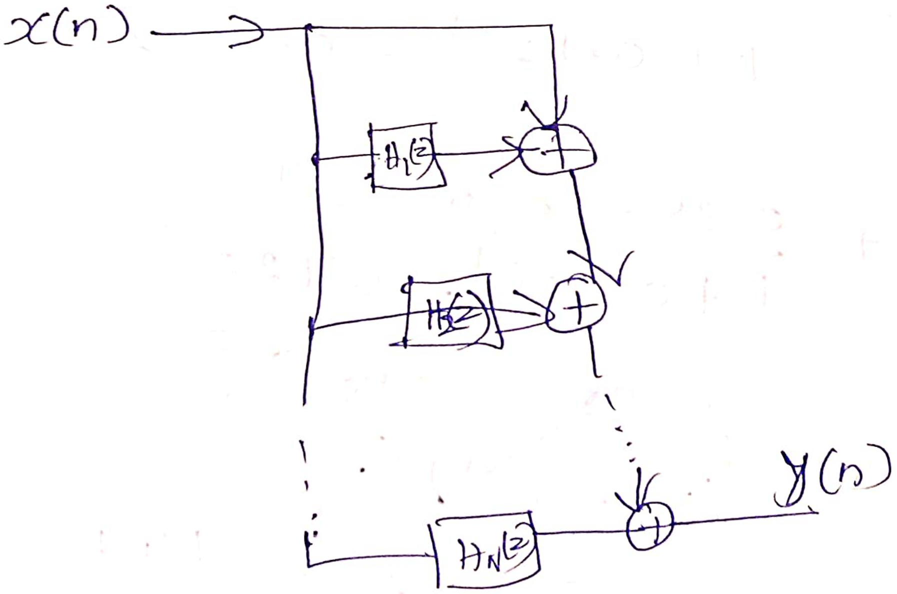
$\{P_k\}$ are the poles

$$H(z) = c + \frac{C_1}{1 - P_1 z^{-1}} + \frac{C_2}{1 - P_2 z^{-2}} + \cdots$$

$$\frac{C_N}{1 - P_N z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = c + H_1(z) + H_2(z) + \cdots H_N(z)$$

$$Y(z) = c X(z) + H_1(z) X(z) + H_2(z) X(z) + \cdots H_N(z) X(z)$$



① Reduce the system given by difference equation

$$y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$$

Take z transform on both side

$$Y(z) = -0.1\,Y(z)z^{-1} + 0.72\,Y(z)z^{-2} + 0.7\,X(z) - 0.25\,X(z)z^{-2}$$

$$Y(z) + 0.1\,Y(z)z^{-1} - 0.72\,Y(z)z^{-2} = (0.7 - 0.25z^{-2})X(z)$$

$$Y(z)\left[1 + 0.1z^{-1} - 0.72z^{-2}\right] = \left[0.7 - 0.25z^{-2}\right]X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.25z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

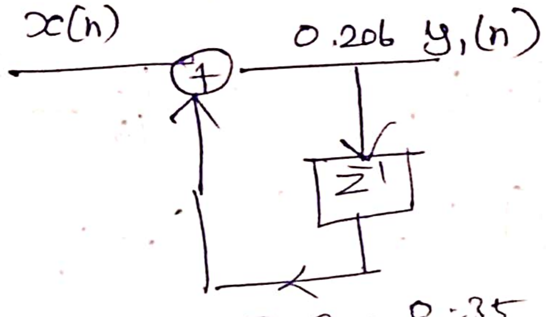
$$= 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

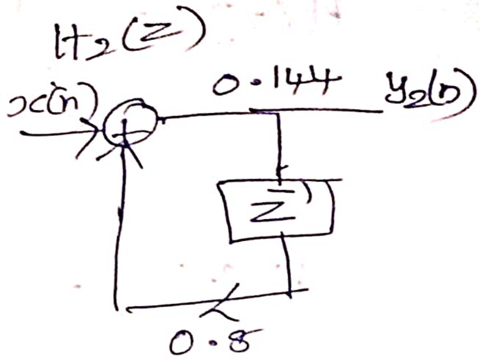
$$= \cancel{0.35} + \cancel{\frac{0.35 - 0.035z^{-1}}{(1+0.9z^{-1})(}}$$

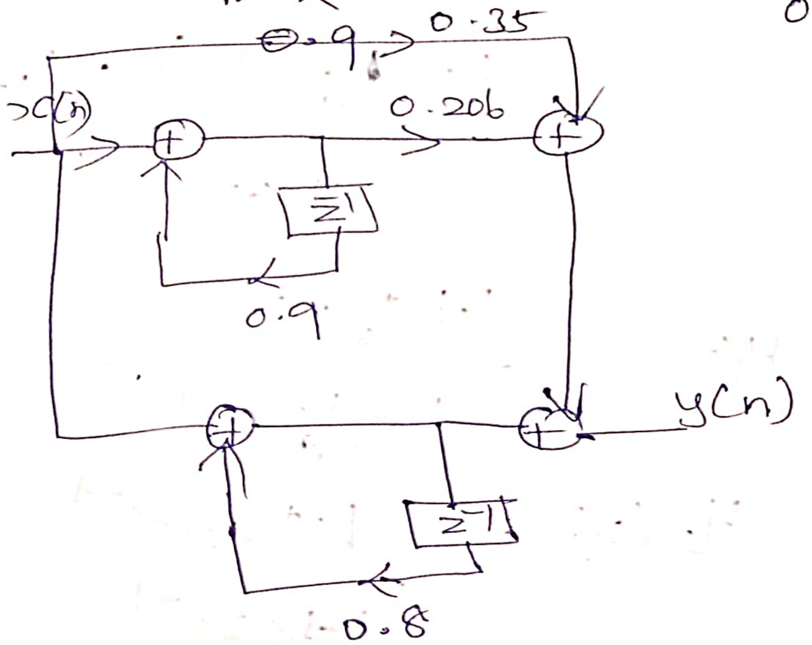
$$= 0.35 + \frac{0.206}{1+0.9z^{-1}} + \frac{0.144}{1-0.8z^{-1}}$$

$$= C + H_1(z) + H_2(z)$$

$H_1(z)$ can be realized in direct form II as







cascade form $H(z) = H_1(z) \cdot H_2(z)$

$$H(z) = \frac{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}{1-\frac{5}{8}z^{-1}+\frac{1}{16}z^{-2}}$$

$\frac{1}{4} + \frac{1}{2}$

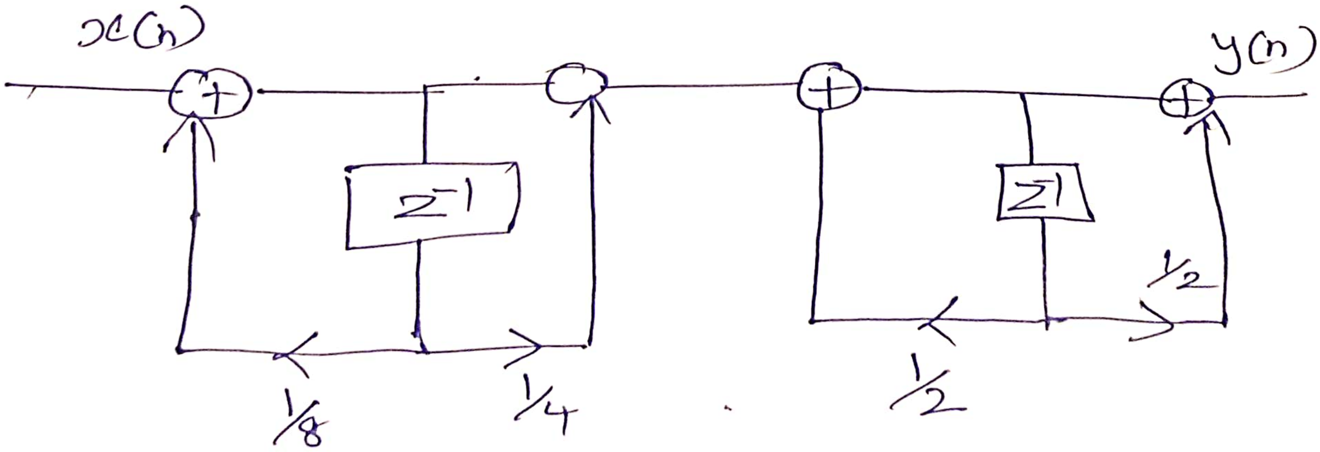
$-\frac{1}{8} + \frac{1}{2}$

$-\frac{5}{8}, \frac{1}{16}$

$$H(z) = \frac{\left(1+\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}{\underbrace{\left(1-\frac{1}{8}z^{-1}\right)}_{H_1(z)} \cdot \underbrace{\left(1-\frac{1}{2}z^{-1}\right)}_{H_2(z)}}$$

$$H_1(z) = \frac{1+\frac{1}{4}z^{-1}}{1-\frac{1}{8}z^{-1}}$$

$$H_2(z) = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}$$



Parallel form

$$y(n) - 3y(n-1) + 2y(n-2) = x(n)$$

Take z transform

$$Y(z) - 3Y(z)z^{-1} + 2Y(z)z^{-2} = X(z)$$

$$Y(z)\left[1 - 3z^{-1} + 2z^{-2}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 3z^{-1} + 2z^{-2}}$$

$$\boxed{H(z) = \frac{z^2}{z^2 - 3z^{+1} + 2}} = \frac{z^2}{(z-1)(z-2)}$$

IN PFE

$$H(z) = \frac{A}{z-1} + \frac{B}{z-2}$$

$$A = \frac{\cancel{(z-1)}\, z^2}{\cancel{(z-1)}(z-2)}\bigg|_{z=1} = \frac{1}{1-2} = -1$$

$$B = \frac{\cancel{(z-2)}\, z^2}{(z-1)\cancel{(z-2)}}\bigg|_{z=2} = \frac{1}{2-1} = 4$$

$$H(z) = \frac{-1}{z-1} + \frac{4}{z-2}$$

$$= \frac{-z^{-1}}{1-z^{-1}} + \frac{4z^{-1}}{1-2z^{-1}}$$

$$H(z) = H_1(z) + H_2(z)$$

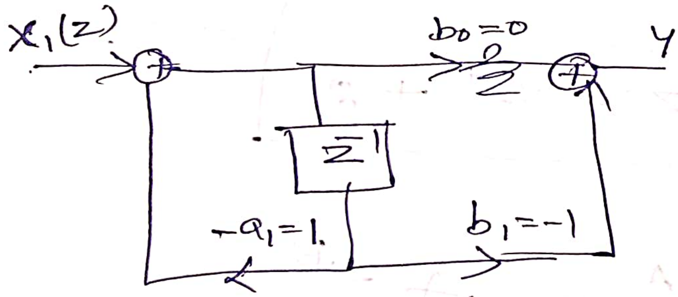
$$H_1(z) = \frac{-z^{-1}}{1-z^{-1}} \qquad H_2(z) = \frac{4z^{-1}}{1-2z^{-1}}$$

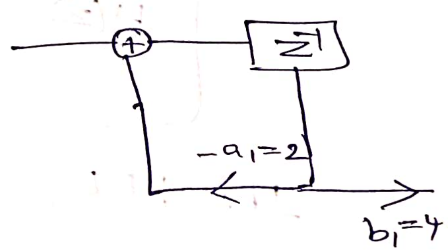
$b_0 = 0 \quad b_1 = -1$

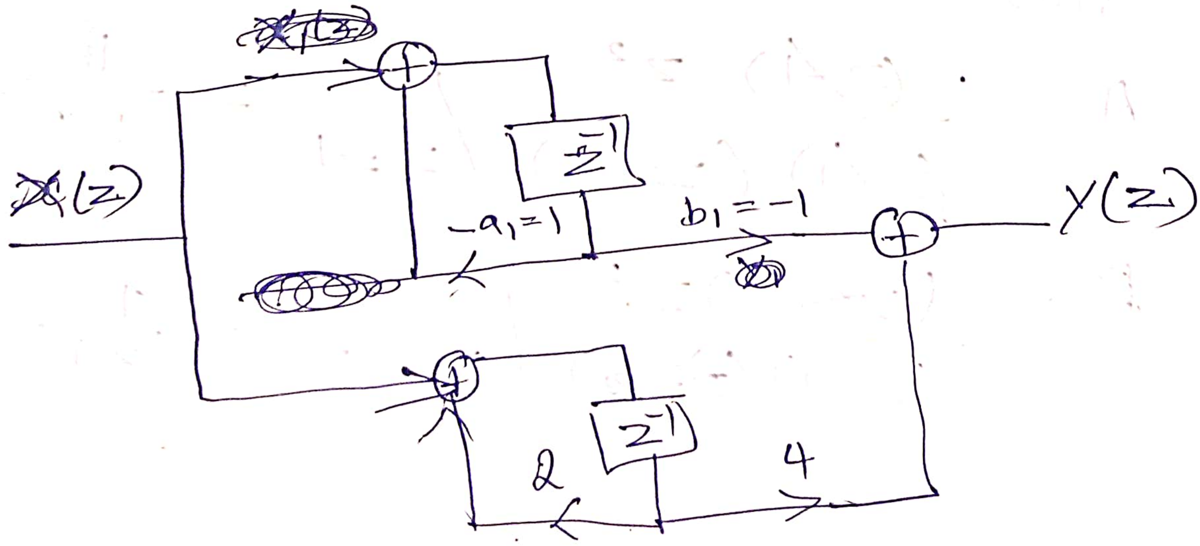
$a_1 = -1$

$b_0 = 0 \quad b_1 = 4$

$a_1 = -2$







① Design a digital butterworth filter Satisfying the constraints.

$$0.707 \leq |H(e^{j\omega})| \leq 1, \text{ for } 0 \leq \omega \leq \pi/2$$

$$|H(e^{j\omega})| \leq 0.2, \text{ for } 3\pi/4 \leq \omega \leq \pi$$

with $T=1$ sec using impulse invariance.

Sol:

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.707 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\frac{1}{0.707} = \sqrt{1+\varepsilon^2}, \qquad \frac{1}{0.2} = \sqrt{1+\lambda^2}$$

$$\left(\frac{1}{0.707}\right)^2 = 1+\varepsilon^2 \qquad \left(\frac{1}{0.2}\right)^2 = 1+\lambda^2$$

$$2.0006 = 1+\varepsilon^2 \qquad 25 = 1+\lambda^2$$

$$\varepsilon^2 = 2.0006-1 \qquad \lambda^2 = 25-1$$

$$\boxed{\varepsilon = 1} \qquad \boxed{\lambda = 4.899}$$

This parameter specifying allowable pass band

This specify allowable stop band.

Freq. of digital filter ω = ΩT ↳ freq. of analog filter.

$W_s = \Omega_s \qquad \omega_p = \Omega_p \qquad \Omega_p = \pi/2$

$T = 1$ sec.

$W_s = \Omega_s \times T = 3\pi/4 \qquad \Omega_s = 3\pi/4$

Step 1 :- $\omega_p = \Omega_p \times T = \pi/2$

Find the order of the filter.

$$N \geq \frac{\log\left(\frac{\cancel{A}}{\cancel{\varepsilon}}\right)}{\log\left(\cancel{\omega_s}/\cancel{\omega_p}\right)}$$

$$= \frac{\log\left(\frac{4.899}{1}\right)}{\log\left(\frac{3\pi/4}{\pi/2}\right)} = \frac{0.6901}{0.1761}$$

$$\boxed{N = 3.918}$$

Step 2 :- Rounding off the order of the filter to next highest integer :

$$\boxed{N = 4}$$

Step 3 :- Find the Tr. fn for $N=4$

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.84775 + 1)}$$

Step 4 :- find the cut off freq. Ω_c

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \frac{\pi/2}{(1)^{1/4}} = 1.57$$

$\Omega_c = 1.57$

Step 5 :- Find the Tr. fn $S \to S/\Omega_c$.

$$H_a(s) = \frac{1}{\left[\left(\frac{S}{1.57}\right)^2 + 0.76537\left(\frac{S}{1.57}\right) + 1\right]\left[\left(\frac{S}{1.57}\right)^2 + 1.8477\left(\frac{S}{1.57}\right) + 1\right]}$$

$$= \frac{1}{(0.4057S^2 + 0.487S + 1)(0.4057S^2 + 1.177S + 1)}$$

$$= \frac{1}{(S + 0.6008 \pm 1.450j)(S + 1.450 \pm 0.6008j)}$$

$$= \frac{1}{[(S + 0.6008 + 1.450j)(S + 0.6008 - 1.450j)][(S + 1.450 + 0.6008j)(S + 1.450 - 0.6008j)]}$$

$$= \frac{A}{(S + 0.6008 + 1.450j)} + \frac{A^*}{S + 0.6008 - 1.450j} + \frac{B}{(S + 1.450 + 0.6008j)} + \frac{B^*}{S + 1.450 - 0.6008j}.$$

Design a digital butterworth filter using impulse invariance method satisfying the constraints Assume $T = 1$ sec.

Nov/DEC 2011

$$0.8 \leq |H(e^{j\omega})| \leq 1 \; ; \quad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.2 \; ; \quad 0.6\pi \leq \omega < \pi$$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$\omega_p = 0.2\pi$ rad/sec $\quad \omega_s = 0.6\pi$ rad/se

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\frac{1}{1+\varepsilon^2} = 0.8^2 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\frac{1}{0.8^2} = 1+\varepsilon^2 \qquad \frac{1}{0.2} = 1+\lambda^2$$

$$\boxed{\varepsilon = 0.75} \qquad \boxed{\lambda = 4.8989}$$

$$\Omega_p = \frac{\omega_p}{T} \qquad \Omega_s = \frac{\omega_s}{T}$$

$$= \frac{0.2\pi}{1} = 0.2\pi \qquad \Omega_s = \frac{0.6\pi}{1}$$

$$= 0.6\pi$$

Step 1 :- Find the order of the filter.

$$N \geq \frac{\log \lambda/\varepsilon}{\log \frac{\Omega_s}{\Omega_p}} = \frac{\log\left(\frac{4.898}{0.75}\right)}{\log \frac{0.6\pi}{0.2\pi}}$$

$$= 1.71.$$

Step 2 :- Round off N to next highest integer

$$N = 2.$$

Step 3 :- For $N = 2$ The transfer fn is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Step 4 :- Calculate the cut off freq.

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \frac{0.2\pi}{(0.75)^{1/2}}$$

$$\boxed{\Omega_c = 0.231\pi}$$

Step 5 :- Find the Tr. fn $H_a(s)$ for the $\Omega_c = 0.231\pi$ $S \rightarrow S/0.231\pi$

$$H_a(s) = \frac{1}{\left(\frac{s}{0.231\pi}\right)^2 + \sqrt{2}\left(\frac{s}{0.231\pi}\right) + 1}$$

$$= \frac{0.5266}{s^2 + 1.03s + 0.5266}$$

$$= \frac{0.5266}{(s+0.5+j0.51)(s+0.5-j0.51)}$$

In partial fraction method

$$H_a(s) = \frac{A}{(s+0.5+j0.51)} + \frac{B}{(s+0.5-j0.51)}$$

$$A = \cancel{(s+0.5+j0.51)} \frac{0.5266}{\cancel{(s+0.5+j0.51)}(s+0.5-j0.51)}\Bigg|_{s=-0.5-j0.51}$$

$$\boxed{A = j0.516}$$

$$\boxed{B = -j0.516}$$

$$H_a(s) = \frac{0.516}{s+0.5+j0.51} + \frac{-j0.516}{s+0.5-j0.51}$$

using impulse invariance technique

$$H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s-P_k}$$

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_k T} z^{-1}}$$

$$= \frac{C_1}{1-e^{P_1 T} z^{-1}} + \frac{C_2}{1-e^{P_2 T} z^{-1}}$$

$T = 1$

$$H(z) = \frac{0.516j}{1-e^{-0.5-j0.518} z^{-1}} - \frac{0.516j}{1-e^{-0.5+j0.51} z^{-1}}$$

$$\cancel{H(z)} = \frac{\cancel{0.516j}}{}$$

$$\boxed{H(z) = \frac{0.3019 z^{-1}}{1-1.048 z^{-1}+0.36 z^{-2}}}$$

Unit - III FIR filters.

⟹ Design of FIR filters
⟹ Symmetric and Anti symmetric FIR filter
⟹ design of Linear phase FIR filters using fourier series method.
⟹ FIR filter design using windows (Rectangular, Hamming and Hanning window).
⟹ Freq. Sampling method.
⟹ FIR filter structures.
⟹ Linear phase ~~filter~~ structure
⟹ Direct form realizations.

Designing of FIR Filter :-

(i) choose an ideal desired freq. response $Hd(e^{jw})$

(ii) Take inverse Fourier transform of $Hd(e^{jw})$ to get $hd(n)$ (or) Sample $Hd(e^{jw})$ at finite no. of points (N) to get $H(k)$.

(iii) If $hd(n)$ is convert the infinite duration $h(n)$ (or) if $H(k)$ is determind take N-point inverse DFT to get $h(n)$.

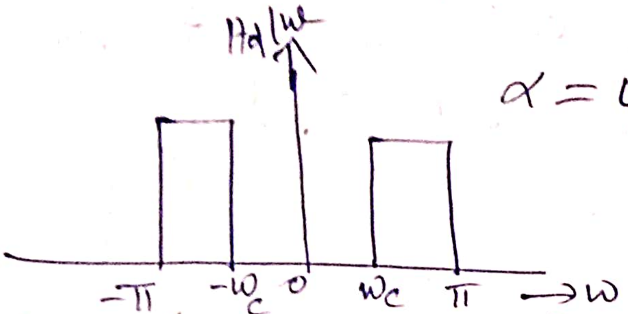
(iv) Take Z-transform of $h(n)$ to get $H(z)$, $H(z)$ is the T.F of the digital filter

(v) choose a suitable structure & realized filter.

Design a HPF with cut off freq. of 1.2 rad/sec with N = 9.

$N = 9 \quad \alpha = \frac{N-1}{2} = \frac{9-1}{2} = 8/2$

$\alpha = 4$



Step 1 :- choose an ideal freq response $H_d(\omega)$

$$H_d(\omega) = e^{-j\alpha\omega} \quad -\pi < \omega < -\omega_c \;\;\&\;\; +\omega_c < \omega < \pi$$

$$0 \quad \text{otherwise}$$

~~Step 2~~

$$\boxed{\begin{aligned} W_H(n) &= 0.54 - 0.46 \cos\frac{2\pi n}{N-1} \qquad n = 0 \text{ to } N-1 \\ & \qquad\qquad\qquad\qquad\qquad\qquad = 0 \text{ to } 8 \\ &= \alpha + (1-\alpha)\cos\frac{2\pi n}{N-1} \\ \alpha &= 0.54 \text{ for Hamming window.} \end{aligned}}$$

Step 2 :- Take inverse Fourier transform To get $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\omega\alpha} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\omega\alpha} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-\omega_c} + \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha}\right]_{\omega_c}^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j\omega_c(n-\alpha)}}{j(n-\alpha)} - \frac{e^{-j\pi(n-\alpha)}}{j(n-\alpha)}\right] + \frac{1}{2\pi}$$

$$\left[\frac{e^{j\pi(n-\alpha)}}{j(n-\alpha)} - \frac{e^{j\omega_c(n-\alpha)}}{j(n-\alpha)}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{+j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}}{2j} - \frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)\right]$$

when $n = \alpha$

$$hd(n) = \underset{(n-\alpha)\to 0}{Lt} \frac{\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)}{\pi(n-\alpha)}$$

$$= \frac{1}{\pi}\left\{\underset{(n-\alpha)\to 0}{Lt} \frac{\sin\pi(n-\alpha)}{(n-\alpha)} - \underset{(n-\alpha)\to 0}{Lt} \frac{\sin\omega_c(n-\alpha)}{n-\alpha}\right]$$

$$= \frac{1}{\pi}\left(\pi - w_c\right)$$

Using L-Hospital rule

$$\lim_{\theta \to 0} \frac{\sin A\theta}{\theta} = A$$

$$= 1 - \frac{w_c}{\pi}$$

Step 3: Determine $h(n)$

$$h(n) = h_d(n)\, w_H(n)$$

$$w_H(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$$ for $n = 0$ to $N-1$

0 otherwise.

$$h(n) = \frac{\sin \pi(n-\alpha) - \sin w_c(n-\alpha)}{\pi(n-\alpha)}\left[0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)\right]$$

for $n \neq \alpha$

$$= \left(1 - \frac{w_c}{\pi}\right)\left(0.54 - 0.46 \cos \frac{2\pi n}{N-1}\right).$$

for $n = \alpha$

$N = 9$, $w_c = 1.2$ rad/sec.

$$\alpha = \frac{N-1}{2} = \frac{9-1}{2} = 4, \quad N-1 = 8$$

* calculate $h(n)$ for $n = 0$ to 8

For

$$h(n) = \frac{-\sin w_c(n-4)}{\pi(n-4)}\left[0.54 - 0.46\cos\frac{2\pi n}{4}\right]$$

for $n \neq 4$

$$= \left(1 - \frac{w_c}{\pi}\right)\left(0.54 - 0.46\cos\frac{n \times \pi}{4}\right] \quad \text{for } n = 4$$

when $n=0$; $h(0) = \dfrac{-\sin(1.2)(0-4)\left[0.54 - 0.46\cos\right.}{\pi(0-4)}$

$= -0.00053$

$n=1$ $h(1) = \dfrac{-\sin(1.2)(1-4)\left[0.54 - 0.46\right.}{\pi(1-4)}$

$$hd(n) = \frac{-\sin w_c(n-\alpha)}{\pi(n-\alpha)}$$

$$hd(0) = \frac{-\sin(1.2)(0-4)}{\pi(0-4)} = -0.0066$$

$$hd(1) = \frac{-\sin(1.2)(1-4)}{\pi(1-4)} = 0.0628$$

$$hd(2) = \frac{-\sin(1.2)(2-4)}{\pi(2-4)} = 0.04188$$

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n} = \sum_{n=0}^{8} h(n)\, z^{-1}$$

$$= h(0) z^{-0} + h(1) z^{-1} + h(2) z^{-2}$$

$$+ h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5}$$

$$+ h(6) z^{-6} +$$

$$+ h(7) z^{-7} + h(8) z^{-8}$$

Rectangular window :-

$$w_R(n) = 1 \quad \text{for } -(N-1)/2 \leq n \leq N-1/2$$
$$0 \quad \text{otherwise.}$$

Hanning window :-

$$w_{Hn}(n) = 0.5 + 0.5 \cos \frac{2\pi n}{N-1}$$
$$\text{for } -(N-1)/2 \leq n \leq \frac{N-1}{2}$$

Hamming window :-

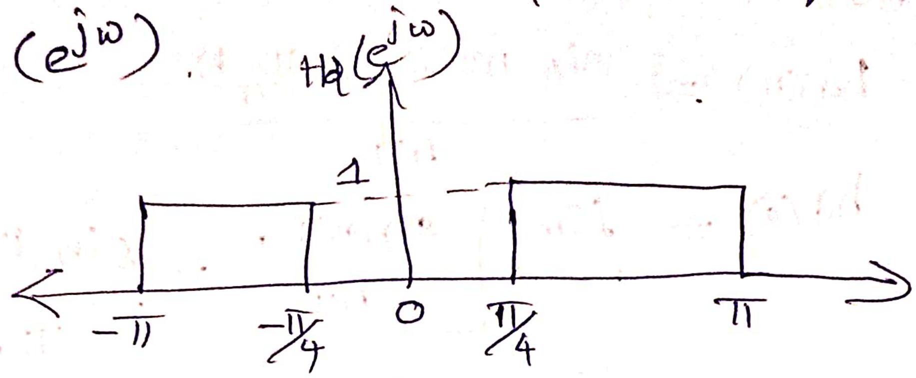
$$w_H(n) = 0.54 + 0.46 \cos 2\pi n/N-1$$
$$\text{for } -(N-1)/2 \leq n \leq \frac{N-1}{2}$$

① Design a HPF with freq. response of

$$H_d(e^{j\omega}) = 1 \quad \pi/4 \leq |\omega| \leq \pi$$
$$0 \quad |\omega| < \pi/4$$

Find $H(z)$ for $N = 11$ using Hamming window and Hanning window and rectangular window.

<u>Step 1</u> Choose an ideal desired freq. response of HPF ~~of~~ $H_d(e^{j\omega})$.



<u>Step 2:</u> Take inverse fourier transform of $H_d(e^{j\omega})$ & find $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{e^{j\omega n}}{jn}\right)_{-\pi}^{-\pi/4} + \left(\frac{e^{j\omega n}}{jn}\right)_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[\left(e^{-jn\pi/4} - e^{-j\pi n}\right) + \left(e^{jn\pi} - e^{jn\pi/4}\right)\right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{jn\pi} - e^{-jn\pi} - \left(e^{+jn\pi/4} - e^{-jn\pi/4}\right)\right]$$

$$= \frac{1}{\pi n}\left[\sin n\pi - \sin n\pi/4\right]$$

L Hospital rule

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\lim_{\theta \to 0} \frac{\sin n\theta}{\theta} = n$

hd(n) to 11 samples

$$hd(n) = \frac{\sin n\pi - \sin \frac{\pi}{4} n}{n\pi}$$

$$hd(0) = \lim_{n \to 0} \left[\frac{\sin n\pi}{n\pi} - \frac{\sin \frac{\pi}{4} n}{\pi n} \right]$$

$$= \lim_{n \to 0} \frac{\sin n\pi}{n\pi} - \lim_{n \to 0} \frac{\sin \frac{\pi}{4} n}{\pi n}$$

$$= 1 - \frac{1}{4} = 0.75$$

$$\boxed{hd(0) = 0.75}$$

$$hd(-1) = hd(1) = \frac{\sin \pi - \sin \frac{\pi}{4}}{\pi} = -0.225$$

$$hd(-2) = hd(+2) = \frac{\sin 2\pi - \sin \frac{\pi}{2}}{2\pi} = -0.159$$

$$hd(-3) = hd(3) = \frac{\sin 3\pi - \sin \frac{3\pi}{4}}{3\pi} = -0.075$$

$$hd(-4) = hd(4) = \frac{\sin 4\pi - \sin \pi}{4\pi} = 0$$

$$hd(-5) = hd(5) = \frac{\sin 5\pi - \sin \frac{5\pi}{4}}{5\pi} = 0.045$$

Step 3 determine Hanning window windowing sequence

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \qquad -\frac{(N-1)}{2} \leq n \leq \frac{N-1}{2}$$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{11-1}$$

$$= 0.5 + 0.5\cos\frac{\pi n}{5} \qquad -5 \leq n \leq 5$$

$$w_{Hn}(0) = 0.5 + 0.5 = 1$$

$$w_{Hn}(1) = w_{Hn}(-1) = 0.5 + 0.5\cos\pi/5 = 0.9045$$

$$w_{Hn}(2) = w_{Hn}(-2) = 0.5 + 0.5\cos 2\pi/5 = 0.655$$

$$w_{Hn}(3) = w_{Hn}(-3) = 0.5 + 0.5\cos 3\pi/5 = 0.345$$

$$w_{Hn}(4) = w_{Hn}(-4) = 0.5 + 0.5\cos 4\pi/5 = 0.095$$

$$w_{Hn}(5) = w_{Hn}(-5) = 0.5 + 0.5\cos\pi = 0$$

Step 4 :- Determine the $h(n)$

$$h(n) = \begin{cases} h_d(n)\, w_{Hn}(n) & \text{for } -5 \leq n \leq 5 \\ 0 & \text{for otherwise} \end{cases}$$

$$h(0) = h_d(0)\, w_{Hn}(0) = 0.75\,(1) = 0.75$$

$$h(-1) = h(1) = h_d(1)\, w_{Hn}(1) = -0.225(0.9045)$$

$$= -0.204$$

$$h(-2) = h(2) = h_d(2)\, w_{Hn}(2) = -0.159 \times 0.655$$

$$= -0.104$$

$$h(-3) = h(3) = h_d(3) \times W_{Hn}(3)$$
$$= -0.015 \times 0.345$$
$$= -0.026$$

$$h(-4) = h(4) = h_d(4) \times W_{Hn}(4)$$
$$= 0 \times 0.8145 = 0$$

$$h(-5) = h(5) = h_d(5) \times W_{Hn}(5) = (0.045)(0)$$
$$= 0$$

Step 5 Find the transfer function :-

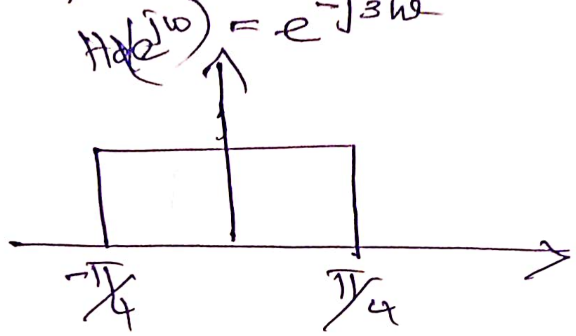
$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$

$$= 0.75 - 0.204(z^{1} + z^{-1}) - 0.104(z^{2} + z^{-2})$$
$$- 0.026(z^{3} + z^{-3})$$

① Given $H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & -\pi/4 \le \omega \le \pi/4 \\ 0 & \text{otherwise} \end{cases}$

Design a FIR filter using Hamming window with $N = 7$

Step 1:- Determine the desired freq response of the filter



Step 2:- Take inverse fourier transform of $H_d(e^{j\omega})$ to get $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} H_d(e^{j\omega})\, e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j3\omega}\, e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega(n-3)} \cdot d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-3)}}{j(n-3)}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\pi/4(n-3)}}{j(n-3)} - \frac{e^{-j\pi/4(n-3)}}{j(n-3)}\right]$$

$$= \frac{1}{\pi(n-3)}\left[\frac{e^{j\pi/4(n-3)} - e^{-j\pi/4(n-3)}}{2j}\right]$$

$$\boxed{h_d(n) = \frac{\sin \pi/4\,(n-3)}{\pi\,(n-3)}}$$

$$h_d(0) = \frac{1}{4}\lim_{n\to 0} \frac{\sin \pi/4\,(n-3)}{\pi/4\,(n-3)}$$

$$h_d(0) = 1/4.$$

$$h_d(-1) = h_d(1) = \frac{\sin \pi/4\,(1-3)}{\pi/4\,(1-3)} = \frac{-1.414}{-1.570}$$

$$= 0.900$$

$$h_d(-2) = h_d(2) = \frac{\sin \pi/4\,(2-3)}{\pi/4\,(2-3)} = \frac{-0.707}{-0.785}$$

$$= 0.900$$

$$hd(3) = hd(-3) = \frac{\sin \pi/4\,(3-3)}{\pi/4\,(3-3)} = 0$$

Step 3 determine the Hamming window fn.

$$w_H(n) = 0.54 + 0.46 \cos \frac{2\pi n}{N-1}$$

-3 to 3

$$w_H(1) = w_H(-1) = 0.54 + 0.46 \cos \frac{2\pi}{6}$$

$$= 0.77$$

$$w_H(2) = w_H(-2) = 0.54 + 0.46 \cos \frac{2\pi}{3}$$

$$= 0.31$$

$$w_H(3) = w_H(-3) = 0.54 + 0.46 \cos \pi$$

$$= 0.08$$

Step 4 Determine the $h(n)$

$$h(1) = h(-1) = hd(1) \times w_H(1) = 0.900 \times 0.77$$

$$= 0.693$$

$$h(2) = h(-2) = 0.900 \times 0.31 = 0.279.$$

$$h(3) = h(-3) = 0 \times 0.08 = 0$$

Step 5 Determine the $H(z)$

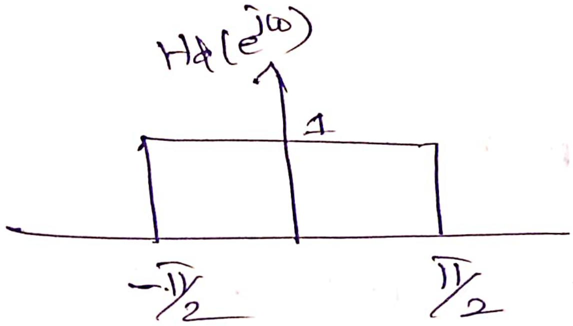
$$H(z) = h(0) + \sum_{n=1}^{3} h(n)(z^{n} + z^{-n})$$

$$= 1/4 + 0.693(z^{1} + z^{-1}) + 0.279(z^{2} + z^{-2})$$

(Q) The desired freq. response of LPF is $H_d(e^{j\omega}) = \begin{cases} 1 & -\pi/2 \leq \omega \leq \pi/2 \\ 0 & \pi/2 \leq \omega \leq \pi \end{cases}$

Determine $h_d(n)$ also determine $h(n)$ using symmetric rectangular window.

Step 1 :- Draw the desired freq response of LPF.



Step 2 :- Take inverse Fourier transform to get $h_d(n)$.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1 \cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi} \left[\frac{e^{jn\pi/2} - e^{-jn\pi/2}}{jn} \right]$$

$$hd(n) = \frac{\sin \frac{n\pi}{2}}{n\pi}$$

$$hd(0) = \frac{1}{2} \lim_{n \to 0} \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}}$$

$$\boxed{hd(0) = \frac{1}{2}}$$

$hd(1) = hd(-1)$

$$= \frac{\sin \pi/2}{\pi} = 0.318$$

$hd(2) = hd(-2)$

$$= \frac{\sin \cancel{2}\pi/\cancel{2}}{2\pi}$$

$= 0$

$$hd(3) = hd(-3) = \frac{\sin 3\pi/2}{3\pi} = -0.10$$

Step 3 :- Determine the $N=7$ rectangular windowing fn.

$W_R(n) = 1$ for -3 to 3

$W_R(1) = W_R(-1) = 1$

$W_R(2) = W_R(-2) = 1$

$W_R(3) = W_R(-3) = 1$

Step 4 :- Determine $h(n)$

$h(n) = hd(m)\, W_R(n)$

$h(1) = h(-1) = 0.318$

$h(2) = h(-2) = 0$

$h(3) = h(-3) = -0.101$

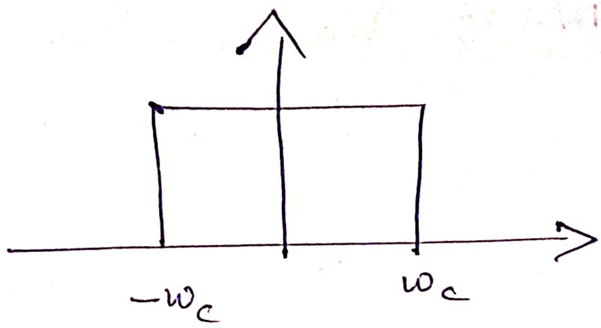
Step 5 :- Determine $H(z)$

$$H(z) = h(0) + \sum_{n=1}^{N-1/2} h(n)(z^n + z^{-n})$$

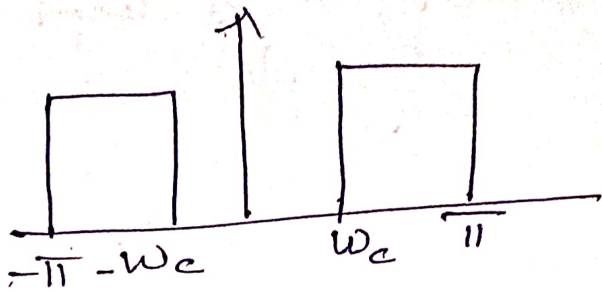
$$= 0.5 + 0.318(z^1 + z^{-1})$$

$$-0.101(z^{+3} + z$$

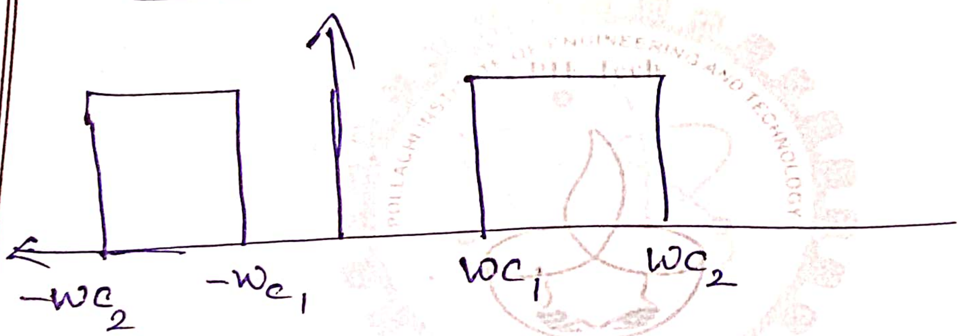
Low pass filter



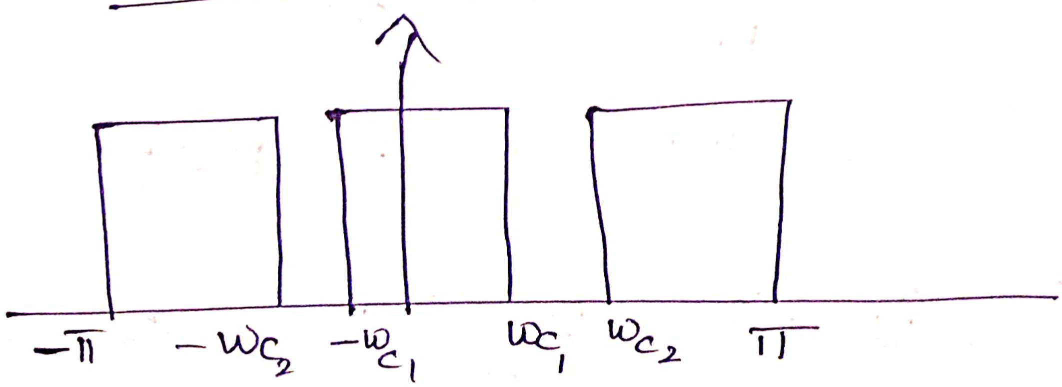
High pass filter



Bandpass filter



Band Stop filter



Freq. sampling method :-

① Determine the coefficients of a linear phase filter of length $N = 15$ has a symmetric unit sample response and freq. response that satisfies the conditions.

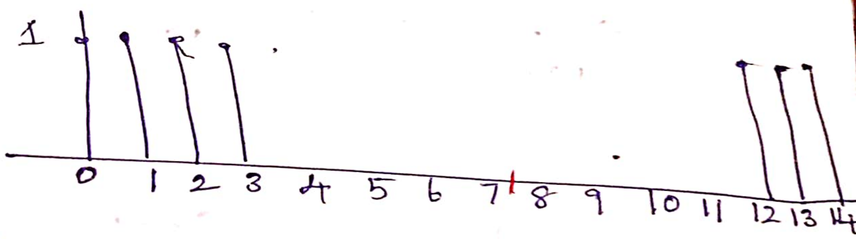
$$H\left(\frac{2\pi k}{15}\right) = 1 \quad k = 0, 1, 2, 3$$

$$= 0 \quad k = 4, 5, 6, 7$$

Solution :-

$$H(k) = 1 \quad \text{for} \quad 0 \leq k \leq 3 \quad \text{and} \quad 12 \leq k \leq 14$$

$$0 \quad \text{for} \quad 4 \leq k \leq 11$$



If N is odd

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \qquad 0 \leq k \leq \frac{N-1}{2}$$

$$= -\frac{14}{15}\pi k \qquad 0 \leq k \leq 7$$

If N is

$$\theta(k) = (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \qquad 8 \leq k \leq 14$$

$$= 14\pi - \frac{14}{15}\pi k$$

$$H(k) = e^{-j14\pi k/15} \quad \text{for } k = 0, 1, 2, 3$$

$$0 \quad \text{for } k = 4, 5, 6, 7, 8, 9, 10, 11$$

$$= e^{-j14\pi(k-15)/15} \quad \text{for } 12 \leq k \leq 14$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{N-1/2} \mathrm{Re}\left[H(k)\, e^{j2\pi nk/N}\right]\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} \mathrm{Re}\left[e^{-j14\pi k/15} \cdot e^{j2\pi nk/15}\right]\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} \mathrm{Re}\left[e^{-j2\pi k(7-n)/15}\right]\right]$$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$e^{+j\theta} = \cos\theta + j\sin\theta$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} \mathrm{Re}\left[\cos\frac{2\pi k(7-n)}{15} - j\sin\frac{2\pi k(7-n)}{15}\right]\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} \cos 2\pi k\frac{(7-n)}{15}\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

$h(0) = h(14) = -0.05$

$h(1) = h(13) = 0.041$

$h(2) = h(12) = 0.0666$

$h(3) = h(11) = -0.0365$

$h(4) = h(10) = -0.1078$

$h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188$

$h(7) = 0.466$

Design a FIR filter by freq. Sampling Technique.

Step 1 :- Choose the ideal freq. response $H_d(e^{jw})$

Step 2 :- Sample $H_d(e^{jw})$ at N-points by taking $w = w_k = 2\pi k/N$ $\quad K = 0, 1, 2 \dots N-1$ to generate $H(K)$

$$H(K) = H_d(e^{jw})\Big|_{w = \frac{2\pi K}{N}}.$$

Step 3 :- Compute the N samples of impulse response $h(n)$ using the following eqn.

N = odd

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{\frac{N-1}{2}} Re\left[H(K)\,e^{j2\pi nk/N}\right]\right]$$

N = even

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{N/2-1} Re\left[H(K)\,e^{j2\pi nk/N}\right]\right]$$

Step 4 :- Take Z-transform of the impulse response $h(n)$ to get filter transfer fn.

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}.$$

(2) Design a Linear phase FIR LPF with a cutoff freq. of 0.5π rad/samp by taking 11 samples of ideal freq. response.

Step 1 :- The desired freq. response of LPF is given by

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & |\omega| \leq \omega_c \\ 0 & |\omega| \geq \omega_c. \end{cases}$$

$$\tau = \frac{N-1}{2} = \frac{11-1}{2} = 5$$

$$\omega_c = \pi/2$$

Step 2 :- To generate $H(K)$

$$H_d(\omega) = \begin{cases} e^{-j\omega 5} & 0 \leq \omega \leq \pi/2 \\ 0 & \text{otherwise.} \end{cases}$$

$$H(K) = e^{-j2\pi k 5/11} \qquad 0 \leq K \leq 5$$

$$= e^{-10\pi K/11} \qquad 0 \leq K \leq 5$$

Step 3 :- compute $h(n)$

$$h(n) = 1/N \left[H(0) + 2 \sum_{K=1}^{5} \mathrm{Re}\left[H(K)\, e^{j2\pi kn/N} \right] \right]$$

$$= \frac{1}{11} \left[H(0) + 2 \sum_{K=1}^{5} \mathrm{Re}\left[e^{-10\pi K/11} \cdot e^{j2\pi kn/11} \right] \right]$$

$$h(n) = \frac{1}{11}\left[H(0) + 2\sum_{k=1}^{5} \mathrm{Re}\left[e^{j2\pi k(n-5)/11}\right]\right]$$

$$= \frac{1}{11}\left[H(0) + 2\sum_{k=1}^{5} \cos\left[\frac{2\pi k(n-5)}{11}\right]\right]$$

$$= \frac{1}{11}\left[1 + 2\cos\frac{2\pi(n-5)}{11} + 2\cos\frac{4\pi(n-5)}{11} + 2\cos\frac{6\pi(n-5)}{11} + 2\cos\left(\frac{8\pi(n-5)}{11}\right)\right]$$

$h(0) = h(10) = \cancel{0.2846}$ 0

$h(1) = h(9) = 0.0962$

$h(2) = h(8) = 0.1063$

$h(3) = h(7) =$

$h(4) = h(6) =$

$h(5) =$

$\omega_0 = 0$

$\omega_1 = \frac{2\pi}{11} = 0.571$

$\omega_2 = \frac{4\pi}{11} = 1.142$

$\omega_3 = \frac{6\pi}{11} = 1.719$

$\omega_4 = 2.285$

$\omega_c = 1.57$

$h(0) = 0.0694$

$h(1) = -0.0540$

$h(2) = -0.1094$

Two marks

① What are the types of Filter based on impulse response?

* IIR Filter
* FIR "

② What are the types of filters based on freq. response?

* LPF
* HPF
* BPF
* BRF.

③ What is the general form of IIR Filter?

$$H(z) = \frac{\sum_{i=0}^{M} b_k z^{-k}}{1 + \sum_{i=1}^{N} a_k z^{-k}}$$

④ What are the advantages & features of FIR filter?

* FIR filters have Linear phase
* " " are always stable
* " " can be realized in both recursive & non recursive structure

What is Gibbs phenomenon?
(or) " oscillations.

One possible way of finding an FIR Filter that approximates $H(e^{j\omega})$ would be to truncate the infinite fourier series at $n = \pm\left(\frac{N-1}{2}\right)$. Abrupt truncation of the series will lead to oscillation in both pass band and stop band. This phenomenon known as Gibbs pheomenon.

Difference between FIR & IIR filter?

FIR	IIR
1. FIR filters can be easily designed have Linear phase	IIR filters do not have Linear phase.
2. FIR filter can be realized recursively and non recursively	IIR fiters are easily realized recursively
3. Greater flexibility	Less flexibility
4. Roundoff noise are less in FIR filters	Roundoff noise are more in IIR filters.

Linear phase FIR filters:

* The T.F of FIR filter is

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

h(n) — impulse respons of the filter.

* The FT of h(n) is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} \quad \text{— ①}$$

which is periodic in freq. with period 2π.

* $$H(e^{j\omega}) = \underbrace{\pm|H(e^{j\omega})|}_{\text{magnitude response}} \underbrace{e^{j\theta(\omega)}}_{\theta(\omega)\text{ phase response}} \quad \text{— ②}$$

* Define the phase delay and groupdelay of filter.

$$\tau_p = \frac{-\theta(\omega)}{\omega}, \quad \tau_g = -\frac{d\theta(\omega)}{d\omega}$$

* For FIR filter with Linear phase we can define

$$\theta(\omega) = -\alpha\omega \qquad -\pi < \omega < \pi$$

α — constant phase delay

$$\tau_p = \frac{-(-\alpha\omega)}{\omega} = +\alpha$$

$$\tau_g = -\frac{d(-\alpha\omega)}{d\omega} = +\alpha$$

equating eqn ① = ②

$$\sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} = \pm |H(e^{j\omega})|\, e^{j\theta(\omega)}$$

$$\sum_{n=0}^{N-1} h(n) \cos \omega n = \pm |H(e^{j\omega})| \cos \theta(\omega) \quad \text{③}$$

$$-\sum_{n=0}^{N-1} h(n) \sin \omega n = \pm |H(e^{j\omega})| \sin \theta(\omega) \quad \text{④}$$

Taking ratio of eqn ③ & ④

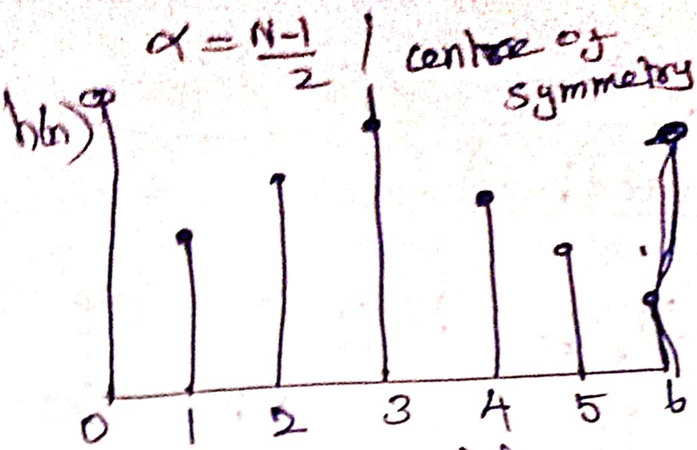
$$\frac{\sum_{n=0}^{N-1} h(n) \sin \omega n}{\sum_{n=0}^{N-1} h(n) \cos \omega n} = \frac{\sin \alpha\omega}{\cos \alpha\omega} \quad \text{⑤}$$

After simplifying eqn ⑤

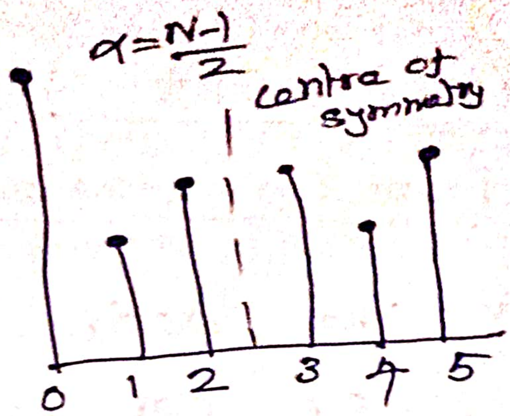
$$\sum_{n=0}^{N-1} h(n) \sin(\alpha - n)\omega = 0$$

$$h(n) = h(N-1-n)$$

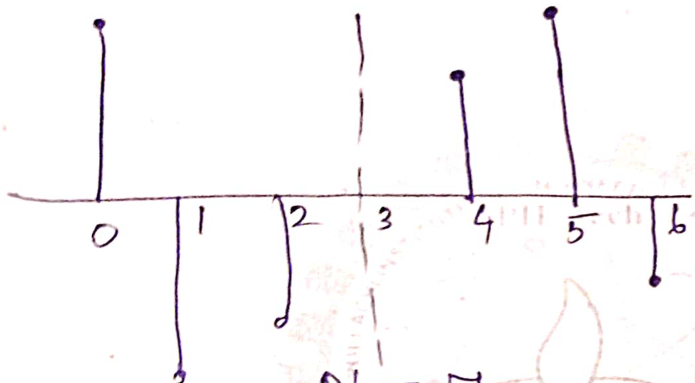
$$\alpha = \frac{N-1}{2}$$



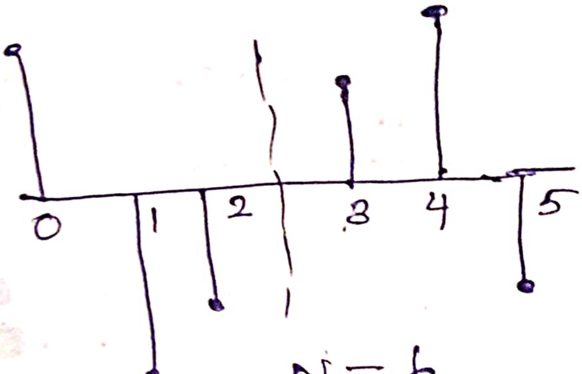
N - odd
(symmetric)



N - even
(symmetric)



N = 7
(antisymmetric)



N = 6
Antisymmetric

Design of FIR filters using Fourier series method.

* The desired freq. response of an FIR filter can be represented by the Fourier series:

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n}$$

* where the fourier coefficients $h_d(n)$ are the desired impulse response of the filter

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega.$$

* The z-transform of the sequence is

$$H(z) = \sum_{n=-\infty}^{\infty} h_d(n)\, z^{-n}$$

3-3

* To get an FIR filter transfer function the series can be truncated by assigning

$$h(n) = h_d(n) \quad \text{for } |n| \leq \frac{N-1}{2}$$

$$0 \quad \text{otherwise}$$

* $$H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n)\, z^{-n}$$

$$= h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\, z^{-n} + h(-n) z^{n} \right]$$

$h(n) = h(-n)$ [For symmetric impulse response)

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

Design an ideal low pass filter with a freq. response

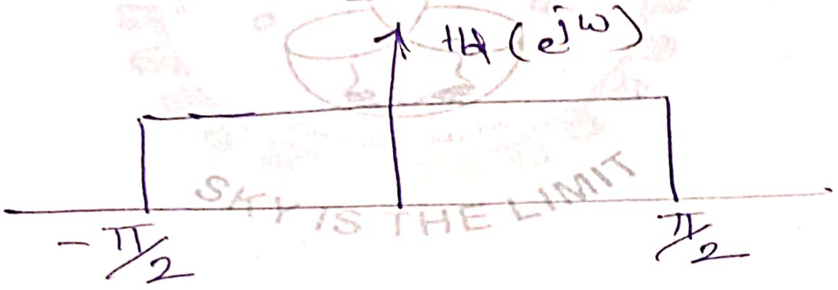
$$H_d(e^{j\omega}) = 1 \quad \text{for } -\pi/2 \le \omega \le \pi/2$$
$$0 \quad \text{for } \pi/2 \le \omega \le \pi$$

Find the values of $h(n)$ for $N = 11$

Find $H(z)$.

$$\omega_c = \pi/2$$

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } -\pi/2 \le \omega \le \pi/2 \\ 0 & \text{for } \pi/2 \le \omega < \pi \end{cases}$$



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} H_d(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi j n}\left[e^{j\omega n}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi n(2j)}\left[e^{j\pi n/2} - e^{-j\pi n/2}\right]$$

$$h_d(n) = \frac{\sin \frac{\pi}{2} n}{\pi n}$$

$$h(n) = \frac{\sin \frac{\pi}{2} n}{\pi n} \quad \text{for} \quad n \leq 5$$

$n=0$

$$h(0) = \lim_{n\to 0} \frac{\sin \frac{\pi}{2} n}{\pi n} = \frac{1}{2} \lim_{n \to 0} \frac{\sin \frac{\pi}{2} n}{\pi n/2}$$

$n=1$

$$= \frac{1}{2}$$

$$h(1) = h(-1) = \frac{\sin \frac{\pi}{2}}{\pi} = \frac{1}{\pi} = 0.3183$$

$n=2$

$$h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$$

$n=3$

$$h(3) = h(-3) = \frac{\sin \frac{3\pi}{2}}{3\pi} = \frac{-1}{3\pi} = -0.106$$

$n=4$

$$h(4) = h(-4) = \frac{\sin \frac{4\pi}{2}}{4\pi} = 0$$

$n=5$

$$h(5) = h(-5) = \frac{\sin \frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.063$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left(z^n + z^{-n}\right)$$

$$= 0.5 + \sum_{n=1}^{5} h(n)\left(z^n + z^{-n}\right)$$

$$= 0.5 + 0.3183(z^{1} + z^{-1}) - 0.106(z^{3} + z^{-3})$$
$$+ 0.0636(z^{5} + z^{-5})$$

The transfer function of the realizable filter

$$H'(z) = z^{-(N-1)/2} H(z)$$
$$= z^{-5}\left[0.5 + 0.3183(z^{1} + z^{-1}) - 0.106(z^{3} + z^{-3}) + 0.0636(z^{5} + z^{-5})\right]$$

$$= 0.5z^{-5} + 0.3183z^{-4} + 0.3183z^{-6} - 0.106z^{-2} - 0.106z^{-8} + 0.06366 + 0.0636z^{-1}$$

$$= 0.06366 - 0.106z^{-2} + 0.3183z^{-4} + 0.5z$$
$$+ 0.3183z^{-6} - 0.106z^{-8} + 0.0636z$$

The filter coefficients of casual filter are

$h(0) = h(10) = 0.06366$

$h(1) = h(9) = 0$ $\quad h(5) = 0$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

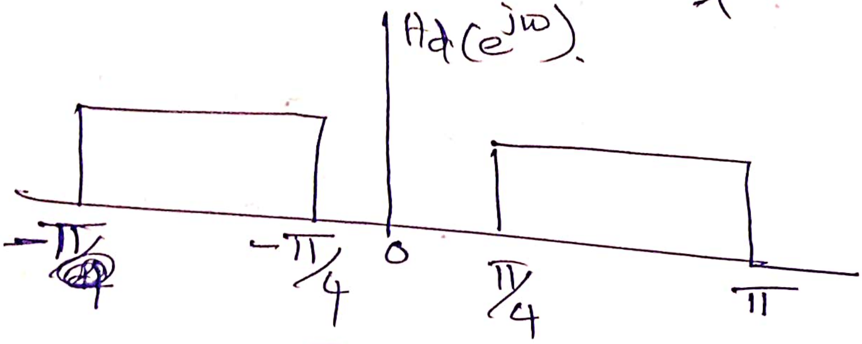
$h(4) = h(6) = 0.3183$

② Design an ideal highpass filter with freq. response

$$H_d(e^{j\omega}) = 1 \quad \pi/4 \leq |\omega| \leq \pi$$

$$0 \quad |\omega| < \pi/4$$

Find the values of $h(n)$ for $N = 11$. Find $H(z)$



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} H_d(e^{j\omega})\cdot e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi j n}\left[\left(e^{j\omega n}\right)_{-\pi}^{-\pi/4}\right] + \frac{1}{2\pi j n}\left[e^{j\omega n}\right]_{\pi/4}^{\pi}$$

$$= \frac{1}{\pi n(2j)}\left[e^{-jn\pi/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi/4 n}\right]$$

$$= \frac{1}{\pi n}\left[\frac{e^{j\pi n} - e^{-j\pi n}}{2j} - \left(\frac{e^{j\pi n/4} - e^{-j\pi n/4}}{2j}\right)\right]$$

$$h_d(n) = \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi n}{4}\right]$$

Truncating $h_d(n)$ to 11 samples

$h(n) = h_d(n)$ for $|n| \leq 5$

$n = 0$

$$h(0) = \lim_{n \to \infty} \frac{\sin \pi n}{\pi n} - \frac{1}{4} \lim_{n \to \infty} \frac{\sin \frac{\pi n}{4}}{\frac{\pi n}{4}}$$

$$= 1 - \frac{\pi}{4} = 0.75$$

$$\boxed{h(0) = 0.75}$$

$n=1$ $\quad h(1) = h(-1) = \dfrac{\sin \pi}{\pi} - \dfrac{\sin \pi/4}{\pi} = -0.2$

$n=2$ $\quad h(2) = h(-2) = \dfrac{\sin 2\pi - \sin \pi/2}{2\pi} = -0.159$

$n=3$ $\quad h(3) = h(-3) = \dfrac{\sin 3\pi - \sin 3\pi/4}{3\pi} = -0.075$

$n=4$ $\quad h(4) = h(-4) = \dfrac{\sin 4\pi - \sin \pi}{4\pi} = 0$

$n=5$ $\quad h(5) = h(-5) = \dfrac{\sin 5\pi - \sin 5\pi/4}{5\pi}$

$= 0.045$

$$H(z) = h(0) + \sum_{n=0}^{N-1/2} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.75 + \sum_{n=0}^{5} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.75 - 0.225(z^{1} + z^{-1}) - 0.159(z^{2} + z^{-2})$$
$$- 0.675(z^{3} + z^{-3}) + 0.045(z^{5} + z^{-5})$$

The transfer fn of the realizable filter is

$$H'(z) = z^{-5} H(z)$$

$$= z^{-5}\left[0.75 - 0.225(z + z^{-1}) - 0.159(z^{2} + z^{-2}) - 0.075(z^{3} + z^{-3}) + 0.045(z^{5} + z^{-5})\right.$$

$$= 0.75z^{-5} - 0.225z^{-4} - 0.225z^{-6} - 0.159z^{-3} - 0.159z^{-7} - 0.075z^{-2} - 0.075z^{-8}$$
$$+ 0.045 + 0.045z^{-10}$$

The filter coefficients of the Casual filters are

$h(0) = h(10) = 0.045$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.075$

$h(3) = h(7) = -0.159$

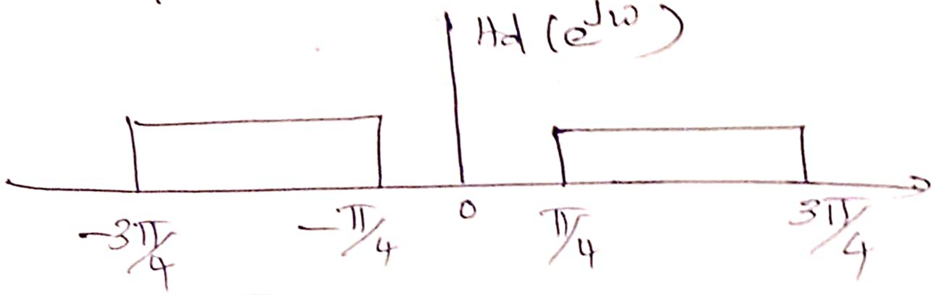
$h(4) = h(6) = -0.225$

$h(5) = 0.75$

3. Design an ideal band pass filter with a freq response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad \pi/4 \leq |\omega| \leq 3\pi/4$$
$$\qquad 0 \quad \text{for otherwise}$$

Find the value of $H(z)$ and coefficients for $N=9$



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{3\pi/4} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{e^{j\omega n}}{jn}\right)_{-3\pi/4}^{-\pi/4} + \left(\frac{e^{j\omega n}}{jn}\right)_{\pi/4}^{3\pi/4}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j\pi/4\, n} - e^{-j3\pi/4\, n} + e^{j3\pi/4\, n} - e^{j\pi/4\, n}\right]$$

$$= \frac{1}{\pi n}\left[\frac{e^{j3\pi/4\, n} - e^{-j3\pi/4\, n}}{2j} - \left(\frac{e^{j\pi/4\, n} - e^{-j\pi/4\, n}}{2j}\right)\right]$$

$$= \frac{1}{\pi n}\left[\sin \frac{3\pi}{4} n - \sin \frac{\pi}{4} n\right]$$

Truncating $hd(n)$ to 9 samples

$h(n) = hd(n)$ for $|n| \leq 4$

$n = 0$

$$h(0) = \lim_{n \to \infty} \frac{\sin \frac{3}{4}\pi n}{\frac{3}{4}\pi n}\left(\frac{3}{4}\right) - \lim_{n \to \infty} \frac{\sin \frac{\pi}{4} n}{\frac{\pi n}{4}}\left(\frac{1}{4}\right)$$

$$= \frac{3}{4} - \frac{1}{4} = 0.5$$

$n = 1$ $\quad h(1) = h(-1) = \dfrac{\sin \frac{3\pi}{4} - \sin \frac{\pi}{4}}{\pi} = 0$

$n = 2$ $\quad h(2) = h(-2) = \dfrac{\sin \frac{3\pi}{2} - \sin \frac{\pi}{2}}{2\pi} = -0.318$

$n = 3$ $\quad h(3) = h(-3) = \dfrac{\sin \frac{9\pi}{4} - \sin \frac{3\pi}{4}}{3\pi} = 0$

$n = 4$ $\quad h(4) = h(-4) = \dfrac{\sin 3\pi - \sin \pi}{4\pi} = 0$

$$H(z) = h(0) + \sum_{n=1}^{N-1/2} h(n)\left[z^{n} + z^{-n}\right]$$

$$H(z) = 0.5 + \sum_{n=1}^{4} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.5 - 0.3183\left[z^{2} + z^{-2}\right]$$

$$H'(z) = z^{-5}\left[0.5 - 0.3183\left(z^{2} + z^{-2}\right)\right]$$

$$= 0.5z^{-5} - 0.3183\, z^{-3}$$

$$- 0.3183 z^{-7}$$

The filter co-efficients of the casual filter are

$h(0) = h(1) = h(9) = h(2) = h(8)$

$h(4) = h(6) = 0$

$h(3) = h(7) = -0.3183$

$h(5) = 0.5$

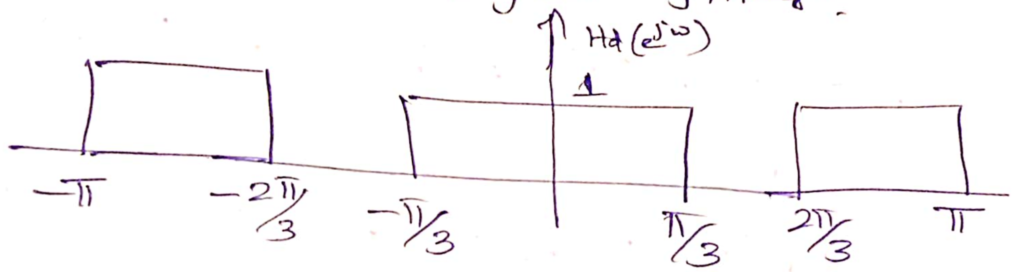
(7) Design an ideal band reject filter with a desired freq. response

$H_d(e^{j\omega}) = 1$ for $|\omega| \leq \pi/3$ and $\omega \geq 2\pi/3$

0 otherwise.

Find the value of $h(n)$ for $N=1$ and Find the coefficients of the filter.

Step 1 :- Find the desired freq. response of the Band reject filter.



Step 2 :- Take Inverse fourier transform find $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{-2\pi/3} 1 \cdot e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} 1 \cdot e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{2\pi/3}^{\pi} 1 \cdot e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi n j}\left[\left(e^{j\omega n}\right)_{-\pi}^{-2\pi/3} + \left(e^{j\omega n}\right)_{-\pi/3}^{\pi/3} + \left(e^{j\omega n}\right)_{2\pi/3}^{\pi}\right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{-j2\pi/3\, n} - e^{-j\pi n} + e^{j\pi/3\, n} - e^{-j\pi/3\, n} + e^{j\pi n} - e^{j2\pi/3\, n}\right)$$

$$= \frac{1}{\pi n}\left[\frac{e^{j\pi n} - e^{-j\pi n}}{2j} + \frac{e^{j\pi/3\, n} - e^{-j\pi/3\, n}}{2j} - \left(\frac{e^{j2\pi/3\, n} - e^{-j2\pi/3\, n}}{2j}\right)\right]$$

$$= \frac{1}{\pi n}\left[\sin n\pi + \sin \pi/3\, n - \sin 2\pi/3\, n\right]$$

Step 3:- Determine the $h(n)$

For $n=0$

$$h(0) = \lim_{n \to 0}\left[\frac{\sin \pi n}{\pi n} + \frac{1}{3}\frac{\sin \pi/3\, n}{n\pi/3} - \frac{2}{3}\frac{\sin 2\pi/3\, n}{2n\pi/3}\right]$$

$$= 1 + 1/3 - 2/3$$

$$= 0.667.$$

$$h(1) = h(-1) = \frac{\sin \pi}{\pi} + \frac{\sin \pi/3}{\pi} - \frac{\sin 2\pi/3}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin 2\pi + \sin 2\pi/3 - \sin 4\pi/3}{2\pi} = 0.2757$$

$$h(3) = h(-3) = \frac{\sin 3\pi + \sin \pi - \sin 2\pi}{3\pi} = 0$$

$$h(4) = h(-4) = \frac{\sin 4\pi + \sin 4\pi/3 - \sin 8\pi/3}{4\pi}$$

$$= -0.1378$$

$$h(4) = h(-5) = \frac{\sin 5\pi + \sin 5\pi/3 - \sin 10\pi/3}{5\pi} = 0$$

Step 4:- Determine the Transfer function H(z).

$$H(z) = h(0) + \sum_{n=1}^{N-1/2} h(n)\left(z^{n} + z^{-n}\right)$$

$$H(z) = 0.667 + 0.2757\left(z^{2}+z^{-2}\right) - 0.1378\left(z^{4}+z^{-4}\right)$$

Step 5:- The transfer function of the realizable filter is

$$H'(z) = z^{-5}\, H(z)$$

$$= z^{-5}\left[0.667 + 0.2757\left(z^{2}+z^{-2}\right) - 0.1378\left(z^{4}+z^{-4}\right)\right]$$

$$= 0.667\, z^{-5} + 0.2757\, z^{-3} + 0.2757\, z^{-7} - 0.1378\, z^{-1} - 0.1378\, z^{-9}$$

Step 5:- The filter coeffients of the Casual filter are

$$h(0) = h(10) = h(2) = h(8) = h(4) = h(6) = 0$$

$$h(1) = h(9) = -0.1378$$

$$h(3) = h(7) = 0.2757$$

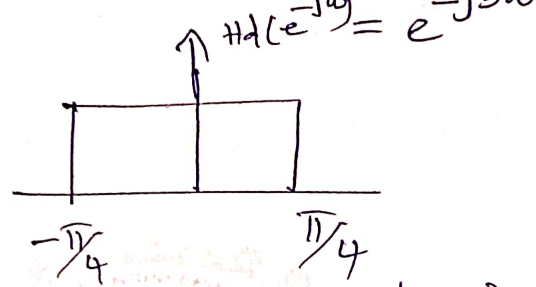
$$h(5) = 0.667$$

Windowing Techniques :-

(1) Design a filter with $H_d(e^{j\omega}) = e^{-j3\omega}$; $-\frac{\pi}{4} \le \omega \le \frac{\pi}{4}$

using Hanning window with $N=7$. 0 for $\frac{\pi}{4} \le |\omega| \le \pi$

Step 1 :- Determine the desired freq. response of filter



Step 2 :- Take inverse fourier transform find $h_d(n)$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j3\omega} \cdot e^{j\omega n} \cdot d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{+j\omega(n-3)} \cdot d\omega$$

$$= \frac{1}{2\pi j} \left[\frac{e^{j\omega(n-3)}}{n-3} \right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi j} \left[\frac{e^{j(n-3)\pi/4} - e^{-j\frac{\pi}{4}(n-3)}}{n-3} \right]$$

$$= \frac{1}{\pi(n-3)} \left[\frac{e^{j(n-3)\pi/4} - e^{-j\pi/4(n-3)}}{2j} \right]$$

$$= \frac{1}{\pi(n-3)} \sin(n-3)\pi/4$$

$$= \frac{\sin(n-3)\pi/4}{\pi(n-3)}$$

For $n=7$ we have

$$h_d(0) = h_d(6) = \frac{\sin \pi/4(-3)}{\pi(-3)} = 0.075$$

$$h_d(1) = h_d(5) = \frac{\sin \pi/4(-2)}{\pi(-2)} = 0.159$$

$$h_d(2) = h_d(4) = \frac{\sin \pi/4(-1)}{\pi(-1)} = 0.225$$

$$h_d(3) = \lim_{n \to \alpha} \frac{1}{4} \frac{\sin(n-3)\pi/4}{\pi/4(n-3)}$$

$$= \frac{1}{4} \cdot 1$$

$$= 0.25$$

$$W_{Hn}(n) = 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \quad \text{for } -3 \leq n \leq 3$$

$$W_{Hn}(0) = 0.5 + 0.5 = 1$$

$$W_{Hn}(-1) = W_{Hn}(1) = 0.5 + 0.5 \cos \pi/3 = 0.75$$

$$W_{Hn}(-2) = W_{Hn}(2) = 0.5 + 0.5 \cos 2\pi/3 = 0.25$$

$$W_{Hn}(-3) = 0.5 + 0.5 \cos \pi = 0$$

$w_{Hn}(0) = w_{Hn}(6) = 0$

$w_{Hn}(1) = w_{Hn}(5) = 0.25$

$w_{Hn}(2) = w_{Hn}(4) = 0.75$

$w_{Hn}(3) = 1$

The filter coefficients using hanning window

$h(n) = hd(n)\, w_{Hn}(n)$

$h(0) = h(6) = hd(0)\, w_H(0) = (0.075)(\phi) = 0.075$

$h(1) = h(5) = hd(1)\, w_H(1) = 0.159\,(0.75) =$

Design a filter for the given freq. response using Hamming window with N=7

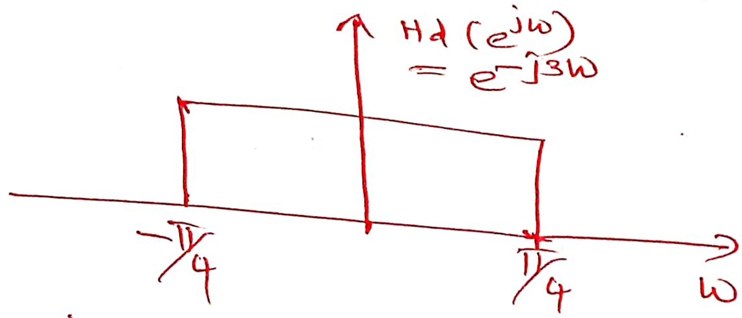
$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} ; & -\pi/4 \leq \omega \leq \pi/4 \\ 0 & \text{otherwise.} \end{cases}$$

Solution :-

N = 7.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & -\pi/4 \leq \omega \leq \pi/4 \\ 0 & \text{otherwise} \end{cases}$$

Step 1 :- determine the desired impulse response



Step 2 :- Take inverse FT of $H_d(e^{j\omega})$ to get $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} H_d(e^{j\omega}) \cdot e^{j\omega n}\, d\omega .$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j3\omega} \cdot e^{j\omega n} \cdot d\omega .$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j(n-3)\omega} \cdot d\omega .$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(n-3)\omega}}{j(n-3)}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{\pi(n-3)}\left[\frac{e^{j(n-3)\pi/4} - e^{-j(n-3)\pi/4}}{2j}\right]$$

$$\boxed{hd(n) = \frac{\sin(n-3)\pi/4}{\pi(n-3)}}$$

The given freq. response is

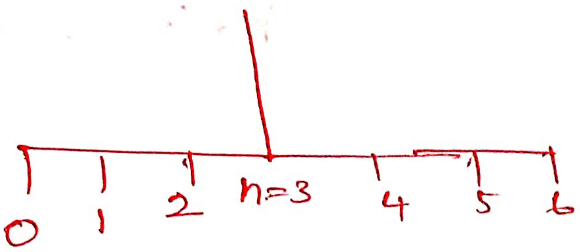
$$e^{-j3\omega} = e^{j\left(\frac{7-1}{2}\right)\omega} = e^{j\left(\frac{N-1}{2}\right)\omega}$$

The impulse response sequence is symmetric about $n = \frac{N-1}{2} = 3$

$hd(0) = hd(6)$

$hd(1) = hd(5)$

$hd(2) = hd(4)$; $hd(3)$



$\underline{n=3}$

$hd(3)$ becomes indeterminant.

$$hd(3) = \lim_{n\to\infty} \frac{1}{4}\frac{\sin(n-3)\pi/4}{\pi/4\,(n-3)}$$ L Hospital rule

$$\boxed{hd(3) = 1/4 = 0.25}$$

$\underline{\underline{n=0}}$

$$hd(0) = hd(6) = \frac{\sin\left(\frac{0-3}{4}\right)\pi}{\pi(0-3)}$$

$$= \frac{\sin(-3\pi/4)}{-3\pi}$$

$$\boxed{hd(0) = hd(6) = +0.075}$$

$\underline{\underline{n=1}}$

$$hd(1) = hd(5) = \frac{\sin\left(\frac{1-3}{4}\right)\pi}{\pi(1-3)}$$

$$= \frac{\sin(-\frac{1}{2}\pi)}{-2\pi}$$

$$\boxed{hd(1) = hd(5) = 0.159}$$

$\underline{\underline{n=2}}$

$$hd(2) = hd(4) = \frac{\sin\left(\frac{2-3}{4}\right)\pi}{\pi(2-3)}$$

$$= \frac{\sin(-\pi/4)}{-\pi}$$

$$\boxed{hd(2) = hd(4) = 0.225}$$

Step 3:- Find Hamming window sequence for $N=7$

$$w_H(n) = \begin{cases} 0.54 + 0.46 \cos \frac{2\pi n}{N-1} & |n| \leq \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$w_H(n) = w_H(-n)$$

$$w_H(n) = 0.54 + 0.46 \cos \frac{2\pi n}{7-1} \quad -3 \le n \le 3$$

$$= 0.54 + 0.46 \cos \frac{\pi n}{3}$$

For $n=0$

$$w_H(0) = 0.54 + 0.46 \cos 0$$
$$= 0.54 + 0.46$$

$$\boxed{w_H(0) = 1}$$

For $n=1$

$$w_H(1) = 0.54 + 0.46 \cos \pi/3$$
$$= 0.77$$

$$\boxed{w_H(1) = w_H(-1) = 0.77}$$

For $n=2$

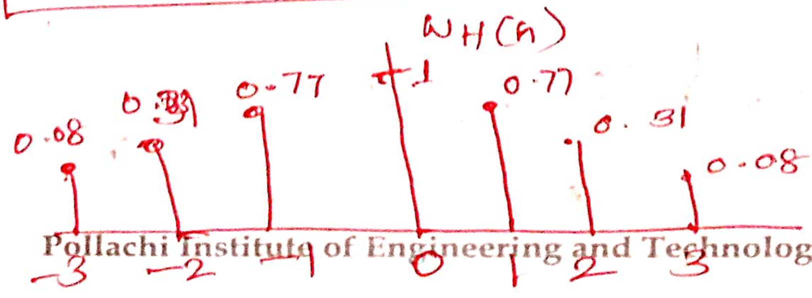
$$w_H(2) = 0.54 + 0.46 \cos \frac{2\pi}{3}$$

$$\boxed{w_H(2) = w_H(-2) = 0.31}$$

$n=3$

$$w_H(3) = 0.54 + 0.46 \cos \pi$$

$$\boxed{w_H(3) = w_H(-3) = 0.08}$$



The casual window can be obtained by shifting the sequence $w_H(n)$ to right by 3

$w_H(0) = 0.08$

$w_H(1) = 0.31$

$w_H(2) = 0.77$

$w_H(3) = 1$

$w_H(4) = 0.77$

$w_H(5) = 0.31$

$w_H(6) = 0.08$.

Step 4 :- Determine $h(n)$

$$h(n) = h_d(n)\, w_H(n)$$

$$h(0) = h_d(0)\, w_H(0)$$

$$= (0.075)(0.08)$$

$$\boxed{h(0) = h(6) = 0.006}$$

$$h(1) = h(5) = h_d(1)\, w_H(1)$$

$$= (0.159)(0.31)$$

$$\boxed{h(1) = h(5) = 0.0493}$$

$h(2) = h(4) = (0.225)(0.77)$

$$\boxed{h(2) = h(4) = 0.17325}$$

$h(3) = (0.25)(1)$

$h(3) = 0.25$

Step 5 :- Determine the T.F of digital FIR filter

$$H(z) = z^{-\left(\frac{N-1}{2}\right)} \left\{ h(0) + \sum_{n=1}^{N-1/2} h(n) \left[z^{n} + z^{-n} \right] \right.$$

$$= z^{-3} \left[0.006 + 0.0493 \left[z^{1} + z^{-1} \right] \right.$$

$$+ 0.17325 \left[z^{2} + z^{-2} \right] \Big) + 0.25 \left[z^{3} + z^{-3} \right]$$

$$= 0.006 z^{-3} + 0.0493 z^{-2} + 0.0493 z^{-4}$$

$$+ 0.17325 z^{-1} + 0.17325 z^{-5}$$

$$+ 0.25 z^{0} + 0.25 z^{-6}$$

$$= 0.25 + 0.17325 z^{-1} + 0.0493 z^{-2}$$

$$+ 0.006 z^{-3} + 0.0493 z^{-4} + 0.1733 z^{-5}$$

$$+ 0.25 z^{-6}$$

Filter coeffients are

$$h(n) = \{\underset{\underset{n=0}{\uparrow}}{0.25},\ 0.1733,\ 0.0493,\ 0.006,\ 0.0493,\ 0.1733,\ 0.25\}$$

---x---x---

Unit-4 Finite word length effects

⇒ Fixed point & floating point representation

⇒ ADC - quantization - truncation and rounding.

⇒ quantization noise - i/p / o/p quantization

⇒ coefficient quantization error

⇒ product quantization

⇒ error - overflow error

⇒ Limit cycle oscillations due to product quantization and summation

⇒ scaling to prevent overflow.

Types of number representation :-

(i) fixed point representation
(ii) floating " "
(iii) Block " "

Fixed point representation :-

* In fixed point arithmetic the position of binary point is fixed.
* The bit to the right represents the fractional part of the number and left represent the integer part.
* For eg.

01·1100 ~~has~~

integer part ← 01 ; 1100 → fractional par

$$01\cdot1100 = 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 0\times2^{-4}$$

$$= 0 + 1\cdot\frac{1}{2} + \frac{1}{4} + 0 + 0$$

$$= 1\cdot75 \text{ (Decimal)}$$

110.010 convert into Decimal

110.010

$0 \times 2^{-3} = 0$

$1 \times 2^{-2} = \frac{1}{4}$

$0 \times 2^{-1} = 0$

$0 \times 2^{0} = 0$

$1 \times 2^{1} = 2$

$1 \times 2^{2} = 4$

6.25

(2) Convert the decimal number 30.275 to binary form

Integer part

2	30	
2	15	0
2	7	1
2	3	1
	1	1

11110

fractional part

$0.275 \times 2 = 0.550 \rightarrow 0$

$0.55 \times 2 = 1.10 \rightarrow 1$

$0.10 \times 2 = 0.2 \rightarrow 0$

$0.2 \times 2 = 0.4 \rightarrow 0$

$0.4 \times 2 = 0.8 \rightarrow 0$

$0.8 \times 2 = 1.6 \rightarrow 1$

$0.6 \times 2 = 1.2 \rightarrow 1$

$0.2 \times 2 = 0.4 \rightarrow 0$

$(30.275)_{10} = (11110.01000110)_2$

⟹ The fixed point arithmatic forms '3 different ways.

(i) sign-magnitude form

(ii) one's complement "

(iii) 2's "

Sign magnitude form:-

⟹ In this representation most significant bit is set to 1 to represent -ve sign

$N_n = 1 + \sum_{i=1}^{B} b_i 2^{-i}$ ⟹ For eg. -1.75 represented as

$-1.75 \Rightarrow 11.110000$

$+1.75 \Rightarrow 01.110000$

with b bits only $2^b - 1$ numbers can be represented

Fraction point:

$N_p = 0 + \sum_{i=1}^{B} b_i 2^{-i}$

One's complement form

$(0.875)_{10} = 0.111000$

$(-0.875)_{10} = 1.000111$

$0.875 \times 2 = 1.75 \to 1$

$0.75 \times 2 = 1.5 \to 1$

$0.5 \times 2 = 1.0 \to 1$

$0 \times 2 = 0 \to 0$

$0 \times 2 = 0 \to 0$

$(0.875)_2 = 0.11100$

One's complement :-

The negative numbers are represents by complementing the binary number

$$(+0.0625)_{10} = (0.0001)_2$$

$$(-0.0625)_{10} = (1.1110)_2$$

0.0625×2.

$= 0.1250 \rightarrow 0$

$0.125 \times 2 = 0.250 \rightarrow 0$

$0.250 \times 2 = 0.500 \rightarrow 0$

$0.5 \times 2 = 1.000 \rightarrow 1$

$$(0.0625)_{10} = (0.0001)_2$$

Two's complement :-

The negative number is represented by ~~complementing the binary numbers~~. forming 2's compliment of the corresponding positive number.

2's complement obtained by

(i) Take one's complement

(ii) Add one to the one's complement

$(0.0625)_{10} = (0.0001)_2$

$(-0.0625)_{10} = ?$

1's complement $= 1.1110$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad 1$

2's complement $= \underline{1.1111}$

$(-0.0625)_{10} = (1.1111)_2$

<u>Floating point number representation</u> :-

* It provides high dynamic range

It is expressed as:

$$x = M \cdot 2^E$$

M – mantissa, it is magnitude of number.

Express the following numbers in floating point representation with 5 bits for matissa and 3 bits for 2^E.

(i) $(+5)_{10}$ (ii) $(-5)_{10}$

(iii) $(+0.125)_{10}$ (iv) $(-0.125)_{10}$

(i) $(+5)_{10} = (101)_2 = 0.1010 \times 2^{+3}$

Mantissa					Exponent		
0	1	0	1	0	0	1	1

sign bit of mantissa   sign bit of exponent

$$(-5)_{10} = (-101)_2 = 1.1010 \times 2^{+3}$$

$$= 1.1010 \times 2^{(11)_2}$$

Mantissa					Exponent		
1	1	0	1	0	0	1	1

Sign bit of mantissa — sign bit of exponent.

(iii) $(0.125)_{10} = (0.001)_2$

$$= 0.1000 \times 2^{-2} = 0.1000 \times 2^{(-10)_2}$$

0	1	0	0	0	1	1	0

sign bit of mantissa — sign bit of exponent

(iv) $(-0.125)_{10} = -(0.001)_2 = 1.1000 \times 2^{-2}$

$$= 1.1000 \times 2^{(-10)_2}$$

1	1	0	0	0	1	1	0

sign bit of mantissa — sign bit of exponent.

① perform $4/8 - 3/8$ by 2's complement arithmetic.

$$\left(\frac{4}{8}\right)_{10} = \left(\frac{1}{2}\right)_{10} \Rightarrow (0.1)_2 = 0.1$$

$$\left(\frac{3}{8}\right)_{10} = (0.011)$$

$\left(\frac{3}{8}\right)_{10}$ in 2's complement

$$\begin{array}{r} 1.100 \\ 1 \\ \hline 1.101 \end{array}$$

↳ sign bit.

$$\left(\frac{4}{8}\right)_{10} \Rightarrow 0.1$$

$$\left(\frac{3}{8}\right)_{10} \Rightarrow \begin{array}{r} 1.101 \\ \hline 1\,0.001 \\ \hline \end{array}$$

↳ discarded

$$(0.001)_2 = \left(\frac{1}{8}\right)_{10}.$$

Addition in floating point number representation :-

Steps

* exponent of the numbers to be added are made equal.

* mantissa are added.

* Finally exponent is rearranged to get mantissa in its proper form.

① Add 5 and 32 floating point representation

$$(5)_{10} = (101)_2 = 0.101(2)^{0011}$$

$$(32)_{10} = (100000)_2 = 0.1(2)^{0110}$$

Step 1 :- make the exponential equal

$$(5)_{10} = (101)_2 = 0.000101(2)^{0110}$$

Step 2 :- Add the two mantissa

mantissa of $(32)_{10} = 0.1$

mantissa of $(5)_{10} = 0.000101$

$$0.100101$$

Step 3 :- Rearrange the mantissa and exponent

$$0.100101(2)^{0110} \Longrightarrow (100101)_2 = (37)_{10}$$

Multiplication in floating point representation :-

Steps :-

* multiply the two messages
* Add two exponents.
* Rearrange the final product in normalised form.

eg :- multiply 3×9 using floating point representation.

$(3)_{10} = (11)_2 = 0.11(2)^{0010}$

$(9)_{10} = (1001)_2 = 0.1001(2)^{0100}$

Step 1 :- multiply two mantissia

$$
\begin{array}{rr}
(3)_{10} \Longrightarrow & 0.11 \\
(9)_{10} \Longrightarrow & 0.1001 \\
\hline
 & 1001 \\
 & 1001 \\
\hline
 & 11011
\end{array}
$$

$\therefore$ total $2+4=6$ bits after decimal point, the Final product is $(0.011011)_2$

Step 2 :- Add two exponents.

$$
\begin{array}{rr}
(3)_{10} = & 0010 \\
(9)_{10} = & 0100 \\
\hline
 & 0110
\end{array}
$$

Step 3: Rearrange the final number

$0.011011 \quad 2^{0110}$

$0.11011 \; (2)^{0101} \Rightarrow (27)_{10}$

Effects due to truncation & rounding

* most of the i/p signals are continuous in time. To convert discrete must be sampled & quantized.

* Quantization will be done by
  (i) Truncation
  (ii) Rounding.

Truncation is the process of quantizing the binary number by ignoring the bits beyond 'b' bits.

$Q_1(x)$ be the value after truncation

$E_t = Q_t(x) - x$

truncation error

x – original value of the number.

eg: Quantize the number $(0.675)_{10}$ by truncation for 2 bits and calculate the quantization error.

$(0.675)_{10} \longrightarrow$

$0.675 \times 2 = 1.35 \longrightarrow 1$

$0.35 \times 2 = 0.7 \longrightarrow 0$

$0.7 \times 2 = 1.4 \longrightarrow 1$

$0.4 \times 2 = 0.8 \longrightarrow 0$

$0.8 \times 2 = 1.6 \longrightarrow 1$

$0.6 \times 2 = 1.2 \longrightarrow 1$

$(0.675)_{10} = (0.1010)_2$

Truncate to 2 bits

$= (0.10)_2 \xrightarrow[\text{Cons}]{\text{Decimal}} 0.5$

Actual number $x = 0.675$

$Q_t(x) = 0.5$

$\mathcal{E}_t = Q_t(x) - x$

$= 0.5 - 0.675$

$= -0.175.$

Rounding :- It is the process of quantizing a binary number to its nearest value. Rounding is done by ~~to~~ retaining the 'b' bits after decimal point and by adding $(b+1)^{th}$ bit to the LSB.

x - original no.

$Q_r(x)$ - rounded value.

$$\varepsilon_r = Q_r(x) - x.$$

eg:- Quantize the number $(0.675)_{10}$ by rounding for 2 bits and calculate the quantization error.

$$(0.675)_{10} \xrightarrow[\text{Conversion}]{\text{Binary}} (0.10101)_2 \xrightarrow[\text{2bits}]{\text{Round to}} 0.11 \rightarrow$$

Decimal $\rightarrow (0.75)_{10}$

$x = 0.675$

$Q_r(x) = 0.75$

$\varepsilon_r = Q_r(x) - x$

$= 0.75 - 0.675$

$= 0.075$

Explain the characteristics of a Limit cycle oscillation w.r.t the the system described by the difference eqn.

$$y(n) = 0.95\, y(n-1) + x(n)$$

Determine the dead Band of the filter.

$$y_r(n) = Q_r[0.95\, y(n-1)] + x(n)$$

* Let $x(n) = 0.75$ for $n=0$
  0 fo $n \neq 0$

* 4 bits are used to represent the quantized product excluding sign bit.

<u>with n=0</u>

$$y_r(n) = Q_r[0.95\, y_r(n-1)] + x(n)$$

$$y_r(0) = Q_r[0.95\, y_r(-1)] + x(0)$$

$y_r(-1) = 0$

$$= Q_r[0.95 \times 0] + 0.75$$

$$= 0.75$$

$$0.75 = [0.11]_2$$

4 bits rounded

$$[0.11]_2 = [0.1100]_2$$

$\boxed{y_r(0) = 0.75}$ after 4 bits rounding.

$0.75 \times 2 = 1.5 \rightarrow 1$

$.5 \times 2 = 1.0 \rightarrow 1$

$0 \times 2 = 0$

$n = 1$

$y_r(1) = Q_r[0.95\, y_r(0)] + x(1)$ $\quad (0.75)_{10} = 0.11$

$= Q_r[0.95 \times 0.75] + 0$

$= Q_r[0.7125]$

$[0.7125]_{10} = [0.1011011001100\ldots]_2$

Round it to 4 bits..

$Q_r[0.7125] = [0.1011]_2$

But decimal equivalent of

$[0.1011]$ is 0.6875

$\boxed{y_r(1) = 0.6875}$

$n = 2$ $\quad y_r(2) = Q_r[0.95\, y_r(1)] + x(2)$

$= Q_r[0.95 \times 0.6875] + 0$

$= Q_r[0.653125]$

$[0.653125]_{10} = [0.1010011100110011]_2$

$$Q_r[0.653125]_{10} = [0.1010]$$

$$\boxed{y_r(2) = 0.625}$$

$\underline{n=3}$

$$y_r(3) = Q_r[0.95\, y_r(2)] + x(3)$$

$$= Q_r[0.95 \times 0.625] + 0$$

$$= Q_r[0.59375]$$

$$[0.59375] = [0.10101]_2$$

$$Q_r[0.59375] = [0.1010]_2$$

$$y_r(3) = 0.625$$

$$y_r(2) = y_r(3) = \ldots = 0.625$$

Thus the system enters into Limit cycle oscillation when $n = 2$.

Dead band:

$$y(n-1) \le \frac{\delta/2}{1-|\alpha|}$$

$$\delta = \frac{1}{2^b} = \frac{1}{2^4} = 0.0625$$

$$y(n-1) \le \frac{0.0625}{1-0.95}$$

$$\le 0.625$$

Dead Band $= \{-0.625, +0.625\}$

Unit – V: **Dsp Applications**

Multirate signal processing: Decimation, Interpolation, Sampling rate conversion by a rational factor – Adaptive Filters: Introduction, Applications of adaptive filtering to equalization-DSP Architecture Fixed and Floating point architecture principles.

Multi-rate signal processing:

The process of converting a signal from a given rate to a different rate is called sampling rate conversion. Systems that employ multiple sampling rates in the processing of digital signals are called multi rate digital signal processing.

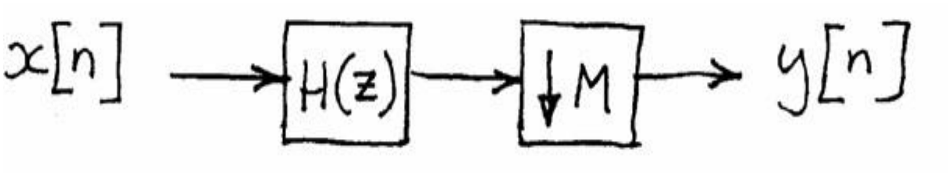
Down-sampling:

The process of reducing the sampling rate by an integer factor(D) is called decimation of the sampling rate. It is also called down sampling by factor(D).Decimator consists of decimation filter to band limit the signal and down sampler to decrease the sampling rate by an integer factor (D).

Decimation

– Reduce the sampling rate of a discrete-time signal.

– Low sampling rate reduces storage and computation requirements.



up-sampling:

Increasing sampling rate of a signal by an integer factor I is known as Interpolation or up-sampling. An increase in the sampling rate by an integer factor I may be done by interpolating (I-1) new samples between successive values of the signals.

• Interpolation

– Increase the sampling rate of a discrete-time signal.

– Higher sampling rate preserves fidelity

Sampling Rate Conversion

Having discussed the special cases of decimation (down sampling by a factor D) and interpolation (upsampling by a factor I), we now consider the general case of sampling rate conversion by a rational factor I/D. Basically, we can achieve this sampling rate conversion by first performing interpolation by the factor I and then decimating the output of the interpolator by the factor D. In other words, a sampling rate conversion by the rational factor I/D is accomplished by cascading an interpolator with a decimator. We emphasize that the importance of performing the interpolation first and the decimation second is to preserve the desired spectral characteristics of x(n).

Sample-rate conversion is the process of changing the sampling rate of a discrete signal to obtain a new discrete representation of the underlying continuous signal. Application areas include image scaling and audio/visual systems, where different sampling rates may be used for engineering, economic, or historical reasons.

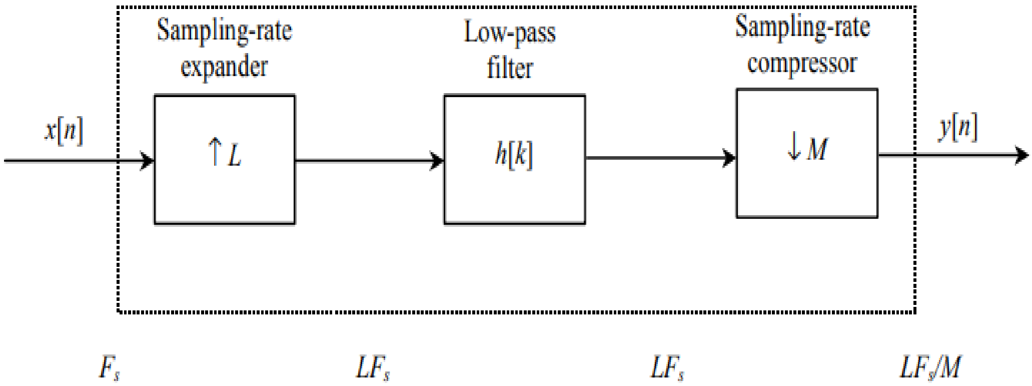


Fig: Sampling-rate conversion by expansion, filtering, and decimation

An example of sampling-rate conversion would take place when data from a CD is transferred onto a DAT. Here the sampling-rate is increased from 44.1 kHz to 48 kHz. To enable this process the non-integer factor has to be approximated by a rational number:

$$\frac{L}{M} = \frac{48}{44.1} = \frac{160}{147} = 1.08844$$

Hence, the sampling-rate conversion is achieved by interpolating by L i.e. from 44.1 kHz to [44.1x160] = 7056 kHz.

Then decimating by M i.e. from 7056 kHz to [7056/147] = 48 kHz.

Multistage Approach

When the sampling-rate changes are large, it is often better to perform the operation in multiple stages, where Mi(Li), an integer, is the factor for the stage i.

$$M = M_1 M_2 \ldots M_I \text{ or } L = L_1 L_2 \ldots L_I$$

An example of the multistage approach for decimation is shown in Figure 9.8. The multistage approach allows a significant relaxation of the anti-alias and anti-imaging filters, with a consequent reduction in the filter complexity. The optimum number of stages is one that leads to the least computational effort in terms of either the multiplications per second (MPS), or the total storage requirement (TSR).

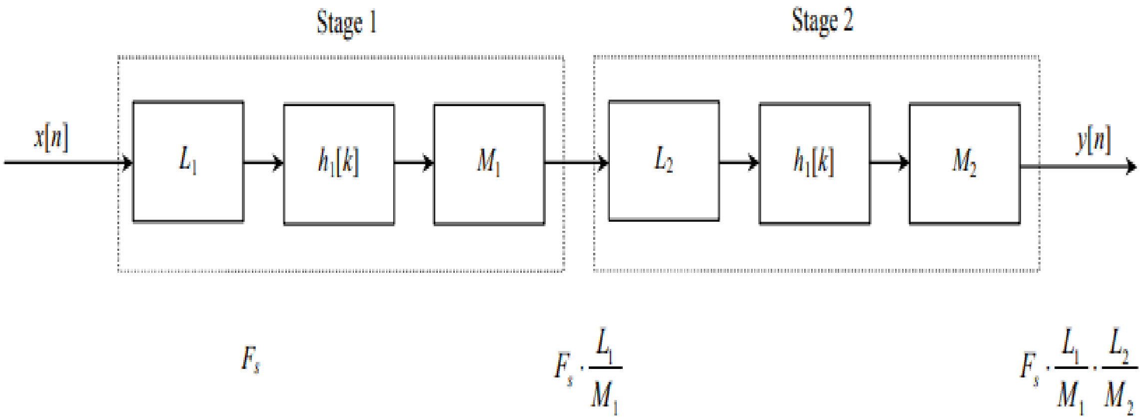


Fig: Multistage approach for the decimation process.

The meth to avoid aliasing:

(i) Pre alias filter must be used to limit band of frequencies of the signal to f_mHz.

(ii) Sampling frequency 'f_s' must be selected such that $f_s > 2 f_m$

The need for anti aliasing filter prior to down sampling:

Anti aliasing filter is used to avoid aliasing caused by down sampling the signal x(n).

The need for anti imaging filter after up sampling a signal?

Anti imaging filter removes the unwanted images that are that are yielded by up sampling.

Applications of multi rate signal processing.

Multirate systems are used in a CD player when the music signal is converted from digital into analogue (DAC). Digital data (16-bit words) are read from the disk at a sampling rate of 44.1 kHz. If this data were converted directly into an analogue signal, image frequency bands centred on multiples of the sampling-rate would occur, causing amplifier overload, and distortion in the music signal. To protect against this, a common technique called oversampling is often implemented nowadays in all CD players and in most digital processing systems of music signals. Fig.3 below illustrates a basic block diagram of a CD player and how oversampling is utilised. It is customary to oversample (or expand) the digital signal by a factor of x8, followed by an interpolation filter to remove the image frequencies. The sampling rate of the resulting signal is now increased up to 352.8 kHz. The digital signal is then converted into an analogue waveform by passing it through a 14-bit DAC. Then the output from this device is passed through an analogue low-pass filter before it is sent to the speakers.

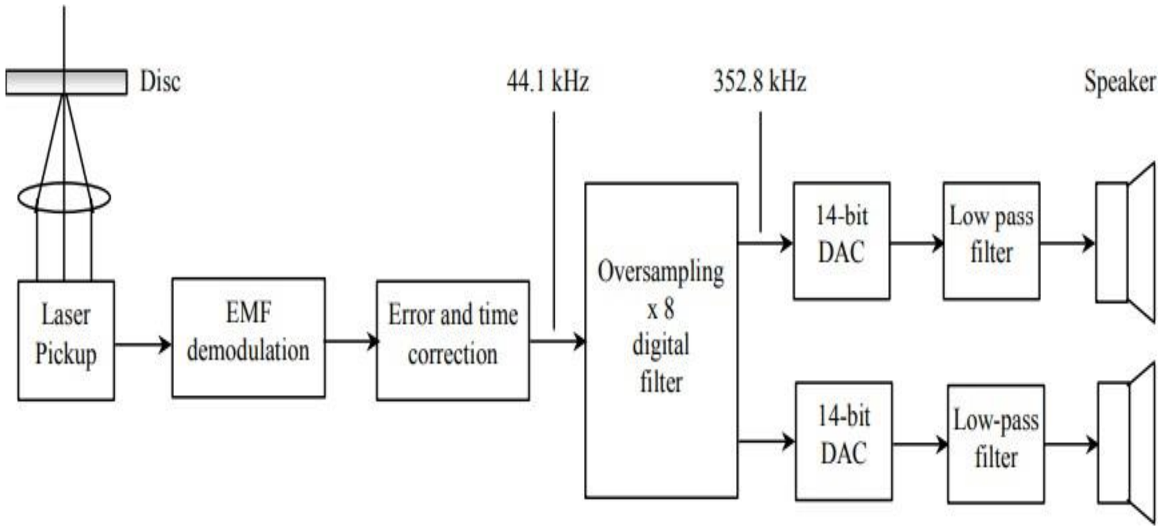


Fig. 3: Digital to analogue conversion for a CD player using x8 oversampling.

Fig. 4 illustrates the procedure of converting a digital waveform into an analogue signal in a CD player using x8 oversampling. As an example, Figure (a) illustrates a 20 kHz sinusoidal signal sampled at 44.1 kHz, denoted by x[n]. The six samples of the signal represent the waveform over two periods. If the signal x[n] was converted directly into an analogue waveform, it would be very hard to exactly reconstruct the 20 kHz signal from this diagram. Now, Figure (b) shows x[n] with an x8 interpolation, denoted by y[n]. Figure (c) shows the analogue signal y(t), reconstructed from the digital signal y[n] by passing it through a DAC. Finally, Figure (d) shows the waveform of z(t), which is obtained by passing the signal y(t) through an analogue low-pass filter.

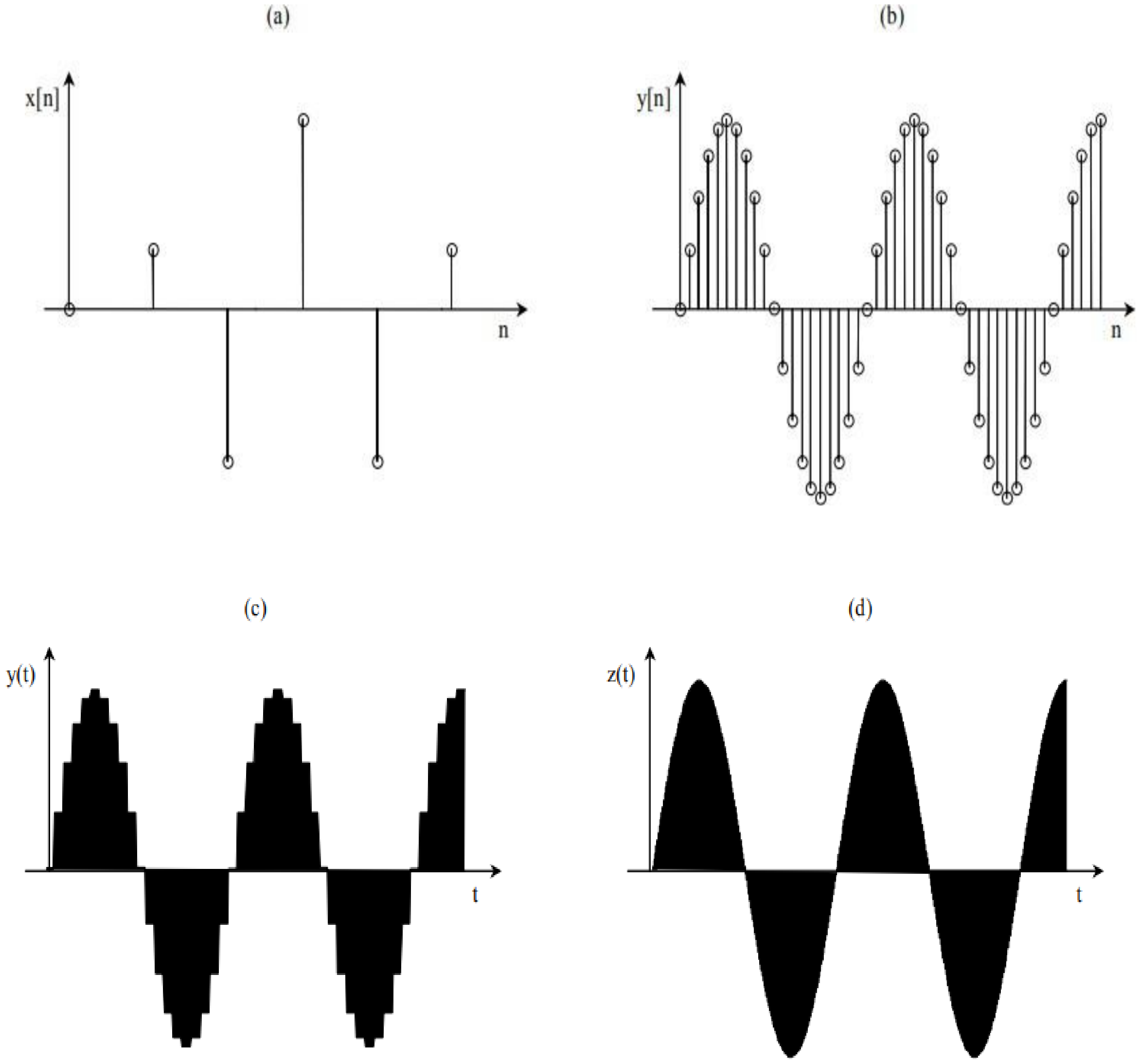


Fig 4: Illustration of oversampling in CD music signal reconstruction.

The effect of oversampling also has some other desirable features. Firstly, it causes the image frequencies to be much higher and therefore easier to filter out. The anti-alias filter specification can therefore be very much relaxed i.e. the cutoff frequency of the filter for the previous example increases from [44.1 / 2] = 22.05 kHz to [44.1x8 / 2] = 176.4 kHz after the interpolation.

1. Design of phase shifters
2. Interfacing of digital systems with different sampling rates
3. Implementation of narrow band LPF & implementation of Digital Filter Bank
4. Sub band coding of speech signals & Quadrature mirror filter
5. Trans multiplexers & Over sampling of A/D and D/A conversion

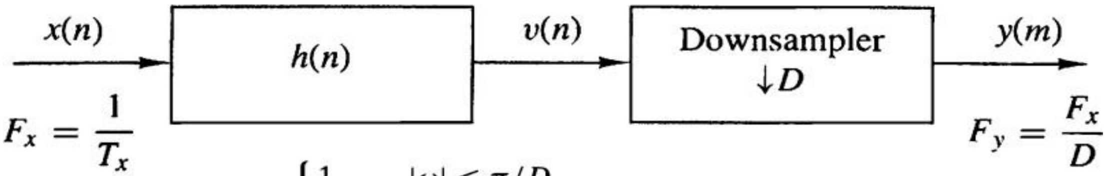
Mention the applications of speech coding.

Digital transmission like telephony, narrow band cellular radio, military communications and secrecy missions, voice mail sent on telephone networks, voice encryption, integrated voice and data transmission over packet networks.

Sub-band coding:

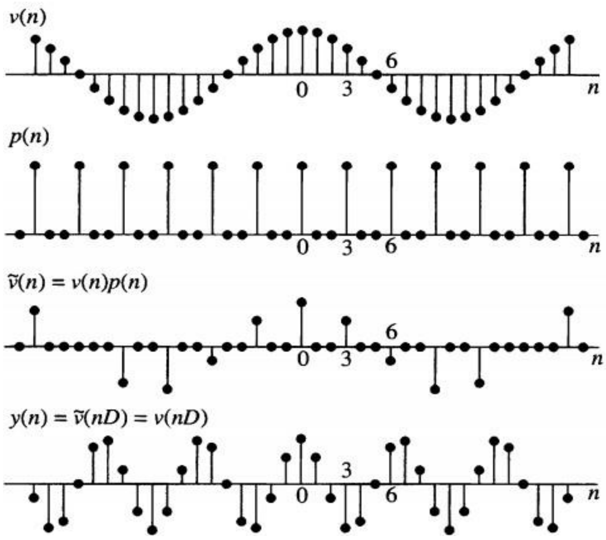
The speech signal is applied to an analysis filter bank consisting of a set of Q band pass filters. This digital filtration divides the speech signal into a non overlapping frequency bands. These filter banks are contiguous in frequency. Hence, by additive recombination of the set of sub band signals, one can approximately generate the original speech signal.

Decimation By A Factor D:



$$H_D(\omega) = \begin{cases} 1, & |\omega| \le \pi/D \\ 0, & \text{otherwise} \end{cases}$$

$$v(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \qquad y(m) = v(mD) = \sum_{k=0}^{\infty} h(k)x(mD-k)$$



$$\tilde{v}(n) = \begin{cases} v(n), & n = 0, \pm D, \pm 2D, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{v}(n) = v(n)p(n)$$

$$p(n) = \frac{1}{D}\sum_{k=0}^{D-1} e^{j2\pi kn/D}$$

$$y(m) = \tilde{v}(mD) = v(mD)p(mD) = v(mD)$$

$$Y(z) = \sum_{m=-\infty}^{\infty} y(m) z^{-m} = \sum_{m=-\infty}^{\infty} \tilde{v}(mD) z^{-m}$$

$$Y(z) = \sum_{m=-\infty}^{\infty} \tilde{v}(m) z^{-m/D}$$

$$\tilde{v}(n) = v(n)p(n)$$

$$p(n) = \frac{1}{D}\sum_{k=0}^{D-1} e^{j2\pi kn/D}$$

$$Y(z) = \sum_{m=-\infty}^{\infty} v(m)\left[\frac{1}{D}\sum_{k=0}^{D-1} e^{j2\pi mk/D}\right] z^{-m/D}$$

$$= \frac{1}{D}\sum_{k=0}^{D-1}\sum_{m=-\infty}^{\infty} v(m)\left(e^{-j2\pi k/D} z^{1/D}\right)^{-m}$$

$$= \frac{1}{D}\sum_{k=0}^{D-1} V\left(e^{-j2\pi k/D} z^{1/D}\right)$$

$$v(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

$$= \frac{1}{D}\sum_{k=0}^{D-1} H_D\left(e^{-j2\pi k/D} z^{1/D}\right) X\left(e^{-j2\pi k/D} z^{1/D}\right)$$

$$H_D(\omega) = \begin{cases} 1, & |\omega| \le \pi/D \\ 0, & \text{otherwise} \end{cases}$$

Evaluate the Z-transform on unit circle with frequency variable $\omega_y = \dfrac{2\pi F}{F_y} = 2\pi F T_y$

$$F_y = \frac{F_x}{D} \qquad \omega_x = \frac{2\pi F}{F_x} = 2\pi F T_x \qquad \omega_y = D\omega_x$$

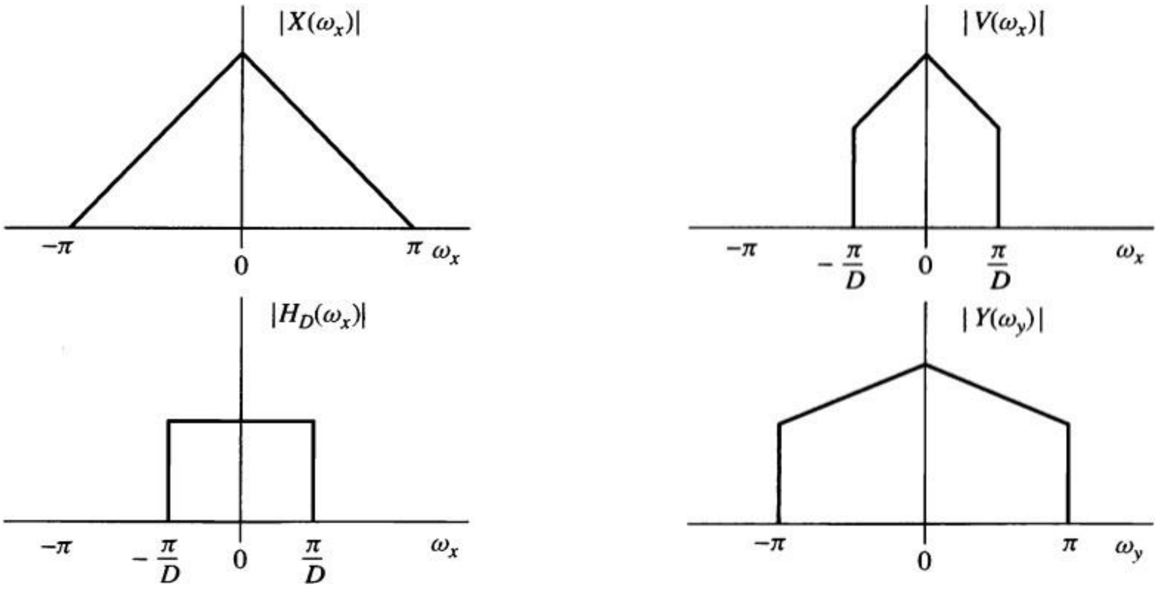
Thus, $0 \le |\omega_x| \le \pi/D$ gets stretched to $0 \le |\omega_y| \le \pi$ by down-sampling

$$Y(z) = \frac{1}{D}\sum_{k=0}^{D-1} H_D\left(e^{-j2\pi k/D} z^{1/D}\right) X\left(e^{-j2\pi k/D} z^{1/D}\right)$$

$$Y(\omega_y) = \frac{1}{D}\sum_{k=0}^{D-1} H_D\left(\frac{\omega_y - 2\pi k}{D}\right) X\left(\frac{\omega_y - 2\pi k}{D}\right)$$

If $H_D(\omega)$ is correctly designed, then aliasing is eliminated and

$$Y(\omega_y) = \frac{1}{D} H_D\left(\frac{\omega_y}{D}\right) X\left(\frac{\omega_y}{D}\right) = \frac{1}{D} X\left(\frac{\omega_y}{D}\right) \qquad \text{for } 0 \le |\omega_y| \le \pi$$



Interpolation By A Factor I:

$$y(m) = x(m/I) \text{ for } m = 0, \pm I, +2I, \ldots$$

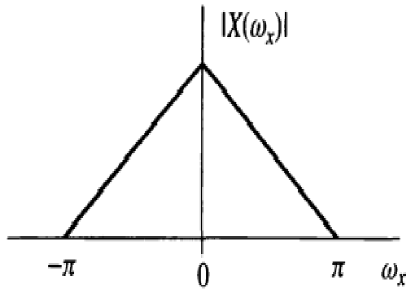
$$x(n) \longrightarrow \quad v(m) = \begin{cases} x(m/I), & m = 0, \pm I, \pm 2I, \ldots \\ 0, & \text{otherwise} \end{cases}$$

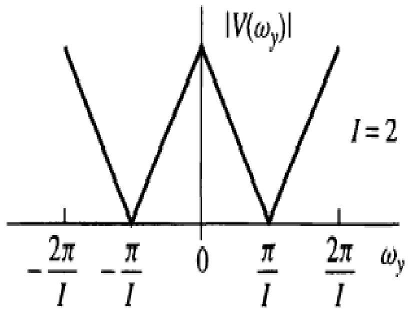
$$F_y = I F_x$$

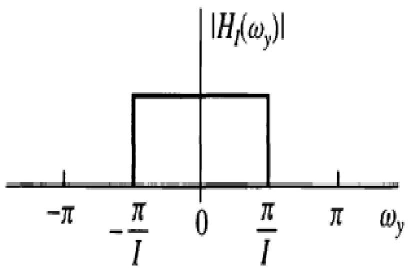
$$V(z) = \sum_{m=-\infty}^{\infty} v(m) z^{-m} = \sum_{m=-\infty}^{\infty} x(m) z^{-mI} = X(z^I)$$

$$\text{DTFT:} \qquad V(\omega_y) = X(\omega_y I)$$

$$\omega_y = 2\pi F/F_y \qquad F_y = I F_x \qquad \Rightarrow \qquad \omega_y = \frac{\omega_x}{I}$$







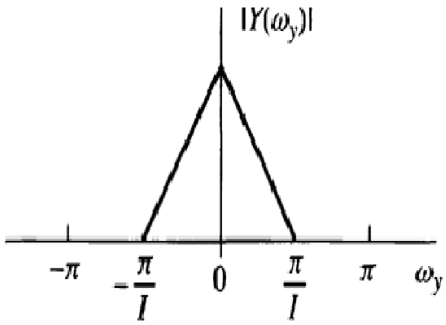
As the frequency component of $x(n)$ are unique in the range $0 \le \omega_y \le \pi/I$ Images beyond that in $v(n)$ should be rejected by low pass filtering

$$H_I(\omega_y) = \begin{cases} C, & 0 \le |\omega_y| \le \pi/I \\ 0, & \text{otherwise} \end{cases}$$

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k)v(k)$$

$$Y(\omega_y) = \begin{cases} CX(\omega_y I), & 0 \le |\omega_y| \le \pi/I \\ 0, & \text{otherwise} \end{cases}$$

$$\boxed{y(m) = \sum_{k=-\infty}^{\infty} h(m-kI)x(k)}$$



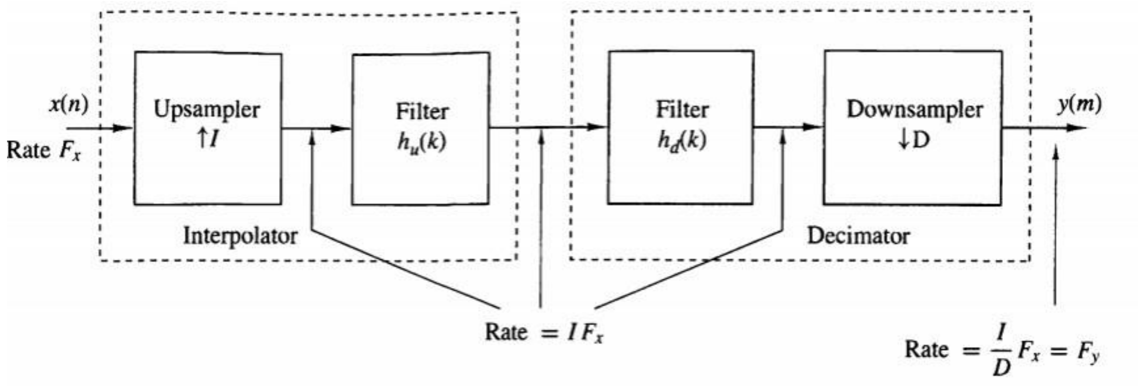
C = ?

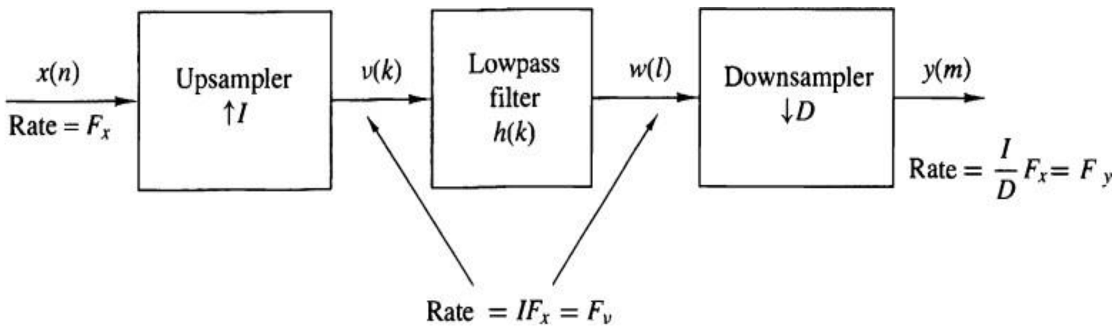
$$y(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(\omega_y)d\omega_y = \frac{C}{2\pi}\int_{-\pi/I}^{\pi/I} X(\omega_y I)d\omega_y$$

$$\omega_y = \omega_x/I, \longrightarrow = \frac{C}{I}\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega_x)d\omega_x = \frac{C}{I}x(0)$$

$C = I$ is the desired normalization factor

Sampling Rate Conversion By A Rational Factor I/D:

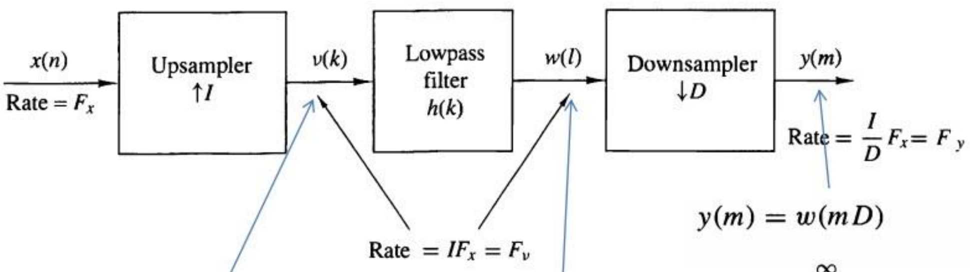




Frequency response of the combined filter

$$H(\omega_v) = \begin{cases} I, & 0 \le |\omega_v| \le \min(\pi/D, \pi/I) \\ 0, & \text{otherwise} \end{cases}$$

$$\omega_v = 2\pi F/F_v = 2\pi F/I F_x = \omega_x/I.$$



$$y(m) = w(mD) = \sum_{k=-\infty}^{\infty} h(mD - kI)x(k)$$

$$w(l) = \sum_{k=-\infty}^{\infty} h(l-k)v(k) = \sum_{k=-\infty}^{\infty} h(l-kI)x(k)$$

$$v(l) = \begin{cases} x(l/I), & l = 0, \pm I, \pm 2I, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$y(m) = w(mD)$$

$$= \sum_{k=-\infty}^{\infty} h(mD - kI)x(k)$$

Change of variable

$$k = \left\lfloor \frac{mD}{I} \right\rfloor - n$$

$$y(m) = \sum_{n=-\infty}^{\infty} h\left(mD - \left\lfloor \frac{mD}{I} \right\rfloor I + nI\right) x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right)$$

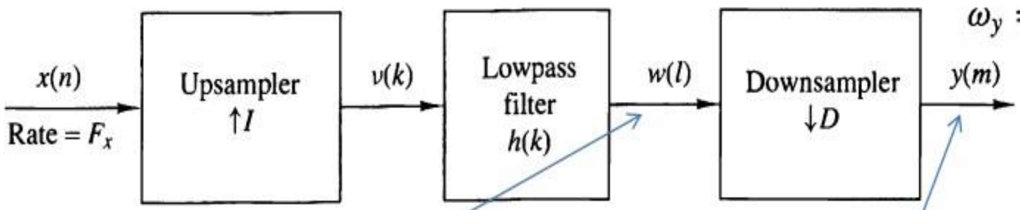
We know

$$mD - \left\lfloor \frac{mD}{I} \right\rfloor I = (mD)_I \Rightarrow y(m) = \sum_{n=-\infty}^{\infty} h(nI + (mD)_I)x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right)$$

$$x(n) \xrightarrow[\substack{g(n,m) = h(nI + (mD)_I),\\ -\infty < m, n < \infty}]{\text{Time-varying filter}} y(m)$$

$$g(n, m + kI) = h(nI + (mD + kDI)_I) = h(nI + (mD)_I) = g(n, m)$$

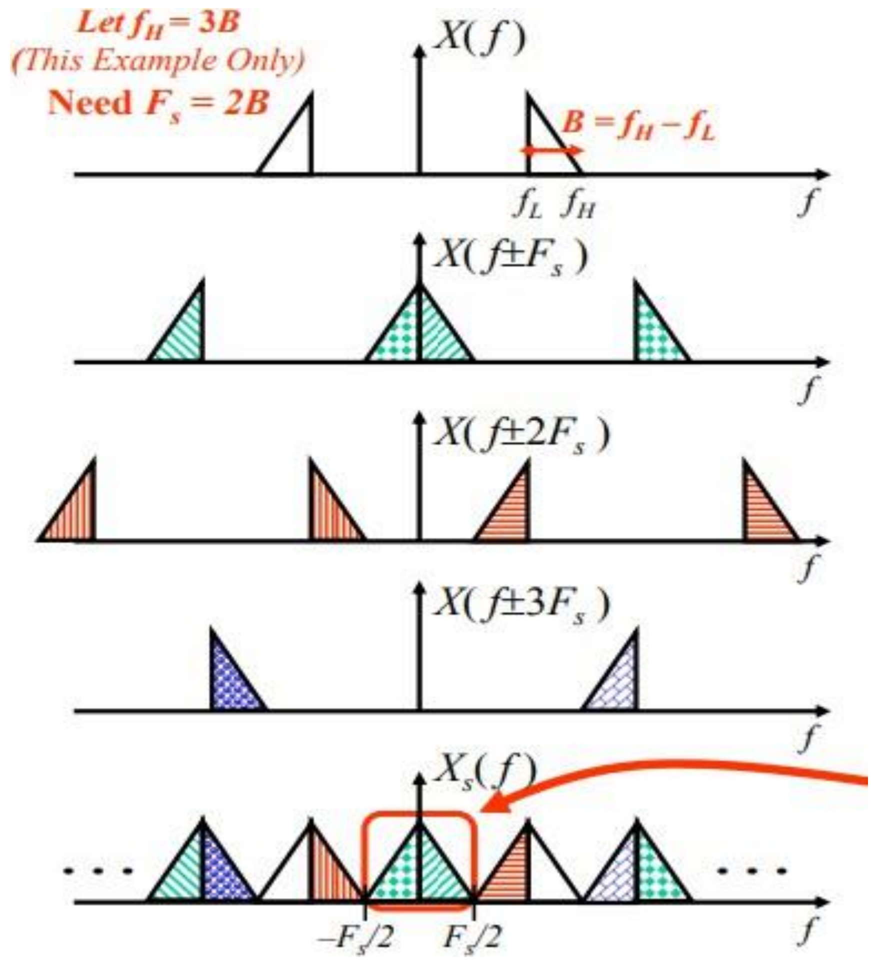
$$\omega_y = D\omega_v$$



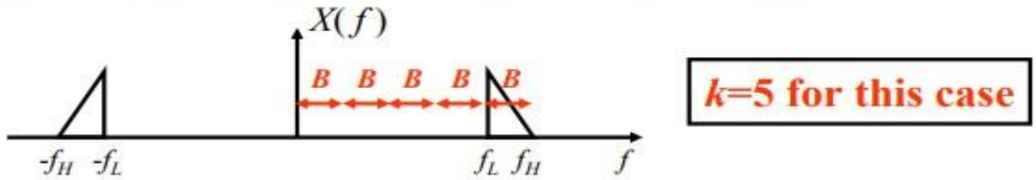
$$H(\omega_v)X(\omega_v I) = \begin{cases} IX(\omega_v I), & 0 \le |\omega_v| \le \min(\pi/D, \pi/I) \\ 0, & \text{otherwise} \end{cases}$$

$$Y(\omega_y) = \begin{cases} \dfrac{I}{D} X\left(\dfrac{\omega_y}{D}\right), & 0 \le |\omega_y| \le \min\left(\pi, \dfrac{\pi D}{I}\right) \\ 0, & \text{otherwise} \end{cases}$$

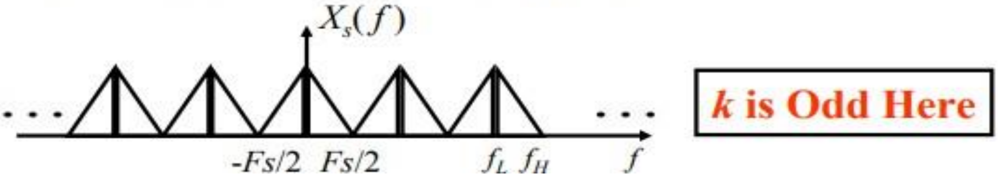
Sampling rate conversion of band pass signals:



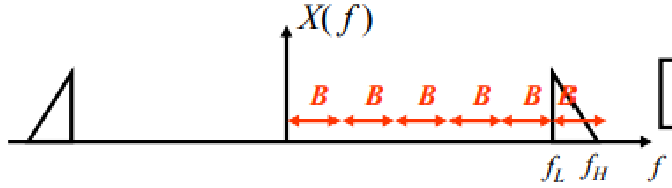
Consider the case where $f_H = kB$ (<u>k an Odd Integer</u>)



Whenever $f_2 = LB$, we <u>can choose $Fs = 2B$</u> to perfectly "interweave" the shifted spectral replicas

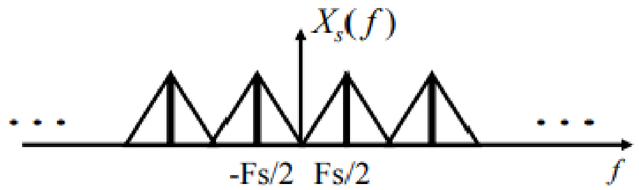


Consider the case where $f_H = LB$ (<u>k an Even Integer</u>)



k=6 for this case

Whenever $f_H = LB$, we <u>can choose $Fs = 2B$</u> to perfectly "interweave" the shifted spectral replicas



k is Even Here

<u>Note</u>: If k is EVEN the spectrum in the 0 to $Fs/2$ range is flipped. This is not usually a problem since the next step after BP sampling is usually to create the lowpass equivalent signal, which can be done in a way that gives either spectral orientation.

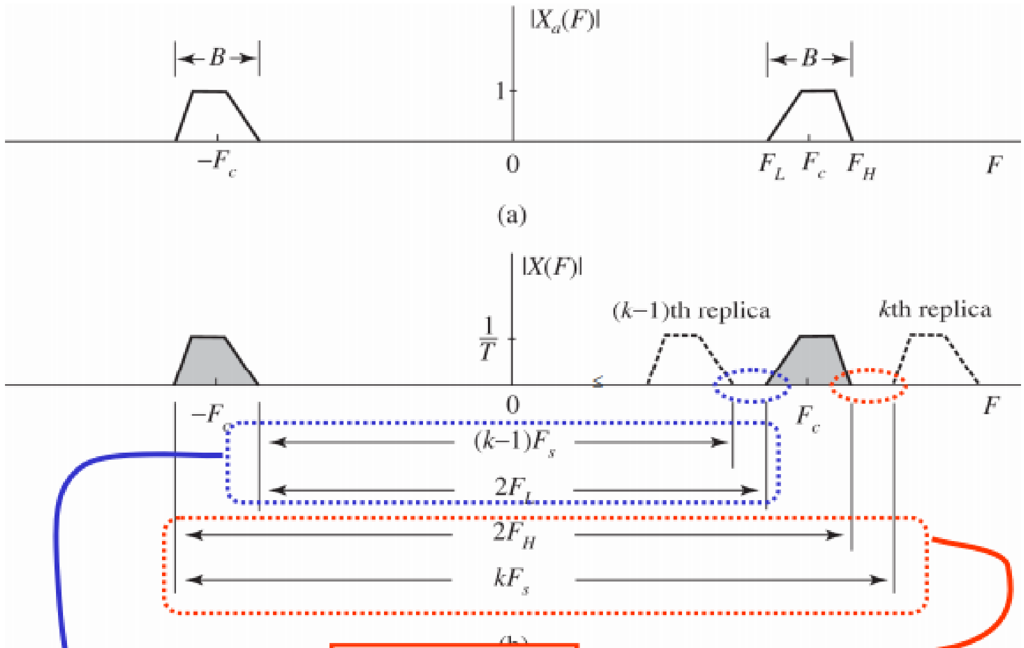


Figure 6.4.2

$$\boxed{2f_H \le kF_s}$$

$$\boxed{(k-1)F_s \le 2f_L}$$

$$\boxed{\frac{2f_H}{k} \le F_s \le \frac{2f_L}{k-1}}$$

To find the required value of *k*… re-write as:

$$\boxed{2f_H \le kF_s}$$
$$\boxed{(k-1)F_s \le 2f_L}$$

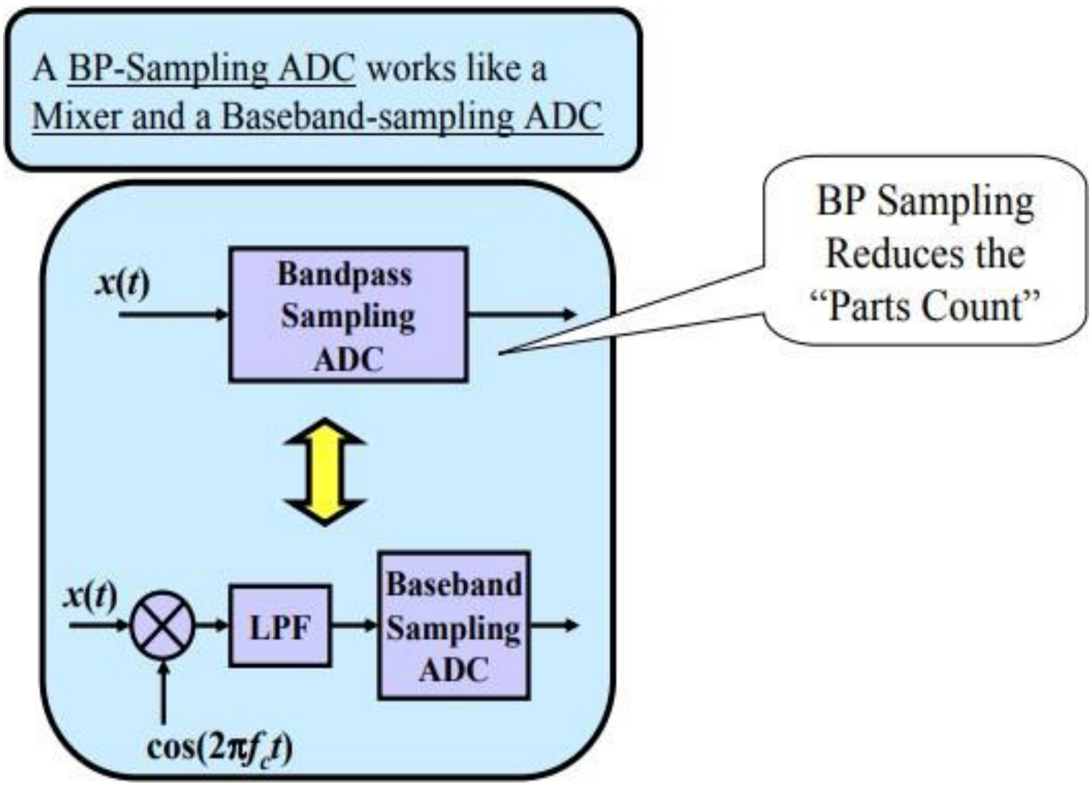
$$\Rightarrow$$

$$\boxed{\frac{1}{F_s} \le \frac{k}{2f_H}}$$
$$\boxed{(k-1)F_s \le 2(f_H - B)}$$

Now… solving these for *k* gives:

$$\boxed{k \le \frac{f_H}{B}} \Rightarrow \boxed{k_{\max} \le \left\lfloor \frac{f_H}{B} \right\rfloor}$$

Advantages of BP Sampling:



Unit – V: **Dsp Applications**

Multirate signal processing: Decimation, Interpolation, Sampling rate conversion by a rational factor – Adaptive Filters: Introduction, Applications of adaptive filtering to equalization-DSP Architecture Fixed and Floating point architecture principles.

Introduction

In general, DSP processors can be classified into two broad categories as general purpose and special purpose. Fixed-point devices such as Texas Instruments TMS320C54x, and Motorola DSP563x processors, and floating- point processors such as Texas Instruments TMS320C4x and Analog Devices ADSP21xxx SHARC processors are included in DSP Processors.

Special purpose hardware are divided into two categories,

1. One type of special- purpose hardware is sometimes called an algorithm-specific digital signal processor. Hardware designed for efficient execution of specific DSP algorithms such as digital filters, Fast Fourier Transform comes under this category.

2. Another type of hardware is sometimes called an application-specific digital signal processor. Hardware designed for specific applications: for example telecommunications, digital audio, or control applications comes under this category.

In most cases application-specific digital signal processors execute specific algorithms, such as PCM encoding/decoding, but they are also required to perform other application-specific operations. Examples of special-purpose DSP processors are Cirrus's processor for digital audio sampling rate converters (CS8420), Intel's multi- channel telephony voice echo canceller (MT9300), FFT processor (PDSPI65I5A) and programmable FIR filter (VPDSP 16256).

Both general-purpose and special-purpose processors can be designed with single chips or with individual blocks of multipliers, ALUs, memories, and so on. First, let us discuss the architectural features of digital signal processors that have made real-time DSP in many possible areas.

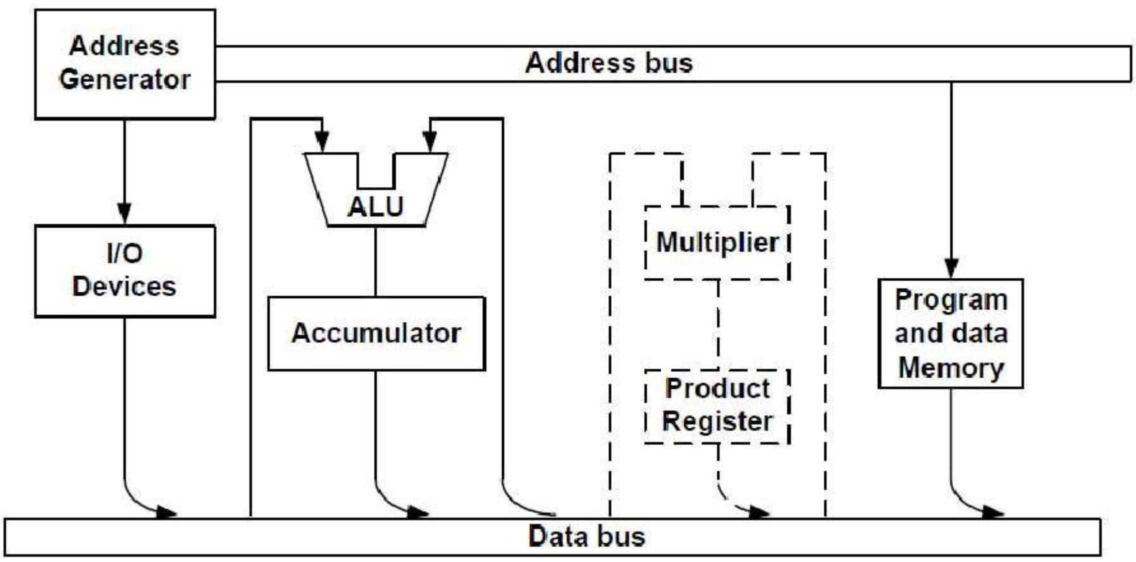


Figure 1. A simplified architecture for standard microprocessor

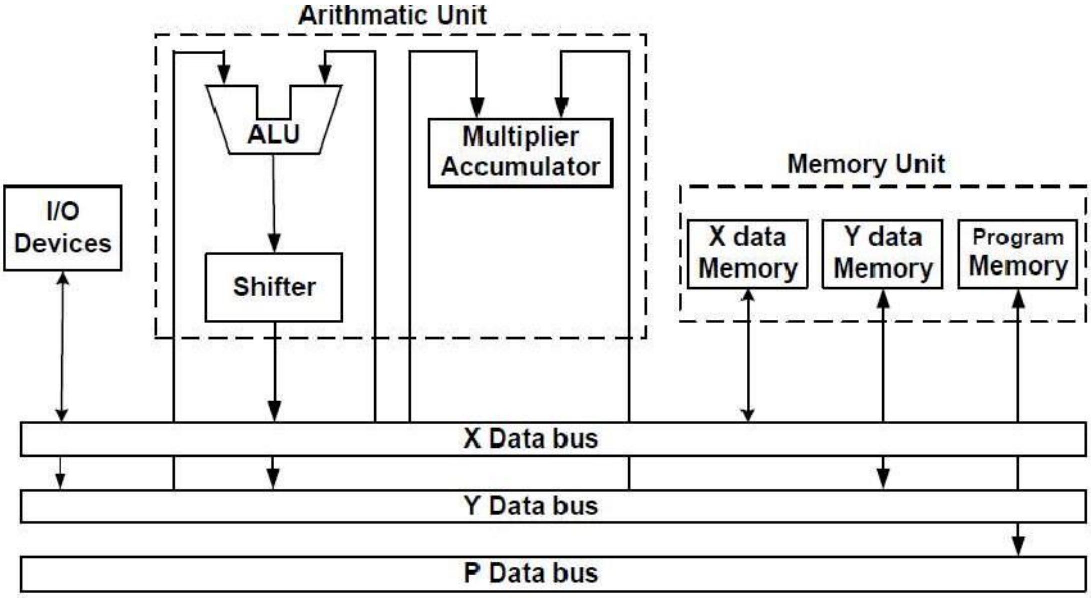


Figure 2. Basic generic hardware architecture for signal processing

Figure 2 shows generic hardware architecture suitable for real time DSP It is characterized by the multiple bus structure with separate memory space for data and program instructions. The data memories hold input data, intermediate data values and output samples, as well as fixed coefficients for example, digital filters or FFTs. The program instructions are stored in the program memory.

The I/O port provides a means of passing data to and from external devices such as the ADC and DAC or for passing digital data to other processors. Direct memory access (DMA), if available, allows for rapid transfer of blocks of data directly to or from data RAM, typically under external control.

Arithmetic units for logical and arithmetic operations include an ALU, a hardware multiplier and shifters (or multiplier--accumulator)

The main necessary of this architecture is that most DSP algorithms (such as filtering correlation and fast Fourier transform) involve repetitive arithmetic operations such as multiply, add, memory accesses, and heavy data flow through the CPU. The architecture of standard microprocessors is not suited for this type of activities. So an important goal in DSP hardware design is to optimize both the hardware architecture and the instruction set for DSP operations. In digital signal processors, this is achieved by making use of the concepts of parallelism. In particular, the following techniques are used:

1. Harvard architecture;
2. pipe-lining;
3. fast, dedicated hardware multiplier/accumulator;
4. special instructions dedicated to DSP;
5. replication;
6. on-chip memory/cache;
7. Extended parallelism — SIMD, VLIW and static superscalar processing.

For successful DSP design, it is important to understand these key architectural features.

Harvard architecture:

The principal feature of the Harvard architecture is that the program and data memories lie in two separate spaces, permitting a full overlap of instruction fetch and execution. Standard microprocessors, such as the Intel 6502, are characterized by a single bus structure for both data and instructions, as shown in Figure 1.

Suppose that in a standard microprocessor if a value op I at address ADR 1 in memory into the accumulator is to be read and then to be stored at two other addresses, ADR2 and *ADR3*. The instructions could be

LDA ADRI load the operand op1 into the accumulator from ADRI STA

ADR2 store op1 in address ADR2

STA ADR3 store op1 in address ADR3

Typically, each of these instructions would involve three distinct steps:

• instruction fetch;

• instruction decode;

• instruction execute.

In our case, the instruction fetch involves fetching the next instruction from memory, and instruction execute involves either reading or writing data into memory. In a standard processor, without Harvard architecture, the program instructions (that is, the program code) and the data (operands) are held in one memory space; see Figure 3. Thus the fetching of the next instruction while the current one is executing is not allowed, because the fetch and execution phases each require memory access.

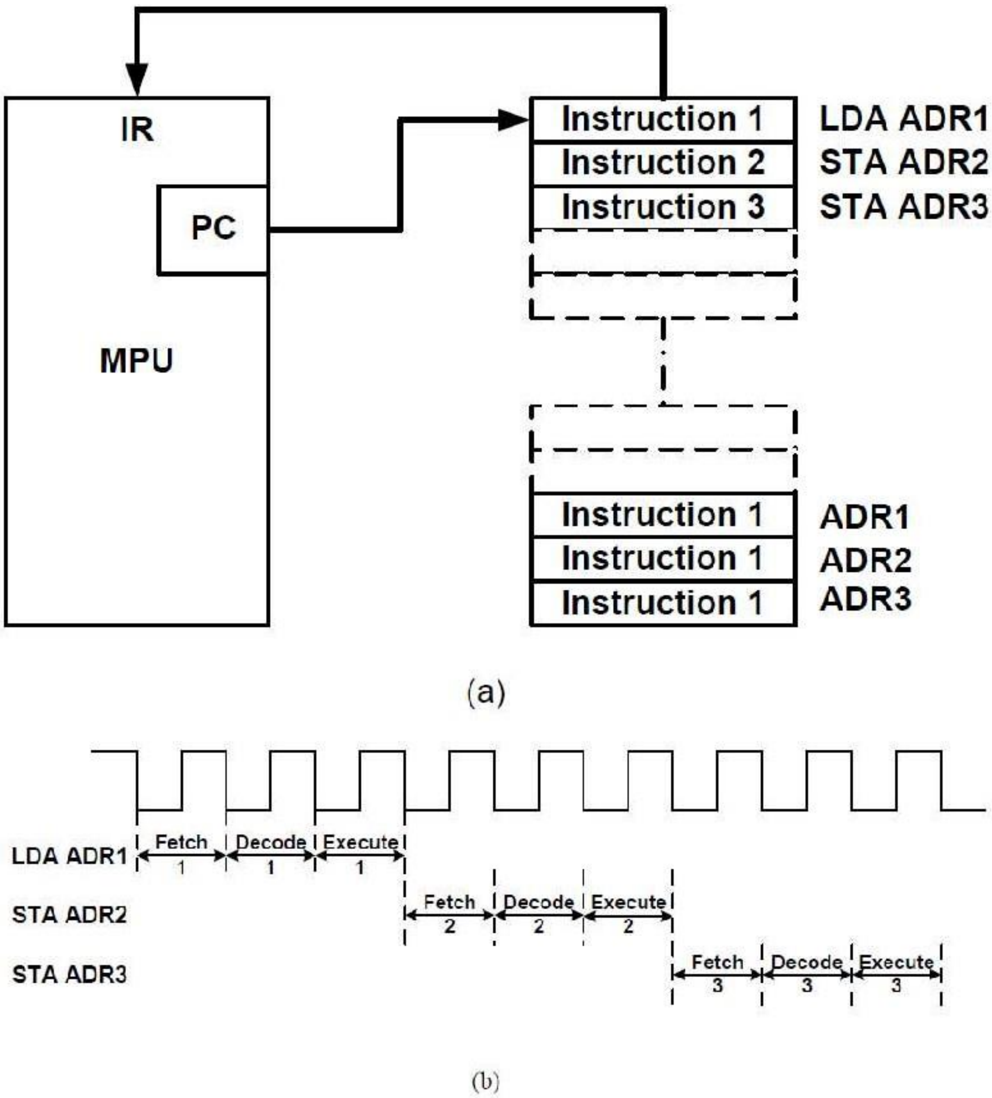


Figure 3. An illustration of instructions fetch, decode, and execute in a Non-Harward architecture with single memory space. (a) instruction fetch from memory (b) timing diagram

In a Harvard architecture (Figure 4), since the program instructions and data lie in separate memory spaces, the fetching of the next instruction can overlap the execution of the current instruction as shown in Figure 5. Normally, the program memory holds the program code, while the data memory stores variables such as the input data samples.

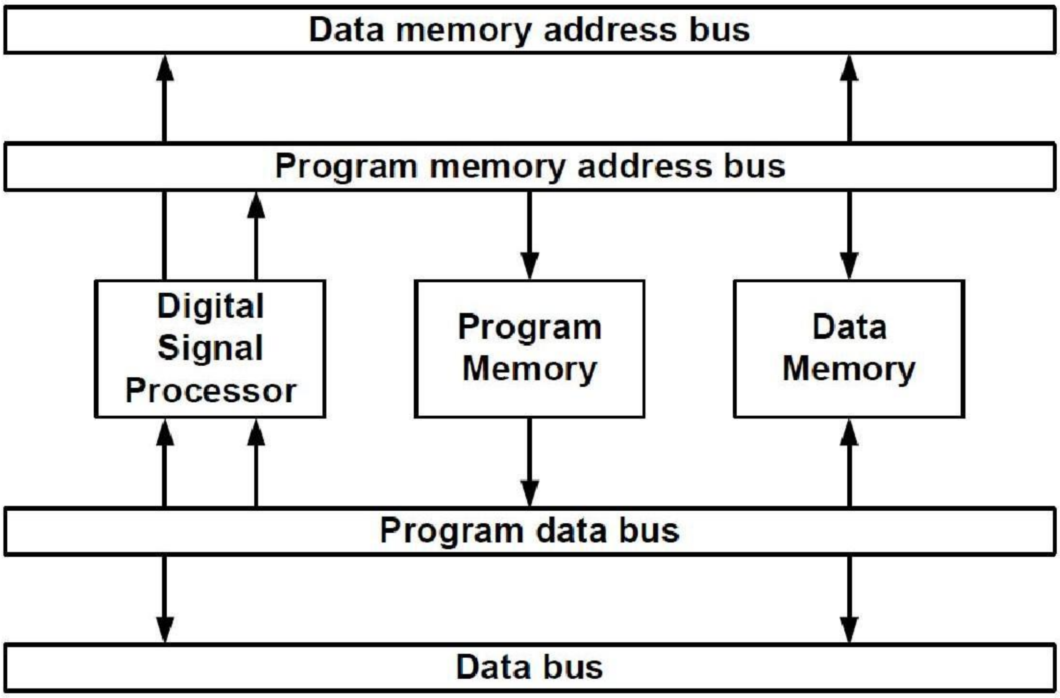


Figure 4. Basic Harvard architecture with separate data and program memory spaces

It may be seen from Figure 4 that data and program instruction fetches can be overlapped as two independent memories are used in the architecture. This is explained with the help of the timing diagram as shown in Figure 5 below.

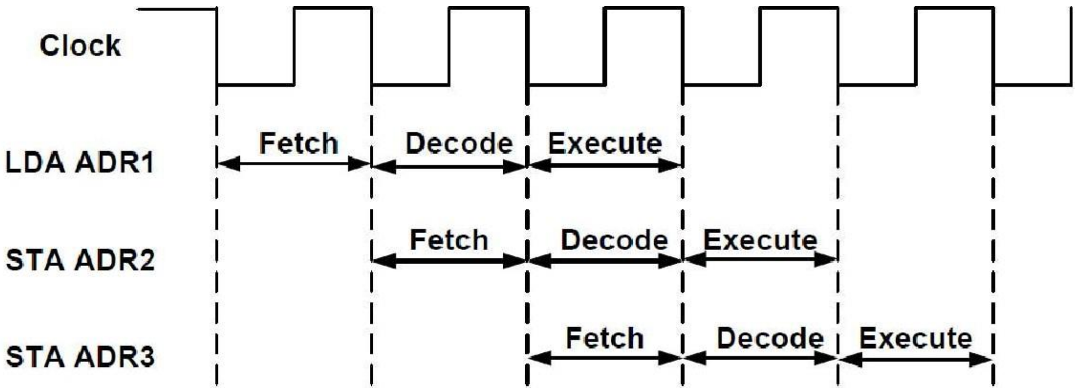


Figure 5. An illustration of instruction overlap made possible by the Harvard architecture

Strict Harvard architecture is used by some digital signal processors (for example Motorola D5P56000), but most use a modified Harvard architecture (for example, the TMS32O family of processors). For example in the modified architecture used by the TMS32O, separate program and data memory spaces are still maintained. But unlike the strict Harvard architecture, communication between the two memory spaces is permitted here.