PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Approved by **AICTE** and Affiliated to **Anna University**)

sky is the limit

Department of Electronics and Communication Engineering

Regulation 2021

II Year – IV Semester

EC3452- Electro Magnetic Fields

Unit–I Introduction

Electromagnetic Model, Units and constants, Review of vector Algebra, Rectangular, cylindrical and spherical coordinate systems, Line, Surface and volume integrals. Gradient of a scalar field, Divergence of vector field, Divergence theorem, Curl of a vector field, Stoke's theorem, Null Identities, Helmholtz's theorem.

⇒ Introduction

Electromagnetics is the study of behaviour of electric charges in ~~the free space~~ rest (or) motion

The effect of charges at rest position is called as electrostatic field (or) electric field 'E'.

The effect of charges at motion is called as Magnetic field. Generally the moving charges produces a current which gives rise to a magnetic field. 'H'

A field is a spatial distribution of any physical Quantity, which may (or) may not be a function of time.

⇒ Electromagnetic Model

It is a mathematical representation of an electromagnetic field.

⇒ Units and constants

Fundamental units

Quantity	Unit
Length	meter (m)
Mass	Kilogram (Kg)
Time	second (s)
Current	ampere (A)

Constants

Universal Constants	symbol	value
velocity of light	c	3×10^{8} m/s
permeability of free space	μ_0	$4\pi\times10^{-7}$ H/m
permittivity of free space	ϵ_0	8.854×10^{-12} F/m

⇒ Review of Vector Algebra

Scalar Quantity

It is a quantity which contain only magnitude.

Ex:- voltage, current, temp etc.

Vector Quantity

It is a quantity which contains both magnitude and direction.

Ex:- force, Displacement & Acceleration.

$\vec{F} = m\vec{A}$

Vector Algebra - Addition, subtraction & multiplication of vectors.

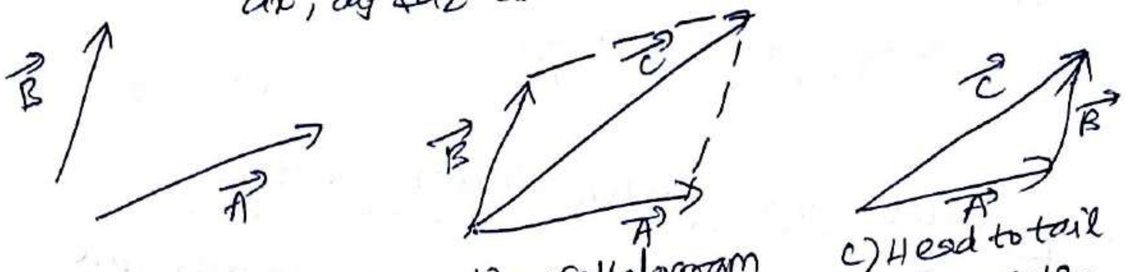
i) Addition of vectors

$\vec{A}$ is a vector & $\vec{B}$ is an another vector.

In General $\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$

$\vec{B} = B_x \vec{a_x} + B_y \vec{a_y} + B_z \vec{a_z}$

Where A_x, A_y & A_z are Magnitudes of vector
$\vec{a_x}$, $\vec{a_y}$ & $\vec{a_z}$ are unit vectors.



(a) Two vectors b) parallelogram rule. c) Head to tail rule

$$\vec{C} = \vec{A} + \vec{B}$$

Vector Addition obeys commutative & Associative laws.

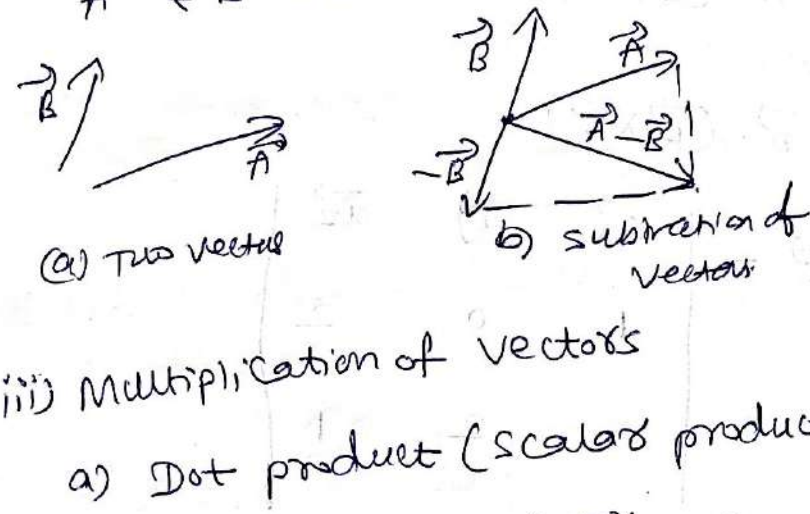
Commutative law: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Associative law: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$.

ii) Subtraction of vectors.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}).$$

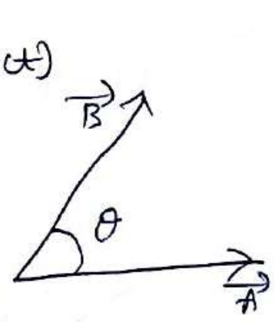
$\vec{A}$ & $\vec{B}$ are two different vectors.



(a) Two vectors b) subtraction of vectors

iii) Multiplication of vectors

a) Dot product (scalar product)



$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Note:- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

b) Cross product (vector product)

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Note:-

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

parallel and perpendicular vectors

a) if $\vec{A}\times\vec{B}=0$ then the two vectors are said to be parallel.

b) if $\vec{A}\cdot\vec{B}=0$ then the two vectors are said to be perpendicular.

problem

① $\vec{A} = \vec{a_x} + \vec{a_y}$

$\vec{B} = \vec{a_x} + 2\vec{a_z}$

$\vec{C} = 2\vec{a_y} + \vec{a_z}$

Find $\vec{A}\cdot(\vec{B}\times\vec{C})$ & $\vec{A}+\vec{B}+\vec{C}$

Sol:-

i) $\vec{A}\cdot(\vec{B}\times\vec{C})$

$$\vec{B}\times\vec{C} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \vec{a_x}(0-4) - \vec{a_y}(1-0) + \vec{a_z}(2-0)$$

$$\vec{B}\times\vec{C} = -4\vec{a_x} - \vec{a_y} + 2\vec{a_z}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{a_x}+\vec{a_y})\cdot(-4\vec{a_x} - \vec{a_y} + 2\vec{a_z})$$

$$= -4 - 1 \qquad \because \vec{a_x}\cdot\vec{a_x} = 1, \quad \vec{a_y}\cdot\vec{a_z} = 0$$

$$\boxed{\vec{A}\cdot(\vec{B}\times\vec{C}) = -5}$$

ii) $\vec{A}+\vec{B}+\vec{C}$

$= (\vec{a_x}+\vec{a_y}) + (\vec{a_x}+2\vec{a_z}) + (2\vec{a_y}+\vec{a_z})$

$= 2\vec{a_x}+3\vec{a_y}+3\vec{a_z}.$

(2) Given $\vec{A} = 4\vec{a_x} - 2\vec{a_y} + 2\vec{a_z}$

$\vec{B} = -6\vec{a_x} + 3\vec{a_y} - 3\vec{a_z}$

find whether the two vectors are parallel (or) perpendicular.

i) parallel condition.

$\vec{A}\times\vec{B} = 0$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 4 & -2 & 2 \\ -6 & 3 & -3 \end{vmatrix}$$

$= \vec{a_x}(6-6) - \vec{a_y}(-12+12) + \vec{a_z}(12-12)$

$\vec{A}\times\vec{B} = 0$. Therefore Two vectors are parallel.

ii) perpendicular condition.

$\vec{A}\cdot\vec{B} = 0$

$\vec{A}\cdot\vec{B} = (4\vec{a_x} - 2\vec{a_y} + 2\vec{a_z})\cdot(-6\vec{a_x} + 3\vec{a_y} - 3\vec{a_z})$

$= -24 - 6 - 6$

$\vec{A}\cdot\vec{B} = -36$

Therefore Two vectors are not perpendicular.

⇒ Vector Calculations.

There are 4 types

i) Differential operator (or) vector operator

ii) Gradiant of a scalar field

iii) Divergence of a vector field

iv) curl of a vector field.

i) Differential operator (or) vector operator

$$\nabla = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$$

ii) Gradient

Gradient of a scalar field produces vector field.

$$\nabla A = \frac{\partial A}{\partial x}\vec{a_x} + \frac{\partial A}{\partial y}\vec{a_y} + \frac{\partial A}{\partial z}\vec{a_z}$$

Gradiant of a scalar is a vector.

iii) Divergence of a vector field.

$$\nabla\cdot\vec{A} = \left(\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right)\cdot\left(A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}\right)$$

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergence of a vector is a scalar.

iv) curl of a vector field.

Curl deals with rotation

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

<u>Note:-</u>

if $\nabla \cdot \vec{A} = 0$ then $\vec{A}$ is solenoidal.

if $\nabla \times \vec{A} = 0$ then $\vec{A}$ is irrotational.

<u>problems</u>

③ Given $\vec{A} = 3y^4z^2\vec{a_x} + 4x^3z^2\vec{a_y} + 3z^2y^2\vec{a_z}$

check whether Given vector is solenoidal.

<u>sol:-</u>

$$\nabla \cdot \vec{A} = \left(\frac{\partial}{\partial x} 3y^4z^2\right) + \left(\frac{\partial}{\partial y} 4x^3z^2\right) + \left(\frac{\partial}{\partial z} 3z^2y^2\right)$$

$$\nabla \cdot \vec{A} = 0 + 0 + 6y^2z$$

$$\nabla \cdot \vec{A} = 6y^2z \neq 0$$

so $\vec{A}$ is not solenoidal.

④ Check whether Given vector is irrotational

$\vec{A} = 2xy\,\vec{a_x} + (x^2 + 2yz)\,\vec{a_y} + (y^2+1)\,\vec{a_z}$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & (x^2+2yz) & (y^2+1) \end{vmatrix}$$

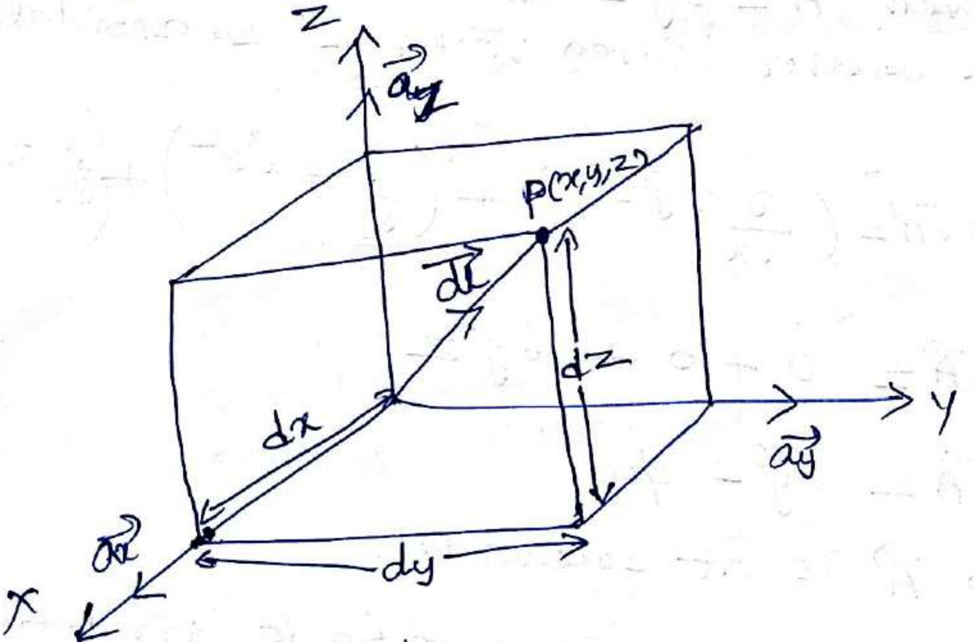
$$= \vec{a_x}\left[\frac{\partial}{\partial y}(y^2+1) - \frac{\partial}{\partial z}(x^2+2yz)\right] - \vec{a_y}\left[\frac{\partial}{\partial x}(y^2+1) - \frac{\partial}{\partial z}(2xy)\right] + \vec{a_z}\left[\frac{\partial}{\partial x}(x^2+2yz) - \frac{\partial}{\partial y}(2xy)\right]$$

$$= \vec{a_x}\left[2y - 2y\right] - \vec{a_y}\left[0 - 0\right] + \vec{a_z}\left[2x - 2x\right]$$

$$= 0$$

$\nabla \times \vec{A} = 0$ ∴ The given $\vec{A}$ is irrotational.

⇒ Rectangular (or) cartesian coordinate system.



→ Differential ~~...~~ line elements

dx, dy & dz

→ Differential Surface element

along x direction $\vec{ds_x} = dy\,dz\,\vec{a_x}$

along y direction $\vec{ds_y} = dx\,dz\,\vec{a_y}$

along z direction $\vec{ds_z} = dx\,dy\,\vec{a_z}$

→ Differential volume

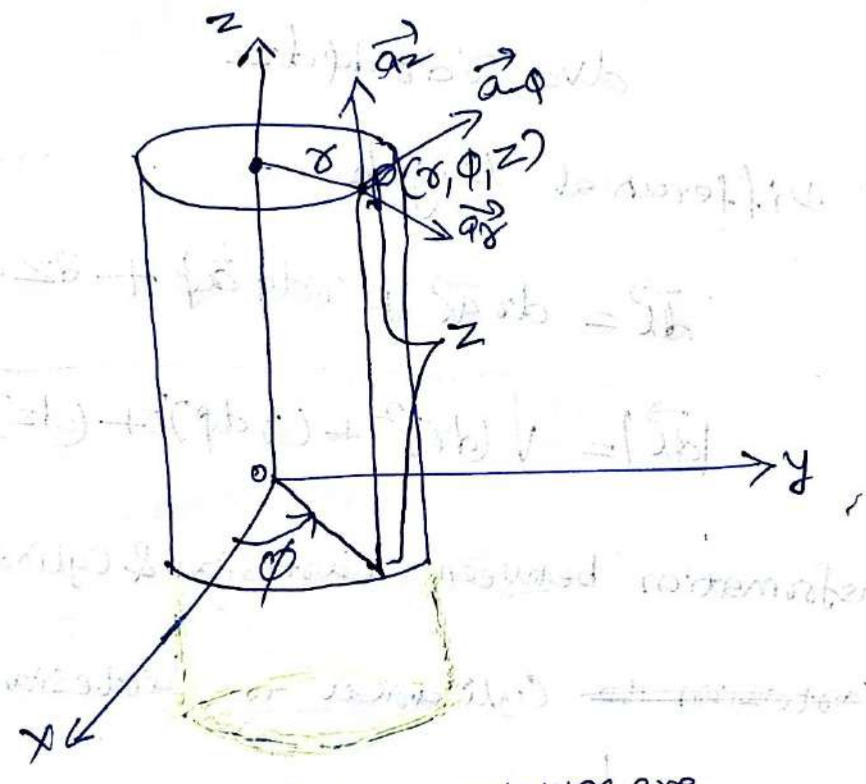
$$dv = dx\,dy\,dz$$

→ Differential length

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$|\vec{dl}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

⇒ Cylindrical coordinate system.



The ranges of the variables are

$$0 \leq r \leq \infty$$

$$0 \leq \phi \leq 2\pi$$

$$-\infty \leq z \leq \infty$$

where r – radius of the cylinder with z axis

φ – angle of the plane w.r.t xz plane

z – height of the plane from origin.

→ differential ~~line~~ line element

$dr,\ r d\phi$ & dz

→ Differential surface element

along r direction $\vec{ds_r} = r\,d\phi\,dz\,\vec{a_r}$

along ϕ direction $\vec{ds_\phi} = dr\,dz\,\vec{a_\phi}$

along z direction $\vec{ds_z} = r\,dr\,d\phi\,\vec{a_z}$

→ Differential volume

$$dv = r\,dr\,d\phi\,dz$$

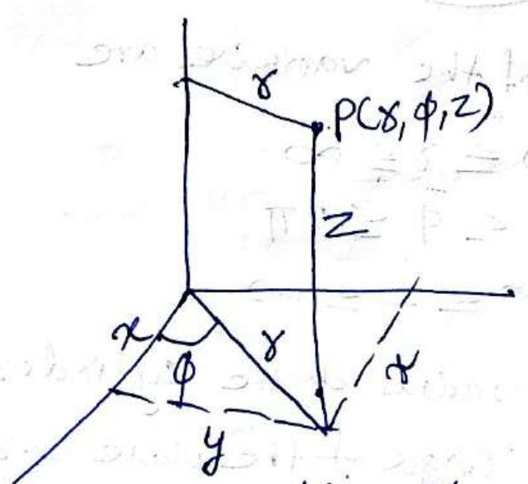
→ Differential length

$$\vec{dl} = dr\,\vec{a_r} + r\,d\phi\,\vec{a_\phi} + dz\,\vec{a_z}$$

$$|\vec{dl}| = \sqrt{(dr)^2 + (r\,d\phi)^2 + (dz)^2}$$

⇒ Transformation between Cartesian & Cylindrical system

i) ~~Cartesian to~~ Cylindrical to Cartesian



$$\cos\phi = \frac{adj}{Hyp} = \frac{x}{r}$$

$$\boxed{x = r\cos\phi}$$

$$\sin\phi = \frac{opp}{Hyp} = \frac{y}{r}$$

$$\boxed{y = r\sin\phi}$$

$$\boxed{z = z}$$

ii) Cartesian to cylindrical

$$x = r\cos\phi \Rightarrow x^2 = r^2\cos^2\phi$$
$$y = r\sin\phi \Rightarrow y^2 = r^2\sin^2\phi$$

$$x^2+y^2 = r^2[\cos^2\phi + \sin^2\phi]$$

$$\therefore r^2 = x^2+y^2$$

$$\boxed{r = \sqrt{x^2+y^2}}$$

$$\frac{y}{x} = \frac{r\sin\phi}{r\cos\phi} = \tan\phi$$

$$\therefore \tan\phi = y/x$$

$$\boxed{\phi = \tan^{-1}(y/x)}$$

$$\boxed{z = z}$$

Problems

(5) At a point $P(3, 90^\circ, 15)$. Convert into Cartesian

Sol:- $r = 3 \quad \phi = 90^\circ \quad z = 15$

$$x = r\cos\phi = 3\cos 90^\circ = 0$$
$$y = r\sin\phi = 3\sin 90^\circ = 3$$
$$z = 15$$

In Cartesian $P(0, 3, 15)$.

(6) At a point $A(x=2, y=3, z=-1)$ convert into cylindrical.

Sol:-

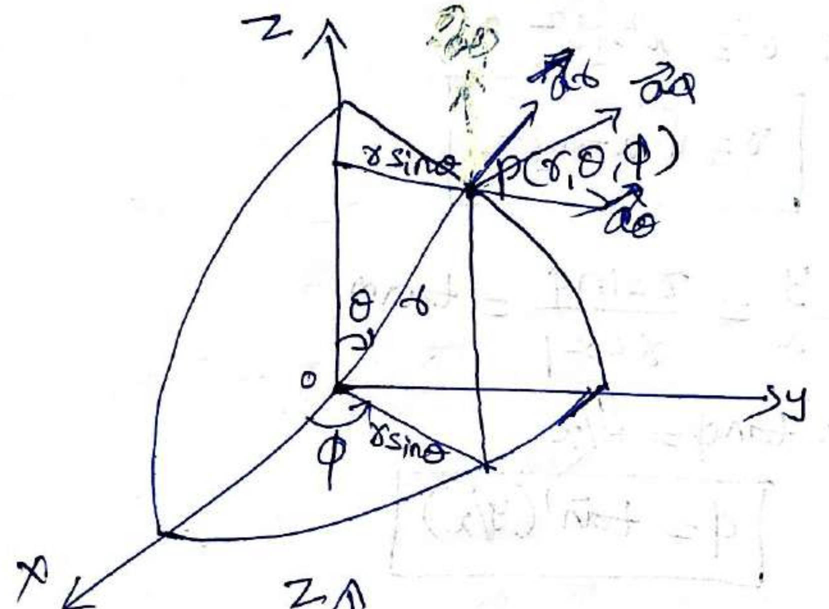
$$r=\sqrt{x^2+y^2}=\sqrt{4+9}=\sqrt{13}=3.6$$

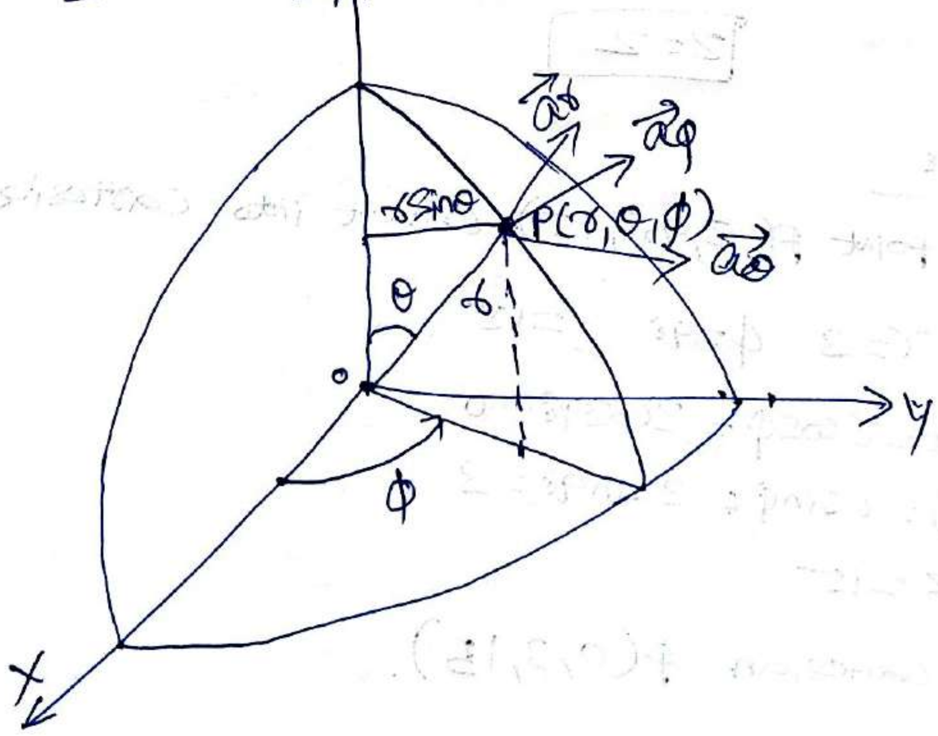
$$\phi=\tan^{-1}(y/x)=\tan^{-1}(3/2)=56.3^\circ$$

$$z=-1$$

In cylindrical $A(r=3.6, \phi=56.3^\circ, z=-1)$

⇒ Spherical coordinate system.





The ranges of the variables are

$$0 \le r \le \infty$$
$$0 \le \theta \le \pi$$
$$0 \le \phi \le 2\pi$$

where r – radius of the sphere.

θ – Half angle of the right circular cone w.r.t z axis

ϕ – angle of the plane w.r.t XZ plane.

→ Differential line element

$$dr,\ r\,d\theta,\ r\sin\theta\,d\phi$$

→ Differential surface element

along r direction $\vec{ds_r} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$

along θ direction $\vec{ds_\theta} = r\sin\theta\, dr\, d\phi\, \vec{a_\theta}$

along ϕ direction $\vec{ds_\phi} = r\, dr\, d\theta\, \vec{a_\phi}$

→ Differential volume

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

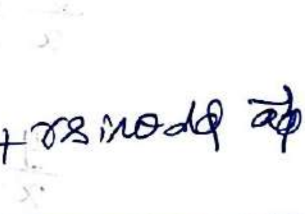
→ Differential length

$$\vec{dl} = dr\,\vec{a_r} + r\,d\theta\,\vec{a_\theta} + r\sin\theta\, d\phi\,\vec{a_\phi}$$

$$|\vec{dl}| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta\, d\phi)^2}$$

⇒ Transformation between Cartesian & spherical system.

1) Spherical to Cartesian.



$$\cos\phi = \frac{x}{r\sin\theta}$$

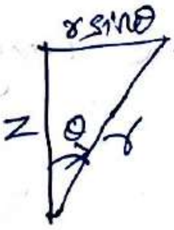
$$\Rightarrow x = r\sin\theta\cos\phi$$

$$\sin\phi = \frac{y}{r\sin\theta}$$

$$\Rightarrow y = r\sin\theta\sin\phi$$

$$\cos\theta = \frac{z}{r}$$

$$\Rightarrow z = r\cos\theta$$



ii) cartesian to spherical

$$x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$$

$$= r^2\sin^2\theta[\sin^2\phi + \cos^2\phi] + r^2\cos^2\theta$$

$$= r^2[\sin^2\theta + \cos^2\theta]$$

$$\Rightarrow r^2 = x^2+y^2+z^2$$

$$\boxed{r = \sqrt{x^2+y^2+z^2}}$$

$$z = r\cos\theta$$

$$\cos\theta = z/r$$

$$\boxed{\theta = \cos^{-1}(z/r)}$$

$$\frac{y}{x} = \frac{r\sin\theta\sin\phi}{r\sin\theta\cos\phi}$$

$$\frac{y}{x} = \tan\phi$$

$$\boxed{\phi = \tan^{-1}(y/x)}$$

problems

(7) A point P(6, 4, 2) into spherical

$x=6 \quad y=4 \quad z=2$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{36+16+4} = \sqrt{56}$

$\boxed{r = 7.48}$

$\theta = \cos^{-1}\left(\frac{2}{7.48}\right)$

$\boxed{\theta = 74.3^\circ}$

$\phi = \tan^{-1}(4/6)$

$\boxed{\phi = 33.69^\circ}$

(8) A point P(8, 110°, 60°) into Cartesian.

$x = r\sin\theta\cos\phi = 8\sin(110^\circ)\cos(60^\circ)$

$= 8 \times 0.939 \times 0.5$

$\boxed{x = 3.75}$

$y = r\sin\theta\sin\phi = 8\sin(110^\circ)\sin(60^\circ)$

$= 8 \times 0.939 \times 0.86$

$\boxed{y = 6.50}$

$z = r\cos\theta = 8\cos(110^\circ)$

$= 8 \times -0.342$

$\boxed{z = -2.73}$

⇒ Divergence Theorem.

The surface integral of the normal component of the vector over the closed surface is equal to the volume integral of a divergence of the vector throughout the volume.

$$\oiint \vec{A} \cdot \vec{ds} = \iiint \nabla \cdot \vec{A}\, dv.$$

<u>Proof</u>

R.H.S

$$\iiint \nabla \cdot \vec{A}\, dv = \iiint \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dx\,dy\,dz$$

∵ In cartesian coordinate system,

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$dv = dx\,dy\,dz$$

$$\iiint \nabla \cdot \vec{A}\, dv = \iiint \frac{\partial A_x}{\partial x} dx\,dy\,dz + \iiint \frac{\partial A_y}{\partial y} dx\,dy\,dz + \iiint \frac{\partial A_z}{\partial z} dx\,dy\,dz$$

$$= \iint A_x\, dy\,dz + \iint A_y\, dx\,dz + \iint A_z\, dx\,dy$$

W.K.T $\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$

$\vec{ds} = ds_x \vec{a_x} + ds_y \vec{a_y} + ds_z \vec{a_z}$

$$= \iint A_x\, ds_x + \iint A_y\, ds_y + \iint A_z\, ds_z$$

$$= \iint \left[A_x \, ds_x + A_y \, ds_y + A_z \, ds_z \right]$$

$$= \oiint \vec{A} \cdot \vec{ds}$$

$\therefore$ R.H.S = L.H.S

Hence proved.

Note:-

~~The~~ Divergence theorem also stated as $\oiint \vec{A} \cdot n \, ds = \iiint \nabla \cdot \vec{A} \, dv$

⇒ Divergence of the vector

→ Cartesian coordinate system

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

→ cylindrical coordinate system

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial (r A_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

→ spherical coordinate system

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

problem.

(9) using divergence theorem, evaluate $\int_S \vec{A} \cdot \vec{ds}$ where $\vec{A} = 2xy\,\vec{a_x} + y^2\,\vec{a_y} + 4yz\,\vec{a_z}$ and S is ~~Bradutbe~~ surface of the cube bounded by

$x=0, x=1, y=0, y=1$ & $z=0, z=1$.

Sol:-

By divergence theorem

$$\oiint \vec{A} \cdot \vec{ds} = \iiint \nabla \cdot \vec{A}\, dV$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= \frac{\partial (2xy)}{\partial x} + \frac{\partial (y^2)}{\partial y} + \frac{\partial (4yz)}{\partial z}$$

$$= 2y + 2y + 4y$$

$$\nabla \cdot \vec{A} = 8y$$

$$\therefore \iiint \nabla \cdot \vec{A}\, dV = \int_0^1\int_0^1\int_0^1 8y\, dx\, dy\, dz$$

$$= \int_0^1\int_0^1 \left[8yx\right]_0^1 dy\, dz$$

$$= \int_0^1\int_0^1 8y\, dy\, dz$$

$$= \int_0^1 \left[\frac{8y^2}{2}\right]_0^1 dz$$

$$= \int_0^1 4\, dz = 4\left[z\right]_0^1 = 4$$

$$\boxed{\therefore \oiint \vec{A} \cdot \vec{ds} = 4}$$

(10) Verify divergence theorem, consider the field vector $\vec{D} = 2xy\,\vec{a_x}$ and Rectangular cube formed by a plane $x=0, x=1, y=0, y=2$ & $z=0, z=3$.

Sol:- Divergence theorem

$$\oiint \vec{D} \cdot \vec{ds} = \iiint \nabla \cdot \vec{D}\, dv$$

<u>R.H.S</u>

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial (2xy)}{\partial x} + \frac{\partial (0)}{\partial y} + \frac{\partial (0)}{\partial z}$$

$$\boxed{\nabla \cdot \vec{D} = 2y}$$

$$\iiint \nabla \cdot \vec{D}\, dv = \int_0^3 \int_0^2 \int_0^1 2y\, dx\, dy\, dz$$

$$= \int_0^1 dx \int_0^2 2y\, dy \int_0^3 dz$$

$$= [x]_0^1 \left[\frac{2y^2}{2}\right]_0^2 [z]_0^3$$

$$= [1][4][3]$$

$$\boxed{\iiint \nabla \cdot \vec{D}\, dv = 12}$$

<u>L.H.S</u>

$$\oiint \vec{D} \cdot \vec{ds} = \iint D_x\, ds_x + \iint \underbrace{D_y}_{0}\, ds_y + \iint \underbrace{D_z}_{0}\, ds_z$$

$$= \int_0^3 \int_0^2 2xy\, dy\, dz + \cancel{\int_0^3 \int_0^1 xy\, dx\, dz}$$

$$= 2x \int_0^2 dy \int_0^3 dz + \cancel{\int_0^1 x^2 dx \int_0^3 dz}$$

$$= 2x [y]_0^2 [z]_0^3 + \cancel{\left[\frac{x^3}{3}\right]_0^1 [z]_0^3}$$

$$= 2x [2][3] + \cancel{[1/3][3]}$$

$$= 12x \cancel{+1}$$

At $x = 1$

$$\boxed{\oint\!\!\oint \vec{D} \cdot \vec{ds} = 12}$$

(11) Given that $\vec{D} = \frac{5r^2}{4} \vec{a_r}$ C/m². Evaluate both the sides of divergence theorem for the volume enclosed by $r = 4m$ & $\theta = \pi/4$.

Sol:-

$r = 4m$, $\theta = \pi/4$ & letus consider $\phi = 0$ to 2π

Divergence theorem

$$\oint\!\!\oint \vec{D} \cdot \vec{ds} = \iiint \nabla \cdot \vec{D} \, dv$$

In spherical coordinate system

$P(r, \theta, \phi)$

$$\vec{ds_r} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r}$$

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

R.H.S

$$\nabla\cdot\vec{D} = \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} \quad \because D_r \text{ present in } \vec{D}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{5r^2}{4}\right)$$

$$= \frac{5}{4r^2}\frac{\partial}{\partial r}(r^4) = \frac{5}{4r^2}\left[4r^3\right]$$

$$\boxed{\nabla\cdot\vec{D} = 5r}$$

$$\iiint \nabla\cdot\vec{D}\, dv = \iiint 5r\left(r^2\sin\theta\, dr\, d\theta\, d\phi\right)$$

$$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^{4} 5r^3\sin\theta\, dr\, d\theta\, d\phi$$

$$= \int_0^{4} 5r^3 dr \int_0^{\pi/4}\sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= 5\left[\frac{r^4}{4}\right]_0^4 \left[-\cos\theta\right]_0^{\pi/4} \left[\phi\right]_0^{2\pi}$$

$$= 5\left[64\right]\left[1-\frac{1}{\sqrt{2}}\right]2\pi$$

$$= 588.8$$

LHS

$$\iint \vec{D}\cdot d\vec{s} = \iint D_r\, ds_r + \iint \underbrace{D_\theta}_{0}\, ds_\theta + \iint \underbrace{D_\phi}_{0}\, ds_\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{5r^2}{4}\cdot r^2\sin\theta\, d\theta\, d\phi$$

$$= \frac{5r^4}{4} \int_0^{2\pi}\int_0^{\pi/4} \sin\theta \, d\theta \, d\phi$$

$$= \frac{5r^4}{4} \int_0^{\pi/4} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{5r^4}{4} \left[-\cos\theta\right]_0^{\pi/4} (\phi)_0^{2\pi}$$

$$= \frac{5r^4}{4} \left[-\frac{1}{\sqrt{2}}+1\right]\left[2\pi\right]$$

$$\iint \vec{D}\cdot\vec{ds} = \frac{5r^4}{4} \left[1-1/\sqrt{2}\right]\left[2\pi\right]$$

at $r=4$

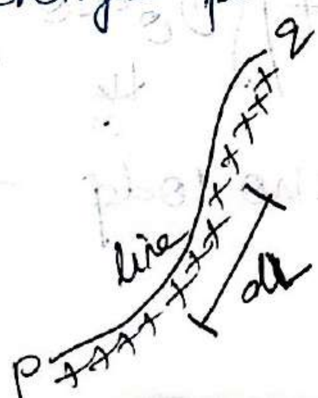
$$\iint \vec{D}\cdot\vec{ds} = 5\left[64\right]\left[1-1/\sqrt{2}\right]\left[2\pi\right]$$

$$= 588.8$$

⇒ Types of Integral related to EM Theory.

i) Line Integral

ρ_L is defined as total number of charges presented throughout entire length 'L'.



Line charge Desity (ρ_L)

$\rho_L = \frac{Q}{L}$ coulomb/meter

Consider a small element

$$\rho_L = \frac{dQ}{dL}$$

$$dQ = \rho_L \, dL$$

$$\boxed{Q = \int \rho_L \, dL}$$

Gauss law

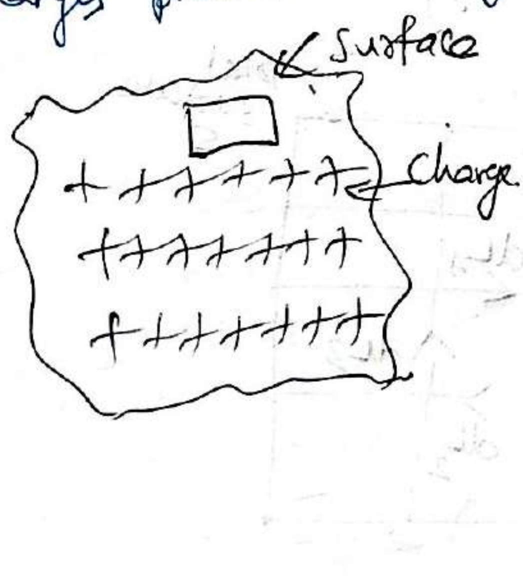
The law states that the electric flux passing through any closed surface is equal to the charge enclosed by the surface.

$$\boxed{\Psi = Q}$$

~~Using Gauss law,~~

ii) Surface Integral

ρ_S is defined as total number of charges presented throughout the entire surface's



Surface Charge Density (ρ_S)

$$\rho_S = \frac{Q}{S} \text{ coulomb}/m^2$$

Consider a small element

$$\rho_S = \frac{dQ}{dS}$$

$$dQ = \rho_S \, dS$$

$$\boxed{Q = \iint \rho_S \, dS}$$

iii) volume Integral

ρ_v is defined as total number of charges presented throughout entire volume.

$$\rho_v = \frac{Q}{V} \quad \frac{\text{Coulomb}}{m^3}$$

Consider small element

$$\rho_v = \frac{dQ}{dV}$$

$$dQ = \rho_v \, dV$$

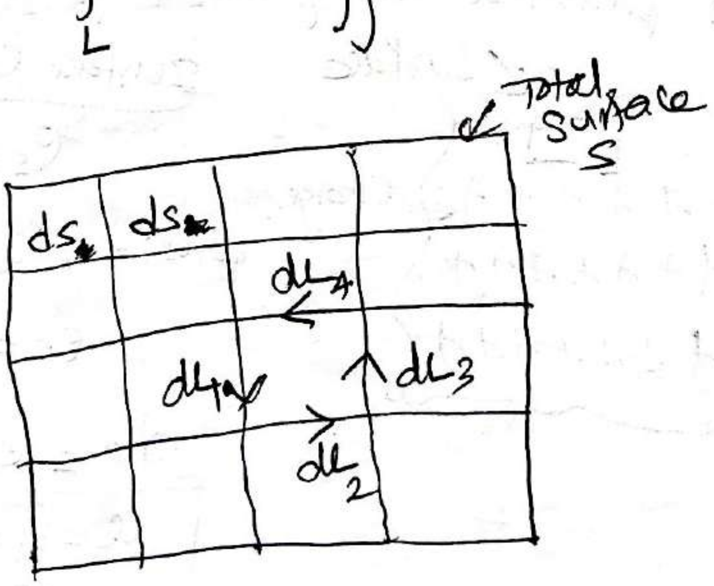
$$\boxed{Q = \iiint \rho_v \, dV}$$

⇒ Stoke's Theorem.

The line integral of any vector around a closed path L is equal to the surface integral of the curl of the vector over a open surfaces enclosed by the closed path L.

$$\oint_L \vec{F} \cdot \vec{dL} = \iint (\nabla \times \vec{F}) \cdot \vec{dS}$$

proof



$$\oint \vec{F}\cdot\vec{dL}_1 + \oint \vec{F}\cdot\vec{dL}_2 + \oint \vec{F}\cdot\vec{dL}_3 + \oint \vec{F}\cdot\vec{dL}_4$$

$$= \iint(\nabla\times\vec{F})\cdot\vec{ds}_1 + \iint(\nabla\times\vec{F})\cdot\vec{ds}_2$$

$$+ \iint(\nabla\times\vec{F})\cdot\vec{ds}_3$$

According to ~~curl~~ the definition, curl is the Amount of rotation ~~is done by dividing open~~ The total surface is divided into small element surface area ds_1, ds_2 etc. Here the vector $\vec{F}$ is moving on a closed path has shown in fig which forms ~~as~~ curl.

→ formula.

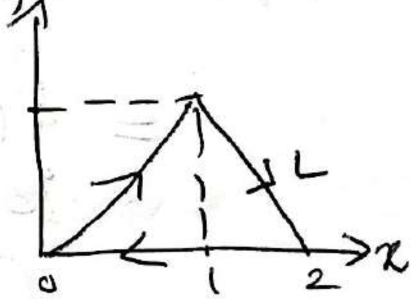
Combining the entire sequences

$$\boxed{\oint \vec{F}\cdot\vec{dL} = \iint(\nabla\times\vec{F})\cdot\vec{ds}}$$

Problem

(12) Given that $\vec{F} = x^2y\,\vec{a_x} - y\,\vec{a_y}$

verify stoke's theorem.



Sol. –

L.14.5

$$\oint \vec{F}\cdot\vec{dl} = \oint_{AB} \vec{F}\cdot\vec{dl} + \oint_{BC} \vec{F}\cdot\vec{dl} + \oint_{CA} \vec{F}\cdot\vec{dl}$$

$$\oint_{AB} \vec{F}\cdot\vec{dl} = \int_2^0 (x^2 y\, \vec{a_x} - y\, \vec{a_y})\cdot dx\, \vec{a_x}$$

$$= \int_2^0 x^2 y\, dx$$

$$\oint_{AB} \vec{F}\cdot\vec{dl} = 0 \quad \because y=0 \text{ for path } AB.$$

$$\oint_{BC} \vec{F}\cdot\vec{dl} = \oint_{BC} (x^2 y\, \vec{a_x} - y\, \vec{a_y})\cdot(dx\,\vec{a_x} + dy\,\vec{a_y})$$

$$= \oint_{BC} (x^2 y\, dx - y\, dy)$$

Equation of path BC is $y=x$ i.e. $dy=dx$

$$= \oint_{BC} (x^3\, dx - x\, dx)$$

$$= \int_0^1 (x^3 - x)\, dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1 = \left[\frac{1}{4} - \frac{1}{2}\right]$$

$$= \frac{-(2-4)}{8} = \frac{-2}{8} = \frac{-1}{4}.$$

$$\oint_{CA} \vec{F} \cdot \vec{dL} = \oint_{CA} (x^2 y \vec{a_x} - y \vec{a_y}) \cdot (dx \vec{a_x} + dy \vec{a_y})$$

$$= \oint_{CA} (x^2 y\, dx - y\, dy)$$

<u>Equation of path CA</u> $\quad C \overset{x_1, y_1}{(1,1)} \quad A \overset{x_2\ y_2}{(2,0)}$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-1}{0-1} = \frac{x-1}{2-1}$$

$$y - 1 = -x + 1$$

$$\boxed{y = 2 - x}$$

$$\oint_{CA} (x^2 y\, dx - y\, dy) = \int_1^2 x^2 (2-x)\, dx - \int_1^0 y\, dy$$

$$= \int_1^2 (2x^2 - x^3)\, dx - \int_1^0 y\, dy$$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_1^2 - \left[\frac{y^2}{2}\right]_1^0$$

$$= \left[\frac{16}{3} - \cancel{\frac{16}{4}}\,4\right] - \left[\frac{2}{3} - \frac{1}{4}\right] - \left[0 - \frac{1}{2}\right]$$

$$= \left[\frac{16-12}{3}\right] - \left[\frac{8-3}{12}\right] + \frac{1}{2}$$

$$= \left[\frac{4}{3} - \frac{5}{12}\right] + \frac{1}{2}$$

$$= \left[\frac{16-5}{12}\right] + \frac{1}{2}$$

$$= \frac{11}{12} + \frac{1}{2}$$

$$= \frac{11+6}{12}$$

$$\oint_{CA} \vec{F} \cdot \vec{dl} = \frac{17}{12}.$$

$$\oint \vec{F} \cdot \vec{dl} = 0 - \frac{1}{4} + \frac{17}{12}$$

$$= \frac{17-3}{12} = \frac{14}{12}$$

<u>R.H.S</u>

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & -y & 0 \end{vmatrix}$$

$$= \vec{a_x}\left(0 + \frac{\partial y}{\partial z}\right) - \vec{a_y}\left(0 - \frac{\partial (x^2y)}{\partial z}\right)$$

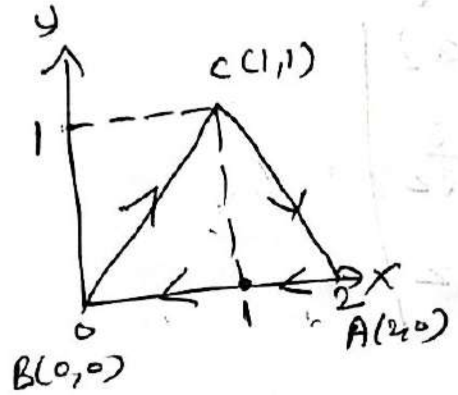
$$+ \vec{a_z}\left(\frac{\partial (-y)}{\partial x} - \frac{\partial}{\partial y}(x^2y)\right)$$

$$= 0 - 0 + \vec{a_z}(-x^2)$$

$$\nabla \times \vec{F} = -x^2 \vec{a_z}$$

$$\iint (\nabla \times \vec{F}) \cdot \vec{ds} = \iint -x^2 \vec{a_z} \cdot dx\,dy\,\vec{a_z}$$

$$= \iint -x^2\, dx\, dy$$



Now split the area into two right angled triangles.

for first triangle, the equation of line is $y=x$, hence use $dy = x$ & x varies from 1 to 0.

for second triangle, the equation of line is $y = 2-x$, hence use $dy = 2-x$ & x varies from 2 to 1.

$$\iint (\nabla \times \vec{F}) \cdot \vec{ds} = \int_{x=1}^{0} -x^2 \cdot x\, dx + \int_{x=2}^{1} -x^2(2-x)\, dx$$

$$= \int_{1}^{0} -x^3 dx + \int_{2}^{1} (-2x^2 + x^3)\, dx$$

$$= \left[\frac{-x^4}{4}\right]_1^0 + \left[-\frac{2x^3}{3} + \frac{x^4}{4}\right]_2^1$$

$$= \left[0 + \frac{1}{4}\right] + \left[\left(-\frac{2}{3} + \frac{1}{4}\right) - \left(-\frac{16}{3} + \frac{16}{4}\right)\right]$$

$$= \frac{1}{4} + \left[\frac{(3-8)}{12} - \frac{(-64+48)}{12}\right]$$

$$= \frac{1}{4} + \left[\frac{(-5)}{12} - \frac{(-16)}{12}\right] = \frac{1}{4} + \left[\frac{16-5}{12}\right]$$

$$= \frac{1}{4} + \frac{11}{12} = \frac{3+11}{12} = \frac{14}{12}.$$

Identities

⇒ curl of the vector

→ Cartesian coordinate system.

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

→ cylindrical.

$$\nabla \times \vec{F} = \frac{1}{r} \begin{vmatrix} \vec{a_r} & r\vec{a_\phi} & \vec{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & F_\phi & F_z \end{vmatrix}$$

→ Spherical.

$$\nabla \times \vec{F} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta\, F_\phi \end{vmatrix}$$

⇒ Null Identities.

There are two vector identities based on curl & divergence of the field, which are called null vector identities.

i) $\nabla \times (\nabla V) = 0$

ii) $\nabla \cdot (\nabla \times \vec{A}) = 0$

→ The curl of the gradient of any scalar field is identically zero.

→ The divergence of the curl of any vector field is identically zero.

⇒ Helmholtz's theorem.

It is based on curl & divergence of vector field. It states that a vector field is ~~~~ uniquely defined within an additive constant by specifying its divergence and curl.

$$\vec{B} = -\nabla U + (\nabla \times \vec{A})$$

U → scalar field

A → vector field.

$\vec{B}$ can be divided into two components

i) Gradient of scalar field U

ii) Curl of the vector field $\vec{A}$.

<u>Divergence of $\vec{B}$</u>

$$\nabla \cdot \vec{B} = \nabla \cdot (-\nabla U) + \nabla \cdot (\nabla \times \vec{A})$$

According to ~~second~~ Null Identities (ii)

$$\nabla \cdot \vec{B} = \nabla \cdot (-\nabla U) \qquad \because \nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\boxed{\nabla \cdot \vec{B} = e} \qquad \because \nabla \cdot (-\nabla U) \neq 0$$

put $\nabla \cdot (-\nabla U) = e$

This is non solenoidal as divergence is non-zero

Curl of $\vec{B}$

$$\nabla \times \vec{B} = \nabla \times (-\nabla U) + \nabla \times (\nabla \times \vec{A})$$

According to Null Identities (i)

$$\nabla \times (\nabla U) = 0$$

then $\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A})$

$$\boxed{\nabla \times \vec{B} = \vec{J}}$$ put $\nabla \times (\nabla \times \vec{A}) = \vec{J}$

This is rotational as the curl is not equal to zero

According to Helmholtz's theorem, four types of fields are defined

i) field is non solenoidal & rotational

$$\nabla \cdot \vec{B} = \rho \ \& \ \nabla \times \vec{B} = \vec{J}$$

ii) field is non solenoidal & irrotational

$$\nabla \cdot \vec{B} = \rho \ \& \ \nabla \times \vec{B} = 0$$

iii) field is solenoidal & rotational

$$\nabla \cdot \vec{B} = 0 \ \& \ \nabla \times \vec{B} = J$$

iv) field is solenoidal & irrotational

$$\nabla \cdot \vec{B} = 0 \ \& \ \nabla \times \vec{B} = 0$$

UNIT-II ELECTROSTATICS

Coulomb's law.

coulomb's law states that the force of attraction or repulsion between two point charges is directly propotional to the product of the charge ($Q_1 Q_2$) and inversly propotional to the square of distance between them.

$+Q_1$ $\longleftarrow R \longrightarrow$ $+Q_2$

$$F \propto \frac{Q_1 Q_2}{R^2}$$

$$F = K\frac{Q_1 Q_2}{R^2} \qquad K = \frac{1}{4\pi\varepsilon}$$

$$\boxed{F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2}}$$ in Newton

ε – permittivity ε

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

ε_0 = permittivity in free space

$= 8.854 \times 10^{-12}$ F/m

ε_r = relative permittivity in medium

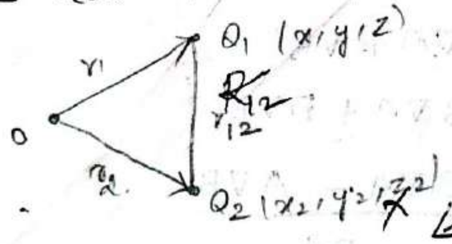
In air medium – $\varepsilon_r = 1$. (

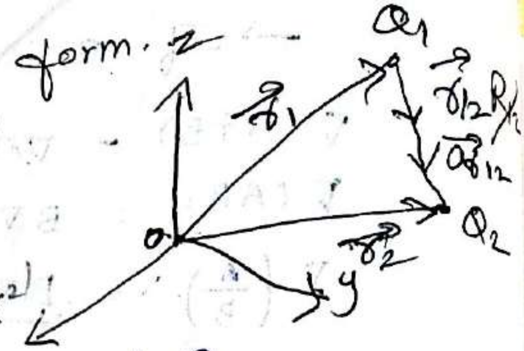
$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q_1 Q_2}{R^2} \qquad \varepsilon_r = 1$$

In air medium

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

Columb's law in vector form.





Distance of vector Q_1 and Q_2

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r_{12}^2}\, \vec{a}_{r_{12}}$$

$$\vec{a}_{r_{12}} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

$$\vec{a}_{r_{12}} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r_{12}^2}\, \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

Electric Field Intensity :- (E)

Q ———————————— Q_t (test charge)

$$\boxed{E = \frac{F}{Q_t}}$$

Electric Field Intensity is defined as force expreciened by test charge (Q_t) move due to main charge is known as Electric field Intensity

Unit is N/C (Force)

$$E = \frac{\dfrac{Q_1 Q_2}{4\pi\epsilon_0 r^2}}{Q_2}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{C}{m} \text{ or } V/m$$

Charge distribution:

→ point charge

Point charge may $+q$ (or) $-q$ is located in free space $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a_r}$ → (1)

→ Line charge.

It is defined as total number of charges presented throughout the entire length (l) line charge becomes

$$\boxed{\rho_l = \frac{Q}{l}}$$

Let us consider a small elemental length

$$\boxed{\rho_l = \frac{dQ}{dl}}$$

$$dQ = \rho_l\, dl$$

$$Q = \int \rho_l\, dl \rightarrow (2)$$

$$E = \frac{\int \rho_l\, dl}{4\pi\varepsilon_0 r^2}\vec{a_r}$$

Unit → C/m

Surface charge.

It is defined as ratio of total no. of charges presented throughout the surface area.

$$\rho_s = \frac{Q}{s}$$

consider small elemental sueface

$$\rho_s = \frac{dQ}{ds}$$

$$dQ = \rho_s\, ds$$

$$Q = \iint \rho_s\, ds = \oint_s \rho_s\, ds \rightarrow (4)$$

sub eqn (4) in (1)

$$\boxed{\vec{E} = \frac{\iint \rho_s \, ds}{4\pi \varepsilon_0 r^2} \vec{a_r}}$$

unit $\Rightarrow C/m^2$

Volume charge

It is defined as ratio of total charges located throughout the entire volume

$$\rho_V = \frac{Q}{V}$$

Consider a small element

$$\rho_V = \frac{dQ}{dV}$$

$$dQ = \rho_V \, dv$$

$$Q = \iiint \rho_V \, dv$$

$$Q = \int_V \rho_V \, dv \rightarrow (5)$$

Sub eqn (5) in (1)

$$\boxed{\vec{E} = \frac{\iiint \rho_V \, dv}{4\pi \varepsilon_0 r^2} \vec{a_r}}$$

Electric ~~field~~ flux Density

It is defined as total number of lines of force in a electric field is called as Electric flux density D

$$D = \frac{Q}{S} \qquad \rho_s = \frac{Q}{S}$$

Total no. of charges per unit area

$$D \propto \vec{E}$$

$$\boxed{D = \varepsilon \vec{E}} = \varepsilon_0 \varepsilon_r \vec{E}$$

In air medium $\epsilon_r = 1$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$= \epsilon_0 \left(\frac{Q}{4\pi\epsilon_0 r^2} \vec{a_r} \right)$$

$$\vec{D} = \frac{Q}{4\pi r^2} \vec{a_r}$$

Properties of Electric flux Density.

The force of lines are parllel to each other

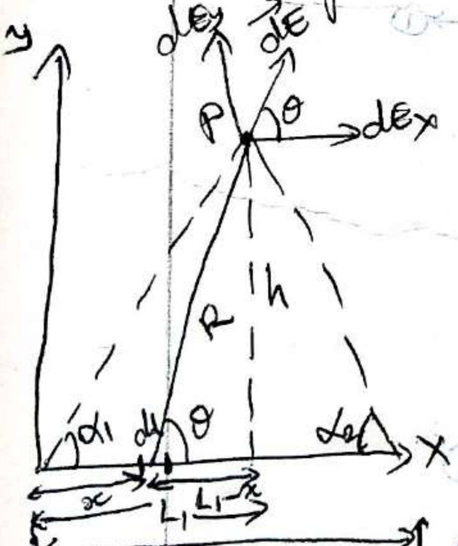
The force of lines are never cross each other

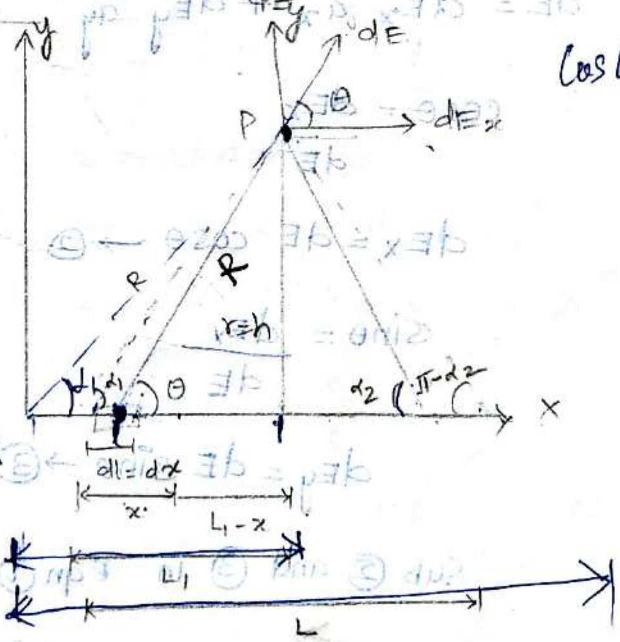
The electric flux lines are entering or leaving normally charged surface.

The electric flux line are strong. the electric field intensity become strong.

Electric field due to line charge distribution.

13m (or) To find the Electric field intensity at a point 'p' ~~where~~ the charges are distributed throughout the entire length





$$\cos\theta = \frac{dE_x}{dE}$$

Let us consider the charges are presented throughout entire length due to point charge in free space.

Here Force Experienced by test charge or point charge due to main charge electric field intensity is Produced.

Hence to find out the electric field intensity in small elemental length dl.

$\theta \rightarrow$ angle between point charge and main charge

h $\rightarrow$ Indicate the equi potential surface in free space

$\alpha_1 \rightarrow$ angle btw starting of the length to point charge.

$(\pi - \alpha_2) \rightarrow$ angle btw end point and point charge.

Case (i)

$$\vec{dE} = dE_x \vec{a_x} + dE_y \vec{a_y} \rightarrow (1)$$

$$\cos\theta = \frac{dE_x}{dE}$$

$$dE_x = dE \cos\theta \rightarrow (2)$$

$$\sin\theta = \frac{dE_y}{dE}$$

$$dE_y = dE \sin\theta \rightarrow (3)$$

Sub (2) and (3) in Eqn (1)

$$\vec{dE} = dE\cos\theta \, \vec{a_x} + dE \sin\theta \, \vec{a_y} \rightarrow (4)$$

According to Electric field intensity in line charge distribution

$$E = \frac{\int \rho_l \, dl}{4\pi \varepsilon_0 R^2} \vec{a_R}$$

from diagram $dl = dx$ $\int = \rightarrow E = dE$

$$dE = \frac{\rho_l \, dl}{4\pi \varepsilon_0 R^2} \vec{a_R} \rightarrow (5)$$

$\vec{a_R} = \vec{a_x} + \vec{a_y}$

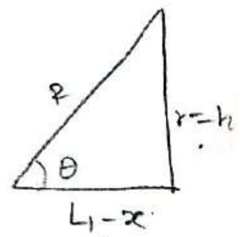
sub eqn (5) in (4).

$$\vec{dE} = \frac{\rho_l \, dx}{4\pi \varepsilon_0 R^2} \cos\theta \, \vec{a_x} +$$

$$\frac{\rho_l \, dx}{4\pi \varepsilon_0 R^2} \sin\theta \, \vec{a_y} \rightarrow (6)$$

Here they are three different parameters 'R', x, θ it may be varied due to that we need a single line integral convert this 3 parameter into one parameter

Let us consider a traingle from the diagram.



$$\cot\theta = \frac{L_1 - x}{h}$$

$$L_1 - x = h \cot\theta$$

$$-dx = h[-\operatorname{cosec}^2\theta \, d\theta]$$

$$dx = h \operatorname{cosec}^2\theta \, d\theta \rightarrow (7)$$

$$\operatorname{cosec}\theta = \frac{R}{h}$$

$$R = h \operatorname{cosec}\theta \rightarrow (8)$$

Sub (7) & (8) in eqn (6)

$$dE = \frac{\rho_l (h \operatorname{cosec}^2\theta \, d\theta)}{4\pi \varepsilon_0 (h^2 \operatorname{cosec}^2\theta)} \cos\theta \, \vec{a_x} +$$

$$\frac{\rho_l (h \operatorname{cosec}^2 d\theta)}{4\pi\varepsilon_0 (h^2 \cos^2 c\,\theta)} \sin\theta \, \vec{a_y}$$

$$d\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 h}\left[\cos\theta\, d\theta\, \vec{a_x} + \sin\theta\, d\theta\, \vec{a_y}\right]$$

Integrate on BS

$$\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 h}\left[\int_{\alpha_1}^{\pi-\alpha_2} \cos\theta\, d\theta\, \vec{a_x} + \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta\, d\theta\, \vec{a_y}\right]$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\left[(\sin\theta)_{\alpha_1}^{\pi-\alpha_2}\vec{a_x} + (-\cos\theta)_{\alpha_1}^{\pi-\alpha_2}\right]$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\left\{(\sin(\pi-\alpha_2) - \sin\alpha_1)\vec{a_x} + (-\cos(\pi-\alpha_2) + \cos\alpha_1)\vec{a_y}\right\}$$

For finite length

$$\boxed{\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 h}\left\{\sin(\pi-\alpha_2) - \sin\alpha_1)\vec{a_x} - \cos(\pi-\alpha_2) + \cos\alpha_1)\vec{a_y}\right.}$$

$$\vec{D} = \varepsilon \vec{E}$$

For free space

$$\vec{D} = \varepsilon_0 \vec{E} \qquad \varepsilon_r = 1$$

$$\vec{D} = \frac{\rho_l}{4\pi h}\left\{\sin(\pi-\alpha_2) - \sin\alpha_1)\vec{a_x} + (-\cos(\pi-\alpha_2 + \cos\alpha_1)\vec{a_y}\right\}$$

Case (ii)

Let us consider an infinite length

$\sin(180°-\theta) = \sin\theta$

$\cos(180°-\theta) = -\cos\theta$

$\alpha_1 = 0 \quad \alpha_2 = 0 \quad$ So,

$\sin(\pi-\alpha_1) = \sin\theta$

$$\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 h}\left\{\sin(\pi-\alpha_2) - \sin\alpha_1)\vec{a_x} + (-\cos(\pi-\alpha_2) + \cos\alpha_1)\vec{a_y}\right\}$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\left\{(\sin\alpha_2 - \sin\alpha_1)\vec{a_x} + (+\cos\alpha_2 + \cos\alpha_1)\right.$$

$\alpha_2 = 0 \quad \alpha_1 = 0$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\{(0-0)\vec{a_x} + (1+1)$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\{0\,\vec{a_x} + 2\,\vec{a_y}\}$$

$\sin(\pi-\theta) = \sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\,2\,\vec{a_y}$$

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 h}\vec{a_y} \rightarrow (11)$$

$$\varepsilon_0\left(\frac{\rho_l}{2\pi\varepsilon_0 h}\right) = \frac{\rho_l}{2\pi h}$$

$D = \varepsilon_0 \vec{E}$ in free space is

$$\vec{D} = \frac{\rho_l}{2\pi h} \rightarrow (12)$$

Case (iii)

Let us consider the point charge 'p' at mid position Hence $\alpha_1 = \alpha_2 = \alpha$

From eqn (9)

$$\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 h}\{(\sin(\pi-\alpha_2) - \sin\alpha_1)\vec{a_x} + (-\cos(\pi-\alpha_2) + \cos\alpha_1)\vec{a_y}\}$$

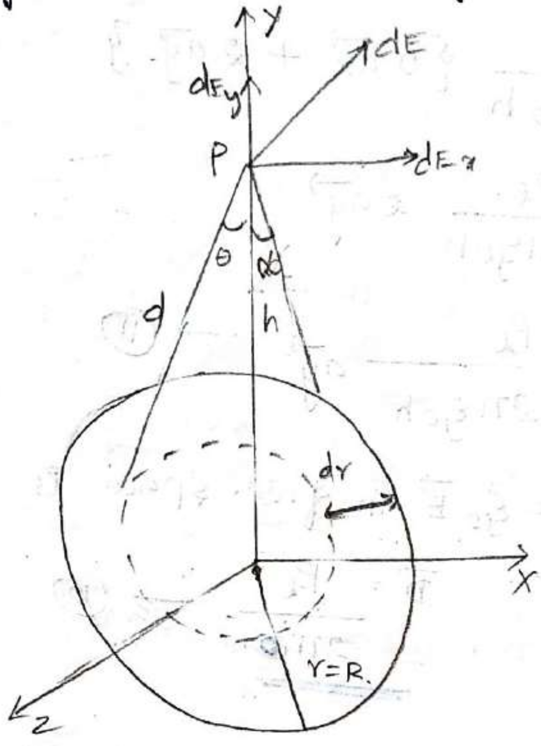
$$\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 h}\{\sin(\pi-\alpha) - \sin\alpha)\vec{a_x} + (-\cos(\pi-\alpha) + \cos\alpha)\vec{a_y}\}$$

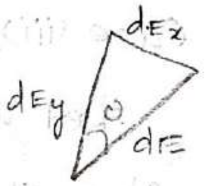
$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\{0 + (\cos\alpha + \cos\alpha)\vec{a_y}\}$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}(2\cos\alpha)\,\vec{a_y}$$

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 h}\cos\alpha\,\vec{a_y} \rightarrow (13)$$

To find Electric field intensity and density on the circular axis. Here the charges are uniformly distributed throughout the circular disc.





$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_y = dE\cos\theta \rightarrow (1)$$

According to surface charge distribution

$$\vec{E} = \frac{\iint \rho_s \cdot ds}{4\pi\varepsilon_0 R^2}\vec{a_R} \rightarrow (2)$$

Diff on B.S

$$\vec{dE} = \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\vec{a_R} \rightarrow (3)$$

sub eqn (3) in (4)

$$d\vec{E_y} = \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\cos\theta\,\vec{a_y}$$

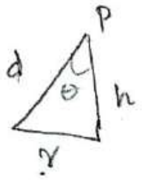
$ds \rightarrow$ differential surface

In circle $S = A = \pi r^2$

$$As = dA = 2\pi r\, dr$$

$$d\vec{E_y} = \frac{\rho_s (2\pi r \cdot dr) \cos\theta}{4\pi \epsilon_0 d^2} \vec{a_y} \rightarrow (4)$$

$d \rightarrow$ distance btw point charge and inner circle



$$\tan\theta = \frac{r}{h} \qquad \cos\theta = \frac{h}{d}$$

$$r = h\tan\theta \qquad d = \frac{h}{\cos\theta}$$

$$dr = h\sec^2\theta\, d\theta \qquad d = h\sec\theta$$

$$d\vec{E_y} = \frac{\rho_s\, 2\pi\, (h\tan\theta)(h\sec^2\theta\, d\theta)\cos\theta}{4\pi\, \epsilon_0 (h^2\sec^2\theta)} \vec{a_y}$$

$$\boxed{d\vec{E_y} = \frac{\rho_s \sin\theta}{2\epsilon_0} \vec{a_y}} \rightarrow (5)$$

Integrate on B.S

$$\vec{E_y} = \int_0^\alpha \frac{\rho_s \sin\theta}{2\epsilon_0} \vec{a_y}$$

$$= \frac{\rho_s}{2\epsilon_0}\left[-\cos\theta\right]_0^\alpha \vec{a_y}$$

$$= \frac{\rho_s}{2\epsilon_0}\left[1 - \cos\alpha\right] \vec{a_y}$$

$$2\theta = \alpha \qquad \theta = \frac{\alpha}{2}$$

$$= \frac{\rho_s}{2\epsilon_0}\left[2\sin^2\frac{\alpha}{2}\right]\vec{a_y} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\boxed{\vec{E_y} = \frac{\rho_s}{\epsilon_0} \sin^2\frac{\alpha}{2}\, \vec{a_y}} \rightarrow (6)$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$2\sin^2\frac{\alpha}{2} = 1-\cos\alpha$$

$$\vec{D} = \epsilon \vec{E}$$

In free space $\vec{D} = \epsilon_0 \vec{E}$

$$\boxed{\vec{D_y} = \rho_s \sin^2\frac{\alpha}{2}\, \vec{a_y}} \rightarrow (7)$$

Case (ii)

The charges are distributed infinitly from Circle to plane of sheet

$$\alpha = 90^\circ$$

$$\vec{E_y} = \frac{\rho_s}{\varepsilon_0} \sin^2 \frac{90}{2} \vec{a_y}$$

$$= \frac{\rho_s}{\varepsilon_0} \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\boxed{\vec{E_y} = \frac{\rho_s}{2\varepsilon_0} \vec{a_y}}$$

$$D = \varepsilon_0 \vec{E} \text{ in free}$$

$$\vec{D_y} = \frac{\rho_s}{2} \vec{a_y} \rightarrow (9)$$

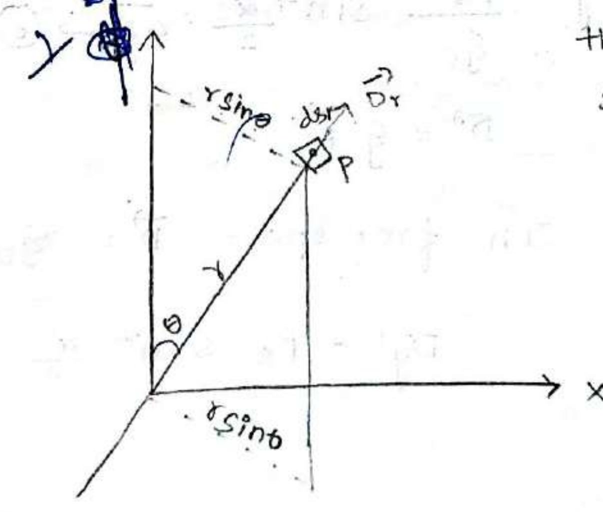
Gauss law

The total number of lines of force leaving from the charging surface area is equal to the total number of charges enclosed by the surface

$$\boxed{Q = \psi}$$

Where $Q \rightarrow$ Total no. of charges
$\psi \rightarrow$ flux lines.

Proof :-



through $\vec{r}$ direction
So ds_r, $\vec{D}_r$

Let us consider spherical coordinate s/y.

$$Q = \psi$$

AS per electric flux density

$$D = \frac{Q}{S} = \frac{dQ}{dS}$$

$$D = \frac{\psi}{S} = \frac{d\psi}{dS}$$

$$d\psi = D_1 \, dS_1$$

$$D_r = \frac{d\psi}{dS_r}$$

~~From diagram~~

$$d\psi = D_r \, dS_r \rightarrow (1)$$

Let us consider the flux lines are produced outwards from the surface.

By point charge

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a_r} \qquad D = \varepsilon_0 \vec{E}$$

$$\vec{D_r} = \frac{Q}{4\pi r^2} \vec{a_r}$$

In spherical coordinate s/m

$$ds_r = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r} \rightarrow (3)$$

sub (2) & (3) into (1)

$$d\psi = \frac{Q}{4\pi r^2} \vec{a_r} \, r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r}$$

$$d\psi = \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

$$\psi = \frac{Q}{4\pi} \left[\int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi \right]$$

$$\psi = \frac{Q}{4\pi} \left[(-\cos\theta)_0^{\pi} \, (\phi)_0^{2\pi} \right]$$

$$= \frac{Q}{4\pi}\left[4\pi\right]$$

$$\boxed{\psi = Q}$$

Hence Gauss law proved

Application :

Gauss law is applied to the surface if the following condition are satisfied

→ Surface is enclosed

→ electric flux density $\vec{D}$ is either normal or tangential to the surface. at each point of the Surface

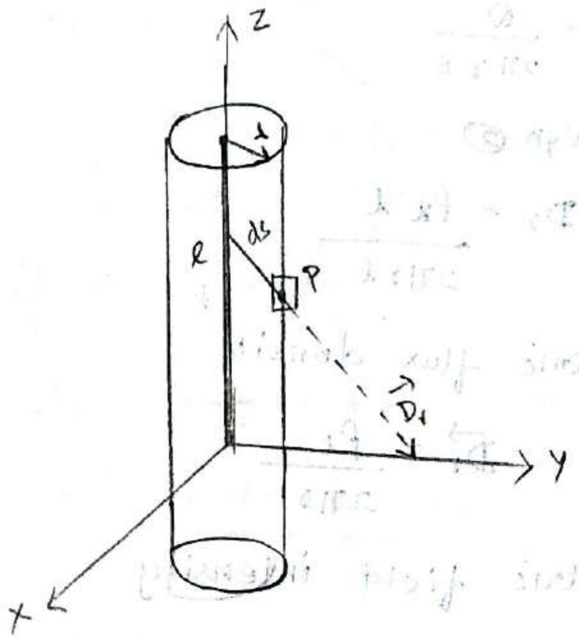
→ $\vec{D}$ is constant over the point P out of the surface where $\vec{D}$ is normal.

To find electric field intensity and density by using Gauss law The charges are uniformly distributed throughout the infinite line
line charge.

Assume that infinite line is in Gaussian surface.

consider cyclindrical coordinate system

↳ Line charge distribution



In cyclindrical.

$$ds_r = r\, d\phi\, dz\, \vec{a_r}$$

Due to line charge distribution the charge density

$$\rho_l = \frac{Q}{l} = \frac{dQ}{dl} \rightarrow ①$$

$$dQ = \rho_l\, dl$$

$\int d \rightarrow$ cancel.

$$Q = \int \rho_l\, dl$$

$$Q = \rho_l\, l \rightarrow ②$$

Electric flux density

$$\vec{D} = \frac{Q}{s} = \frac{dQ}{ds} = \frac{d\psi}{ds}.$$

$$dQ = \vec{D}\, ds$$

$$Q = \iint \vec{D_r}\, ds_r$$

$$Q = \iint \vec{D_r}\vec{a_r}\left[r\, d\phi\, dz\, \vec{a_r}\right].$$

$$= \int_0^l \int_0^{2\pi} D_r\, r\, d\phi\, dz$$

$$= D_r\, r\, [\phi]_0^{2\pi}\, [z]_0^l$$

$$= D_r\, r\, (2\pi)\, l$$

$$Q = 2\pi r l\, \vec{D_r}$$

$2\pi r l$

Sub eqn ②

$$D_r = \frac{\rho_l\, l}{2\pi r l}$$

Electric flux density

$$\boxed{\vec{D_r} = \frac{\rho_l}{2\pi r}}$$

Electric field intensity

$$\vec{E} = \frac{\vec{D_r}}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho_l}{2\pi r \varepsilon_0}}$$

To find Electric field intensity and density when the charges in cartessian rectangular box or pill box Assume the pill box is infinite sheet of charges

Surface charge distribution consider cartessian coordinate system.

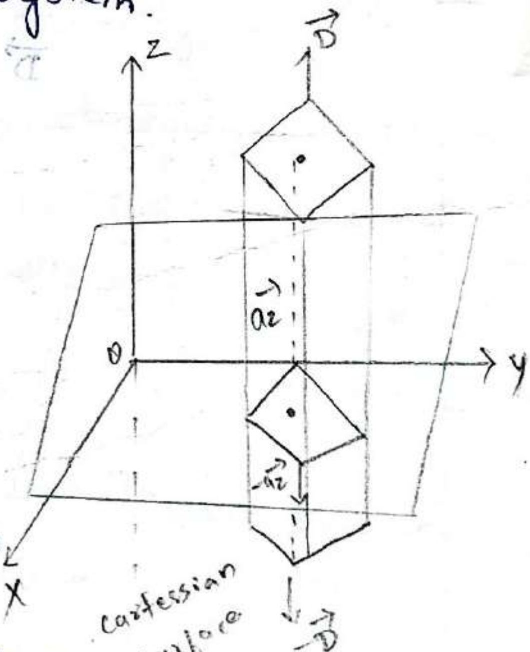
Electric flux Density

$$D = \frac{Q}{S} = \frac{dQ}{ds}$$

$$dQ = D \cdot ds$$

$$Q = \iint \vec{D}\, d\vec{s}$$

Let us consider electric flux density along z direction



$$Q = \int_{Top} \vec{D_z}\, ds_z + \int_{side} \vec{D_z}\, ds_z + \int_{Bottom} \vec{D_z}\, ds_z$$

Let $\int_{Side} \vec{D_z}\, ds_z = 0.$

$$Q = \iint_{Top} \vec{D_z}\, ds_z + \iint_{Bot} D_z\, ds_z$$

$$= \iint_{Top} \vec{D_z}\,\vec{a_z}\,[dx\,dy\,\vec{a_z}] + \iint \vec{D_z}\,-\vec{a_z}\,[dx\,dy\,(-\vec{a_z})]$$

$$= \iint \vec{D_z}\, dx\,dy + \iint \vec{D_z}\, dx\,dy$$

$$= 2\iint \vec{D_z}\, dx\,dy$$

$$= 2\,\vec{D_z}\,[x]\,[y] \qquad \underset{l\ \ b}{xy} \to \text{Area}$$

$$= 2\,\vec{D_z}\,(xy)$$

$$Q = 2\,\vec{D_z}\,(A) \rightarrow ①$$

$$\rho_s = \frac{Q}{A}$$

$$\rho_s A = Q \rightarrow ②$$

Sub ② in ①

$$\rho_s A = 2\,\vec{D_z}\,A$$

$$\rho_s = 2\,\vec{D_z}$$

$$\boxed{\vec{D_z} = \frac{\rho_s}{2}}$$

Electric field Intensity $E = \frac{\vec{D}}{\varepsilon_0}$ in free space

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}}$$

Absolute Electric Potential (V)

Electric potential is defined as work done moving a unit positive charge Q from infinite point to the given point is known as potential denoted by V

$$E = \frac{Q}{4\pi\epsilon_0 r} \text{ volt}$$

$$E = \frac{F}{Qt}$$

$\frac{F}{q_x}$

$$W = F \times d$$

$W = F \cdot d$

$$W = (E\,Qt)\,d$$

work done on unit positive charge also called as potential

$$V = \frac{W}{q} \quad \text{unit } J/C$$

$V = \frac{Q}{4\pi\epsilon_0 r}$

$$V = -\int_{r_1}^{r_2} \vec{E}\, ds$$

$E = \frac{Q}{4\pi\epsilon_0 r^2}$

$V = -\int_{r_1} \vec{E}\, ds$

$$-\int_{r_1}^{r_2} \frac{Q}{4\pi\epsilon_0 r^2}\, dr$$

$$= \frac{-Q}{4\pi\epsilon_0} \int_{r_1}^{r_2} \frac{1}{r^2}\, dr$$

$\frac{r^{-2+1}}{-2+1}$

$$= \frac{-Q}{4\pi\epsilon_0} \left[\frac{-1}{r}\right]_{r_1}^{r_2}$$

$-r^{-1}$

$\left[-\frac{1}{r}\right]$

$$= \frac{-Q}{4\pi\epsilon_0} \left[\frac{-1}{r_2} + \frac{1}{r_1}\right]$$

$$V = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\boxed{V = -\int \vec{E}\, dl \rightarrow (2)}$$

$$\Delta V = -E\,\Delta l$$

$$E = -\frac{\Delta V}{\Delta l}$$

$$\boxed{\begin{array}{l} E = -\text{grad}\, V \\ \vec{E} = -\nabla V \longrightarrow (3) \\ E = \frac{V}{d} \text{ or } \frac{V}{l} \end{array}}$$

d - distance

Potential Difference:-

It is defined as Work done moving a unit positive charge from one point to another point

$$V_A = \frac{+Q}{4\pi\varepsilon_0 r_A}$$

$$V_B = \frac{-Q}{4\pi\varepsilon_0 \cdot r_B}$$

$$V_{AB} = V_A - V_B$$

$$= \frac{Q}{4\pi\varepsilon_0 r_A} - \frac{-Q}{4\pi\varepsilon_0 r_B}$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r_A} + \frac{Q}{4\pi\varepsilon_0 r_B}$$

Find $\vec{E}$ at $(1,1,1)$ if potential $V = xyz^2 + x^2yz + xy^2z$

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}\right]$$

$$= \cancel{-\left[y^2\vec{a_x} + x^2z\vec{a_y} +\right.}$$

$$= -\frac{\partial}{\partial x}(xyz^2 + x^2yz + xy^2z) + \frac{\partial}{\partial y}$$

$$= -\left[yz^2 + 2xyz + y^2z\right] + \left[xz^2 + x^2z + 2xyz\right]\vec{a_y}$$

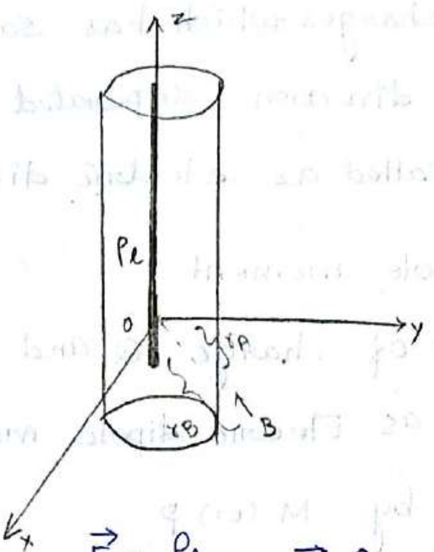
$$+ \left[2xyz + x^2y + xy^2\right]\vec{a_z}$$

$$= -\left[1+2+1\right] + \left[1+1+2\right]\vec{a_y} + \left[4\right]\vec{a_z}$$

$$= -4\vec{a_x} - 4\vec{a_y} - 4\vec{a_z}$$

$$= -4\left[\vec{a_x} + \vec{a_y} + \vec{a_z}\right] \ \text{V/m}$$

Potential Difference for different configuration



$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 r}\vec{a_r} \text{ in } xy \text{ plane}$$

(line charge distribution along cyclindrical coordinate system)

$$V = -\int \vec{E}\, dl$$

For cyclindrical coordinate system

$$dl = dr\,\vec{a_r} + r\,d\phi\,\vec{a_r} + dz\,\vec{a_z}$$

$$V = -\int \frac{\rho_l}{2\pi\varepsilon_0 r}\vec{a_r}\left[dr\,\vec{a_r} + r\,d\phi\,\vec{a_\phi} + dz\,\vec{a_z}\right]$$

$$= -\int_{r_B}^{r_A} \frac{\rho_l}{2\pi\varepsilon_0 r}\, dr$$

$$= -\frac{\rho_l}{2\pi\varepsilon_0 r}\int_{r_B}^{r_A}\frac{1}{r}\, dr$$

$$= \frac{-\rho_l}{2\pi\varepsilon_0 r}\left[\log r\right]_{r_B}^{r_A}$$

$$= \frac{-\rho_l}{2\pi\varepsilon_0 r}\left[\log r_A - \log r_B\right]$$

$$= \frac{-\rho_l}{2\pi\varepsilon_0 r}\left(\log\left(\frac{r_A}{r_B}\right)\right)$$

$$= \frac{\rho_l}{2\pi\varepsilon_0 r}\log\left(\frac{r_B}{r_A}\right)$$

2m Electric dipole:

Two point charges which has same magnitude but opposite direction seperated by small distance is called as electric dipole.

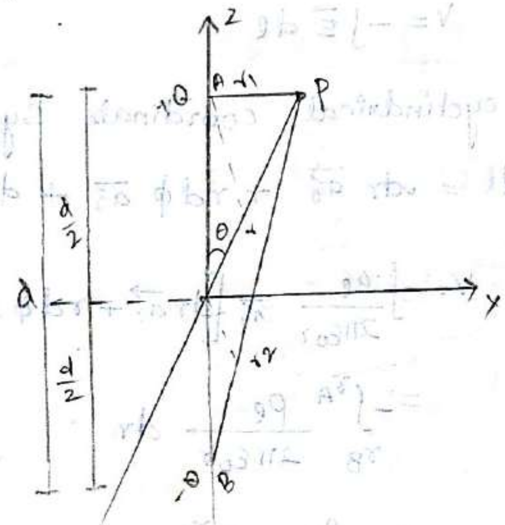
2m Electric dipole moment

The product of charge Q and displacement 'd' is called as Electric dipole moment.

It denoted by M (or) P

$$M = Qd$$

Relationship between Electric dipole moment and potential



$$V_1 = \frac{Q}{4\pi\epsilon_0 r_1}$$

same magnitude
↔ direction.

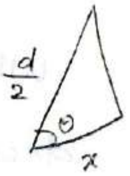
$$V_2 = \frac{-Q}{4\pi\epsilon_0 r_2}$$

$$V = V_1 + V_2$$

$$= \frac{Q}{4\pi\epsilon_0 r_1} + \frac{-Q}{4\pi\epsilon_0 r_2}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \rightarrow ①$$

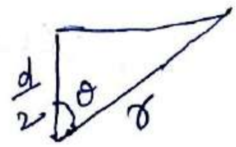
$$\cos\theta = \frac{x}{d/2}$$



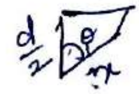
$$x = \frac{d}{2}\cos\theta$$

$$r_1 = r - x$$

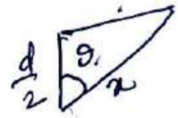
$$r_2 = r + x$$



$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r-x} - \frac{1}{r+x}\right]$$



$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{r+x-r+x}{r^2-x^2}\right]$$



$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{2x}{r^2-x^2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{2\,\frac{d}{2}\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}\right]$$

$\frac{d^2}{4} << 1$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{d\cos\theta}{r^2}\right]$$

$$= \frac{(Qd)\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}}$$

6m **Continuity Equation**

According to law of conservation enegy of charges cannot be created nor destroyed

Let us assume cubical box the charges are present in the cubical box

The charges are slowly move from box to the outwards, the charges are decreasing hence $\boxed{I = -\frac{dQ}{dt}}$

Where $(-) \rightarrow$ indicates the charges are decreasing

Current Density :-

Current Density $J = \frac{I}{A} = \frac{dI}{dA} = \frac{dI}{ds}$

$$dI = J \cdot ds$$

$$I = \int_S \vec{J}\, ds \rightarrow ①$$

By using divergance theorom

$$\oint_S \vec{J} \cdot ds = \int_V (\nabla \cdot J)\, dv$$

$$\rho_v = \frac{Q}{V} = \frac{dQ}{dv}$$

$$dQ = \rho_v\, dv$$

$$Q = \int_V \rho_v\, dv \rightarrow ②$$

By continuity eqn

$$I = -\frac{dQ}{dt} = \int_S \vec{J}\, ds \rightarrow ③$$

$$\frac{-dQ}{dt} = \int_V (\nabla \cdot \vec{J})\, dv$$

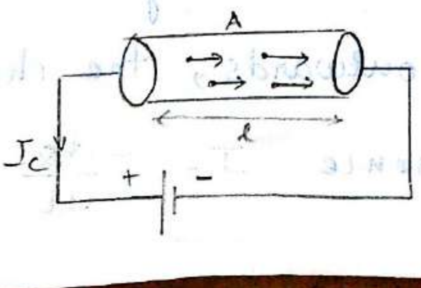
sub ② in where

$$-\frac{d}{dt}\left(\int_V \rho_v\, dv\right) = \int_V \nabla \cdot \vec{J}\, dv$$

$$-\frac{\partial \rho_v}{\partial t} = \nabla \cdot \vec{J}$$

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

Current and current Density :



$$I = \frac{Q}{t} = \frac{dQ}{dt}$$

$I = \frac{V}{R}$

By ohm's law

$$I_c = \frac{V}{R}$$

WKT $R = \frac{\rho l}{A}$ $\quad \frac{1}{\rho}$ = conductivity

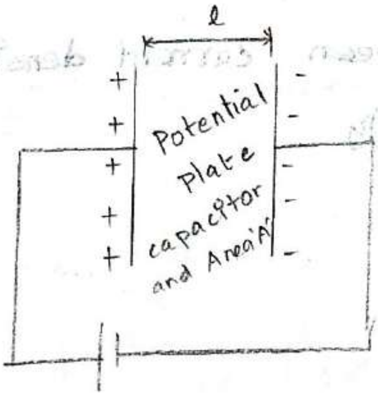
$$I_C = \frac{VA}{\rho l}$$

$$\frac{I_c}{A} = \frac{V}{\rho l} \qquad \frac{V}{l} = \vec{E}$$

$$\frac{I_c}{A} = \frac{1}{\rho}\vec{E}$$

$$\boxed{J = \sigma \vec{E}}$$

Displacement current and Displacement current Density



Displacement current:

It is defined as current flowing through the capacitor when the Voltage is applied across the capacitor is called Displacement currend Id.

$$I_D = \frac{Q}{t} = \frac{dQ}{dt} \rightarrow ①$$

$$C = \frac{Q}{V}$$

$$Q = CV \rightarrow ②$$

$$I_D = \frac{d(CV)}{dt}$$

$$I_D = C\frac{dV}{dt} \rightarrow (3)$$

$E = \frac{V}{d}$

We know that $C = \frac{\epsilon A}{d}$ $\qquad E = \frac{V}{d}$

Ed $\qquad V = Ed.$

$$I_D = \frac{\epsilon A}{d}\frac{dV}{dt}$$

$$= \frac{\epsilon A}{d}\frac{d(Ed)}{dt}$$

$$= \frac{\epsilon A}{d} d \frac{dE}{dt}$$

$$I_D = \epsilon A \frac{dE}{dt}$$

$I_d = J.ds$

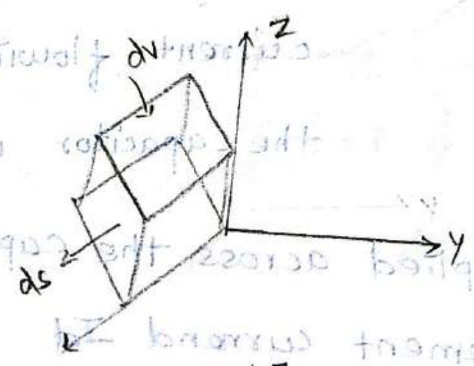
$$\frac{I_D}{A} = \epsilon \frac{d(E)}{dt}$$

electric field flux $\vec{D} = \epsilon E$

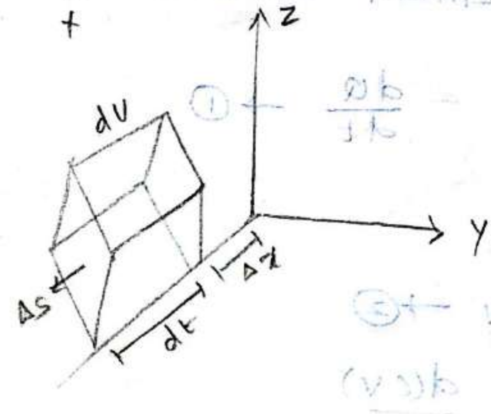
$$J_D = d(\epsilon E)/dt$$

$$\boxed{J_D = \frac{dD}{dt}}$$

Relationship between current density and Volume charge density



Potential $= \frac{x}{t}$



Volume charge distribution

$$\rho_v = \frac{Q}{V} = \frac{dQ}{dv}$$

$$dQ = \rho_v \, dv \rightarrow (1)$$

$$I = \frac{Q}{t} = \frac{dQ}{dt} \rightarrow (2)$$

$$\Delta v = \Delta s \cdot \Delta x \longrightarrow (3)$$

Sub (3) in (1)

$dv = \Delta v$

$ds = \Delta s$

$$dQ = \rho v \, (\Delta s \cdot \Delta x)$$

$$I = \frac{\rho v \, (\Delta s \cdot \Delta x)}{dt}$$

$$I = \rho_v \, \Delta s \left(\frac{\Delta x}{dt}\right) \qquad \frac{dx}{dt} = v_x$$

$$I = \rho_v \, \Delta s \, (v_x) \rightarrow (4)$$

$$J = \frac{I}{A} = \frac{I}{S} = \frac{I}{\Delta s} \text{ from dia}$$

$$I = J \cdot \Delta s \rightarrow (5)$$

Equate (4) & (5)

$$J \cdot \Delta s = \rho_v \, \Delta s \, (v_x)$$

$$\boxed{J = \rho_v \, v_x}$$

8m * **Poission's and Laplace Equations.**

Poission's Eqn:-

Assume point form to Gauss law

$$\nabla \cdot D = \rho_v$$

WKT $\quad D = \varepsilon E$

$$\nabla (\varepsilon E) = \rho_v$$

Relationship between E and V

$$E = -\nabla V$$

$$\nabla (\varepsilon (-\nabla V)) = \rho_v$$

$$-\varepsilon (\nabla^2 V) = \rho_v$$

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$$

Cartesian coordinate

$$\nabla^2 V = \frac{\partial V}{dx}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z} = \frac{-\rho_v}{\varepsilon}$$

Cylindrical coordinate

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho_v}{\varepsilon}$$

Spherical coordinate

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right)$$

$$+ \frac{1}{r\sin\theta}\left(\frac{\partial^2 V}{\partial \phi^2}\right) = \frac{-\rho_v}{\varepsilon}$$

Lapalace Equation :-

In steady state condition there is no rate of flow of charge, which means there is no current

By continivity equation

$$\nabla \cdot J = \frac{-d\rho_v}{dt}$$

$$\frac{dQ}{dt} = 0$$

current $= 0$ $\qquad I = 0$

$$\nabla \cdot J = 0.$$

$$J = \sigma E$$

$$\nabla (\sigma E) = 0$$

$$\sigma (\nabla E) = 0$$

Relationship btw E and V

$$E = -\nabla V$$

$$\sigma\,(\nabla\cdot(-\nabla V)) = 0$$

$$\sigma\,(-\nabla^2 V) = 0$$

where

$$\sigma \neq 0\,;\; -\nabla^2 V = 0$$

$$\nabla^2 V = 0$$

Cartessian :-

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2}\vec{a_x} + \frac{\partial^2 V}{\partial y^2}\vec{a_y} + \frac{\partial^2 V}{\partial z^2}\vec{a_z} = 0$$

cyclindrical :

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0$$

Spherical :-

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r\sin\theta}\left(\frac{\partial^2 V}{\partial\phi^2}\right) = 0$$

Applications of poissions and lapalace equation

Poission and lapalace equation are used to solve the electrostatic problem involving a set of conductors maintaining at different potential.

Both poission and lapalace are used to find the electric field intensity and capacitance value in different condition

Point form (differential form) and Integral from of ohm's law

$$V \propto I$$

$$V = IR$$

$$R = \frac{\rho l}{A} = \frac{1}{\sigma} \cdot \frac{l}{A} \qquad \rho = \frac{1}{\sigma}$$

$$V = \frac{Il}{\sigma A} \qquad \frac{I}{A} = J$$

$$\frac{V}{l} = \frac{I}{\sigma A}$$

$$E = \frac{J}{\sigma}$$

$$\boxed{J = E\sigma}$$

Point from differential form of ohm's law

$$J = \frac{I}{A} = \frac{I}{S} = \frac{dI}{ds}$$

$$dI = J\,ds$$

$$\boxed{\begin{aligned} I &= \iint J \cdot ds \\ I &= \oint_S J \cdot ds \end{aligned}}$$ Integral form of ohm's law

Properties of Conductor

* The conductor surface is an equi potential surface
* The charge density is always zero within the conductor.
* The charge can exist on the surface of the conductor which gives rise to the surface charge density.
* The conductivity of ideal conductor is infinite Hence it is known as superconductor
* No charges and no electric field can exit at any point within the conductor

* The conductivity of the material depends on the temperature

$$\sigma \propto \frac{1}{T}$$

$$\boxed{J = \sigma E = \frac{I}{A}}$$

For aluminum:

$\sigma = 3.82 \times 10^{17}$

For copper $\sigma = 5.8 \times 10^{7}$

The conductors of the material which have no fordiboen gap btw Valence band and conduction band

In perfect conduction

$E = 0$ (inside the conductor)

$E = \infty$ (surface of conductor)

Properties of Dielectrics:

* Dielectrics does not contain any free electrons which all the charges are well bounded and cannot be in motion easily

* The charges in dielectric medium are bounded by finite force Hence it is called bounded charges

* In dielectric medium the charges are bounded there is no free electrons so they cannot contribute to the conduction process

* Dielectrics are the material for which have large forbidden gap btw valence and conduction band

* In perfect dielectric material the conductivity $\sigma = 0$

* The dielectric does not contain any current which oppose the flow of current

* The volume charge density $\rho_v = 0$

* Applied the electric field the bounded charges becomes slowly breakdown

* It produces the free charges it slowly starts to change their position. Hence dielectric store the charges.

2m ✱ Dielectric strength

The minimum value of applied electric field at which dielectric breakdown occurs is called Dielectric strength of the material

* Dielectric become conducting due to dielectric breakdown.

* The electric field outside and inside the dielectrics gets modified due to the induced electric dipole.

Dielectric is classified into two types

* Polar Dielectric
* Non polar Dielectric

Polar Dielectric

In a polar type both (+) ve and (−) ve charges are seperated by small distance 'd' which act as a dipole, where exist a dipole moment m (or) p.

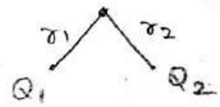
Non - polar:

In a non-polar dielectric both (+)ve and (-)ve charges are coincides hence there is no dipole and dipole moment.

For such a material is placed in a electric field the centre of positive and negative are displacement by small distance now there exist a dipole moment.

Since non-polar dielectric material becomes polar dielectric material

Find the Force on a charge Q_1 20 μC at (0,1,2) m due to Q_2 300 μC at (2,0,0) m



$$r = r_2 - r_1$$

$$r = (2-0)\vec{a_x} + (0-1)\vec{a_y}\ (0-2)\vec{a_z}$$

$$\vec{r} = 2\vec{a_x} - \vec{a_y} - 2\vec{a_z}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\vec{a_r}$$

$$\vec{a_r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{a_r} = \frac{2\vec{a_x} - \vec{a_y} - 2\vec{a_z}}{\sqrt{(2)^2+(-1)^2+(-2)^2}}$$

$$= \frac{2\vec{a_x} - \vec{a_y} - 2\vec{a_z}}{\sqrt{9}}$$

$$\vec{a_r} = \frac{2\vec{a_x} - \vec{a_y} - 2\vec{a_z}}{3}$$

$$F = \frac{(20\times10^{-6})\ (300\times10^{-6})}{4\pi\ (8.85\times10^{-12})\times(3)^2}$$

$V_2 = 31.81$ volt

$V_{total} = V_1 + V_2$

$= 23.135 + 31.81$

$= 8.675$ volt

Capacitance :

A capacitor is a electronic device which consist of two conductors seperated by dielectric medium.

The capacitance of two conducting planes is defined as the ratio of magnitude of charge on either side of the conductor to the potential difference between conductor

$$\boxed{C = \frac{Q}{V}}$$ unit C/V

consider a capacitance composed of two conducting plates of area 'A' seperated by small dielectric medium

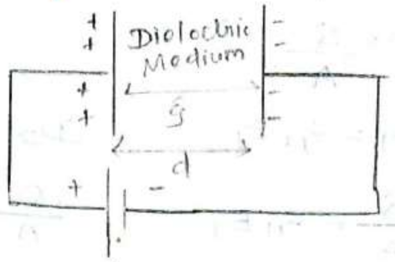
$$C = \frac{\epsilon A}{d}$$ unit is Farad.

If potential V is applied across the plate, the positive charge Q is deposited. and negative charge Q is deposited on another plate since net charge equal to zero

capacitance is used to store the electric charge

1

Capacitance of conductor (parallel plate capacitance)



Due to (+) ve and (−) ve charges the charge seperated by small distance where there is dipole moment

$$C = \frac{Q}{V}$$

$$D = \varepsilon E \rightarrow (1)$$

$$D = \frac{Q}{S} = \frac{Q}{A} \rightarrow (2)$$

Sub eqn (2) in (1)

$$\frac{Q}{A} = \varepsilon E$$

$$Q = \varepsilon A E$$

Relationship between E and V

$$E = \frac{V}{d}$$

$$Q = \varepsilon A \left(\frac{V}{d}\right)$$

$$\frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$C = \frac{\varepsilon A}{d} \qquad \boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$$

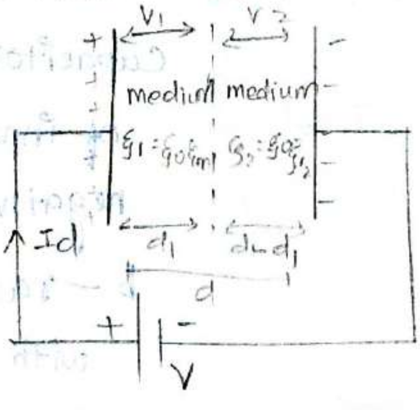
Capacitance are connected in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

capacitance are connected in parllel.

$$C = C_1 + C_2 + C_3 + \cdots$$

Capacitance with Two dielectric Medium



$d = \epsilon_1 A$

$$D = \frac{Q}{A}$$

$$D_1 = \epsilon_1 E_1 \qquad D_2 = \epsilon_2 E_2$$

$$\frac{Q}{A} = \epsilon_1 E_1 \qquad \frac{Q}{A} = \epsilon_2 E_2$$

$$E_1 = \frac{Q}{\epsilon_1 A} \qquad E_2 = \frac{Q}{\epsilon_2 A}$$

R/w btw E and V_i

$$E_1 = \frac{V_1}{d_1} \qquad E_2 = \frac{V_2}{d-d_1}$$

$$V_1 = E_1 d_1 \qquad V_2 = E_2 (d-d_1)$$

$$V_1 = \frac{Q d_1}{\epsilon_1 A} \qquad V_2 = \frac{Q (d-d_1)}{\epsilon_2 A}$$

$$V = V_1 + V_2$$

$$= \frac{Q d_1}{\epsilon_1 A} + \frac{Q (d-d_1)}{\epsilon_2 A}$$

$$V = Q \left[\frac{d_1}{\epsilon_1 A} + \frac{d-d_1}{\epsilon_2 A} \right]$$

$$C = \frac{Q}{V} = \frac{1}{\frac{d_1}{\epsilon_1 A} + \frac{(d-d_1)}{\epsilon_2 A}}$$

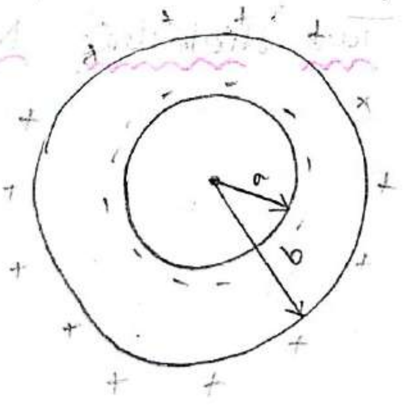
$$C = \frac{\epsilon_1 A}{d} + \frac{\epsilon_2 A}{(d-d_1)}$$

For n no. of medium

$$\boxed{C = \frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2} + \frac{\epsilon_3 A}{d_3} + \ldots}$$

Capacitance on concentric sphere (or)

Capacitance on spherical



Let us assume spherical capacitor small 'a' – radius of inner spherical with negative charge

b – radius of outer spherical with (+)ve charges

Let us consider (+)ve charges and (-)ve charges are seperated by small distance so it act as spherical capacitor.

$$c = \frac{Q}{V}$$

Let us assume voltage is applied to the capacitor no. of flux line are produced across the spherical capacitor

$$V = -\int \vec{E}\, dl \rightarrow ①$$

WKT

$$\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \vec{a_r}$$

In spherical coordinate

$$dl = dr\, \vec{a_r} + r d\theta\, \vec{a_\theta} + r\sin\theta\, \vec{d\phi}$$

$$V = -\int \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a_r} \left[dr \vec{a_r} + r\, d\theta\, \vec{a_r} + r\sin\theta d\vec{\phi} \right]$$

$$V = \frac{-Q}{4\pi\varepsilon_0} \int_b^a \frac{1}{r^2} dr$$

$$= \frac{-Q}{4\pi\varepsilon_0} \left[\frac{-1}{r} \right]_b^a$$

$$= \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$\frac{1}{c} = \frac{V}{Q} = \frac{1}{4\pi\varepsilon_0} \left[\frac{b-a}{ab} \right]$$

$$\boxed{c = \frac{4\pi\varepsilon_0 (ab)}{b-a}} \quad F$$

Capacitance on coaxial cable :- / cyclindriical capacitance

$$c = \frac{Q}{V}$$

$$V = -\int_b^a \vec{E}\, dl$$

Let consida cyclindrical coordinals

$$dl = dr\, \vec{ar} + r\, d\phi\, \vec{a\phi} + dz\, \vec{a_z}$$

For line charge distribution

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 r}\vec{a_r}$$

$$V = -\int_b^a \frac{\rho_l}{2\pi\epsilon_0 r}\vec{a_r}\left[dr\,\vec{a_r} + r\,d\phi\,\vec{a_\phi} + dz\,\vec{a_z}\right]$$

$$= \frac{-\rho_l}{2\pi\epsilon_0 r}\int_b^a \frac{1}{r}\,dr$$

$$V = \frac{-\rho_l}{2\pi\epsilon_0 r}\left[\log r\right]_b^a$$

$$= \frac{-\rho_l}{2\pi\epsilon_0 r}\left[\log a - \log b\right]$$

$$= \frac{\rho_l}{2\pi\epsilon_0 r}\left[\log b - \log a\right]$$

$$V = \frac{\rho_l}{2\pi\epsilon_0 r}\left(\log b/a\right)$$

WKT $C = \frac{Q}{V}$ $\qquad Q = \rho_l l$

$\qquad \frac{Q}{\rho_l} = l$

$$C = \frac{\frac{Q}{\rho_l}\, l}{\frac{\rho_l}{2\pi\epsilon_0 r}\log(b/a)}$$

$$C = \frac{2\pi\epsilon_0 r l}{\log(b/a)} \text{ F}$$

13m (*) Boundary Condition:

The condition existing between two media when Electric field $\vec{E}$ passes from one medium to another medium such condition are called as boundary condition

(i) ⇒ $\vec{E} = 0$ and $\vec{D} = 0$ within the conductor

(ii) No charge can exist within the conductor

(iii) The charges are appeared on the surface of the conductor in the form of surface charge distribution ρ_s

→ volume charge density $\rho_v = 0$ within the conductor.

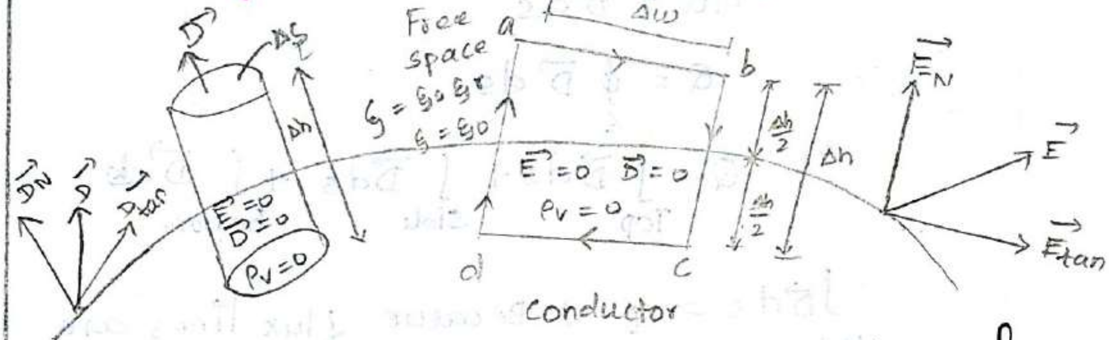
Classification:-

→ The boundary condition btw freespace and conductor

→ The boundary condition btw dielectric and conductor

→ The boundary condition btw perfect dielectric material (or) two dielectric material (ϵ_1 and ϵ_2)

Boundary Condition btw freespace & conductor:-



$$\oint \vec{E}\cdot d\vec{l} = 0 \qquad \oint \vec{E}\cdot dl = \rho_v = 0 \quad (\because \rho_v = 0)$$

$$\vec{E} = \vec{E_N} + \vec{E_{tan}}$$

$\boxed{\int dl = \Delta w}$

$$\int_a^b \vec{E}\cdot dl + \int_b^c \vec{E}\, dl + \int_c^d \vec{E}\, dl + \int_d^a \vec{E}\, dl = 0$$

$$\int_a^b \vec{E}_{tan}\,\Delta w + \int_b^c \vec{E}_N \frac{\Delta h}{2} - \int_b^c (0)\frac{\Delta h}{2} + 0 + \int_d^a (0)\frac{\Delta h}{2} + \int_d^a \cancel{\vec{E}_N \frac{\Delta h}{2}} = 0$$

cancel

at conductor ⇒ zero

$$\int_d^b E_{tan}\,\Delta w = 0$$

$$E_{tan}\,\Delta w = 0$$

$$E_{tan}\,\Delta w \neq 0$$

$$\Delta w \neq 0 \qquad \boxed{\vec{E}_{tan} = 0}$$

Relationship btw E and R

$$D = \epsilon E$$

$$\vec{D}_{tan} = \varepsilon_0 \vec{E}_{tan}$$

$$\vec{E}_{tan} = \frac{\vec{D}_{tan}}{\varepsilon_0} = 0$$

$$\varepsilon_0 \neq 0$$

$$\boxed{\vec{D}_{tan} = 0}$$

Let us consider cyclindrical Gaussian Surface

$$\vec{D} = \frac{Q}{S} = \frac{\psi}{S} = \frac{dQ}{ds}$$

$$dQ = \vec{D}\, ds$$

$$Q = \oint_S \vec{D}\, ds$$

$$Q = \int_{Top} \vec{D}\, ds + \int_{Side} \vec{D}\, ds + \int_{Bottom} \vec{D}\, ds$$

$\int_{Bot} \vec{D}\, ds = 0$; Because flux lines are within the conductor $= 0$

The flux lines leaving from lateral side surface $= 0$

$$\Delta h <<< \Delta s$$

$$\Delta h \to 0$$

$$\int_{Side} \vec{D}\, \frac{\Delta h}{2} = 0$$

$$Q = \int_{Top} \vec{D}\, ds + 0 + 0$$

$$Q = \int_{Top} \vec{D}_N\, dx$$

$$\vec{D}_x = D_N$$

$$Q = D_N \cdot S$$

$$\int dx = S$$

Surface charge distribution

$$\rho_s = \frac{Q}{S}$$

$$Q = S\, \rho_s$$

$$\rho_s \cdot S = D_N \cdot S$$

$$\boxed{\rho_s = D_N}$$

$$\vec{D_N} = \epsilon \vec{E_N}$$

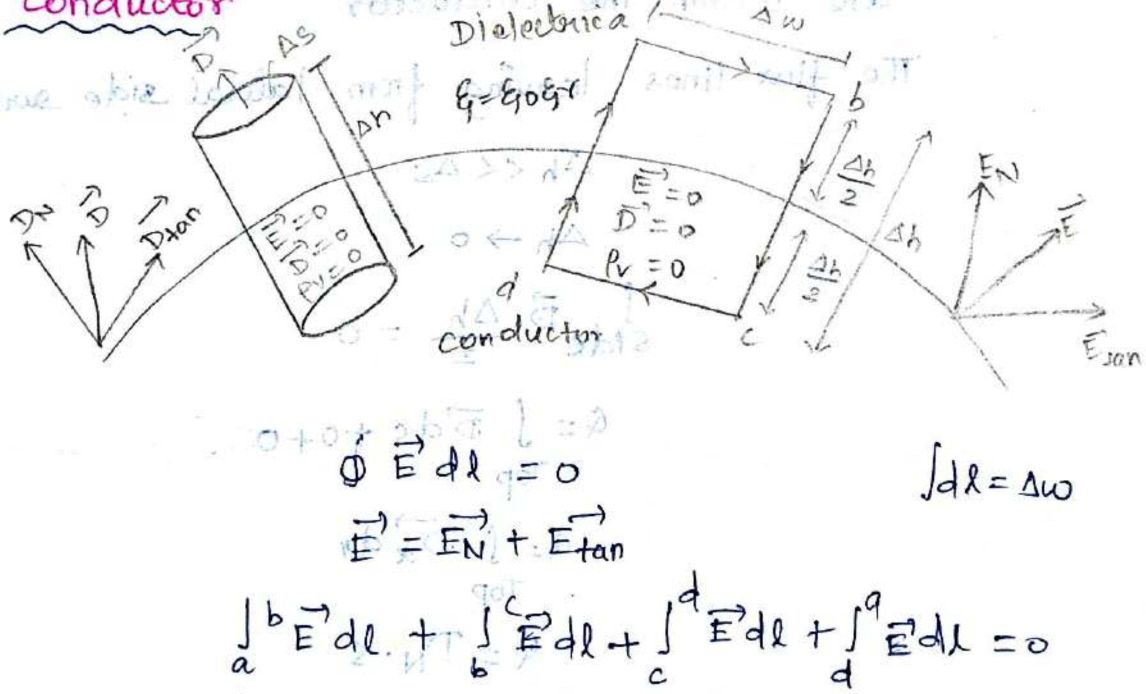
$$\vec{D_N} = \epsilon_0 \vec{E_N} \text{ in freespace}$$

$$\vec{E_N} = \frac{\vec{D_N}}{\epsilon_0}$$

$$\boxed{\vec{E_N} = \frac{\rho_s}{\epsilon_0}}$$

1. $\vec{E_{tan}} = 0$
2. $\vec{D_{tan}} = 0$
3. $\vec{E_N} = \rho_s/\epsilon_0$
4. $\vec{D_N} = \rho_s$

Boundary condition between Dielectric and conductor



$$\oint \vec{E}\, dl = 0 \qquad \int dl = \Delta w$$

$$\vec{E} = \vec{E_N} + \vec{E_{tan}}$$

$$\int_a^b \vec{E}\,dl + \int_b^c \vec{E}\,dl + \int_c^d \vec{E}\,dl + \int_d^a \vec{E}\,dl = 0$$

$$\vec{E_{tan}}\,\Delta w - \vec{E_N}\,\frac{\Delta h}{2} - (0)\,\frac{\Delta h}{2} + 0 + 0 + \vec{E_N}\,\frac{\Delta h}{2} = 0$$

$$E_{tan}\,\Delta w = 0$$

$$E_{tan}\,\Delta w = 0$$

$$\Delta w \neq 0.$$

$$\boxed{\vec{E_{tan}} = 0}$$

Relationship b/w $\vec{E}$ and $\vec{R}$

$$D = \varepsilon E$$

$$\vec{D}_{tan} = \varepsilon \vec{E}$$

$$\vec{E} = \frac{\vec{D}_{tan}}{\varepsilon_0} = 0$$

$$\varepsilon \neq 0.$$

$$\boxed{\vec{D}_{tan} = 0}$$

Let us consider cyclindrical Gaussian surface

$$\vec{D} = \frac{Q}{s} = \frac{\psi}{s} = \frac{dQ}{ds}$$

$$dQ = \vec{D}\,ds$$

$$Q = \int_{Top} \vec{D}\,ds + \int_{side} \vec{D}\,ds + \int_{Bot} \vec{D}\,ds$$

$\int_{Bot} \vec{D}\,ds = 0$ Because flux lines are within the conductor

The flux lines leaving from lateral side surface = 0

$$\Delta h << \Delta s$$

$$\Delta h \rightarrow 0$$

$$\int_{side} \vec{D}\,\frac{\Delta h}{2} = 0$$

$$Q = \int_{Top} \vec{D}\,ds + 0 + 0$$

$$Q = \int_{Top} \vec{D_x}\,dx$$

$$Q = D_N \cdot S$$

Surface charge distribution

$$\rho_S = \frac{Q}{S}$$

$$Q = S\,\rho_S$$

$$\rho_S \cdot S = D_N \cdot S$$

$$\rho_s = D_N$$

$$\vec{D_N} = \varepsilon \vec{E_N}$$

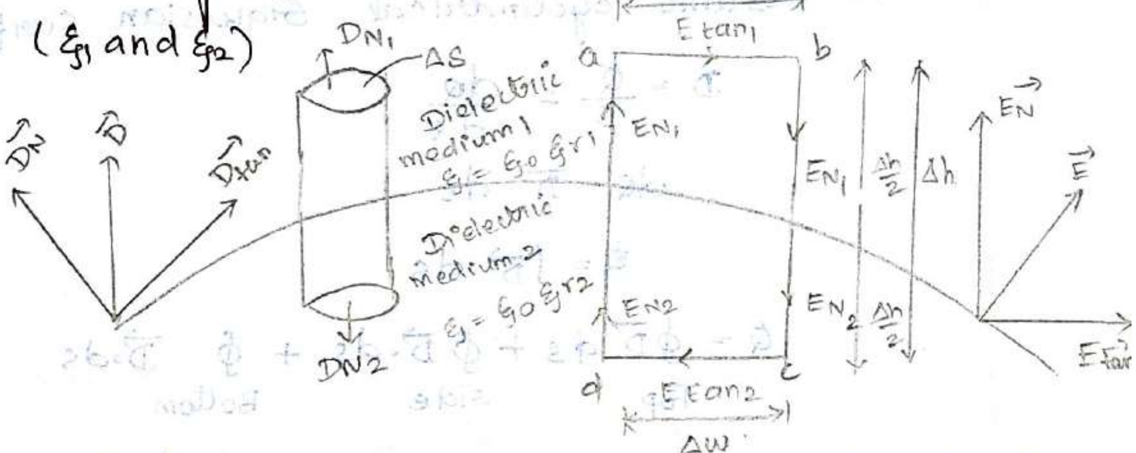
$$\vec{D_N} = \varepsilon \vec{E_N}$$

$$\vec{E_N} = \frac{\vec{D_N}}{\varepsilon}$$

$$\boxed{\vec{E_N} = \frac{\rho_s}{\varepsilon}}$$

$$\boxed{\begin{aligned} \vec{E_{tan}} &= 0 \\ \vec{D_{tan}} &= 0 \\ \vec{E_N} &= \frac{\rho_s}{\varepsilon} \\ \vec{D_N} &= \rho_S \end{aligned}}$$

Boundary Condition between two dielectric medium (ε_1 and ε_2)



$\oint \vec{E} \cdot dl = 0$ which means this closed contour is zero ∴ workdone in carrying a unit positive charge along a closed path is zero

$$\oint \vec{E} \cdot dl = 0$$

$$\int_a^b \vec{E} \cdot dl + \int_b^c \vec{E} \cdot dl + \int_c^d \vec{E} \cdot dl + \int_d^a \vec{E} \cdot dl = 0$$

$$\vec{E}_{tan_1} \Delta w - \vec{E_{N_1}} \frac{\Delta h}{2} - \vec{E_{N_2}} \frac{\Delta h}{2} - \vec{E}_{tan_2} \Delta w + \vec{E_{N2}} \frac{\Delta h}{2} + \vec{E_{N_1}} \frac{\Delta h}{2} = 0$$

$$\vec{E_{tan_1}} \Delta w - \vec{E_{tan_2}} \Delta w = 0$$

$$\vec{E_{tan_1}} \Delta w - E_{tan_2} \Delta w = 0$$

$$(\vec{E_{tan_1}} - E_{tan_2}) \Delta w = 0$$

$$\Delta w \neq 0$$

$$\vec{E}_{tan1} = \vec{E}_{tan2}$$

$$\vec{D}_{tan1} = \varepsilon_1 \vec{E}_{tan1}$$

$$\vec{D}_{tan2} = \varepsilon_2 \vec{E}_{tan2}$$

$$\frac{\vec{D}_{tan1}}{\vec{D}_{tan2}} = \frac{\varepsilon_1 \vec{E}_{tan1}}{\varepsilon_2 \vec{E}_{tan2}} \qquad E_{tan1} = E_{tan2}$$

$$\boxed{\frac{\vec{D}_{tan1}}{\vec{D}_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}}$$

Let us assume cyclindrical Gaussian surface

$$D = \frac{Q}{S} = \frac{dQ}{ds}$$

$$dQ = \vec{D} \cdot ds$$

$$Q = \int \vec{D} \cdot ds$$

$$Q = \oint_{Top} \vec{D}\, ds + \oint_{side} \vec{D} \cdot ds + \oint_{Bottom} \vec{D} \cdot ds$$

The flux leaving from the lateral surface $= 0$.

$$\Delta h <<< \Delta s$$

$$\Delta h \to 0.$$

$$Q = \int_{Top} \vec{D}\, ds + \int_{Bot} \vec{D} \cdot ds.$$

$$Q = D_{N1} \Delta S + D_{N2} \Delta(-S)$$

$$Q = \Delta S(\vec{D}_{N1} - \vec{D}_{N2})$$

$$Q = S(\vec{D}_{N1} - \vec{D}_{N2})$$

WKT

$$\rho_S = \frac{Q}{S}$$

$$Q = \rho_S \cdot S$$

$$\rho_S \cdot S = S(\vec{D}_{N1} - \vec{D}_{N2})$$

when $\rho_S = 0$ because in a dielectric material only bounded charges are present

$$\boxed{\overrightarrow{D_{N1}} = \overrightarrow{D_{N2}}}$$

$$\overrightarrow{D_{N1}} = \varepsilon_1 \overrightarrow{E_{N1}}$$

$$\overrightarrow{D_{N2}} = \varepsilon_2 \overrightarrow{E_{N2}}$$

$$\frac{E_{N1}}{E_{N^2}} = \frac{\overrightarrow{D_{N1}}/\varepsilon_1}{\overrightarrow{D_{N2}}/\varepsilon_2} \qquad \boxed{\overrightarrow{D_{N1}} = \overrightarrow{D_{N2}}}$$

$$\boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}}$$

$$\boxed{\begin{gathered} \overrightarrow{E}_{tan1} = \overrightarrow{E}_{tan2} \\ \frac{\overrightarrow{D}_{tan1}}{\overrightarrow{D}_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2} \\ \overrightarrow{D}_{N1} = \overrightarrow{D}_{N2} \\ \frac{\overrightarrow{E}_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} \end{gathered}}$$

(6m) ## Energy density in Electrostatic field :-

When a unit positive charge is moved from infinite to a point in a field, the work is done by External source and the energy is expanded.

This energy gets to store in the form of potential Energy. (it means Electrostatic energy)

When external source removed, the potential Energy gets converted into Kinectic Energy

$$W_E = \frac{1}{2} \sum_{m=1}^{n} Q_m V_m \quad \text{Joule}$$

For line charge ρ_l

$$W_E = \frac{1}{2} \int \rho_l \, dl \cdot V$$

For surface charge ρ_s,

$$W_S = \frac{1}{2}\int_S \rho_s \, ds \cdot V$$

For volume charge ρ_v,

$$W_E = \frac{1}{2}\int_V \rho_v \, dv \cdot V$$

Energy stored interms of $\vec{E}$ and $\vec{D}$

Volume charge distribution along charge density ρ_v in C/m^2

$$W_E = \frac{1}{2}\iiint \rho_v \cdot dv \cdot V \quad \text{Joule}$$

Gauss divergance theorem $\quad \nabla \cdot D = \rho_v$

$$W_E = \frac{1}{2}\iiint (\nabla \cdot \vec{D}) dv \cdot V$$

(⟶ infinite limit)

$$= \frac{1}{2}\iiint \vec{D}(-\nabla V) dv$$

$$W_E = \frac{1}{2}\iiint \vec{D}\vec{E} \, dv \quad \text{in Joule} \rightarrow (1)$$

$$W_E = \frac{1}{2}\iiint \frac{D^2}{\epsilon_0} dv \qquad D = \epsilon_0 E$$

$$E = \frac{D}{\epsilon_0}$$

$$W_E = \frac{1}{2}\iiint \epsilon_0 E^2 dv$$

$$\frac{D^2 = \epsilon_0^2 E^2}{\epsilon_0}$$

Diff (1) $\quad dW_E = \frac{1}{2}\vec{D}\vec{E} \, dv$

$$D^2 = \epsilon_0 E^2$$

Energy density $\dfrac{dW_E}{dv} = \dfrac{1}{2}\vec{D}\vec{E} \quad J/m^3$

Energy stored interms of capacitance:

The capacitance (or) store the electrostatic Energy is equal to workdone to build up the charges

If a voltage is connected across the capacitor, the capacitor charges

Potential difference is defined as workdone per unit charge

$$V = \frac{W}{Q} = \frac{dw}{dQ} \qquad V = \frac{w}{Q}$$

$$dw = V \cdot dQ.$$

$$W = \int V \cdot dQ$$

$$\boxed{V = \frac{Q}{C}} \quad \boxed{Q = CV}$$

$$W = \int \frac{Q}{C} \, dQ$$

$$= \frac{1}{C} \int Q \, dQ$$

$$= \frac{1}{C} \left[\frac{Q^2}{2} \right]$$

$$= \frac{1}{2} \frac{1}{C} \left[C^2 V^2 \right]$$

$$\boxed{W = \frac{1}{2} C V^2}$$

Polarization :-

polarization is defined as dipole moment per unit volume

polarization increases the electric flux density

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n} Q_i d_i}{\Delta V} \quad C/m^2$$

$$\vec{D} = \varepsilon_0 \vec{E} \text{ in free space}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\text{polarization } \vec{P} = \psi_e \, \varepsilon_0 \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \psi_e \varepsilon_0 \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} (1 + \psi_e) \rightarrow (1)$$

WKT $D = \varepsilon_0 \varepsilon_r E \rightarrow (2)$

compare (1) & (2)

$$\varepsilon_r = 1 + \psi_e$$

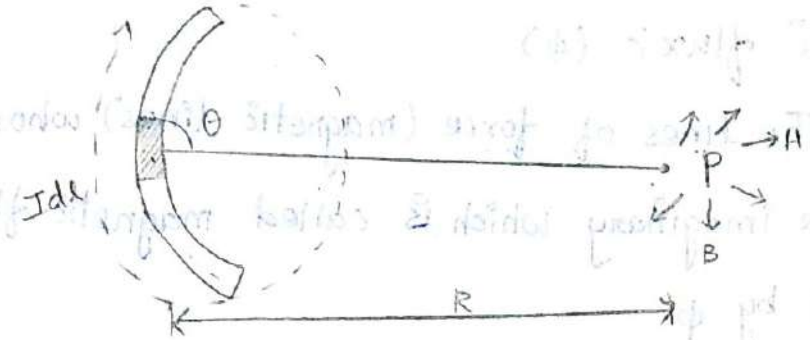
$$\boxed{\psi_e = \varepsilon_r - 1}$$

ψ_e = suscepitibility in dielectric medium

UNIT- 3

Magnetostatics

Bio - Savarat Lahl :-



According to Bio-savart law, the magnetic field intensity produces due to current carrying a conductor

The no. of charges moving from one end to another end of conductor which gives arise to current

This current carrying conductor produce the magnetic field such a magnetic field is called steady magnitic field

Statement

The magnetic field Intensity H is directly Propotional to product of the current 'I' differential length dl and sin of angle btw line joining point 'P' to the current element and Inversely propotional to the square of distance btw the point P to differential length dl. is expressed as

$$dH \propto \frac{I dl \sin\theta}{R^2}$$

$$dH = \frac{K I dl \sin\theta}{R^2}$$

$$K = \frac{1}{4\pi}$$

$$dH = \frac{I\,dl \sin\theta}{4\pi R^2} \quad A/m$$

Magnetic flux:- (ϕ)

The lines of force (magnetic lines) whose lines are imaginary which is called magnetic flux denoted by ϕ

Unit is weber.

Magnetic Field Intensity:-

Magnetic strength or weakness of flux lines interms of number of flux lines are produced due to the current is known as Magnetic Field Intensity It is denoted as 'H'

Unit is A/m.

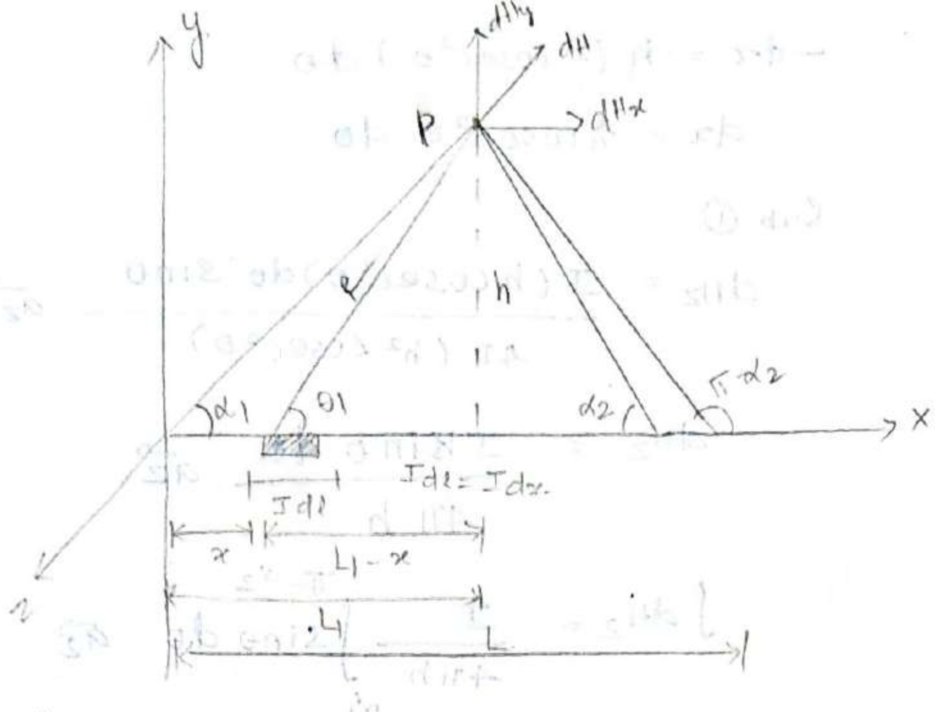
Magnetic Flux Density:-

It is defined as magnetic flux is passing through unit area denoted by the

$$B = \frac{\text{no. of flux lines}}{\text{Unit area}} = \frac{\phi}{A} \quad w/m^2$$

Estimation of magnetic field Intensity and Density.

(i) To find the magnetic field intensity and density at a point 'p' in current carrying conductor for finite line and infinit line.



According to Bio-savart law

$Idl = Idx$

$$dH = \frac{I dl \sin\theta}{4\pi R^2}$$

$$= \frac{I dx \sin\theta}{4\pi R^2}$$

$$\vec{dH} = dH_x + dH_y + dH_z$$

Let us assume the magnetic flux are moving on z direction

$$dH_z = \frac{I dx \sin\theta}{4\pi R^2} \vec{a_z} \rightarrow (1)$$

Case (i)

H for finite line conductance

$$\sin\theta = \frac{h}{R}$$

$$R = \frac{h}{\sin\theta}$$

$$\boxed{R = h\,cosec\,\theta}$$

$$\tan\theta = \frac{h}{L_1 - x}$$

$$L_1 - x = \frac{h}{\tan\theta} \Rightarrow h\cot\theta$$

$$-dx = h(-\operatorname{cosec}^2\theta)\, d\theta$$

$$dx = h\operatorname{cosec}^2\theta\, d\theta$$

Sub ①

$$dH_z = \frac{I(h\operatorname{cosec}^2\theta)d\theta\,\sin\theta}{4\pi(h^2\operatorname{cosec}^2\theta)}\,\vec{a_z}$$

$$dH_z = \frac{I\sin\theta\, d\theta}{4\pi h}\,\vec{a_z}$$

$$\int dH_z = \frac{I}{4\pi h}\int_{\alpha_1}^{\pi-\alpha_2}\sin\theta\, d\theta\ \vec{a_z}$$

$$H_z = \frac{I}{4\pi h}\left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$\cos(\pi-\theta) = -\cos\theta$
$$= \frac{I}{4\pi h}\left[-\cos(\pi-\alpha_2)+\cos\alpha_1\right]$$

$$= \frac{I}{4\pi h}\left[-(-\cos(\alpha_2)+\cos\alpha_1\right]\vec{a_z}$$

$$H_z = \frac{I}{4\pi h}\left[\cos\alpha_1+\cos\alpha_2\right]\vec{a_z}\ \text{A/m}$$

$$B_z = \mu H_z$$

$$B_z = \frac{\mu I}{4\pi h}\left[\cos\alpha_1+\cos\alpha_2\right]\vec{a_z}\ \text{Wb/m}^2$$

case (ii)

H infinite line conducatance

For infinite line conductance

$$\alpha_1 = 0 \quad \alpha_2 = 0$$

$$\cos\alpha_1 = \cos\alpha_2 = \cos 0 = 1.$$

$$H_z = \frac{I}{4\pi h}\left[\cos\alpha_1+\cos\alpha_2\right]\vec{a_z}$$

$$= \frac{I}{4\pi h}\left[\cos(0)+\cos(0)\right]\vec{a_z}$$

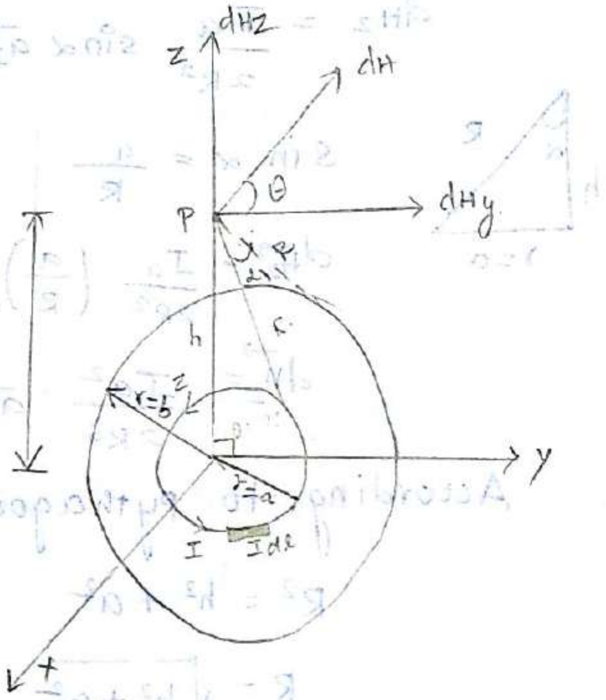
$$= \frac{I}{4\pi h}\left[1+1\right] \vec{a_z}$$

$$= \frac{I}{4\pi h}\left[2\right] \vec{a_z} \quad A/m$$

$$H_z = \frac{I}{2\pi h} \vec{a_z} \quad A/m^2$$

$$\boxed{B_z = \frac{\mu I}{2\pi h} \vec{a_z} \quad w/m^2}$$

(ii) To find Magnetic Field Intensity and density for circular conductor.



According to Bio- savarat law

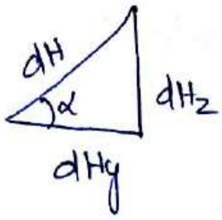
$$dH = \frac{Idl\sin\theta}{4\pi R^2}$$

The magnetic field is produced on the axis $\theta = 90°$

$$\sin\theta = \sin 90° = 1$$

$$dH = \frac{Idl}{4\pi R^2}$$

Here the point P is on z axis which is right angle to the circular loop ($\theta = 90°$)



$$\sin\alpha = \frac{dH_z}{dH}$$

$$dH_z = dH \sin\alpha$$

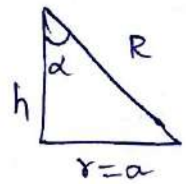
$$dH_z = \frac{I\,dl}{4\pi R^2} \sin\alpha \,\vec{a_z}$$

circumferance $\rightarrow dl = 2\pi r$

$r = a$ in inner circular disc

$$\vec{dH_z} = \frac{I\,(2\pi a)}{4\pi R^2} \sin\alpha \,\vec{a_z}$$

$$\vec{dH_z} = \frac{Ia}{2R^2} \sin\alpha \,\vec{a_z}$$



$$\sin\alpha = \frac{a}{R}$$

$$\vec{dH_z} = \frac{Ia}{2R^2}\left(\frac{a}{R}\right)\vec{a_z}$$

$$\vec{dH_z} = \frac{Ia^2}{2R^3}\,\vec{a_z}$$

According to pythagoras theorem

$$R^2 = h^2 + a^2$$

$$R = \sqrt{h^2+a^2}$$

$$R^3 = (h^2+a^2)^{3/2}$$

$$\vec{dH_z} = \frac{Ia^2}{2(h^2+a^2)^{3/2}}\,\vec{a_z}$$

when $h=0$ according to equipotential surface from ground level to the freespace because the magnetic field flux lines are produced at freespace $H=0$

$$\vec{dH_z} = \frac{Ia^2}{2(a^2)^{3/2}}\,\vec{a_z}$$

$$= \frac{I\, a^2}{2\, a^3} \vec{a_z}$$

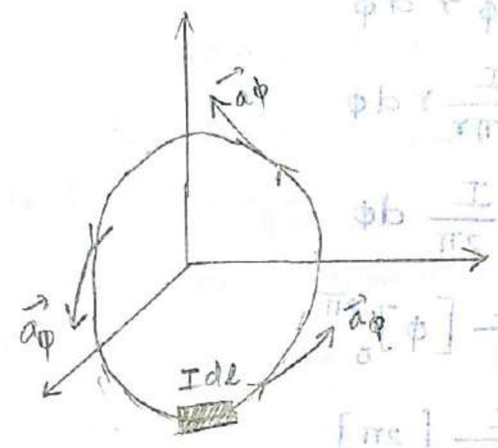
$$d\vec{H_z} = \frac{I}{2a} \vec{a_z}$$

Magnetic flux density

$$B = \mu H$$

$$\boxed{B_z = \frac{\mu I}{2a} \vec{a_z}} \quad w/m^2$$

Ampere's Circuital Law:-



$$\boxed{\oint \vec{H} \cdot dl = I_{enclosed}}$$

The line integral of magnetic field intensity produced by an closed path is equal to the current enclosed by that path.

Proof :-

Letus consider cyclindrical coordinate system according to ampere circuital law, the current carrying conductor which produces magnetic flux lines.

Assume magnetic flux lines are produced in ϕ direction

$$\oint H \cdot dl \qquad dl = dr\,\vec{a_r} + r d\phi\, \vec{a\phi} + dz\,\vec{a_z}]$$

$$I = \oint H_\phi\, a_\phi\, [dr\,\vec{a_r} + r d\phi\, \vec{a_\phi} + dz\,\vec{a_z}]$$

$$\boxed{I = \oint H_\phi r \, d\phi}$$

$$= H\phi \; r \int_0^{2\pi} d\phi$$

$$= H\phi \; r \; [\phi]_0^{2\pi}$$

$$I = H_\phi \, r \, [2\pi]$$

$$H\phi = \frac{I}{2\pi r}$$

R.H.S

$$= \oint H\phi \, r \, d\phi$$

$$= \oint \frac{I}{2\pi r} r \, d\phi$$

$$= \oint \frac{I}{2\pi} d\phi$$

$$= \frac{I}{2\pi} [\phi]_0^{2\pi}$$

$$= \frac{I}{2\pi} [2\pi]$$

$$= I$$

L.H.S = R.H.S

Hence proved.

Point form / Differential form of Amper's Circuital Law

Interim of magnetic field intensity 'H'

$$\oint \vec{H} \, dl = I_{enclosed}$$

According to stoke's theorem

$$\oint \vec{H} \cdot dl = \iint (\nabla \times H) \, ds \rightarrow ①$$

$$I = \iint (\nabla \times H) \, ds \rightarrow ②$$

$$J = \frac{I}{A} = \frac{I}{S} = \frac{dI}{ds}$$

$$dI = J \cdot ds$$

$$I = \iint \vec{J}\, ds$$

Sub (2)

$$\iint \vec{J}\, ds = \iint (\nabla \times H)\, ds$$

$$\boxed{\vec{J} = \nabla \times \vec{H}}$$

Interm of magnetic flux density B

$$B = \frac{\phi}{A} = \frac{d\phi}{ds}$$

$$d\phi = B \cdot ds$$

$$\phi = \iint \vec{B}\, ds$$

There is no circuit there is an isolated electric charge

Hence there is no magnetic field

$$\phi = \iint \vec{B}\, ds = 0$$

According Divergance theorem

$$\iint \vec{B} \cdot ds = \iiint (\nabla \cdot B)\, dv = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Application of ampere circuital law :-

(1) Line :-

To find magnetic field intensity for

For finite line

$$\vec{H} = \frac{I}{4\pi h}\left[\cos\alpha_1 + \cos\alpha_2\right]$$

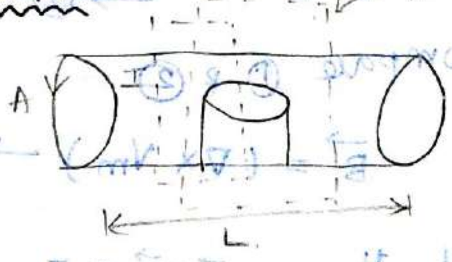
For infinite line

$$\vec{H} = \frac{I}{2\pi h}.$$

(2) Solenoidal

No. of turns (N)

$I \propto$ No. of turns



To find magnetic field intensity

According to Ampere's Law

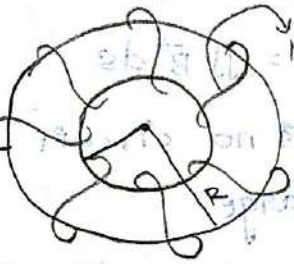
$$\oint \vec{H}\cdot dl = I \text{ enclosed.}$$

$$H \cdot L = I$$

$$H \cdot L = NI$$

$$\boxed{\vec{H} = \frac{NI}{L}}$$

(3) Toroid :-



$$\oint \vec{H}\cdot dl = I_{enclosed}$$

$$\vec{H}\,(2\pi R_m)\; I$$

$$R_m = \frac{R_1 + R_2}{2} \text{ mean Radius}$$

$$\vec{H}\,(2\pi R_m)\, NI$$

$$\vec{H} = \frac{NI}{2\pi R_m}$$

$\nabla \times (\nabla m) = 0$

$\nabla \cdot (\nabla \times \vec{A}) = 0$

$\vec{H} = -\nabla V_m$

Scalar and Vector Magnetic potential :-

Vectors Magnetic potential (or) poisons eqn

$$\nabla \cdot (\nabla \times V_m) = 0 \rightarrow ①$$

$$\nabla \cdot (\nabla \times \vec{H}) = 0$$

Pt from ampere's interm of 'B'

$$\nabla \cdot \vec{B} = 0 \rightarrow ②$$

compare ① & ②

$$\vec{B} = (\nabla \times V_m) \rightarrow ③$$

Interm of H , $\nabla \times \vec{H} = J \rightarrow ④$

Relationship btw B and H

$$B = \mu H$$

$$H = \frac{B}{\mu}$$

$$\nabla \times \left(\frac{B}{\mu}\right) = J$$

$$\nabla \times B = \mu J \rightarrow (5)$$

compare eqn (3) and (5)

$$\nabla \times (\nabla \times V_m) = \mu J$$

Formula:- $\nabla \times (\nabla \times A) = (\nabla \times A).\nabla - (\nabla \cdot \nabla) A$

$$(\nabla \times V_m).\nabla - (\nabla \cdot \nabla) V_m = \mu J$$

$$(\vec{B} \cdot \nabla) - \nabla^2 V_m = \mu J$$

$$0 - \nabla^2 V_m = \mu J$$

For steady state condition due to isolated electric charge

$$\boxed{\vec{B} = \nabla \times V_m = 0.}$$

$$-\nabla^2 V_m = \mu J$$

$$\boxed{\nabla^2 V_m = -\mu J}$$

Cartesian $\nabla^2 V_m = \frac{\partial^2 V_m}{\partial x^2}\vec{a_x} + \frac{\partial^2 V_m}{\partial y^2}\vec{a_y} + \frac{\partial^2 V_m}{\partial z^2}\vec{a_z} = -\mu J$

cylindrical $\nabla^2 V_m = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V_m}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V_m}{\partial \phi^2}\right) + \frac{\partial^2 V_m}{\partial z^2} = -\mu J$

spherical $\nabla^2 V_m = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V_m}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V_m}{\partial \theta}\right) + \frac{1}{r \sin\theta}\left(\frac{\partial^2 V_m}{\partial \phi^2}\right) = -\mu J$

Scalar Magnetic potential (or) Lapalace eqn

$$\nabla \times (\nabla V_m) = 0 \rightarrow (1)$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

R/w btw H and Vm

$$\boxed{H = -\nabla V_m}$$

Sub

$$\nabla \times (-\vec{H}) = 0$$

$$\nabla \times \vec{H} = 0$$

μ form of ampere's law

$$\nabla \times \vec{H} = J$$

Current density $J = 0$

For an isolated electric charge

$$\boxed{\nabla \cdot \vec{B} = 0}$$

$$B = \mu H$$

$$\boxed{H = -\nabla V_m}$$

$$\nabla \cdot (\mu H) = 0$$

$$\cancel{\mu(\nabla \cdot (\mu H))}$$

$$\mu(\nabla \cdot H) = 0$$

$$\mu(\nabla \cdot (-\nabla V_m)) = 0.$$

$$\mu(-\nabla^2 V_m) = 0$$

$$\mu \neq 0$$

$$-\nabla^2 V_m = 0$$

$$\boxed{\nabla^2 V_m = 0}$$

cartesian:

$$\nabla^2 V_m = \frac{\partial^2 V_m}{\partial x^2}\vec{a_x} + \frac{\partial^2 V_m}{\partial y^2}\vec{a_y} + \frac{\partial^2 V_m}{\partial z^2}\vec{a_z} = 0$$

Cyclindrical:

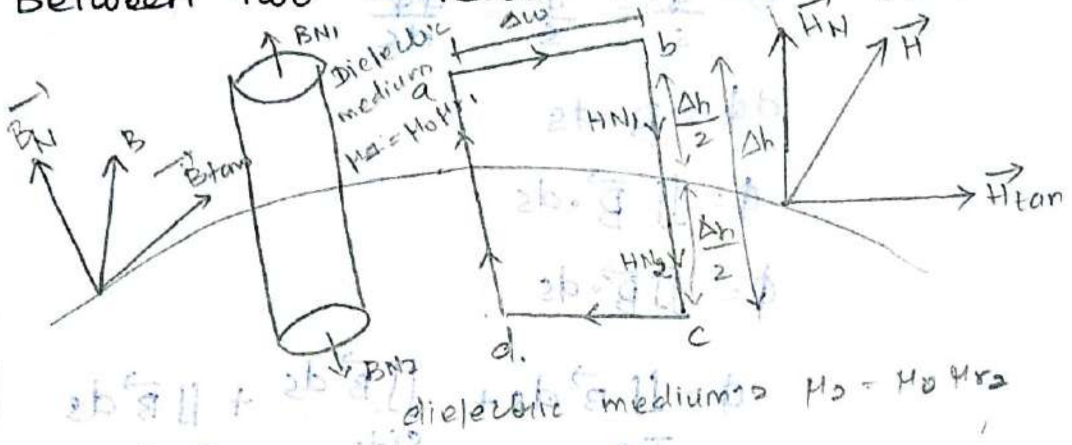
$$\nabla^2 V_m = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_m}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V_m}{\partial \phi^2}\right) + \frac{\partial^2 V_m}{\partial z^2} = 0$$

Spherical:-

$$\nabla^2 V_m = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V_m}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V_m}{\partial \theta}\right) + \frac{1}{r\sin\theta}\left(\frac{\partial^2 V_m}{\partial \theta^2}\right) = 0$$

Magnetic Boundary Condition

Between two dielectric Medium (μ_1, μ_2).



According to ampere circuital law

$$\oint \vec{H} \cdot dl = I_{enclosed}$$

$$\oint_A^B \vec{H} \cdot dl + \oint_B^C \vec{H}\, dl + \oint_C^D \vec{H}\, dl + \oint_D^A \vec{H}\, dl = I$$

$$H_{tan_1} \Delta w + (-H_{N_1}) \frac{\Delta h}{2} - H_{N_2} \frac{\Delta h}{2} - H_{tan_2} \Delta w +$$

$$H_{N_1} \frac{\Delta h}{2} + H_{N_2} \frac{\Delta h}{2} = I.$$

$$H_{tan_1} \Delta w - H_{tan_2} \Delta w = I$$

$$\Delta w (\vec{H}_{tan_1} - \vec{H}_{tan_2}) = I.$$

Let us consider a closed surface and.

current $I = 0$

$$\Delta w (\vec{H}_{tan_1} - \vec{H}_{tan_2}) = 0$$

$$\Delta w \neq 0$$

$$\vec{H}_{tan_1} - \vec{H}_{tan_2} = 0$$

$$\boxed{\vec{H}_{tan_1} = \vec{H}_{tan_2}}$$

$$B_{tan_1} = \mu_1 H_{tan_1}$$

$$B_{tan_2} = \mu_2 H_{tan_2}$$

$$\boxed{\frac{B_{tan_1}}{B_{tan_2}} = \frac{\mu_1}{\mu_2}}$$

Let us consider cylindrical Gaussian Surface

$$B = \frac{\Phi}{s} = \frac{\psi}{s} = \frac{d\Phi}{ds}$$

$$d\Phi = B \cdot ds$$

$$\Phi = \iint \vec{B} \cdot ds$$

$$\Phi = \iint \vec{B} \cdot ds$$

$$\Phi = \iint_{Top} \vec{B}\, ds + \iint_{Side} \vec{B}\, ds + \iint_{Bottom} \vec{B}\, ds$$

$\Phi = \psi$ $B = \frac{\Phi}{s}$ $D = \frac{\psi}{s}$

$$\Delta h <<< \Delta s, \quad \iint_{side} \vec{B}\, ds = 0.$$

$$\Phi = \iint_{Top} \vec{B}\, ds + \iint_{Bot} \vec{B}\, ds$$

$$\Phi = B_{N1}\, \Delta s - B_{N2}\, \Delta s$$

Let us consider steady state $\Phi = 0$

$$0 = \Delta s\,(B_{N1} - B_{N2})$$

$$\Delta s \neq 0 \qquad \rho_s = \frac{Q}{S}$$

$$\boxed{B_{N1} = B_{N2}}$$

$$B_{N1} = \mu_1 H_{N1}$$

$$B_{N2} = \mu_2 H_{N2}$$

$$\boxed{\frac{H_{N1}}{H_{N2}} = \frac{\mu_2}{\mu_1}}$$

$$\vec{H}_{tan1} = \vec{H}_{tan2}$$

$$\frac{B_{tan1}}{B_{tan2}} = \frac{\mu_1}{\mu_2}$$

$$B_{N1} = B_{N2}$$

$$\frac{H_{N1}}{H_{N2}} = \frac{\mu_2}{\mu_1}$$

Steady Magnetic field.

The force experienced by the charge moving with a velocity which produce the current due to magnetic flux is produced. which is known as steady magnetic Field.

Electric force:

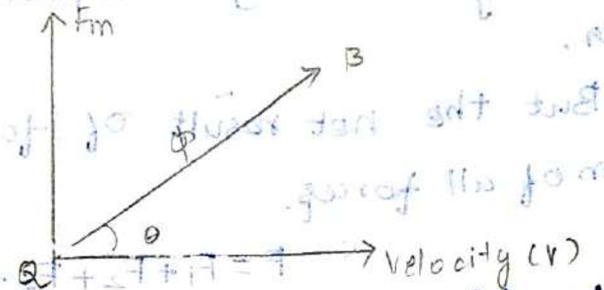
The force experienced by test charge which exists the electric field.

$$\vec{E} = \frac{F_e}{Qt}$$

$$\boxed{\vec{F_e} = \vec{E} \cdot Qt}$$

Where $\vec{E}$ is static Electric Field.

Magnetic Force / Force on a point charge / moving charge



In a steady state magnetic field, the force experienced by the charge moving with a Velocity V and produce magnetic flux density 'B'

Here Force is directly propotional to the magnetic flux density 'B' and

$\theta \rightarrow$ angle btw 'V' and 'B'

The magnetic Force F_m is directly propotional to Both 'V' and 'B'

$$F \propto B$$

$$F_m = Q(\vec{V} \times \vec{B})$$

$$F_m = Q|\vec{V}||\vec{B}|\sin\theta$$

$\boxed{\theta = 90°}$ $\sin 90° = 1$

Let consider $\sin 90° = 1$

$$\boxed{F_m = Q\,|\vec{V}|\,|\vec{B}|}$$

Lorentz's Force Eqn:

The Force on moving particle, due to combine electric and magnetic Field.

$$\vec{F} = \vec{F_e} + \vec{F_m}$$

F_e for electric force $= \vec{E}\,Q_t$

F_m for magnetic force $= Q\,V\vec{B}$

$$\boxed{F = \vec{E}\,Q_t + Q\,V\vec{B}}$$

The two or more forces F_1 and F_2, F_3 ... etc that they are acting independently with each other.

But the net result of force is equal to sum of all forces.

$$F = F_1 + F_2 + F_3 + \cdots$$

Force acting on differential current element (or) current carrying conductor

By Lorentz force eqn

$$\vec{F} = \vec{F_e} + \vec{F_m}$$

only magnetic Force is charging because current carrying conductor $\boxed{F_e = 0}$

$$F = F_m$$

$$F = Q\,(\vec{V} \times \vec{B})$$

$$F = dQ\left(\frac{dl}{dt} \times \vec{B}\right)$$

$$= \frac{dQ}{dt}\,(dl \times \vec{B})$$

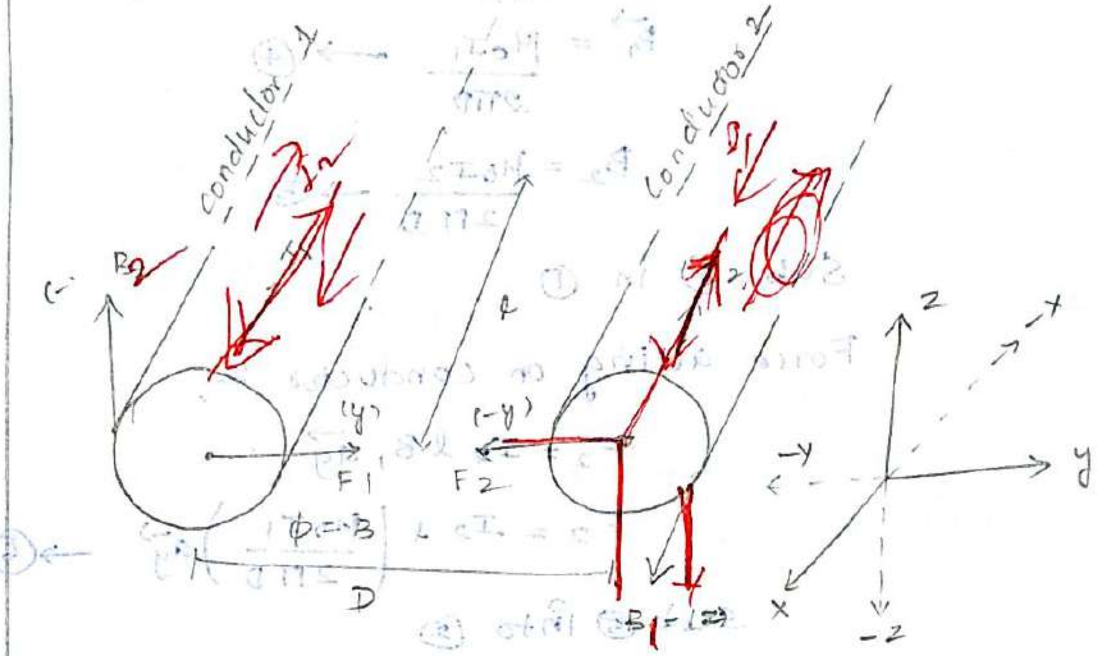
$$F = dI\,(dl \times \vec{B})$$

$$F = dI\,|dl|\,|\vec{B}|\sin\theta$$

$$\theta = 90^\circ$$

$$\boxed{F = I l \vec{B}}$$ Force acting on total length of a conductor

Force btw the two current element (or) Two parellel conductor



$a_x \times a_z = -a_y$

$a_z \times a_x = -a_y$

Force acting on conductor 2

$$F_2 = I_2 l B_1$$

$$\vec{F_2} = I_2(-a_x)\,l\,\vec{B_1}\,(-\vec{a_z})$$

$a_x a_z = a_y$

$$F_2 = I_2\,l\,B_1(-\vec{a_y}) \rightarrow ①$$

Force acting on conductor 1

$$F_1 = I_1 l \vec{B_2}$$

$$= I_1(\vec{a_z})\,l\,\vec{B_2}\,(+\vec{a_z})$$

$$F_1 = I_1\,l\,\vec{B_2}\,(+)\vec{a_y} \rightarrow ②$$

For infinite line straight line conductor the magnetic field Intensity

$$\vec{H} = \frac{I}{2\pi h}.$$

$h \rightarrow$ distance $= D$

Here the Force btw two parellel plate

Conductors seperated by a distance 'D'

$$H = \frac{I}{2\pi D} \rightarrow (3)$$

R/w btw B and H

$B = \mu_0 H$ for free space.

$$\vec{B} = \frac{\mu_0 I}{2\pi D}$$

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi D} \longrightarrow (4)$$

$$B_2 = \frac{\mu_0 I_2}{2\pi D} \rightarrow (5)$$

Sub (4) in (1)

Force acting on conductor 2

$$F_2 = I_2 \, l \, B_1 \, \vec{a_y}$$

$$F_2 = I_2 \, l \left(\frac{\mu_0 I_1}{2\pi D} \right) \vec{a_y} \longrightarrow (6)$$

Sub (5) into (2)

Force acting on conductor 1

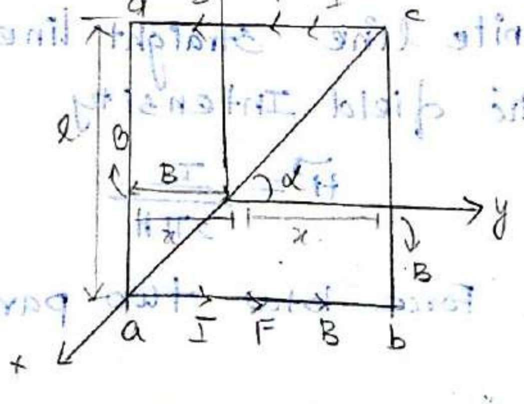
$$\vec{F_1} = I_1 \, l \, B_2 (-\vec{a_y})$$

$$|\vec{F_1}| = I_1 \, l \left(\frac{\mu_0 I_2}{2\pi D} \right) (\vec{a_y})$$

H = t.

$$|\vec{F_1}| = |\vec{F_2}|$$

Force or Torque on a closed loop conductor (or) closed loop circuit (or) Torque on a loop carrying a current I



'α' is the angle between the axis of rotation and magnetic flux line (or) magnetic density

r – distance between reference point to the force acting on the conductor.

L – Length of the conductor

w – width of the conductor

Definition of Torque:

It is defined as rate of change of angle moment (or) movement of rotation (or) how much of force acting on a closed loop conductor. due to dispo. this force. The conductor starts to rotate with an angle 'α' from the reference point is known as Torque.

$$\vec{T} = r \times \vec{F}$$

Force acting on A to B.

$$\vec{F} = IlB \sin\theta$$

Here Force are parallel line to each other

$\theta = 0^\circ$ $\boxed{\vec{F} = 0}$

· Force acting on B to C

$$F = IlB \sin\theta$$

Here Force are acting perpedicular to each other. $\theta = 90^\circ$ $\sin 90^\circ = 1$.

$$\boxed{\vec{F} = IlB}$$

Force acting on C to D

$$F = IlB \sin\theta \qquad \theta = 0^\circ$$

Here Force are parallel line to each other.

$\theta = 0^\circ$ $\boxed{\vec{F} = 0}$

Force acting on D to A

$$F = I l B \sin\theta$$

$$\theta = 90^\circ$$

$$F = I l B.$$

Here Force acting on ⊥r to conductor

Total Force $\vec{F} = 0 + IlB + 0 + IlB$

$$\boxed{\vec{F} = 2IlB}$$

$$\vec{T} = r \times \vec{F}$$

$$\vec{T} = |r|\,|\vec{F}|\sin\alpha.$$

$$= \frac{w}{2}(2IlB)\sin\alpha.$$

$$\vec{T} = wl\,I\vec{B}\sin\alpha \qquad wl = N$$

N → No. of turns

$$\vec{T} = NIA\vec{B}\sin\alpha$$

~~Force acting on B to C.~~

~~$F = IlB\sin\theta$~~

~~Here Force acting on ⊥~~

Let $\alpha = 90^\circ$

$$\boxed{\vec{T} = NIA\vec{B}}$$

Inductance

It is used to store magnetic Energy it is defined as ratio of magnetic flux produced by the coil to the current flowing through a coil.

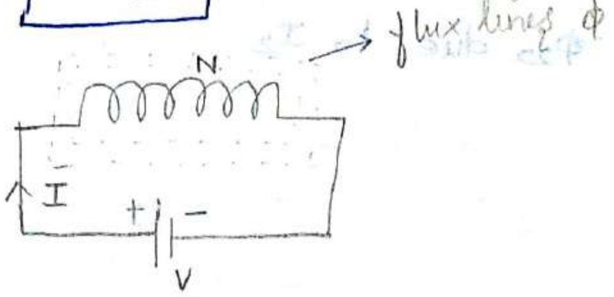
$$\boxed{L = \frac{\phi}{I}}$$

$$\phi \propto I$$

N – No. of turns

$I \propto N\phi$

$$\boxed{L = \frac{N\phi}{I}}$$



(2m) Statement :-

The coil consist of 'N' number of turns the current flowing the each turn which produce the magnetic flux line ϕ those flux lines are imaginary lines which is stored by inductane 'L'

$$\boxed{L = \frac{N\phi}{I}}$$

Self Inductance :-

It is defined as flux produced by the coil due to current carrying the coil Here the coil which linked with coil itself

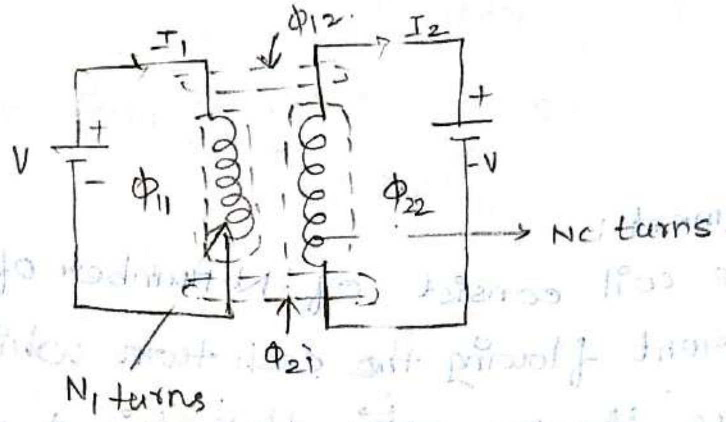
Mutal Inductanee :-

→ Let us consider the magnetic coil with difference M_1 & M_2 with different current I_1, I_2 flowing through the coil 1, 2

→ Due to that magnetic flux lines are Produced it is coupling with each other it is Known as Mutal Inductance

→ Let the coil N_1 turns with Inductance L_1 carrying a current I_1 which produce magnetic flux ϕ_{11} due to I_1

Ily the coil N_2 turns with inductance L_2 carrying a current I_2 which produce magnetic flux ϕ_{22} due to I_2



Magnetic flux produced by the coil with current I_1 which linked with another coil (coil 2)

Due to this magnetic flux leakage from ϕ_{12}

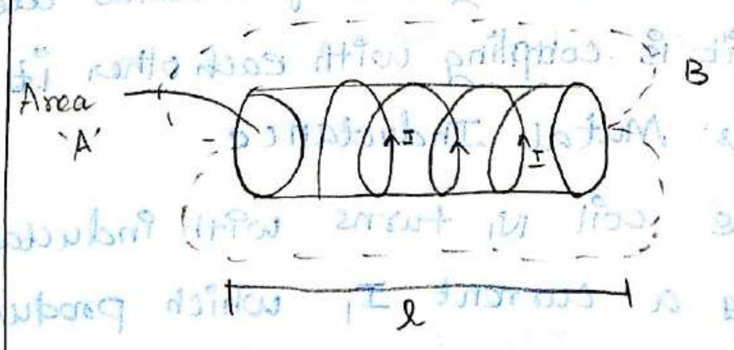
$$M_{12} = \frac{N_2 \phi_{12}}{I_1}$$

Magnetic flux produced by the coil with current I_2 which linked with another coil (coil 1)

Due to this magnetic flux leakage from ϕ_{21}

$$M_{21} = \frac{N_1 \phi_{21}}{I_2}$$

Inductance of solenoidal :-



$$L = \frac{N\phi}{I}$$

Magnetic flux densily

$$B = \frac{\phi}{A}$$

$$\phi = BA$$

$$L = \frac{N(BA)}{I}$$

WKT $B = \mu H$

$$L = \frac{N\mu HA}{I}$$

Magnetic field intensity for solenoid

$$H = \frac{NI}{l}$$

$$L = \frac{N\mu A}{I}\left(\frac{NI}{l}\right)$$

$$L = \frac{N^2\mu AI}{Il}$$

$$L = \frac{N^2\mu A}{l}$$

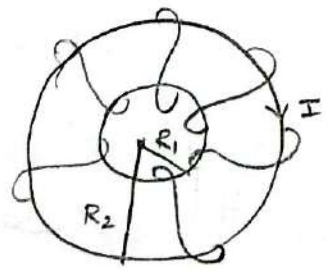
L → inductance of solenoidal

A → area of solenoidal

l - length of solenoidal

μ - permeability of magnetic Field

Inductance of Toreoid:-



$$L = \frac{N\phi}{I}$$

Magnetic flux densily $B = \frac{\phi}{A}$

$$\phi = BA \rightarrow (2)$$

$$L = \frac{N(BA)}{I}$$

WKT, $B = \mu H$

$$L = \frac{N\mu H A}{I} \rightarrow (3)$$

Magnetic field Intensity for Toroid

$$H = \frac{NI}{2\pi R_m} \qquad A = \pi r^2$$

$$L = \frac{N\mu(\pi r^2)}{I}\left(\frac{NI}{2\pi R_m}\right)$$

$$= \frac{N^2\mu(\pi r^2)I}{2\pi R_m I}$$

$$L = \frac{N^2 \mu r^2}{2R_m}$$

$L \rightarrow$ Inductance of Toroid

r – radius of Toroid

R_m – Mean, radius $R_m = \frac{R_1 + R_2}{2}$

μ – Permeability in magnetic field

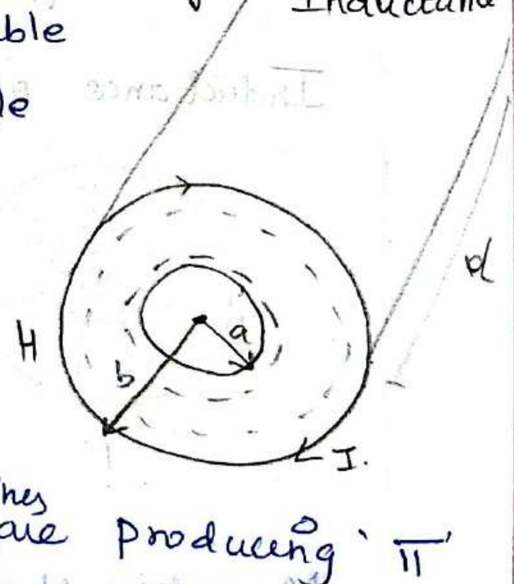
Inductance of co-axial cable / cyclindrical Inductance

$a \rightarrow$ inner radius of co-axial cable

$b \rightarrow$ outer radius of co-axial cable

d – co-axial cable

μ – permeability



$$L = \frac{\phi}{I} \rightarrow (1)$$

$$B = \mu H \rightarrow (2)$$

Let us assume magnetic flux lines are producing 'π' direction as per cyclindrical co-ordinate

System differential line element $dr, rd\phi, dz$

For infinite straight line $H = \frac{I}{2\pi h} \rightarrow (3)$

$$\vec{B} = \frac{\mu I}{2\pi h} \vec{a_\phi}$$

$$B = \frac{\Phi}{A} = \frac{d\phi}{dA} = \frac{d\phi}{ds}$$

$$d\phi = \vec{B} \, dA$$

$$\phi = \iint \vec{B} \, ds$$

$$= \iint \frac{\mu I}{2\pi h} \vec{a_\phi} \, (dr \cdot dz \, \vec{a_\phi})$$

$$= \int_0^d \int_a^b \frac{\mu I}{2\pi h} \, dr \, dz \quad [\because h = r]$$

$$= \frac{\mu I}{2\pi} \int_a^b \frac{1}{r} dr \int_0^d dz$$

$$= \frac{\mu I}{2\pi} [\log r]_a^b \, [z]_0^d$$

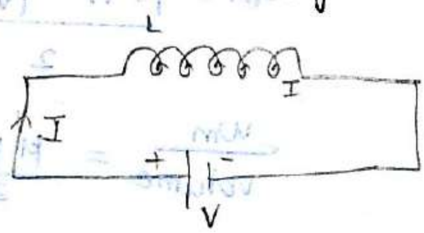
$$\phi = \frac{\mu I}{2\pi} \log b/a \; d$$

$$L = \frac{\phi}{I}$$

$$L = \frac{\frac{\mu I d}{2\pi} \log \frac{b}{a}}{I}$$

$$\boxed{L = \frac{\mu d}{2\pi} \log \frac{b}{a}}$$

Energy stored in a magnetic field.



Power / Energy $P = E = VI$

$$\frac{dw}{dt} = VI$$

$$V = L\frac{dI}{dt}$$

Sub ② in ①

$$\frac{dw}{dt} = \left(L\frac{dI}{dt}\right)I$$

$$dw = LI\,dI$$

$$w = \int LI\,dI$$

$$= L\int I\,dI$$

$$= L\frac{I^2}{2}$$

$$w_m = \frac{LI^2}{2}$$

Interm of H and B

Energy density $w_m = \frac{1}{2}LI^2$

Let us consider the solenoidal

$$w_m = \frac{1}{2}\left(\frac{N^2\mu A}{l}\right)I^2 \qquad L = \frac{N^2\mu A}{l}$$

multiply and ÷ by l.

$$w_m = \frac{1}{2}\left(\frac{N^2\mu A I^2}{l}\right)\left(\frac{l}{l}\right)$$

$$= \frac{N^2I^2}{l^2}\left(\frac{\mu A l}{2}\right)$$

H for solenoidal $H = \frac{NI}{l}$

$$w_m = \frac{\mu H^2 A l}{2}$$

$(A\,l) = $ volum

$$w_m = \frac{\mu H^2 (\text{volume})}{2}$$

$$\frac{w_m}{\text{volume}} = \frac{\mu H^2}{2}$$

$$\boxed{\text{Energy density} = \frac{1}{2}\mu H^2}$$

$$= \frac{1}{2} \mu \frac{B^2}{\mu^2}$$

$$\boxed{\text{Energy Density} = \frac{1}{2} \frac{B^2}{\mu}}$$

$B = \mu H$

$\mu = \frac{B}{H}$

$$= \cancel{\frac{1}{2}} \cancel{(\mu^2 \mu)}$$

$$= \frac{1}{2} \frac{B^2}{B/H}$$

$$\boxed{\text{Energy Density} = \frac{1}{2} BH}$$

A Ferrite Material as $\mu r = 10$ operate with sufficiently low flux Density $B = 0.02$ T. Find 'H'

$$B = \mu_0 \mu_r H$$

$$B = 4\pi \times 10^{-7} \times 10 \times H$$

$$H = \frac{B}{4\pi \times 10^{-7} \times 10}$$

$$= \frac{0.02}{4 \times \pi \times 10^{-7} \times 10}$$

$$= \frac{0.02}{1.25 \times 10^{-5}}$$

$$= \cancel{15.9 \times 10^{-12}} \ A/m$$

$H = 1.592$ K.A/m $\quad H = \cancel{185.92 P A/m}$

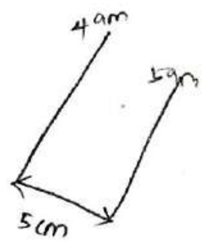
A long straight wire carries a current 5 Amps at which distance magnetic field 6 A/m

$$I = 5 \quad |H| = 6 \text{ A/m}$$

$$H = \frac{I}{2a} = \frac{5}{2a}$$

$$a = \frac{I}{2H} = \frac{5}{2 \times 6} = 0.416.$$

2 wires carrying current in same direction 4 Amps and 10 Amps are placed with their axis 5 cm apart free space permeability calculate the force between them in (N/m) $\rightarrow \frac{F}{l}$



$$\boxed{\frac{F}{l} = \frac{I_1 I_2 \mu_0}{2\pi d}}$$

$$= \frac{4 \times 10 \times \overset{2}{\cancel{4\pi}} \times 10^{-7}}{2\cancel{\pi} \times 5 \times 10^{-2}}$$

$$= \frac{8 \times 10 \times 10^{-7}}{5 \times 10^{-2}}$$

$$\frac{F}{l} = 1.6 \times 10^{-4} \text{ N/m}$$

A current of 3 amps flowings through inductor of 100 mH what is energy stored inductor

$$W_m = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \times 100 \times (3)^2$$

$$= \frac{1}{2} \times 100 \times 10^{-3} \times 9$$

$$= \frac{0.9}{2}$$

$$W_m = 0.45 \text{ J}$$

If point P $A_x = 4x + 3y + 2z$, $A_y = 5x + 6y + 3z$, $A_z = 2x + 3y + 5z$ Determine magnetic flux density and also state the nature of field.

$$B = \nabla \times A = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x+3y+2z & 5x+6y+3z & 2x+3y+5z \end{vmatrix}$$

$$= \vec{a_x}\left(\frac{\partial}{\partial y}(2x+3y+5z) - \frac{\partial}{\partial z}(5x+6y+3z)\right)$$

$$-\vec{a_y}\left(\frac{\partial}{\partial x}(2x+3y+5z) - \frac{\partial}{\partial z}(4x+3y+2z)\right)+$$

$$\vec{a_z}\left(\frac{\partial}{\partial x}(5x+6y+3z) - \frac{\partial}{\partial y}(3)\right)$$

$$= \vec{a_x}(3-3) - \vec{a_y}(2-2) + \vec{a_z}(5-3)$$

$$= 0 - \vec{a_y}(2-2) + 2\vec{a_z}$$

$$B = 2\vec{a_z} \neq 0$$

The field is irrotational

It is not conservative Vector field

In a perfect conducting surface in xy plane, magnetic field intensity $\vec{H} = 3\cos x\,\vec{a_x} + z\cos x\,\vec{a_y}$ $z>0$ and $z<0$ find current density of conducting surface

$$\vec{J} = \nabla \times \vec{H}$$

$$= \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3\cos x & z\cos x & 0 \end{vmatrix}$$

$$= \vec{a_x}(0 - \cancel{\sin x}\cos x) - \vec{a_y}(0-0) + \vec{a_z}$$

$$(z\sin x - 0)$$

$$\vec{J} = -\cos x\,\vec{a_x} - z\sin x\,\vec{a_z} \text{ A/m.}$$

If $\vec{A} = (3y-z)\vec{a_x} + 2xz\,\vec{a_y}$ w/m in a certain region of free space Determine $\vec{B}, \vec{H}, \vec{J}$

$$B = \nabla \times A$$

$$=$$

$$B = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3y-z) & 2xz & 0 \end{vmatrix}$$

$$= \vec{a_x}\left(0 - 2x\right) - \vec{a_y}\left(0 - (-1)\right) + \vec{a_z}\left(2z - 3\right)$$

$$\vec{B} = -2x\vec{a_x} - \vec{a_y} + (2z-3)\vec{a_z} \quad W/m^2$$

$$\vec{H} = \frac{B}{\mu_0}$$

$$= \frac{-2x\vec{a_x} - \vec{a_y} + (2z-3)\vec{a_z}}{4\pi \times 10^{7}}$$

$$\vec{H} = 795.7 \times 10^3 \left(-2x\vec{a_x} - \vec{a_y} + (2z-3)\vec{a_z}\right)$$

$$J \Rightarrow \nabla \times \vec{H}$$

$$795.7 \times 10^3 \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2x & -1 & 2z-3 \end{vmatrix}$$

$$= \vec{a_x}(0-0) - \vec{a_y}(0-0) + \vec{a_z}(0-0)$$

$$\vec{J} = 795.7 \times 10^3 (0)$$

$$\vec{J} = 0 \quad A/m^2$$

In a cylindrical co-ordinate system $\vec{B} = 2\pi r\,\vec{a_\phi}$ determine the magnetic flux passing through the Plane Surface in the range of $0.5 \le r \le 2.5$ $0 \le z \le 2$ $\quad \phi = ?$

$$B = \frac{\phi}{A} = \frac{\phi}{s} \quad \frac{d\phi}{ds}$$

$$d\phi = \vec{B} \cdot ds$$

$$\phi = \iint \vec{B}\, ds$$

$$= \iint \frac{2}{r} \vec{a_\phi} \, dr \, dz \, \vec{a_\phi}$$

$$= \int_0^2 \int_{0.5}^{2.5} \frac{2}{r} \, dr \cdot dz$$

$$= \int_0^2 2 \left[\log r \right]_{0.5}^{2.5} dz$$

$$= 2 \int_0^2 \left[\log(2.5) - \log(0.5) \right) dz$$

$$= 2 \left[0.395 + 0.301 \right] (2)$$

$$\boxed{= 4 [0.696]}$$

$$\phi = 2.784.$$

If $\vec{B} = 2.5 \sin\left(\frac{\pi x}{2}\right) e^{-2y} \vec{a_z}$ find the total Magnetic flux crossing the line where

$z = 0$, $y \geq 0$, $0 < x \leq 2m$

$\int_0^\infty \int_0^2$

$$d\phi = \vec{B} \cdot ds$$

$$\phi = \iint \vec{B} \cdot ds$$

$$= \int_0^\infty \int_0^2 2.5 \sin\left(\frac{\pi x}{2}\right) e^{-2y} \, dx \, dy$$

$$= \int_0^\infty 2.5 - \cos\left(\frac{\pi x/2}{\pi/2}\right)_0^{2.5m} \left[\frac{e^{-2y}}{-2}\right]_0^\infty$$

$$= 2.5 \left(\frac{2}{\pi}\right) \left[-\cos\left(\pi/2\right)(2m) + \cos 0\right]$$

$$\left[\frac{e^{-2(\infty)}}{-2} + \frac{e^0}{2}\right]$$

$$= 2.5 \left(\frac{2}{\pi}\right) \left[-\cos\pi + \cos 0\right] \left[0 + \tfrac{1}{2}\right]$$

$$= 2.5 \left(\frac{2}{\pi}\right) [2] \left[\tfrac{1}{2}\right]$$

$$= 2.5 \, \frac{2}{\pi}$$

$$\phi = 1.592 \text{ weber.}$$

The magnetic field strength 200 turns coil carrying a current of 2 amps. the length of Solenoid is 0.2m Find H

$$H = \frac{NI}{l}$$

$$= \frac{200 \times 2}{0.2}$$

$$\boxed{H = 2000\ A/m}$$

$\vec{B} = 0.05$ Telsa (T) and $\mu_r = 50$. Find H

$$H = \frac{B}{\mu_0 \mu_r}$$

$$= \frac{0.05}{4\pi \times 10^{-7} \times 50}$$

$$\boxed{H = 795.77\ A/m}$$

A circular coil of radius 2cm, $B = 10\ w/m^2$ In a plane of circular coil ~~deveto~~ ⊥r to the field deteimine the total flux around the coil.

$$\phi = \cancel{\frac{B}{A}} = \cancel{\frac{B}{\pi r^2}} = \cancel{\frac{10}{3.14 \times 4 \times 10^{-4}}}$$

$$B = \frac{\phi}{A}$$

$$\phi = BA$$

$$= 10 \times (2 \times 10)^2 \times 3.14$$

$$\boxed{\phi = 0.0125\ weber}$$

In a cylindrical co-ordinate system $50r^2 \vec{a_z}$ wb/m² is a magnetic potential. In a certain region of free space find H, B, I

$0 \leq r \leq 1$

$0 \leq \phi \leq 2\pi$

$$B = \nabla \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 50r^2 \end{vmatrix}$$

$$= \frac{1}{r} \begin{vmatrix} \vec{a_r} & r\vec{a_\phi} & \vec{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & 50r^2 \end{vmatrix}$$

$$= \frac{1}{r}\left[0 - r\vec{a_\phi}\,(100r) + 0\right]$$

$$= \frac{1}{r}\, r\,\vec{a_\phi}\,(-100r)$$

$$\boxed{\vec{B} = -100\, r\, \vec{a_\phi} \quad w/m^2}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{-100 r\, \vec{a_\phi}}{4\pi \times 10^{-7}}$$

$$\boxed{\vec{H} = -7.95\, r\, \vec{a_\phi} \times 10^{+7} \quad A/m}$$

$$J = \nabla \times \vec{H}$$

$$= \frac{1}{r} \begin{vmatrix} \vec{a_r} & r\vec{a_\phi} & \vec{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & -7.95 \times 10^{+7} r & 0 \end{vmatrix}$$

$$= \frac{1}{r}\left[0 - r\, a_\phi\,(0) + \vec{a_z}\,(-7.95 \times 10^{7})\right]$$

$$\boxed{J = \frac{-7.95 \times 10^{7}}{r}\, \vec{a_\phi} \quad A/m^2}$$

$$I = \iint J_z \, ds_z$$

$$= \int_0^{2\pi}\int_0^1 -\frac{7.95 \times 10^{7}}{r}\, \vec{a_z}\,(r\, dr\, d\phi\, \vec{a_z})$$

$$= -7.95 \times 10^{7} \int_0^1 dr \int_0^{2\pi} d\phi$$

$$= -7.95 \times 10^{7}\,[1]\,[2\pi]$$

$$J = -49.95 \times 10^{7}$$

$= 499 \times 10^{6}$

$= -500 \cdot M\ Amps$

$I \simeq -500\ M\ Amp$

Find $\vec{H}$ on origin due to current element $3\pi(\vec{a_x} + 2\vec{a_y} + 3\vec{a_z})$ at $(3,4,5)$

$$Idl = 3\pi(\vec{a_x} + 2\vec{a_y} + 3\vec{a_z})$$

$$dH = \frac{Idl \sin\theta}{4\pi r^2}$$

$$r = \sqrt{3^2 + 4^2 + 5^2}$$

$$= \sqrt{9 + 16 + 25}$$

$$= 7.071$$

$\theta = 90^\circ$

$\sin 90 = 1$

$$d\vec{H} = \frac{3\pi(\vec{a_x} + 2\vec{a_y} + 3\vec{a_z})}{4\pi(7.071)^2}$$

$$\vec{dH} = 0.015(\vec{a_x} + 2\vec{a_y} + 3\vec{a_z})$$

$$\boxed{\vec{dH} = 0.015\vec{a_x} + 0.03\ \vec{a_y} + 0.045\ \vec{a_z}\ \frac{A}{m}}$$

A circular loop located on $x^2 + y^2 = 25$, $z = 0$ carrys a direct current of 10 Amps along $\vec{a_\phi}$ determine $\vec{H}$ at $(0,0,2)$ and $(0,0,-2)$

$$x^2 + y^2 = a^2$$

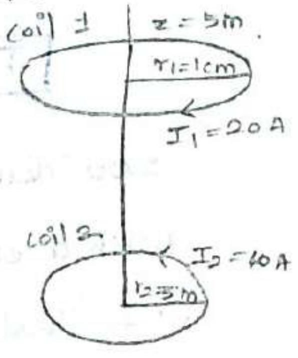
$$\boxed{a = 5}$$

$$I = 10A$$

$$H = \frac{I}{2a} = \frac{10}{2 \times 5} = 1\ A/m.$$

A 2 circular coil is located at $z=0$ and $z=5m$ centred about the z axis the first coil has radius of 1m and carrys a current of 20 amp, while

Second coil has radius of 0.5 m Carries the current of 40 amps calculate the H. at (0,0,3) m.



$$H = H_1 + H_2$$

$$\vec{H_1} = \frac{I_1 r^2}{2(r_1^2+z_1^2)^{3/2}} \vec{a_z}$$

$$= \frac{20(1)^2}{2(1^2+5^2)^{3/2}}$$

$$= \frac{20}{2(26)\sqrt{26}} \vec{a_z}$$

$$\vec{H_1} = 0.079 \ \vec{a_z}$$

26.5.14

$$H_2 = \frac{I_2 r_2^2}{2(r_2^2+z_2^2)} \vec{a_z}$$

$$= \frac{40 \times (0.5)^2}{2(0.5^2+0)^{3/2}}$$

$$= \frac{40 \ (0.25)}{2(0.5)^2}$$

$$H_2 = 40. \ \vec{a_z}$$

$$H = H_1 + H_2$$

$$= 40 + 0.079$$

$$H = 40.079 \ \vec{a_z}$$

What is maximum Torque on a square loop 1000 turns in a field of flux density 2 Tesla. the loop as 10 cm sides carrying the current of 3 Amps

$$T = N I A \vec{B}$$

$$N = 1000, \quad I = 3A, \quad B = 1T.$$

$$A = \text{side} \times \text{side}$$

$$= 10 \text{ cm} \times 10 \text{ cm}$$

$$= 10 \times 10^{-2} \times 10 \times 10^{-2}$$

$$A = 0.01$$

$T = NIA\vec{B}$

$= 1000 \times 3 \times 0.01 \times 1$

$\boxed{T = 30N}$

200 Turns Rectangular coil with a area 30cm × 15cm with a current of 5 amps is uniformly field of 0.2 Tesla find Torque and magnetic moment

$N = 200 \quad A = 30 \times 15\,cm \quad I = 5 \quad B = 0.2$

$= 0.045$

$T = NIAB$

$\boxed{T = 9N}$

$M = IA = 0.225$

$\boxed{M = 0.225}$

A conductor 6m long lies along z direction with current of 2 amps a_z find force experienced by the conductor If $\vec{B} = 0.08\,T\,\vec{a_x}$

$I = 2\vec{a_z} \quad l = 6m \quad B = 0.08\,\vec{a_x}$

$F = IlB \sin\theta$

$= 2 \times 6 \times 0.08$

$\boxed{F = 0.96\,\vec{a_y}\,N}$

Calculate the inductance of solenoid $N = 2000$ turns bounded uniformly over a length 0.5m on a cylindrical cube of diameter 4cm in free space

$L = \frac{N^2 \mu A}{l}$ $\quad \pi r^2$

$N = 2000 \quad l = 0.5m \quad \mu = \mu_0 = 4\pi \times 10^{-7}$

$A = \pi r^2$

$= \pi (2 \times 10^{-2})^2$

$= 1.256 \times 10^{-3}$

$$= \frac{63.13 \times 10^3}{0.5}$$

$$= 0.0126 \text{ H}$$

Magnetisation :-

$$M = \frac{\text{Magnetic moment}}{\text{Volume}}$$

$$= \frac{Q.d}{V}$$

$V = A \times d.$
$= m^2 \times m$

$$= \frac{Q.d}{m^2.m}$$

$$= \frac{Q\ d}{A.d}$$

$$\boxed{M = \frac{Q}{A}}$$

Magnetic Susceptbility (ψ_m)

$$\psi_m = \frac{M}{H}$$

$$M = \psi_m . H$$

$B = \mu H$ at free space

$$B = \mu_0 (H + M)$$

$$= \mu_0 (H + \psi_m H)$$

$$B = \mu_0 H (1 + \psi_m) \rightarrow ①$$

$$B = \mu_0 \mu_r H. \rightarrow ②$$

compare ① & ②

$$\mu_r = 1 + \psi_m$$

$$\psi_m = \mu_r - 1 \quad \boxed{\psi_m = \mu_r - 1}$$

$\mu_r \rightarrow$ Relative permeability

$\psi_m \rightarrow$ magnetic susceplity.

TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Fundamental Relations For electrostatic & Magnetostatic fields

- Electrostatic corresponds to stationary charges whereas magnetostatics corresponds to steady currents.

- Two important quantities that define static electric fields are electric field intensity E & electric flux density D.

$$D = \varepsilon E$$

- The fundamental differential equations that govern the static electric fields are

$\nabla \times E = 0$ (conservative property of electrostatics)

$\nabla \cdot D = \rho_v$ (Gauss law for electrostatics)

ρ_v - Volume charge density

The 2 important quantities that define static magnetic fields are magnetic field intensity H & magnetic flux density B.

$$B = \mu H$$

The fundamental differential equations that govern the static magnetic fields are

$\nabla \cdot B = 0$ (Gauss law for magnetostatics)

$\nabla \times H = J$ (Ampere's circuital law)

J - current Density

Fundamental relations	Electrostatics	Magnetostatics
Sources	stationary charges	steady current
Static field condition	$\frac{dQ}{dt} = 0$	$\frac{dI}{dt} = 0$
Field quantities	E & D	H & B
constitutive parameters	ε & σ	μ
constitutive relations	$D = \varepsilon E$	$B = \mu H$
Field equations in differential or point form	$\nabla \cdot D = \rho_v$ $\nabla \times E = 0$	$\nabla \cdot B = 0$ $\nabla \times H = J$
Field equation in integral form	$\int D \cdot ds = Q$ $\int E \cdot dl = 0$	$\int B \cdot ds = 0$ $\int H \cdot dl = I$
Force on charge Q	$F_e = QE$	$F_m = QVB$
Flux	$\psi = \int D \cdot ds$ $\psi = Q = CV$	$\phi = \int B \cdot ds$ $\phi = LI$
Potential	$E = -\nabla V$	$H = -\nabla V_m$
Energy density	$W_e = \frac{1}{2} \varepsilon E^2$	$W_m = \frac{1}{2} \mu H^2$

$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$ $\nabla^2 A = -\mu J$

Circuit elements R & C L

③

Faraday's law of Electromagnetic induction

- when the magnetic flux linking a circuit changes an emf is always induced in it. The ~~mg~~ magnitude of such an emf is proportional to the rate of change of flux linkages.

- when a conductor moves through a magnetic field by cutting its flux, an emf is induced in it.

III'y when the magnetic flux cuts a stationary conductor an emf is induced.

In either case the induced emf & the rate of change of the magnetic flux are related in differential form as

$$V = \frac{-d\phi}{dt} \rightarrow ①$$

V - Total EMForce in Volt ϕ - total magnetic flux Wb

t - time in second.

The Process of inducing an emf in a conductor in the presence of time varying magnetic field

- The -ve sign in eqn 1 indicates that the induced emf opposes the flux producing it. This is known as Lenz's law, which states that any induced emf will circulate a current in such a direction so as to oppose the cause producing it.

Equation ① is applicable to single-turn loop. For a multi-turn loop where all turns are associated with the same flux ϕ, Faraday's law may be expressed as

$$V = -N \frac{d\phi}{dt} \rightarrow ②$$

N - No of turns of the loop

If every turn is not associated with the same value of flux. Then the faraday's law may be expressed as

$$V = \frac{-d\lambda}{dt} \rightarrow ③$$

λ - total flux linkage in weber-turns. Hence for N turns the linkage is

$$\lambda = \phi_{m1} + \phi_{m2} + \dots \phi_{mN} \rightarrow ④$$

ϕ_{m1} - flux associated with the 1st turn

ϕ_{m2} - " " 2nd turn

The magnetic flux ϕ passing through a loop is defined as the surface integral of normal component of magnetic flux density B over the surface area of the loop as given by (5)

$$\phi = \iint B\,ds \rightarrow (6)$$

The induced emf can be defined in terms of electric field intensity E as

$$V = \int E \cdot dl \rightarrow (6)$$

l - closed path of integration. eqn can be expressed in terms of E & B as

$$V = \int E \cdot dl = \frac{-d}{dt}\iint B \cdot ds = -\iint \frac{dB}{dt} \cdot ds \rightarrow (7)$$

eqn (7) is known as the integral form of Faraday's law.

Transformer & motional Electromotive force

An emf is induced in a coil or conductor whenever there is a change in flux linkages. ∴ an emf can be produced in a closed conducting circuit by the following ways

The magnetic flux ϕ passing through a loop is defined as the surface integral of normal component of magnetic flux density B over the surface area of the loop as given by (5)

$$\phi = \iint B\,ds \longrightarrow (5)$$

The induced emf can be defined in terms of electric field intensity E as

$$V = \int E \cdot dl \longrightarrow (6)$$

l - closed path of integration. eqn can be expressed in terms of E & B as

$$V = \int E \cdot dl = \frac{-d}{dt} \iint B \cdot ds = -\iint \frac{dB}{dt} \cdot ds \longrightarrow (7)$$

eqn (7) is known as the integral form of Faraday's law.

Transformer & motional Electromotive force

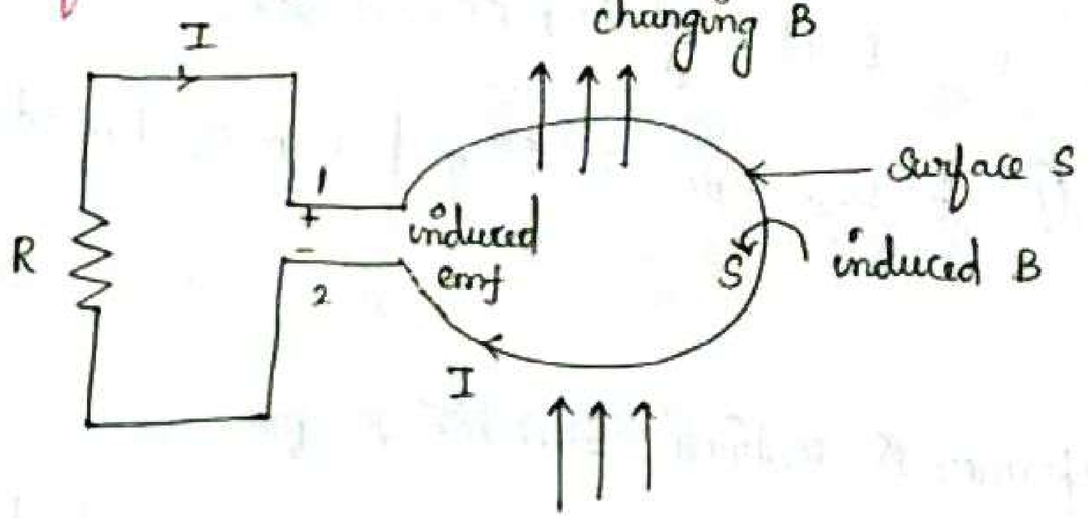
An emf is induced in a coil or conductor whenever there is a change in flux linkages. $\therefore$ an emf can be produced in a closed conducting circuit by the following ways

i) By placing a stationary conductor in a time varying magnetic field (the induced emf in this case is called transformer emf)

ii) By placing a moving conductor in a static magnetic field (the induced emf in this case is called motional emf)

iii) By placing a moving conductor in a time varying field.

Stationary conductor in time varying magnetic field (transformer emf)



- A stationary conductor carrying current I is placed in a time varying magnetic field of flux density B

- The induced current flows in such a way to satisfy Lenz's law so that a magnetic field is produced which opposes a change in value of B.

As per faraday's law

$$V = \oint_l E \cdot dl = -\int_s \frac{\partial B}{\partial t} ds \longrightarrow (1)$$

(1)

- The emf given by the above equation is termed as transformer emf which is due to the time varying current generating time varying magnetic field in a stationary loop. This effect is mainly due to transformer action.

Using stoke's theorem

$$\oint_s (\nabla \times E) ds = -\int_s \frac{\partial B}{\partial t} ds \longrightarrow (2)$$

comparing the surface integrals on both sides

$$\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow (3)$$

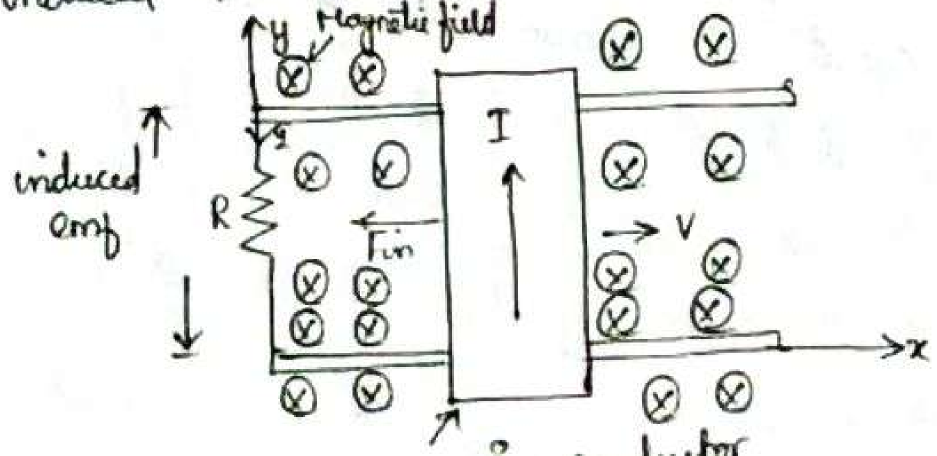
The above eqn is in a differential form which relates the field quantities at any point in space, whether or not a physical circuit exist at that point. This equation is known as the differential form of faraday's law which states that a time varying magnetic field induces an electric field E whose curl is equal to the -ve of the time derivation of B.

Eqn ③ is one of the Maxwell's equations for time varying field & it also shows that time (8)
varying electric field is not conservative in nature
i.e $\nabla \times E \neq 0$ since the work done in moving a
charge on a closed path in a time varying electric
field is due to energy from the time varying
magnetic field laws of energy conservation are thus
satisfied. Suppose if B is time indepent i.e $\frac{dB}{dt} = 0$
the equation ① & ③ reduced to the electrostatic
equations

$$\nabla \times E = 0 \quad \& \quad \oint_l E.dl = 0$$

Moving conductor in static magnetic field (Motional emf)

consider a moving conductor carrying current
I is placed in a static magnetic field. An emf is
induced in the loop.



- The force F_m on a charge Q moving with an uniform Velocity placed in a magnetic field of flux density B is given by

(9)

$$F = Q(V \times B) \longrightarrow ①$$

∴ the electric field intensity is given by

$$E = \frac{F}{Q} = V \times B \rightarrow ②$$

The field produced by the motion of the charged particle is known as motional electric field & its direction is normal to the plane containing V & B.

If we assume that a large no. of free electrons moving with an uniform Velocity V is present in a conducting loop, then the emf induced in the loop is given by

$$V = \oint_l E \cdot dl = \oint_l (V \times B) dl \longrightarrow ③$$

The eqn is termed as motional emf or flux cutting emf, as it is caused by motional effect. The motional emf is present in generators & motors.

Applying stoke's theorem

$$\int (\nabla \times E) \cdot ds = \nabla \times (V \times B) \cdot ds \rightarrow ④$$

Moving conductor in time varying magnetic field

If a moving conductor carrying current I is placed in a time varying magnetic field, then the induced emf is the sum of both transformer emf & motional emf.

$$V = \oint_l E \cdot dl = \text{Transformer emf} + \text{motional emf}$$

$$V = -\int_S \frac{\partial B}{\partial t} \cdot ds + \oint_l (v \times B) dl$$

Differential & integral form of Maxwell's equations

Maxwell's equation I

From Ampere's circuital law:-

It states that line integral of magnetic field intensity H on any closed path is equal to current enclosed by that path.

$$\oint H \cdot dl = I = \int_S J \cdot ds$$

current involves both conduction current & displacement current.

A current through resistive element is called condition current where as current through capacitive element is called displacement current

through a conductor of resistance R is

But

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} \qquad \sigma = \frac{1}{\rho}$$

$A \rightarrow$ Area of cross section

σ - conductivity

ρ - Resistivity

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$$I_c = \frac{V\sigma A}{l}$$

if E is electric field then $V = El$

$$I_c = \frac{El\sigma A}{l} = \sigma E A$$

$$\frac{I_c}{A} = \sigma E = J_c$$

current through a capacitor is

$$I_D = dQ/dt$$

$$Q = CV$$

$$I_D = C\frac{dV}{dt}$$

W.K.T

$$C = \varepsilon A/d$$

ε - Permittivity of medium

A - Area of $||^{el}$ plate capacitor

d - Distance b/w 2 plates

$$I_D = \frac{\varepsilon A}{d}\frac{dV}{dt}$$

$$V = Ed$$

$$I_D = \frac{\varepsilon A}{d} d \frac{dE}{dt} \qquad I_D = \varepsilon A\frac{dE}{dt}$$

$$\frac{I_D}{A} = \varepsilon \frac{\partial E}{\partial t} = \frac{dD}{dt}$$

$$J_D = \frac{\partial D}{\partial t}$$

$$\oint H \cdot dl = \iint (J_c + J_D)\, ds$$

$$\oint H \cdot dl = \iint \left(\sigma E + \frac{\partial D}{\partial t}\right) ds$$

$$\oint H \cdot dl = \iint \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) ds$$

$$\boxed{\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds} \longrightarrow (1)$$

eqn (1) is Maxwell's equations I in integral form from ampere's circuital law

By applying stoke's theorem

$$\oint H \cdot dl = \iint (\nabla \times H) \cdot ds \longrightarrow (2)$$

Comparing eqn (1) & (2)

$$\nabla \times H = J + \frac{\partial D}{\partial t} \longrightarrow (3)$$

eqn (3) is Maxwell's eqn I in point or differential form

Maxwell's equation II

Faraday's law: It states EM force induced in a circuit is equal to rate of ↓ of magnetic flux linkage in the circuit.

$$V = \frac{-d\phi}{dt}$$

$$V = \frac{-d}{dt}\left(\iint B.ds\right)$$

$$V = \oint E.dl$$

$$\oint E.dl = \frac{-d}{dt}\iint B.ds$$

$$\oint E.dl = -\iint \frac{\partial B}{\partial t}.ds$$

$$= -\mu \iint \frac{\partial H}{\partial t}.ds \longrightarrow (4)$$

eqn (4) is maxwell's eqn II in integral form. By applying stoke's theorem

$$\oint E.dl = \iint (\nabla \times E) ds \longrightarrow (5)$$

comparing eqn (4) & (5)

$$\iint (\nabla \times E).ds = -\mu \iint \frac{\partial H}{\partial t}.ds$$

$$\boxed{\nabla \times E = -\mu \frac{\partial H}{\partial t} = -\frac{\partial B}{\partial t}} \longrightarrow (6)$$

eqn (6) is maxwell's eqn II in point or differential

Maxwell's equation III

Electric Gauss law: It states that electric flux passing through any closed surface is equal to charge enclosed by that surface

$$\psi = Q$$

$$\iint D.ds = Q \text{ (or) } \iiint \rho_v \, dv = Q$$

$$\iint D.ds = \iiint \rho_v \, dv \rightarrow (7)$$

eqn (7) is maxwell's eqn III in integral form. By applying Divergence theorem

$$\iint D.ds = \iiint \nabla.D \, dv \rightarrow (8)$$

Comparing eqn (7) & (8)

$$\iiint \nabla.D \, dv = \iiint \rho_v \, dv$$

$$\boxed{\nabla.D = \rho_v = \rho} \rightarrow (9)$$

eqn (9) is maxwell's equation III in point or differential form

Maxwell's equation IV

Magnetic Gauss law: It states that total magnetic flux through any closed surface is equal to zero

$$\phi = 0$$

$$\iint B \cdot ds = 0 \rightarrow (10)$$

eqn (10) is maxwell's equations IV in integral form.

By applying Divergence theorem

$$\iint B \cdot ds = \iiint \nabla \cdot B \, dv \rightarrow (11)$$

comparing eqn (10) & (11)

$$\iiint \nabla B \, dv = 0$$

$$\boxed{\nabla \cdot B = 0} \rightarrow (12)$$

eqn (12) is Maxwell's equation IV in point or differential form

*) If $\vec{D} = 10x\,\vec{a_x} - 4y\,\vec{a_y} + kz\,\vec{a_z}$ $\mu C/m^2$ & $\vec{B} = 2\vec{a_y}$ mT Find the Value of k to satisfy the Maxwell's equations for region $\sigma = 0$, $\rho_v = 0$

Sol $\vec{D} = 10x\,\vec{a_x} - 4y\,\vec{a_y} + kz\,\vec{a_z}$ $\mu C/m^2$

$\vec{B} = 2\vec{a_y}$ mT, $\sigma = 0$, $\rho_v = 0$

$$\left(\frac{d}{dx}\vec{a_x}+\frac{d}{dy}\vec{a_y}+\frac{d}{dz}\vec{a_z}\right)\cdot\left(10x\vec{a_x}-4y\vec{a_y}+Kz\vec{a_z}\right)=$$

16

$$\frac{d}{dx}(10x)-\frac{d}{dy}(4y)+\frac{d}{dz}(Kz)=0$$

$$10-4+K=0$$

$$K=-6\mu C/m^2$$

*) if the magnetic field $\vec{H}=(3x\cos\beta+6y\sin\alpha)\vec{a_z}$. Find current density $\vec{J}$ if fields are invarient with time.

$$\vec{H}=(3x\cos\beta+6y\sin\alpha)\vec{a_z}$$

From Maxwell's 2nd eqn

$$\nabla\times H=\vec{J}+\frac{d\vec{D}}{dt}$$

The fields are invarient with time so $\frac{dD}{dt}=0$

$$\nabla\times H=\vec{J}$$

$$\vec{J}=\begin{vmatrix}\vec{a_x} & \vec{a_y} & \vec{a_z}\\ d/dx & d/dy & d/dz\\ 0 & 0 & 3x\cos\beta+6y\sin\alpha\end{vmatrix}$$

$$=\vec{a_x}\left[\frac{d}{dy}(3x\cos\beta+6y\sin\alpha)\right]-\vec{a_y}\left[\frac{d}{dx}(3x\cos\beta+6y\sin\alpha)\right]+\vec{a_z}(0\cdot0)$$

$$\vec{J}=6\sin\alpha\,\vec{a_x}-3\cos\beta\,\vec{a_y}\ A/m^2$$

*) For 1 Ampere conductor current in copper wire, find displacement current at 100 MHZ. Assume for copper $\sigma = 5.8 \times 10^{7}$ ℧/m

(17)

<u>Sol</u> $I_c = 1$ Ampere

$f = 100$ MHZ

$\sigma = 5.8 \times 10^{7}$ ℧/m

Conduction current $I_c = J_c A = 1$ Ampere

$$J_c = I/A$$

$$\sigma E = 1/A$$

$$E = \frac{1}{\sigma A} = \frac{1}{5.8 \times 10^{7} \times A}$$

$$E = \frac{0.172 \times 10^{-7}}{A} \text{ V/m}$$

Displacement current $I_D = \omega \epsilon E A$

$$= \omega \epsilon_0 \epsilon_r E A$$

$$= 2\pi f \epsilon_0 \epsilon_r E A$$

$$= 2\pi \times 100 \times 10^{6} \times 8.854 \times 10^{-12} \times 1 \times \frac{0.172 \times 10^{-7}}{A} \times A$$

$$I_D = 9.556 \times 10^{-11} \text{ Ampere}$$

Potential functions

For static EM field, the electric scalar potential (18)

is given by

$$V = \int_V \frac{\rho_v \, dv}{4\pi \varepsilon R} \rightarrow (1)$$

& magnetic vector potential is

$$A = \int_V \frac{\mu J dv}{4\pi R} \rightarrow (2)$$

W.K.T $\quad B = \nabla \times A \rightarrow (3)$

$$\nabla \times E = -\frac{\partial B}{\partial t} \rightarrow (4)$$

Sub eqn (3) in eqn (4)

$$\nabla \times E = -\frac{\partial}{\partial t}(\nabla \times A) \rightarrow (5)$$

$$\nabla \times \left(E + \frac{\partial A}{\partial t}\right) = 0 \rightarrow (6)$$

Since curl of gradient of a scalar field is identically zero, the solution to eqn (6) is

$$E + \frac{\partial A}{\partial t} = -\nabla V \rightarrow (7)$$

or

$$E = -\nabla V - \frac{\partial A}{\partial t} \rightarrow (8)$$

From eqn (3) & (8) we can determine the vector fields provided the potentials A & V are known.

W.K.T $\nabla \cdot D = \rho_v$ is valid for time varying

conditions. By taking the divergence of eqn (8)

(19)

$$\nabla \cdot E = \nabla \cdot \left(-\nabla V - \frac{\partial A}{\partial t}\right)$$

$$\nabla \cdot E = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot A) \longrightarrow (9)$$

& making use of $D = \varepsilon E$ & $\nabla \cdot D = \rho_v$ eqn (9)

becomes

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot A)$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\rho_v/\varepsilon \longrightarrow (10)$$

Taking the curl of equation (3)

$$\nabla \times \nabla \times A = \nabla \times B \longrightarrow (11)$$

W.K.T $B = \mu H \longrightarrow (12)$

Sub eqn (12) in eqn (11)

$$\nabla \times \nabla \times A = \mu(\nabla \times H)$$

$$= \mu\left(J + \frac{\partial D}{\partial t}\right) = \mu\left[J + \varepsilon \frac{\partial E}{\partial t}\right]$$

$$\nabla \times \nabla \times A = \mu J + \mu \varepsilon \frac{\partial E}{\partial t} \longrightarrow (13)$$

Sub eqn (8) in eqn (13)

$$\nabla \times \nabla \times A = \mu J + \mu \varepsilon \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial A}{\partial t}\right)$$

$$\mu J - \mu \varepsilon \nabla\left(\frac{\partial V}{\partial t}\right) - \mu \varepsilon \frac{\partial^2 A}{\partial t^2} \longrightarrow (14)$$

Where $D = \varepsilon E$ & $B = \mu H$ have been assumed

By applying Vector identity

$$\nabla \times \nabla \times A = \nabla(\nabla . A) - \nabla^2 A \quad \text{to eqn (14)} \longrightarrow (15)$$

$$\nabla^2 A - \nabla(\nabla . A) = -\mu J + \mu\varepsilon \nabla\left(\frac{\partial V}{\partial t}\right) + \mu\varepsilon \frac{\partial^2 A}{\partial t^2} \longrightarrow (16)$$

A vector field is uniquely defined when its curl & divergence are specified

$$\nabla . A = -\mu\varepsilon \frac{\partial V}{\partial t} \longrightarrow (17)$$

The above equation relates A & V & it is called Lorentz condition for potentials. Sub eqn (17) in eqn

$$\nabla^2 V + \frac{\partial}{\partial t}\left(-\mu\varepsilon \frac{\partial V}{\partial t}\right) = \frac{-\rho_v}{\varepsilon}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t} = -\rho_v/\varepsilon \longrightarrow (18)$$

Sub eqn (17) in eqn (16)

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J \longrightarrow (19)$$

eqn (18) & (19) are called wave equation for pote[ntials]

It can be shown that the solution to equation (1[8]) (19)

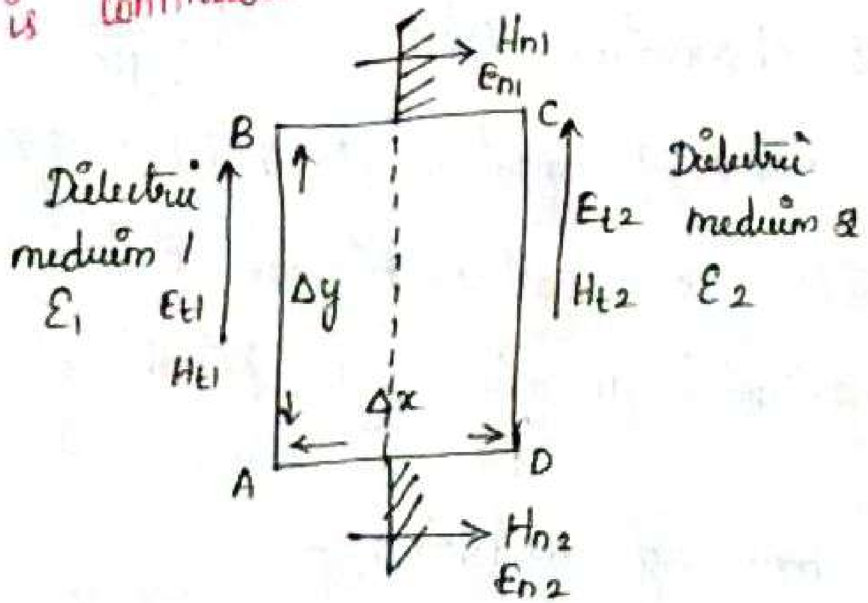
$$V = \int_v \frac{[\rho_v] dv}{4\pi\varepsilon R} \longrightarrow (20)$$

&

$$V = \int \frac{[\rho_v] dv}{4\pi\varepsilon R} \longrightarrow (21)$$

3. The normal component of electric flux density D continuous if there is no surface charge density. D is discontinuous by an amount equal to surface charge density.

4. The normal component of magnetic flux density B is continuous at the surface of discontinuity.



consider a rectangle of length Δy & width Δx at the boundary of 2 dielectric media.

$$V = \oint E \cdot dl = 0$$

Apply this to the rectangular path ABCD in which AB just inside the medium 2

$$\oint E \cdot dl = E_{t1}\Delta y + E_{n1}\Delta x - E_{t2}\Delta y - E_{n2}\Delta x$$

E_{t1} & E_{t2} are tangential component of E along the path AB & CD

E_{n1} & E_{n2} are normal component of E along the path

The side AB & CD are brought closer together the length BC & AD approaches zero. $\Delta x \to 0$

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$$E_{t1}\Delta y - E_{t2}\Delta y = \oint E \cdot dl = 0$$

$$E_{t1} = E_{t2}$$

The integral form of 1st maxwell's equation is

$$\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) \cdot ds$$

Apply this to the rectangular path ABCD

$$H_{t1}\Delta y + H_{n1}\Delta x - H_{t2}\Delta y - H_{n2}\Delta x = \iint \left(J + \frac{\partial D}{\partial t}\right) \cdot \Delta x \Delta y$$

H_{t1} & H_{t2} are tangential component of H along path AB & CD

H_{n1} & H_{n2} are normal component of H along the path BC & AD.

As $\Delta x \to 0$ then $H_{t1}\Delta y - H_{t2}\Delta y = 0$

$$H_{t1} = H_{t2}$$

For a perfect conductor a HF current will flow in a thin sheet near the surface. In a current sheet a linear current density J_l flows in a sheet of depth Δx.

$$\lim_{\Delta x \to 0} J \cdot \Delta x = J_l$$

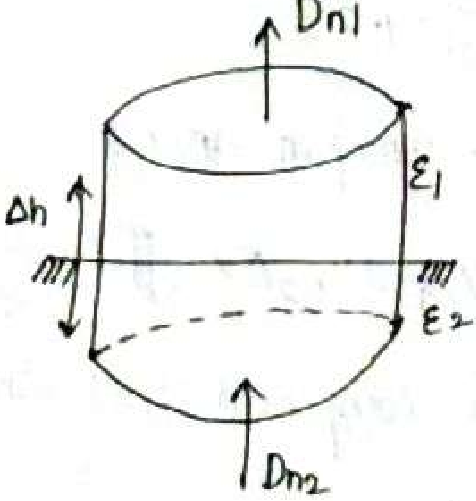
if the Maxwell's 1st equation is applied to the rectangle

$$H_{t1}\Delta y + H_{n1}\Delta x - H_{t2}\Delta y - H_{n2}\Delta x = J\Delta x \Delta y + \frac{\partial D}{\partial x}\Delta x \Delta y$$

$$H_{t1}\,\Delta y - H_{t2}\,\Delta y = J_l\,\Delta y$$

$$H_{t1} - H_{t2} = J_l$$

The tangential component of H is discontinuous by an amount of linear current density at the surface of perfect conductor.



Consider a pill box at boundary of 2 dielectric dielectric constant ε_1 & ε_2

The integral form of Maxwell's 3rd equation

$$\oint\!\!\oint_S D.ds = \iiint_V \rho\, dv$$

Assume there are no free charges on boundary.

Apply Gauss law to the pill box at the bounda

$$\int_S (D_{n1}ds - D_{n2}ds) = 0$$

D_{n1} - normal component of electric flux density in medium 1

Normal component of electric flux density in

$D_{n1}\,ds - D_{n2}\,ds = 0$ 95

$$D_{n1} = D_{n2}$$

The normal component of D is continuous if there is no surface charge density

If the charges are enclosed by pill box $\Delta h \to 0$

$$\int D \cdot ds = Q$$

$$D_{n1}\,ds - D_{n2}\,ds = Q$$

$$D_{n1} - D_{n2} = \frac{Q}{ds} = \rho_s$$

$$D_{n1} - D_{n2} = \rho_s$$

The normal component of D is discontinuous across the boundary by the amount of surface charge density. The integral form of Maxwell's 4^{th} eqn is

$$\iint B \cdot ds = 0$$

Apply to the pill box at the boundary

$$B_{n1}\,ds - B_{n2}\,ds = 0$$

$$B_{n1} = B_{n2}$$

The normal component of magnetic flux density B is continuous across the boundary.

Electromagnetic wave equation

Wave equation for conducting medium

The maxwell's equation from Faraday's law in point form is given by

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (1)$$

Taking curl on both sides

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}(\nabla \times H) \rightarrow (2)$$

Maxwell's eqn from ampere's law in point form is given by

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \frac{\partial E}{\partial t} \rightarrow (3)$$

Sub eqn (3) in eqn (2)

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$= -\mu \sigma \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (4)$$

But from vector identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E \rightarrow (5)$$

$$\nabla \cdot E = \frac{\nabla \cdot D}{\varepsilon}$$

Since there is no net charge within the conductor, the charge density $\rho = 0$

$$\nabla \cdot D = 0$$

eqn (5) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \rightarrow (6)$$

Comparing eqn (4) & (6)

$$-\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (7)$$

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \rightarrow (8)$$

This is the wave eqn in terms of electric field E.

The wave equation in terms of magnetic field H is obtained in a similar manner as follows.

The Maxwell's eqn from Ampere's law in point form is given by

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \frac{\partial E}{\partial t} \rightarrow (9)$$

Taking curl on both sides

$$\nabla \times \nabla \times H = \sigma \nabla \times E + \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \rightarrow (10)$$

But from Faraday's law

$$\nabla \times E = \frac{-\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (11)$$

Sub eqn (11) in eqn (10)

$$\ldots \mu\sigma \frac{\partial H}{\partial t} \ldots \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \rightarrow (12)$$

Maxwell's equation from Ampere's law in point form is given by

$$\nabla \times H = J + \frac{dD}{dt} = \sigma E + \varepsilon \frac{dE}{dt} \rightarrow (3)$$

Sub eqn (3) in eqn (2)

$$\nabla \times \nabla \times E = -\mu \frac{d}{dt}\left(\sigma E + \varepsilon \frac{dE}{dt}\right)$$

$$= -\mu\sigma \frac{dE}{dt} - \mu\varepsilon \frac{d^2E}{dt^2} \rightarrow (4)$$

But from vector identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E \rightarrow (5)$$

$$\nabla \cdot E = \frac{\nabla \cdot D}{\varepsilon}$$

Since there is no net charge within the conductor, the charge density $\rho = 0$

$$\nabla \cdot D = 0 \qquad \nabla \cdot E = 0$$

eqn (5) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \rightarrow (6)$$

compare eqn (4) & (6)

$$-\nabla^2 E = -\mu\sigma \frac{dE}{dt} - \mu\varepsilon \frac{d^2E}{dt^2}$$

$$\nabla^2 E = \mu\sigma \frac{dE}{dt} + \mu\varepsilon \frac{d^2E}{dt^2} \rightarrow (7)$$

$$\nabla^2 E - \mu\sigma \frac{dE}{dt} - \mu\varepsilon \frac{d^2E}{dt^2} = 0 \rightarrow (8)$$

This is the wave eqn in terms of electric field E.

The Maxwell's eqn from Ampere's law in point form

given by

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \frac{\partial E}{\partial t} \rightarrow (9)$$

Taking curl on both sides

$$\nabla \times \nabla \times H = \sigma \nabla \times E + \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \rightarrow (10)$$

But from Faraday's law

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (11)$$

Sub eqn (11) in eqn (10)

$$\nabla \times \nabla \times H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \rightarrow (12)$$

From vector identity

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H \rightarrow (13)$$

$$\nabla \cdot B = \mu \nabla H = 0$$

eqn (13) becomes

$$\nabla \times \nabla \times H = -\nabla^2 H \rightarrow (14)$$

Comparing eqn (12) & (14)

$$-\nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0}$$

... is the wave equation in terms of magnetic

From Vector identity

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H \rightarrow (13)$$

$$\nabla \cdot B = \mu \nabla \cdot H = 0$$

eqn (13) becomes

$$\nabla \times \nabla \times H = -\nabla^2 H \rightarrow (14)$$

comparing eqn (12) & (14)

$$-\nabla^2 H = -\mu\sigma \frac{dH}{dt} - \mu\varepsilon \frac{d^2 H}{dt^2}$$

$$\nabla^2 H = \mu\sigma \frac{dH}{dt} + \mu\varepsilon \frac{d^2 H}{dt^2}$$

$$\boxed{\nabla^2 H - \mu\sigma \frac{dH}{dt} - \mu\varepsilon \frac{d^2 H}{dt^2} = 0}$$

This is the wave eqn in terms of magnetic field H.

Wave equation for free space

For free space the conductivity of the me[dium] is zero (i.e $\sigma = 0$) & there is no charge contain[ed] in it (i.e $\rho = 0$)

The Maxwell's equation from Faraday's law for f[ree] space in point form is

$$\nabla \times E = -\frac{dB}{dt} = -\mu \frac{dH}{dt} \rightarrow (1)$$

Taking curl on both sides

$$\rightarrow (2)$$

Wave equation for free space

$\sigma = 0$ & $\rho = 0$

The Maxwell's eqn from faraday's law for free space in point form is

$$\nabla \times E = \frac{-\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (1)$$

Taking curl On both sides

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t} (\nabla \times H) \rightarrow (2)$$

The Maxwell's equation from Ampere's law for free Space in point form is

$$\nabla \times H = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \rightarrow (3)$$

Sub eqn (3) in eqn (2)

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial E}{\partial t} \right)$$

$$= -\mu \varepsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (4)$$

From Vector identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\nabla \cdot E = \frac{\nabla \cdot D}{\varepsilon} = \frac{\rho}{\varepsilon} = 0$$

$$\nabla \times \nabla \times E = -\nabla^2 E \rightarrow (5)$$

Compare eqn (4) & eqn (5)

$$\nabla^2 E = \mu \varepsilon \frac{d^2 E}{dt^2}$$

$$\nabla^2 E - \mu \varepsilon \frac{d^2 E}{dt^2} = 0 \rightarrow (6)$$

This is the wave equation for free space interms of electric field.

The wave equation for free space interms of magnetic field H is obtained in a similar manner follows.

The Maxwell's eqn from Ampere's law for free space in point form is given by

$$\nabla \times H = \varepsilon \frac{dE}{dt} \rightarrow (7)$$

Taking curl on both sides

$$\nabla \times \nabla \times H = \varepsilon \frac{d}{dt} (\nabla \times E) \rightarrow (8)$$

The Maxwell's eqn from faraday's law

$$\nabla \times E = -\mu \frac{dH}{dt} \rightarrow (9)$$

Sub eqn (9) in eqn (8)

$$\nabla \times \nabla \times H = -\mu \varepsilon \frac{d}{dt} \left(\frac{dH}{dt} \right)$$

$$\nabla \times \nabla \times H = -\mu \varepsilon \frac{d^2 H}{dt^2} \rightarrow (10)$$

From Vector identity

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H$$

$$\nabla \cdot H = \frac{1}{\mu} \nabla \cdot B = 0$$

$$\nabla \times \nabla \times H = -\nabla^2 H \longrightarrow (11)$$

Compare eqn (10) & (11)

$$-\nabla^2 H = -\mu \varepsilon \frac{d^2 H}{dt^2}$$

$$\nabla^2 H = \mu \varepsilon \, d^2 H / dt^2$$

$$\nabla^2 H - \mu \varepsilon \frac{d^2 H}{dt^2} = 0 \longrightarrow (12)$$

This is the wave eqn for free space interm of H

For free space $\mu_r = 1$ & $\varepsilon_r = 1$

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{d^2 H}{dt^2} = 0$$

or

$$\boxed{\nabla^2 H = \mu_0 \varepsilon_0 \frac{d^2 H}{dt^2}}$$

Solution of wave equation

considering a plane wave propagating in x direction.

The wave eqn for free space is

$$\frac{d^2 E}{dx^2} = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

The general solution of this differential equation is of the form

$$E = f_1(x - V_0 t) + f_2(x + V_0 t)$$

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \text{Velocity of propagation of free space.}$$

f_1 & f_2 are any function of $(x - v_0 t)$ & $(x + v_0 t)$ respectively.

The solution of wave equation consist of 2 ... One travelling in +ve direction & other travelling in -ve direction. consider the wave travel in +ve direction alone.

$$f_2(x + V_0 t) = 0$$

The general solution of wave equation becomes

$$E = f_1(x - V_0 t) = f(x - V_0 t)$$

$$\nabla \times E = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \vec{a_x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \vec{a_y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \vec{a_z}\left(\cdots - \frac{\partial E_x}{\partial y}\right)$$

Since the wave travelling in x direction, E & H are independent of y & z.

$$E_x = H_x = 0 \quad \& \quad \frac{\partial E}{\partial y} = \frac{\partial E}{\partial z} = 0$$

$$\nabla \times E = \frac{-\partial E_z}{\partial x}\vec{a_y} + \frac{\partial E_y}{\partial x}\vec{a_z}$$

Similarly

$$\nabla \times H = \frac{-\partial H_z}{\partial x} \vec{a_y} + \frac{\partial H_y}{\partial x} \vec{a_z}$$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t}$$

$$-\frac{\partial H_z}{\partial x} \vec{a_y} + \frac{\partial H_y}{\partial x} \vec{a_z} = \varepsilon \left[\frac{\partial E_y}{\partial t} \vec{a_y} + \frac{\partial E_z}{\partial t} \vec{a_z} \right]$$

Equating $\vec{a_y}$ & $\vec{a_z}$ terms

$$-\frac{\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_y}{\partial x} = \varepsilon \frac{\partial E_z}{\partial t}$$

From Maxwell's eqn for free space

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times E = -\frac{\partial E_z}{\partial x} \vec{a_y} + \frac{\partial E_y}{\partial x} \vec{a_z}$$

$$-\frac{\partial E_z}{\partial x} \vec{a_y} + \frac{\partial E_y}{\partial x} \vec{a_z} = -\mu \left[\frac{\partial H_y}{\partial t} \vec{a_y} + \frac{\partial H_z}{\partial t} \vec{a_z} \right]$$

Equating $\vec{a_y}$ & $\vec{a_z}$ terms

$$\frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}$$

Let the solution of this eqn is given by

$$E_y = f(x - V_0 t)$$

$$\frac{dE_y}{dt} = \frac{df}{d(x - V_0 t)} \cdot \frac{d(x - V_0 t)}{dt}$$

$$= f'(x - V_0 t)(-V_0)$$

$$f'(x - V_0 t) = f'$$

$$\frac{dE_y}{dt} = -V_0 f'$$

$$-\frac{dH_z}{dx} = \varepsilon \frac{dE_y}{dt}$$

$$\frac{dH_z}{dx} = -\varepsilon(-V_0 f') = \varepsilon V_0 f' = \frac{1}{\sqrt{\mu\varepsilon}} \varepsilon f'$$

$$= \sqrt{\frac{\varepsilon}{\mu}} f'$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} \int f' \, dx$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} f = \sqrt{\frac{\varepsilon}{\mu}} E_y$$

$$\frac{E_y}{H_z} = \sqrt{\mu/\varepsilon}$$

$$\frac{E_z}{H_y} = -\sqrt{\frac{\mu}{\varepsilon}}$$

if E is the total electric field

$$E = \sqrt{E_y^2 + E_z^2}$$

& H is the total magnetic field

$$H = \sqrt{H_y^2 + H_z^2}$$

$$E/H = \sqrt{\mu/\varepsilon}$$

Characteristic impedance of medium

$$\eta = \frac{E}{H} = \sqrt{\mu/\varepsilon}$$

free space $\mu_r = \varepsilon_r = 1$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}/1}{36\pi \times 10^{9}}}$$

$$\eta_0 = 120\pi \text{ or } 377\,\Omega$$

Uniform plane wave

If the phase of wave is same for all points on a plane surface it is called as plane wave. If the amplitude is also constant in a plane wave it is called as uniform plane wave.

The properties of uniform plane wave are given as follows

* At every point in E & H are $\perp^r$ to each other & to the direction of travel

* The fields are very harmonically with time at the same frequency, everywhere in space

* Each field has same direction, magnitude & Phase at every point in any plane $\perp^r$ to the direction of wave travel

Poyntings theorem

- EMW an energy can be transported from transmitter to receiver. The energy stored in an electric & magnetic field is transmitted at a certain rate of energy flow which can be calculated with help of Poynting theorem

E is electric field expressed in V/m. H is magnetic field expressed in A/m.

- If we take the product of 2 fields it gives a new quantity which is expressed as W/unit area is called power density. As E & H both are vectors, to get power density we carry out either dot or cross product. The result of dot product is always

Time Harmonic fields

A time harmonic field is one that varies periodically or sinusoidally with time. Sinusoids are easily expressed in phasors.

A phasor is a complex number that contains amplitude & phase of sinusoidal oscillation. As a complex number, a phasor z can be represented as

$$z = x + jy = r\angle\phi \rightarrow ①$$

or

$$z = re^{j\phi} = r(\cos\phi + j\sin\phi) \rightarrow ②$$

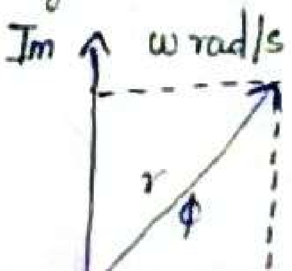
where $j = \sqrt{-1}$, x is real part of z, y is imaginary part of z, r is the magnitude of z, given by

$$r = |z| = \sqrt{x^2 + y^2} \rightarrow ③$$

& ϕ is phase of z given by

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow ④$$

The phasor z can be represented in rectangular form or in polar form.



The phasor z can be represented in rectangular former in polar form. The 2 forms of representing z (50)

Addition & subtraction of phasors are performed in in rectangular form.

$\times$ & $\div$ are performed in polar form

Given complex numbers

$z = x + jy = r\angle\phi$, $z_1 = x_1 + jy_1 = r_1\angle\phi_1$

The following properties should be noted

Addition : $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \rightarrow (5)$

Subtraction : $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \rightarrow (6)$

Multiplication : $z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2 \rightarrow (7)$

Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\angle\phi_1 - \phi_2 \rightarrow (8)$

Square root $\sqrt{z} = \sqrt{r}\angle\phi/2 \rightarrow (9)$

complex conjugate $z^* = x - jy = r\angle-\phi = re^{-j\phi} \rightarrow (10)$

To introduce time element let $\phi = \omega t + \theta \rightarrow (11)$

θ is a function of time or space coordinates or a constant. The real & imaginary part of $re^{j\phi} = re^{j\theta}e^{j\omega t}$ are given by

$R_e(re^{j\phi}) = r\cos(\omega t + \theta) \rightarrow (12)$

In general a phasor could be a scalar or vector. If a vector $A(x,y,z,t)$ is a time harmonic field, the phasor form of A is $A_s(x,y,z)$ 51

$$A = Re(A_s e^{j\omega t}) \rightarrow (14)$$

For example if $A = A_0 \cos(\omega t - \beta x) a_y$ we can write A as

$$A = Re(A_0 e^{-j\beta x} a_y e^{j\omega t}) \rightarrow (15)$$

comparing eqn (14) & (15) the phasor form of A is

$$A_s = A_0 e^{-j\beta x} a_y$$

From eqn (14)

$$\frac{dA}{dt} = \frac{d}{dt} Re(A_s e^{j\omega t})$$

$$= Re(j\omega A_s e^{j\omega t}) \rightarrow (16)$$

showing that taking the time derivative of the instantaneous quantity is equivalent to x its phasor form by $j\omega$. That is

$$\frac{dA}{dt} \rightarrow j\omega A_s$$

$||^{ly}$

$$\int A\, dt \rightarrow A_s / j\omega$$

The phasor concept is applied to time varying EMF.

In phasor form Maxwell's for time harmonic EM fields in a linear, isotropic & homogeneous medium.

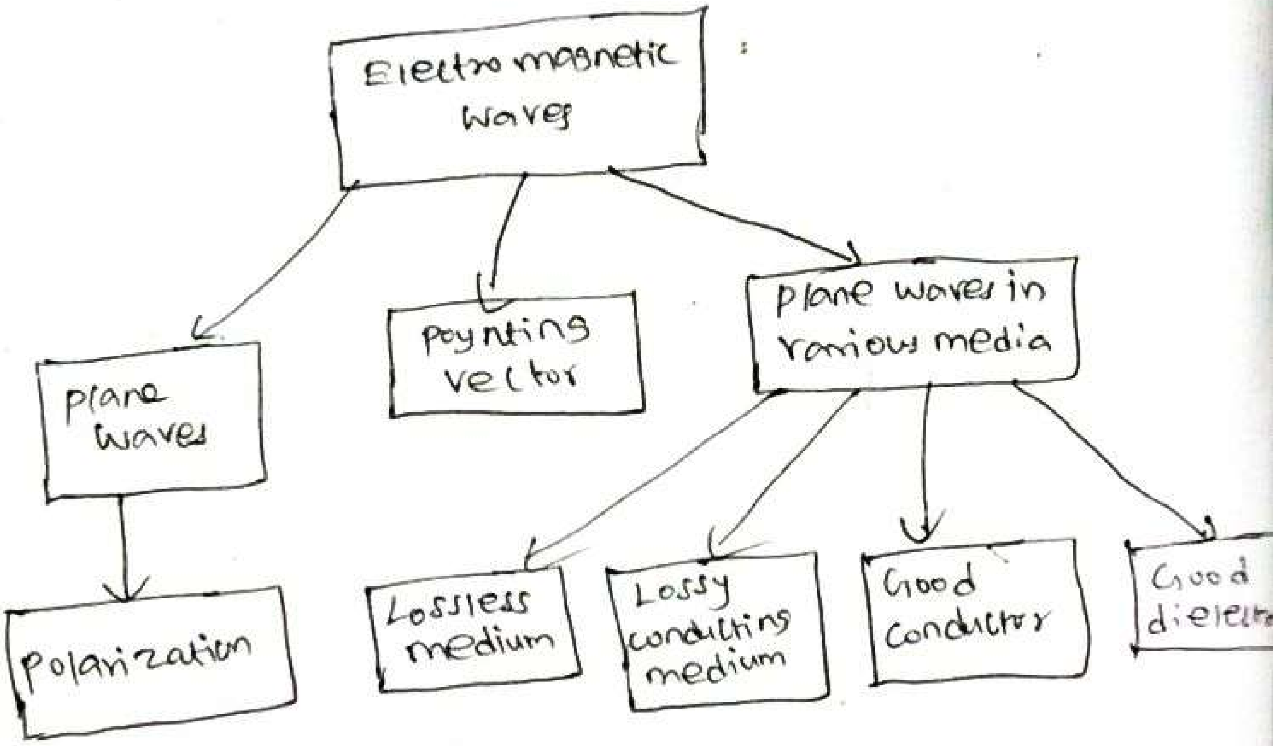


Fig. plane waves

UNIT-V PLANE ELECTROMAGNETIC WAVES

UNIT-V PLANE ELECTROMAGNETIC WAVES

Introduction:

In free space the source-free wave equation. for E is

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \rightarrow (1)$$

where, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \simeq 3\times10^8 \,(m/s) = 300\,(Mm/s) \rightarrow (2)$

It is the velocity of wave propagation in free space. The solutions of eqn (1) represent waves. The study of the behaviour of waves that have a one-dimensional spatial dependence (plane waves) is the main concern of this unit

The propagation of time-harmonic plane wave fields in an unbounded homogeneous medium. medium parameters such as intrinsic impedance, attenuation constant and phase constant.

Skin depth:

✓ The depth of wave penetration into a good conductor. Electromagnetic waves carry with them electromagnetic power.

Uniform plane wave:

It is a particular solution of maxwell's equations with E assuming the same direction, same magnitude and same phase in infinite planes perpendicular to the direction of prop

It is does not exist in practice because a source infinite in extent would be required to creat it, and practical wave sources are always finite in extent.

The characteristics of uniform plane waves are particularly simple, and their study is of fundamental theoretical, as well as practical, importance

plane waves in lossless media:-

The source free wave equation for free space becomes a homogeneous vector Helmholtz's equation.

$$\boxed{\nabla^2 E + K_0^2 E = 0} \longrightarrow (3)$$

where K_0 is the free space wavenumber

$$K_0 = \omega\sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{c} \quad (\text{rad/m}) \rightarrow (4)$$

In Cartesian Co-ordinates, equ (3) is equivalent to three scalar Helmholtz's equations, one each for the components E_x, E_y and E_z. writing it for the component E_x, we have

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2\right) E_x = 0 \longrightarrow (5)$$

Consider a uniform plane wave characterized by ... are plane surfaces perpendicular to i.e.,

$$\frac{\partial^2 E_x}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^2 E_x}{\partial y^2} = 0 \qquad (3)$$

equ (5) simplifies to

$$\frac{d^2 E_x}{dz^2} + K_0^2 E_x = 0 \longrightarrow (6)$$

which is an ordinary differential equation because E_x, a phasor, depends only on z.

The solution of Eq (6) is readily seen to be.

$$E_x(z) = E_x^+(z) + E_x^-(z)$$

$$= E_0^+ e^{-jK_0 z} + E_0^- e^{jK_0 z} \longrightarrow (7)$$

where, E_0^+ and E_0^- are arbitrary constants that must be determined by boundary conditions

Plane waves in various media:-

A media in electromagnetics is characterized by three parameters. ε, μ and σ

1. Lossless medium:-

In a lossless medium, ε and μ are real $\sigma = 0$, so β is real $\therefore \gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$$\gamma^2 = j^2 \omega^2 \mu\varepsilon = (j\beta)^2 \Rightarrow \beta = \omega\sqrt{\mu\varepsilon}$$

Assume the electric field with

*) only x-component,

*) No variation along x-and y-axis and

*) propagation along z-axis.

i.e., $\frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial y} = 0$

Helmholtz wave equation reduces to

$$\frac{\partial^2}{\partial z^2} E_x + \beta^2 E_x = 0$$

whose solution gives wave in one dime[nsion] as follows.

$$E_x = E^+ e^{-j\beta z} + E^- e^{+j\beta z}$$

where, E^+ and E^- are arbitrary consta[nts]

$$\vec{H} = -\frac{\nabla \times \vec{E}}{j\omega\mu} = \frac{j\,\nabla \times \vec{E}}{\omega\mu}$$

$$= \frac{j}{\omega\mu}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E^+e^{-j\beta z}+E^-e^{+j\beta z} & 0 & 0 \end{vmatrix}$$

$$= \hat{y}\frac{j}{\omega\mu}\left\{\frac{\partial}{\partial z}\left(E^+e^{-j\beta z}+E^-e^{+j\beta z}\right)\right\}$$

$$\vec{H} = \frac{-j\beta\left(E^+e^{-j\beta z}\right)+\left(E^-e^{+j\beta z}\right)j\beta}{\omega\mu}(j)\,\hat{y}$$

$$= \frac{-j\beta\left\{\left(E^+e^{-j\beta z}\right)-\left(E^-e^{+j\beta z}\right)\right\}}{\omega\mu}(j)\,\hat{y}$$

$$= \frac{\beta\left\{\left(E^+e^{-j\beta z}\right)-\left(E^-e^{+j\beta z}\right)\right\}}{\omega\mu}\,\hat{y}$$

$$\boxed{\vec{H} = \frac{1}{\eta}\left[E^+e^{-j\beta z}-E^-e^{+j\beta z}\right]\hat{y}}$$

where, η is the wave impedance of the plane wave

$$\eta = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}} = \frac{|E_x|}{|H_y|}$$

For free space,

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\,\Omega$$

Putting in the time dependence and taking real part, we get, (5)

$$E_x(z,t) = E^+ \cos(\omega t - \beta z) + E^- \cos(\omega t + \beta z)$$

For constant phase,

$$\omega t - \beta z = \text{constant} = b \text{(say)}$$

Since phase velocity.

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - b}{\beta}\right) = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$= \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$$

$$\boxed{\therefore \beta = \omega \sqrt{\mu \varepsilon}}$$

For free space,

$$v_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3 \times 10^8 \text{ m/s}$$

which is the speed of light in free space.

This emergence of speed of light from electromagnetic considerations is one of the main contributions from maxwell's theory.

The magnetic field can be obtained from the source free maxwell's curl equation.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

2. Lossy conducting medium:-

If the medium is conductive with a conductivity σ, then the maxwell's curl equation can be written as

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}.$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \sigma\vec{E} = (j\omega\varepsilon + \sigma)\vec{E} = j\omega\varepsilon_{eff}\vec{E}$$

$$\varepsilon_{eff}(\omega) = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - \frac{j\sigma}{\omega} = \varepsilon\left(1 - \frac{j\sigma}{\omega\varepsilon}\right)$$

The effect of the conductivity has been absorbed in the complex frequency dependent effective permittivity

$$\nabla^2\vec{E} + \omega^2\mu\,\varepsilon_{eff}(\omega)\vec{E} = \nabla^2\vec{E} + (j\gamma)^2\vec{E} = 0$$

we can define a complex propagation constant

$$\gamma = j\omega\sqrt{\mu\varepsilon_{eff}(\omega)} = \alpha + j\beta$$

where, α is the attenuation constant and

β is the phase constant

What is implication of complex wave vector?

- The wave is exponentially decaying
- The dispersion relation for a conductor

(usually non-magnetic) μ

$$\gamma = j\omega\sqrt{\mu\varepsilon_{eff}(\omega)} = j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{\frac{\varepsilon_{eff}(\omega)}{\varepsilon_0}}$$

$$= j\omega\sqrt{\mu_0\varepsilon_0}\,n_{eff}(\omega) = j\frac{\omega}{c}\,n_{eff}(\omega)$$

where, η_{eff} is the complex refractive inde

✓ 1-D wave equation for general lossy medium becomes

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

whose solution is 1-D plane waves as follows.

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{+\gamma z}$$

$$= E^+ e^{-\alpha z} e^{-j\beta z} + E^- e^{\alpha z} e^{j\beta z}$$

putting the time dependence and taking real parts we get.

$$E_x(z,t) = E^+ e^{-\alpha z} \cos(\omega t - \beta z) + E^- e^{\alpha z} \cos(\omega t +$$

The magnetic field can be found out from maxwells equations as in the previous sect

$$H_y(z) = \frac{1}{\eta_{eff}} [E^+ e^{-\gamma z} - E^- e^{\gamma z}]$$

where useful expression for intrinsic impedance is

$$\eta_{eff} = \frac{j\omega\mu_0}{\gamma} = \frac{j\omega\mu_0}{j\omega\sqrt{\mu_0 \varepsilon_{eff}(\omega)}}$$

$$\boxed{\eta_{eff} = \sqrt{\frac{\mu_0}{\varepsilon_{eff}(\omega)}}}$$

the electric field and magnetic field (9)
are no longer in phase as ε_{eff} is complex

poynting vector on power flow for this wave inside the lossy conducting medium is.

$$\vec{S} = \vec{E} \times \vec{H}^*$$

$$= E^+ e^{-\alpha z} e^{-j\beta z} \hat{x} \times \left(\frac{E^+ e^{-\alpha z} e^{-j\beta z}}{\eta_{eff}} \right)^* \hat{y}$$

$$= |E^+|^2 e^{-\alpha z} e^{-j\beta z} \times \frac{e^{-\alpha z} e^{+j\beta z}}{\eta^*_{eff}} \hat{z}$$

$$\boxed{\vec{S} = \frac{|E^+|^2}{\eta^*_{eff}} e^{-2\alpha z} \hat{z}}$$

It is decaying in terms of square of an exponential function.

3. Good dielectric / Conductor:-

✓ Note that $\sigma/\omega\varepsilon$ is defined as loss tangent of a medium

✓ A medium with $\sigma/\omega\varepsilon < 0.01$ is said to be a good insulator

✓ whereas a medium with $\sigma/\omega\varepsilon > 100$ is said to be a good conductor

For good dielectric,

$\therefore \sigma << \omega\varepsilon \therefore \gamma = j\omega\sqrt{\mu\varepsilon}\left(\sqrt{1-\frac{j\sigma}{\omega\varepsilon}}\right)$

It canbe approximated using Taylor's series expansion obtain α and β as follow

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}, \quad \boxed{\beta = \omega\sqrt{\mu\varepsilon}}$$

For a good Conductor,

$$\boxed{\sigma >> \omega\varepsilon}$$

$$\therefore \gamma \cong (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\boxed{\Rightarrow \alpha = \beta \cong \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Skin effect:-

*) The field do attenuate as they trav in agood dielectric medium

*) α in a good dielectrics is very sma in comparison to that of a good conductor

*) As the amplitude of the wave varies with $e^{-\alpha z}$

*) The wave amplitude reduces it b by 1/e on 37% times over a distance of

$$\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi f\mu\sigma}} \quad (11)$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}}$$

which is also known as skin depth.

This means that in a good conductor

a) higher the frequency, lower is the skin depth

b) higher is the conductivity, lower is the skin depth &

c) higher is the permeability, lower is the skin depth.

Let us assume an EM wave which has x-component and travelling along the z-axis.

Then, it can be expressed as.

$$E_x(z,t) = E_0 e^{-\alpha z} e^{-j(\beta z - \omega t)}$$

taking the real part we have,

$$E_x(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

Substituting the values of α and β for good conductors, we have,

$$E_x(z,t) = E_0 e^{-\sqrt{\pi f\mu\sigma}z} \cos(\omega t - \sqrt{\pi f\mu\sigma}z)$$

Now using the point form of ohm's law for conductors, we can write.

$$J_x = \sigma E_x(z,t) = \sigma E_0 e^{-\sqrt{\pi f\mu\sigma}z} \cos(\omega t - \sqrt{\pi f\mu\sigma}z)$$

what is the phase velocity and, wavelength inside a good conductor?

$$v_p = \frac{\omega}{\beta} = \omega\delta ;\ \lambda = \frac{2\pi}{\beta} = 2\pi\delta.$$

ELECTROMAGNETIC POWER FLOW & POYNTING VECTOR

The rate of energy flow per unit area in a plane wave is described by a vector termed as poynting vector which is basically curl of electric field intensity vector and magnetic field intensity vector.

$$\vec{S} = \vec{E} \times \vec{H}^*$$

The magnitude of poynting vector is the power flow per unit area and It points along the direction of wave propagation vector.

The average power per unit area is often called the intensity of EM wave and it is given by

$$\vec{S}_{avg} = \frac{1}{2} Re(\vec{E} \times \vec{H}^*).$$

Let us try to derive the point form of poynting theorem from two maxwell's curl equations.

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}$$

From vector analysis,

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

$$= \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) - \vec{E} \cdot \left(\varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}\right)$$

we can further simplify.

$$\vec{A} \cdot \frac{\partial \vec{A}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} (\vec{A} \cdot \vec{A})$$

$$\therefore \nabla \cdot (\vec{E} \times \vec{H}) = -\frac{\mu}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) - \frac{\varepsilon}{2} \frac{\partial}{\partial t} (\vec{E} \cdot \vec{E}) - \vec{E} \cdot \vec{J}$$

Basically a point relation.

It should be valid at every point in space at every instant of time

The power is given by the integral of this relation of poynting vector over a volume as follows.

$$\int_V \nabla \cdot (\vec{E} \times \vec{H}) dv = \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s}$$

$$= \oint_S \vec{S} \cdot d\vec{s}$$

$$\oint_S \vec{S} \cdot d\vec{s} = -\frac{\mu}{2} \int_V \frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) dv - \frac{\varepsilon}{2} \int_V \frac{\partial}{\partial t} (\vec{E} \cdot \vec{E}) dv - \int_V \vec{E} \cdot \vec{J} dv$$

we can interchange the volume integral and partial derivative with respect to time

$$\boxed{\oint_s \vec{S} \cdot \vec{ds} = \frac{-\partial}{\partial t} \int_v \frac{1}{2} \mu H^2 dv - \frac{\partial}{\partial t} \int_v \frac{1}{2} \varepsilon E^2 dv - \int_v \sigma E^2 dv}$$

This is the integral form of poynting vector and power flow in Em fields

Poynting theorem states that "the power coming out of the closed volume is equal to the total decrease in Em energy per unit time" i.e. Power loss from the volume which constitutes of

✓ Rate of decrease in magnetic energy stored in the volume

✓ Rate of decrease in electric energy stored in the volume

✓ ohmic power loss (energy converted into heat energy per unit time) in the volume.

Now going back to the last four points of plane waves:

k) The direction of propagation is in the same direction as of poynting vector

Note that the direction of Poynting vector is also in the z-direction same as that of the wave vector.

The average value of the Poynting vector

$$\vec{S}_{avg} = \frac{1}{2} Re\left(\vec{E} \times \vec{H}^*\right)$$

$$= \frac{1}{2} Re\left(\frac{|\vec{E_0}|^2 \hat{z}}{\eta_0}\right) = \frac{|\vec{E_0}|^2 \hat{z}}{2\eta_0}$$

Stored Electric energy: $W_e = \frac{1}{2}\varepsilon_0 E^2$

Stored magnetic energy:

$$W_m = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\mu_0 \frac{E^2}{\eta_0^2} = \frac{1}{2}\mu_0 \frac{\varepsilon_0}{\mu_0} E^2$$

$$= \frac{1}{2}\varepsilon_0 E^2 = W_e$$

Group velocity:

The relation between phase velocity (u_p) and the phase constant β is

$$\boxed{u_p = \frac{\omega}{\beta} \ (m/s)} \rightarrow ①$$

For plane wave in a lossless medium $\beta = \omega\sqrt{\mu\varepsilon}$ is a linear function of ω.

The phase velocity $u_p = \frac{1}{\quad}$ is a constant

*) The instantaneous value of the poynting vector is given by

E^2/η_0 (or $H^2\eta_0$

*) The average value of the poynting vector is given by $E^2/2\eta_0$ (or $H^2\eta_0/2$.

*) The stored electric energy is equal to the stored magnetic energy at any instant.

Let us assume a plane wave travelling in the +z direction in free space, then

$$\vec{E} = \vec{E_0} e^{-j\beta z} = \vec{E_0} e^{-jk_0 z}$$

$$\vec{H} = \frac{\hat{z} \times \vec{E_0}}{\eta_0} e^{-j\beta z}$$

The instantaneous value of the poynting vector:

$$\vec{S} = \vec{E} \times \vec{H}^* = \left(\vec{E_0} e^{-j\beta z}\right) \times \left(\frac{\hat{z} \times \vec{E_0}}{\eta_0} e^{j\beta z}\right)$$

$$= \frac{1}{\eta_0} \left(\vec{E_0}\right) \times \left(\hat{z} \times \vec{E_0}\right)$$

$$\vec{S} = \frac{\hat{z}\left(\vec{E_0} \cdot \vec{E_0}\right) - \vec{E_0}\left(\vec{E_0} \cdot \hat{z}\right)}{\eta_0} = \frac{\hat{z}\left(\vec{E_0} \cdot \vec{E_0}\right)}{\eta_0} = \frac{|\vec{E_0}|^2 \hat{z}}{\eta_0}$$

that is independent of frequency.

The phenomenon of signal distortion ~~in the~~ ~~signal~~ caused by a dependence of the phase velocity on frequency is called dispersion.

A group velocity is the velocity of propagation of the wave-packet envelope (of a group of frequencies).

Consider the simplest case of a wave packet that consists of two travelling waves having equal amplitude and slightly different angular frequencies $\omega_0 + \Delta\omega$ and $\omega_0 - \Delta\omega$ $(\Delta\omega << \omega_0)$

Let the phase constants corresponding to the two frequencies be $\beta_0 + \Delta\beta$ and $\beta_0 - \Delta\beta$ we have

$$E(z,t) = E_0 \cos[(\omega_0 + \Delta\omega)t - (\beta_0 + \Delta\beta)z] +$$

$$E_0 \cos[(\omega_0 - \Delta\omega t) - (\beta_0 - \Delta\beta)z] \rightarrow (1)$$

$$E(z,t) = 2E_0 \cos(t\Delta\omega - z\Delta\beta)\cos(\omega_0 t - \beta_0 z).$$

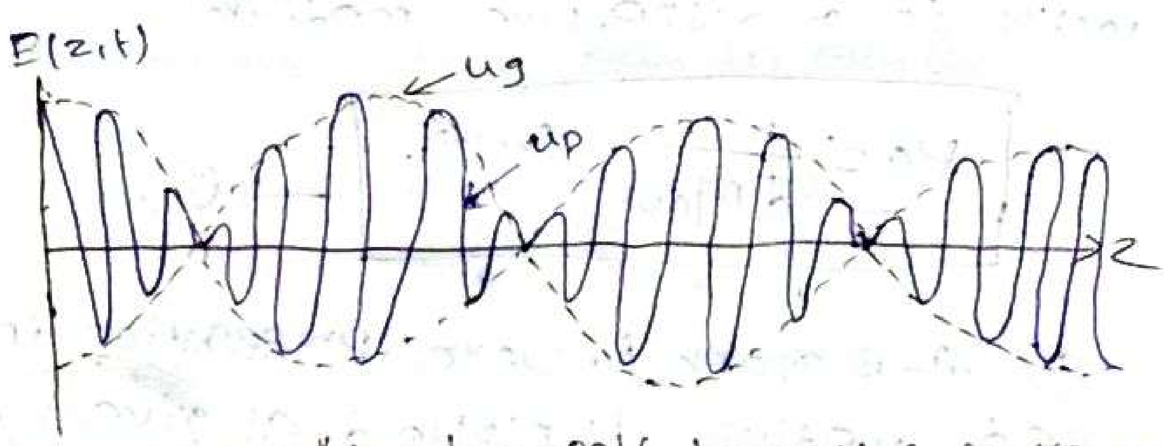


fig: sum of two time harmonic traveling waves of eq..

since, $\Delta\omega \ll \omega_0$. The eqn ② represents a rapi[dly]
oscillating wave having an angular frequen[cy]
ω_0 and an amplitude that varies slowly
with an angular frequency $\Delta\omega$.

The wave inside the envelop propagat[es]
with a phase velocity found by setting

$\omega_0 t - \beta_0 z =$ constant.

$$\mu_p = \frac{dz}{dt} = \frac{\omega_0}{\beta_0}$$

The group velocity (u_g) can be determin[ed]
by setting the argument of the first cosin[e]
factor in eqn ② equal to a constant

$$t\Delta\omega - z\Delta\beta = \text{constant}.$$

From which we obtain

$$u_g = \frac{dz}{dt} = \frac{\Delta\omega}{\Delta\beta} = \frac{1}{\Delta\beta/\Delta\omega}$$

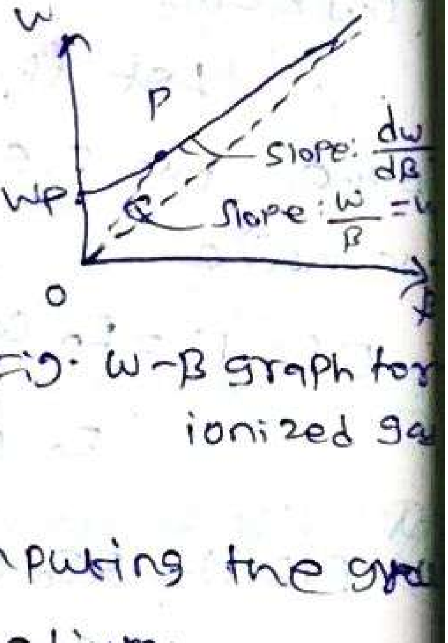


Fig. ω-β graph for ionized ga[s]

In the limit that $\Delta\omega \to 0$,

we have the formula for computing the gro[up]
velocity in a dispersive medium.

$$\boxed{\mu_g = \frac{1}{d\beta/d\omega} \;(\text{m/s}).} \rightarrow ③$$

In ω-β graph for wave propagation in a[n]
ionized medium is plotted as given by

$$\beta = \omega\sqrt{\mu\varepsilon_0}\sqrt{1-\left(\frac{f_p}{f}\right)^2}$$

$$= \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2} \rightarrow (4)$$

At $\omega = \omega_p$ (the cutoff angular frequency), $\beta = 0$. For $\omega > \omega_p$, wave propagation is possible, and

$$u_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}} \rightarrow (5)$$

Sub eqn (4) in eqn (3), we have

$$u_g = c\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2} \rightarrow (6)$$

$u_p \geq c$ and $u_g \leq c$ then $u_p u_g = c^2$.

A general relation between the group and phase velocities may be obtained by combining eqn (1) & (3). From eqn (1) we have

$$\frac{d\beta}{d\omega} = \frac{d}{d\omega}\left(\frac{\omega}{u_p}\right) = \frac{1}{u_p} - \frac{\omega}{u_p^2}\frac{du_p}{d\omega}$$

Sub of the above eqn in eqn (3)

$$u_g = \frac{u_p}{1 - \frac{\omega}{u_p}\frac{du_p}{d\omega}} \rightarrow (7)$$

From eqn (7) we see three possible cases.

a) No dispersion:-

$$\frac{du_p}{d\omega} = 0$$ (up independent of ω, β a linear function of ω)

$$\boxed{u_g = u_p}$$

b) Normal dispersion:-

$$\frac{du_p}{d\omega} < 0$$ (up decreasing with ω)

$$\boxed{u_g < u_p}$$

c) Anomalous dispersion:

$$\frac{du_p}{d\omega} > 0$$ (up increasing with ω)

$$\boxed{u_g > u_p}$$

Pblm: A narrow band signal propagates in a lossy dielectric medium which has a loss tangent 0.2 at 550 (kHz), the carrier frequency of the signal. The dielectric constant of the medium is 2.5. (a) Determine α and β. b) Determine up and ug. Is the medium dispersive?

Solution:

a) since loss tangent $\epsilon''/\epsilon' = 0.2$ and $\epsilon''^2/8\epsilon'^2 <$

$$\epsilon'' = 0.2\epsilon' = 0.2 \times 2.5\epsilon_0$$

$$= 4.42 \times 10^{-12} \text{ (F/m)}$$

$$\alpha = \frac{\omega \varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \pi(550\times10^3)\times(4.42\times10^{-12})\times\frac{377}{\sqrt{2.5}}$$

$$\boxed{\alpha = 1.82\times10^{-3}\ (\mathrm{Np/m})}$$

$$\beta = \omega\sqrt{\mu\varepsilon'}\left[1+\frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right].$$

$$= 2\pi(550\times10^3)\frac{\sqrt{2.5}}{3\times10^8}\left[1+\frac{1}{8}(0.2)^2\right]$$

$$\boxed{\beta = 0.0182\times1.005 = 0.0183\ (\mathrm{rad/m}).}$$

b) Phase velocity.

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon'}\left[1+\frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]} \simeq \frac{1}{\sqrt{\mu\varepsilon'}}\left[1-\frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]$$

$$= \frac{3\times10^8}{\sqrt{2.5}}\left[1-\frac{1}{8}(0.2)^2\right]$$

$$\boxed{u_p = 1.888\times10^8\ (\mathrm{m/s})}$$

c) Group velocity.

$$\frac{d\beta}{d\omega} = \sqrt{\mu\varepsilon'}\left[1+\frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]$$

$$u_g = \frac{1}{(d\beta/d\omega)} \simeq \frac{1}{\sqrt{\mu\varepsilon'}} \simeq u_p$$

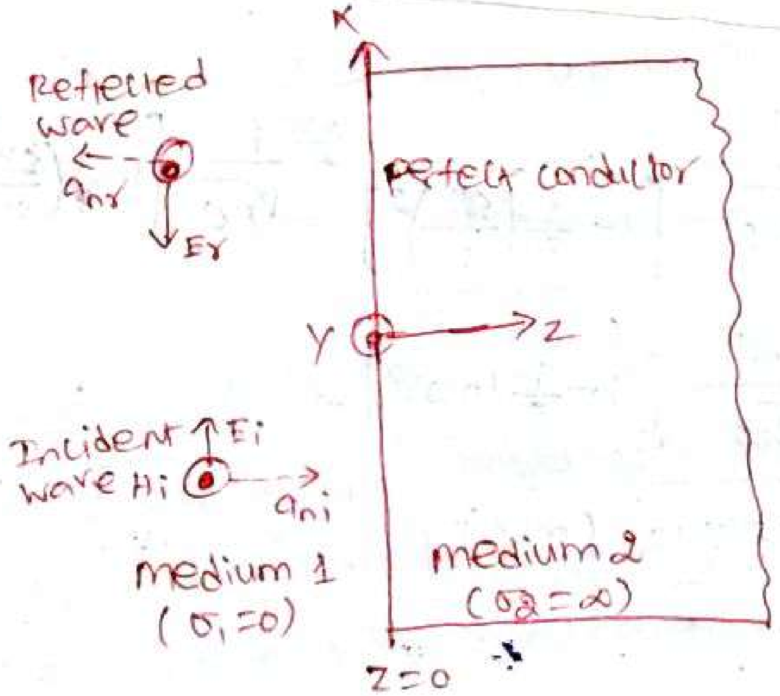
Thus a low-loss dielectric is nearly nondispersi[ve]. Here we have assumed ε'' to be independent of frequency.

For a high-loss dielectric, ε'' will be a fun of ω and may have a magnitude comparable to ε'.

Normal Incidence at a Plane Conducting Boundary

*) The incident wave travels in a lossless medium

*) The boundary is an interface with a perfect conductor.



Incident wave (inside medium 1)

$$\vec{E}_1(z) = \hat{a}_x E_{i0} e^{-j\beta_1 z}$$

$$\vec{H}_i(z) = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

Where, E_{i0} is the magnitude of $\vec{E}_i$

β_1 is the phase constant

η_1 is the intrinsic impedance of medium 1

Inside wave in medium 2, both electric and magnetic field vanish. $\vec{E_2} = 0$, $\vec{H_2} = 0$

No wave is transmitted across the boundary into the $z > 0$

Reflected wave (inside medium 1)

$$\vec{E_r}(z) = \hat{a_x} E_{ro} e^{+j\beta_1 z}$$

$$\vec{H_r}(z) = \frac{1}{\eta_1} \hat{a_{nr}} \times \vec{E_r}(z)$$

$$= \frac{1}{\eta_1} (-\hat{a_z}) \times \vec{E_r}(z)$$

$$\vec{H_r}(z) = -\hat{a_y} \frac{1}{\eta_1} E_{ro} e^{+j\beta_1 z}$$

Total wave in medium 1

$$\vec{E_1}(z) = \vec{E_i}(z) + \vec{E_r}(z)$$

$$= \hat{a_x} (E_{io} e^{-j\beta_1 z} + E_{ro} e^{+j\beta_1 z})$$

Continuity of tangential component of the E field at the boundary $z = 0$

$$\vec{E_1}(0) = \hat{a_x} (E_{io} + E_{ro}) = E_2(0) = 0$$

$$\Rightarrow E_{ro} = -E_{io}$$

$$\therefore \vec{E_1}(z) = \hat{a_x} E_{io} (e^{-j\beta_1 z} - e^{+j\beta_1 z}) = -\hat{a_x} j 2 E_{io} \sin \beta_1 z$$

$$\therefore \vec{H_1}(z) = \vec{H_i}(z) + \vec{H_r}(z)$$

$$= \hat{a_y} 2 \frac{E_{io}}{\eta_1} \cos \beta_1 z$$

$\therefore \vec{E}_1(z) = \vec{a_x}\, E_{io}\left(e^{-j\beta_1 z} - e^{+j\beta_1 z}\right)$

$= -\hat{a}_x\, j2\, E_{io} \sin \beta_1 z$

$\therefore \vec{H}_1(z) = \vec{H}_i(z) + \vec{H}_r(z)$

$= \hat{a}_y\, 2\, \frac{E_{io}}{\eta_1} \cos \beta_1 z$

The space-time behavior of the total field in medium1

$\vec{E}_1(z,t) = Re\left[\vec{E}_1(z)\, e^{j\omega t}\right] = \hat{a}_x 2 E_{io} \sin \beta \sin \omega t$

$\vec{H}_1(z,t) = Re\left[\vec{H}_1(z)\, e^{j\omega t}\right] = \hat{a}_y\, 2\, \frac{E_{io}}{\eta_1} \cos \beta_1 \cos \omega t$

zeros of $\vec{E}_1(z,t)$
maxima of $\vec{H}_1(z,t)$ } occur at $\beta_1 z = -n\pi$ (or)

$z = -n\frac{\lambda}{2}, \quad n = 0, 1, 2, 2 \ldots$

maxima of $\vec{E}_1(z,t)$
zeros of $\vec{H}_1(z,t)$ } occur at $\beta_1 z = -(2n+1)\frac{\pi}{2}$

(or) $z = -(2n+1)\frac{\lambda}{4}, \quad n = 0, 1, 2, \ldots$

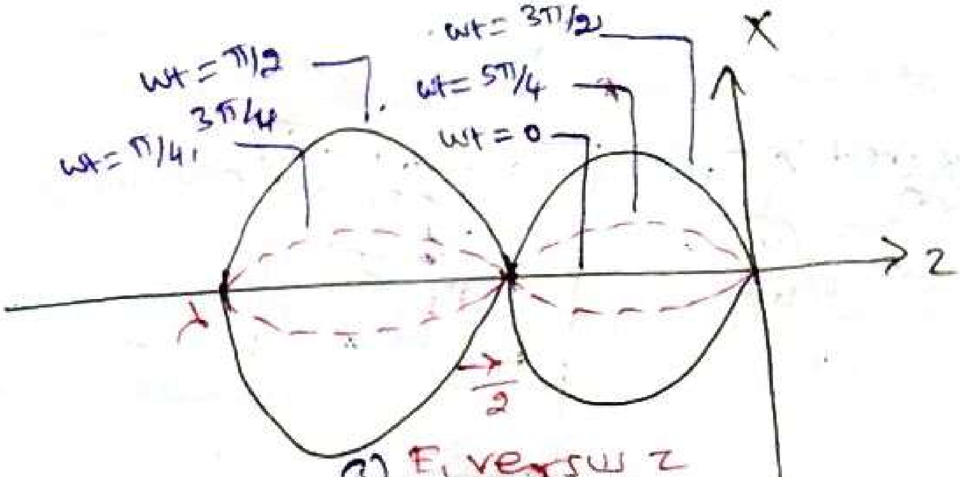
$\vec{E}_1(z,t) = Re\left[\vec{E}_1(z)\, e^{j\omega t}\right]$

$= \hat{a}_x\, 2 E_{io} \sin \beta_1 z \sin \omega t$

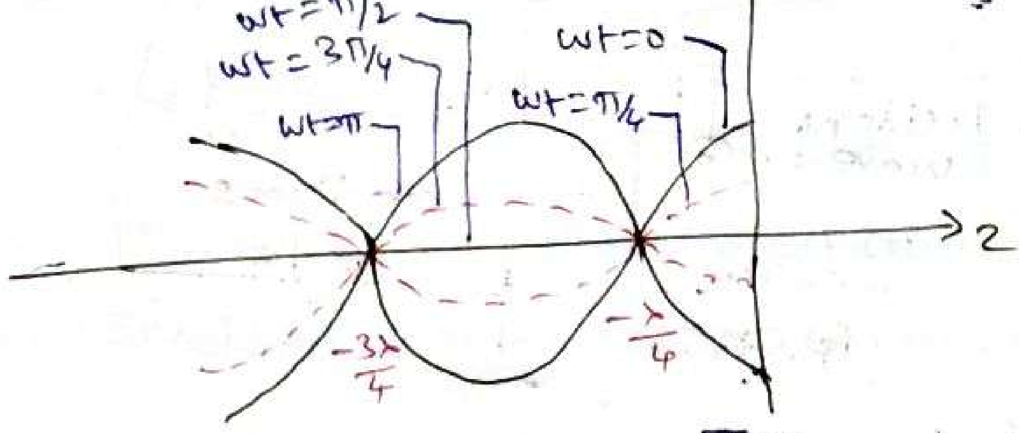
$\vec{H}_1(z,t) = Re\left[\vec{H}_1(z)\, e^{j\omega t}\right]$

$= \hat{a}_y\, 2 \cdot \frac{E_{io}}{\eta_1} \cos \beta_1 z \cos \omega t$

The total wave in medium1 is not a trav wave. Standing wave



a) E_1 versus z



b) H_1 versus z

Fig: Standing waves of $E_1 = a_x E_1$ and $H_1 = a_y H_1$ for several values of ωt

Note following three points

(i) E vanishes on the conducting boundary

(ii) $\vec{H}$ a maximum on the conducting boundary

(iii) The standing waves of $\vec{E}_1$ and $\vec{H}_1$ are in time quadrature (90° phase difference

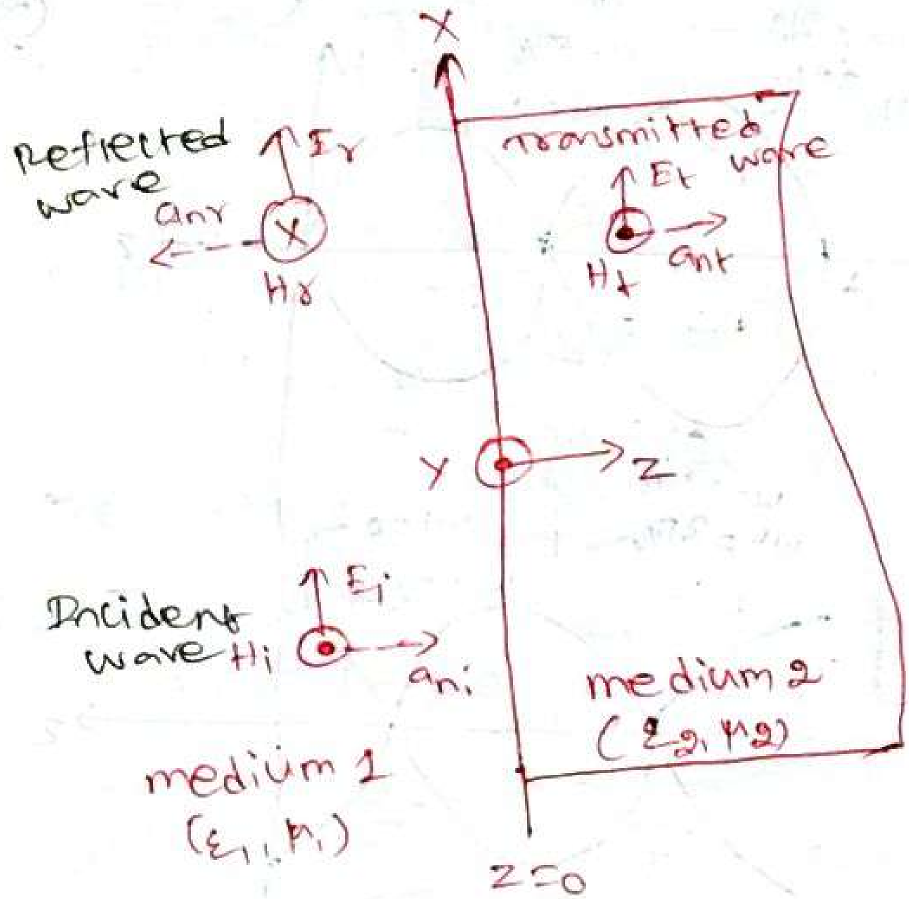
Normal incidence at a plane dielectric boundary

$\sigma_1 = \sigma_2 = 0, \quad \varepsilon_1 \neq \varepsilon_2$

Incident wave (inside medium 1)

$$\vec{E}_1(z) = \hat{a}_x \, E_{i0} \, e^{-j\beta_1 z}$$

$$\vec{H}_1(z) = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$



Reflected wave (inside medium 1)

$$\vec{E}_r(z) = \hat{a}_x E_{r0} e^{-j\beta_1 z}$$

$$\vec{H}_r(z) = (-\hat{a}_z) \times \frac{1}{\eta_1} \vec{E}_r(z) = -\hat{a}_y \frac{E_{r0}}{\eta_1} e^{j\beta_1 z}$$

Transmitted wave (inside medium 2)

$$\vec{E}_t(z) = \hat{a}_x E_{t0} e^{-j\beta_2 z}$$

$$\vec{H}_t(z) = \hat{a}_z \times \frac{1}{\eta_2} \vec{E}_t(z) = \hat{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z}$$

The tangential components (the x-components) of the electric and magnetic field intensities must be continuous. (at interface z=0)

$$E_{1tan} = E_{2tan} \quad , \quad H_{1tan} = H_{2tan}$$

$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0) \Rightarrow E_{io} + E_{ro} = E_{to}$

$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0) \Rightarrow \frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}$

$\therefore E_{ro} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{io} \qquad E_{to} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{io}$

reflection co-efficient $(+ \text{ or } -) \leq 1$

$\eta_1 = \eta_2 \therefore \Gamma = 0$ $\qquad \boxed{\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$

$\eta_2 = 0$ (short) $\Gamma = -1$ E/H, E=0 perfect conductor!!

$\eta_2 = \infty$ (open) $\Gamma = 1$ H(I) = 0 No current!!

$$E_{to} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{io}$$

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} \qquad \boxed{1 + \Gamma = \tau}$$

transmission co-efficient (+always)

If medium 2 → perfect conductor $\eta_2 = 0$

$\Gamma = -1$, $\tau = 0 \Rightarrow E_{ro} = -E_{io}$, $E_{to} = 0$

→ Totally reflected. Standing wave produced in medium 1.

If medium 2 is not a perfect conductor, partial reflection will result.

$\vec{E}_1(z) = \vec{E}_i(z) + \vec{E}_r(z) = \hat{a}_x E_{io}(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z})$

$= \hat{a}_x E_{io}[(1+\Gamma)e^{-j\beta_1 z} + \Gamma(e^{j\beta_1 z} - e^{-j\beta_1 z})]$

$(z<0)$

$$= \hat{a}_x E_{i0} [\underbrace{(1+\Gamma)e^{-j\beta_1 z}}_{\text{traveling}} + \underbrace{\Gamma (j2\sin\beta_1 z)}_{\text{standing}}]$$

$$\Rightarrow \vec{E}_1(z) = \hat{a}_x E_{i0}\, e^{-j\beta_1 z}(1+\Gamma e^{j2\beta_1 z}) \quad (z<$$

$$|\vec{E}_1(z)| = E_{i0} \{(1+\Gamma e^{-j2\beta_1 z})(1+\Gamma e^{+j2\beta_1 z})$$

$$\boxed{|\vec{E}_1(z)| = E_{i0}(1+\Gamma^2+2\Gamma\cos 2\beta_1 z)^{1/2}}$$

For dissipationless media $\eta_1, \eta_2, \tau, \Gamma$ are re

However, Γ can be positive (or) negative

(i) $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} > 0 \quad (\eta_2 > \eta_1)$

✓ maximum value of $|\vec{E}_1(z)|$ is $E_{i0}(1+\Gamma)$, which occures when $2\beta_1 z_{max} = -2n\pi$ $(n=0,$

$$\therefore z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2}, \quad n=0,1,2\ldots$$

✓ minimum value of $|\vec{E}_1(z)|$ is $E_{i0}(1-\Gamma)$, which occures when $2\beta_1 z_{min} = -(2n+1)\pi$ $(n=$

$$\therefore z_{min} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4}, \quad n=0,1,2$$

(ii) $\Gamma < 0 \quad (\eta_2 < \eta_1)$

✓ maximum value of $|\vec{E}_1(z)|$ is $E_{i0}(1-\Gamma)$,

at $z_{min} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4}, \quad n=0,1,$

✓ minimum value of $|\vec{E_1}(z)|$ is $E_{io}(1+\Gamma)$. (29)

at $z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2}$, $n = 0, 1, 2 \cdots$

$$\boxed{S = \frac{|E|_{max}}{|E|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$ Standing wave Ratio (SWR)

$$\boxed{|\Gamma| = \frac{S-1}{S+1}}$$ $(-1 \leq \Gamma \leq 1, \ 1 \leq S \leq \infty)$

if $\Gamma = 0$, $S = 1$, No reflection, full powe transmission

if $\Gamma = 1$, $S = \infty$, Total reflection, no powe transmission