



PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

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Department of Civil Engineering

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III Year – V Semester

CE3501 Design of Reinforced Concrete Structural Elements

UNIT I INTRODUCTION

Methods Of Design Of Concrete Structures

Introduction:

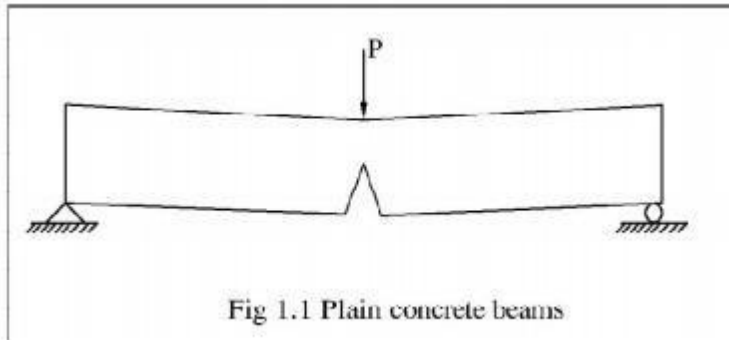
A structure refers to a system of connected parts used to support forces (loads). Buildings, bridges and towers are examples for structures in civil engineering. In buildings, structure consists of walls floors, roofs and foundation. In bridges, the structure consists of deck, supporting systems and foundations. In towers the structure consists of vertical, horizontal and diagonal members along with foundation.

A structure can be broadly classified as (i) sub structure and (ii) super structure. The portion of building below ground level is known as sub-structure and portion above the ground is called as super structure. Foundation is sub structure and plinth, walls, columns, floor slabs with or without beams, stairs, roof slabs with or without beams etc are super structure.

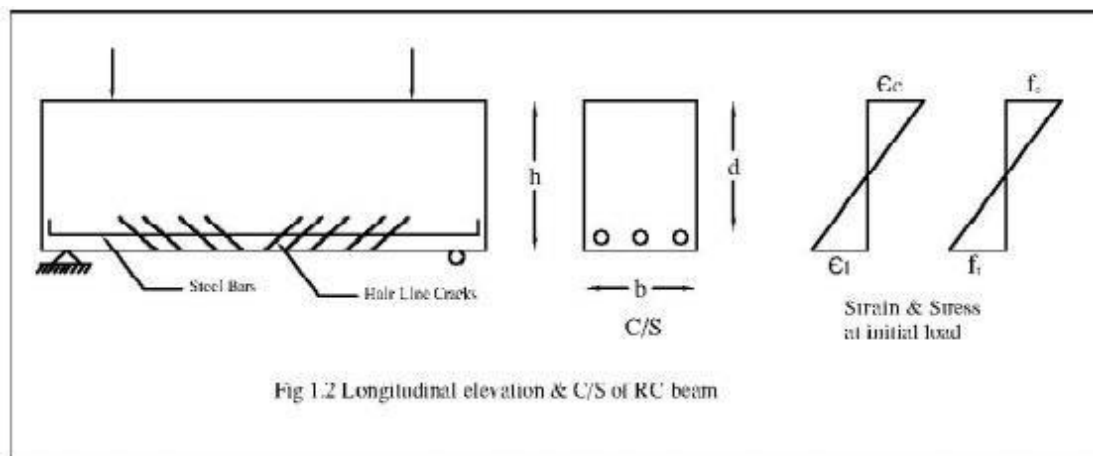
Many naturally occurring substances, such as clay, sand, wood, rocks natural fibers are used to construct buildings. Apart from this many manmade products are in use for building construction. Bricks, tiles, cement concrete, concrete blocks, plastic, steel & glass etc are manmade building materials.

Cement concrete is a composites building material made from combination of aggregates (coarse and fine) and a binder such as cement. The most common form of concrete consists of mineral aggregate (gravel & sand), Portland cement and water. After mixing, the cement hydrates and eventually hardens into a stone like material. Recently a large number of additives known as concrete additives are also added to enhance the quality of concrete. Plasticizers, super plasticizers, accelerators, retarders, pazolonic materials, air entertaining agents, fibers, polymers and silica furies are the additives used in concrete. Hardened concrete has high compressive strength and low tensile strength. Concrete is generally strengthened using steel bars or rods known as rebars in tension zone. Such elements are '**reinforced concrete**' concrete can be moulded to any complex shape using suitable form work and it has high durability, better appearance, fire resistance and economical. For a strong, ductile and durable construction the reinforcement shall have high strength, high tensile strain and good bond to concrete and thermal compatibility. Building components like slab walls, beams, columns foundation & frames are constructed with reinforced concrete. Reinforced concreted can be in-situ concreted or precast concrete.

For understanding behavior of reinforced concrete, we shall consider a plain concrete beam subjected to external load as shown in Fig. 1.1. Tensile strength of concrete is approximately one-tenth of its compressive strength.



Hence use of plain concrete as a structural material is limited to situations where significant tensile stresses and strains do not develop as in solid or hollow concrete blocks, pedestal and in mass concrete dams. The steel bars are used in tension zone of the element to resist tension as shown in Fig 1.2. The tension caused by bending moment is chiefly resisted by the steel reinforcements, while concrete resists the compression. Such joint action is possible if relative slip between concrete and steel is prevented. This phenomena is called 'bond'. This can be achieved by using deformed bars which has high bond strength at the steel-concrete interface. Rebars impart 'ductility' to the structural element, i.e. RC elements have large deflection before it fails due to yielding of steel, thus it gives ample warning before its collapse.



Design Loads

For the analysis and design of structure, the forces are considered as the 'Loads' on the structure. In a structure all components which are stationary, like wall, slab etc., exert forces due to gravity, which are called as 'Dead Loads'. Moving bodies like furniture, humans etc exert forces due to gravity which are called as 'Live Loads'. Dead loads and live loads are gravity forces which act vertically down ward. Wind load is basically a horizontal force due to wind pressure exerted on the structure. Earthquake load is primarily a horizontal pressure exerted due to movement of the soil on the foundation of a structure. Vertical earthquake force is about 5% to 10% of horizontal earthquake force. Fig. 1.3 illustrates the loads that are considered in analysis and design.

IS875 -1987 part 1 gives unit weight of different materials, Part - 2 of this code describes live load on floors and roof. Wind load to be considered is given in part 3 of the code. Details of earthquake load to be considered is described in 1893 - 2002 code and combination of loads is given in part 5 of IS875 - 1987.

Materials for Reinforced Concrete

Concrete

Concrete is a composite material consists essentially of

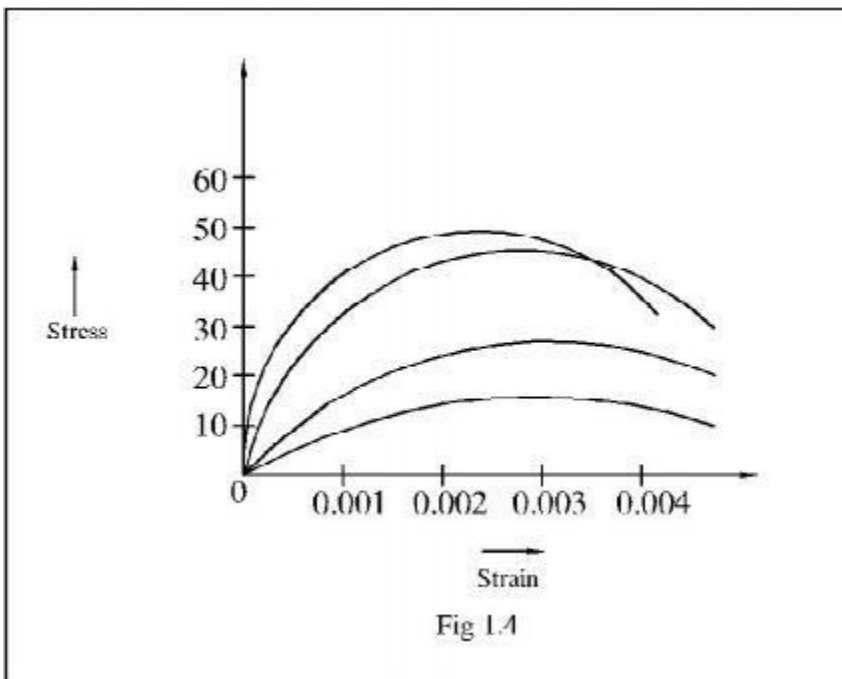
- a) A binding medium cement and water called cement paste
- b) Particles of a relatively inert filler called aggregate

The selection of the relative proportions of cement, water and aggregate is called 'mix design' Basic requirement of a good concrete are workability, strength, durability and economy. Depending upon the intended use the cement may be OPC (33,43 & 53 Grade), Rapid hardening cements Portland slag, Portland pozzolona etc. High cement content give rise to increased shrinkage, creep and cracking. Minimum cement content is 300Kg/m^3 and maximum being 450Kg/m^3 as per Indian code. Mineral additives like fly ash , silica fume, rice husk ash, metakoline and ground granulated blast furnace slag may be used to reduce micro cracks . The aggregate used is primarily for the purpose of providing bulk to the concrete and constitutes 60 to 80 percent of finished product. Fine aggregates are used to increase the workability and uniformity of concrete mixture. Water used for mixing and curing shall be clean and free from oil, acids, alkalis, salts, sugar etc. The diverse requirements of mixability, stability, transportability place ability, mobility, compatibility of fresh concrete are collectively referred to as workability.

Compressive strength of concrete on 28th day after casting is considered as one of the measure of quality. At least 4 specimens of cubes should be tested for acceptance criteria.

Grade of concrete

Based on the compressive strength of concrete, they are designated with letter H followed by an integer number represented characteristic strength of concrete, measured using 150mm size cube. Characteristic strength is defined as the strength of material below which not more than 5% of test results are expected to fall. The concrete grade M10, M15 and M20 are termed as ordinary concrete and those of M25 to M55 are termed as standard concrete and the concrete of grade 60 and above are termed as high strength concrete. The selection of minimum grade of concrete is dictated by durability considerations which are based on kind of environment to which the structure is exposed, though the minimum grade of concrete for reinforced concrete is specified as M20 under mild exposure conditions, it is advisable to adopt a higher grade. For moderate, severe, very severe and extreme exposure conditions, M25, M30, M35 & M40 grades respectively are recommended. Typical stress-strain curves of concrete are shown in Fig.1.4



Reinforcing steel

Steel bars are often used in concrete to take care of tensile stresses. Often they are called as rebars, steel bar induces ductility to composite material i.e reinforced concrete steel is stronger than concrete in compression also. Plain mild steel bars or deformed bars are generally used. Due to poor bond strength plain bars are not used. High strength deformed bars generally cold twisted

(CTD) are used in reinforced concrete. During beginning of 21st century, Thermo-mechanical treated (TMT) bars which has ribs on surface are used in reinforced concrete. Yield strength of steel bars are denoted as characteristic strength. Yield strength of mild steel is 250MPa, yield strength of CTD & TMT bars available in market has 415 MPa or 500 MPa or 550MPa. TMT bars have better elongation than CTD bars. Stress-strain curve of CTD bars or TMT bars do not have definite yield point, hence 0.2% proof stress is used as yield strength. Fig 1.5 shows stress strain curve of different steel grades. Steel grades are indicated by Fe followed by yield strength. In the drawings of RCC, denotes MS bar and # denotes CTD or TMT bars.

Concrete: Design codes and Hand books

A code is a set of technical specifications intended to control the design and construction. The code can be legally adopted to see that sound structure are designed and constructed code specifies acceptable methods of design and construction to produce safe and sound structures.

National building code have been formulated in different countries to lay down guidelines for the design and construction of structures. International building code has been published by international code council located in USA. National building code (NBC - 2005) published in India describes the specification and design procedure for buildings.

For designing reinforced concrete following codes of different countries are available

India - IS456 - 2000 - Plain and reinforced concrete code practice.

USA - ACI 318-2011 - Building code requirements for Structural concrete (American concrete institute)

UK - BS8110 -part1 - structural use of concrete -code of practice for design and construction. (British standard Institute)

Europe - EN 1992(Euro code 2) - Design of concrete structures

Canada - CAN/CSA - A23.3-04 - Design of concrete structures (Reaffirmed in 2010),

Australia - AS 3600 -2001 - concrete structures.

Germany - DIN 1045 - Design of concrete structures

Russia - SNIP

China - GB 50010 -2002 code for design of concrete structures to help the designers, each country has produced 'handbook'. In India following hand books called special publication are available.

SP - 16-1980- Design Aid for Reinforced concrete to IS456-1978

SP - 23-1982- Hand book on concrete mixes

Sp - 24 -1983 - Explanatory hand book on IS456 - 1978

SP - 34-1987 - Hand book on concrete reinforcement and detailing.

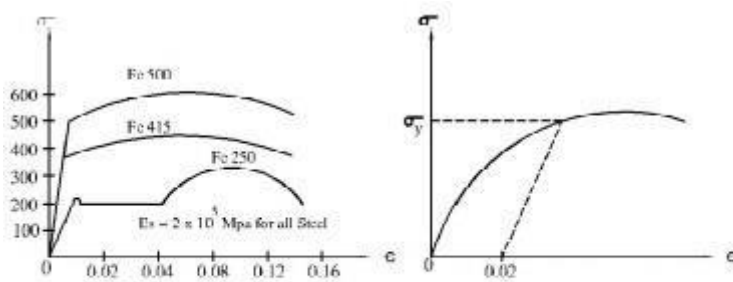


Fig 1-5 stress –strain curve

Concrete: Design Philosophies

Structural design is process of determining the configuration (form and proportion) of a structure subject to a load carrying performance requirement. Form of a structure describes the shape and relative arrangements of its components. The determination of an efficient form is basically a trial and error procedure.

In the beginning of 20th century (1900 to 1960) to late 50's of this century, members were proportioned so that stresses in concrete and steel resulting from service load were within the allowable stresses. Allowable stresses were specified by codes. This method of design is called 'working stress method' (WSM). This method of design resulted in conservative sections and was not economical. This design principle satisfies the relation $\frac{R}{S} > L$.

Where R is resistance of structural element, RS is factor of safety and L is applied external load.

In 1950's ultimate load method or load factor method was developed. In this method, using non linear stress - strain curve of concrete and steel, the resistance of the element is computed. The safety measure in the design is introduced by an appropriate choice of the load factor (ultimate

load/working load). Different load factors are assigned for different loads. Following equations are used for finding ultimate load as per IS456 - 1964

$$U = 1.5 \text{ DL} + 2.2 \text{ LL}$$

$$U = 1.5 \text{ DL} + 2.2 \text{ LL} \text{ to } 5 \text{ WL} \text{ or } 1.5 \text{ DL} + 0.5 \text{ LL} + 2.2 \text{ WL}$$

Here DL = Dead load, LL = Live load WL= wind load or earthquake load. The design principle should satisfy $R \geq LF \cdot L$ or $R \geq U$, Where, R= Resistance, LF= Load factor, L= load. Ultimate load method generally results in more slender section, but leads to larger deformation. Due to the disadvantage of larger deflection, this method was discontinued. To overcome the disadvantages of working stress method and ultimate method, a probabilistic design concept called as 'Limit state method', was developed during 1970's. IS456 -1978 recommended this method and is continued in 2000 version also. This method safeguards the risk of both collapse and unserviceability. Limit state method uses multiple safety factor format, which attempt to provide adequate safety at ultimate loads as well as to denote serviceability at service loads by considering all limit states. The acceptable limit for safety and serviceability requirements before failure or collapse is termed as 'Limit state'. Two principal limit states are considered i.e 1. Limit state of collapse 2. Limit state of serviceability. The limit state of collapse include one or more of i) flexure, II) shear, III) torsion and IV) compression the limit state of collapse is expressed as $R \geq \sum X_i L_i$ Where, X_i and L_i are partial safety factors, Here $X_i < 1$ & $L_i > 1$. The most important limit state considered in design are of deflection, other limit state of serviceability are crack and vibration. For deflection $\delta_{max} \leq \delta_{allow}$ - where deflection, l =span δ_{allow} is an integer numbers. For over all deflection δ is 250 and for short term deflection $\delta = 350$. ❖❖❖

Partial safety factor

To account for the different conditions like for material strength, load etc. Different partial factors are used for material and load. M indicate safety factor for material & for load

$$\text{Design strength} = \text{Characteristic strength} / \gamma_m$$

$$\text{Design Load} = \gamma_f \times \text{Characteristic load}$$

As per clause 36.4.2 page 68 of IS 456, $\gamma_m = 1.5$ for concrete and $\gamma_m = 1.15$ for steel. Similarly clause 36.4.1 page 68 of code gives γ_f in table 18 for different values for different load combinations and different limit states.

IS 456 - 2000 Recommendations

(i) Partial safety factors for materials to be multiplied with characteristic strength is given below.

Values of partial safety factor γ_m

Material	Limit state		
	Collapse	Deflection	Cracking
Concrete	1.5	1.0	1.3
Steel	1.15	1.0	1.0

$$\text{Design strength } f_m = \frac{f_{ck \text{ or } f_y}}{\gamma_m}$$

(ii) The code has suggested effective span to effective depth ratios as given below

Value of partial safety factors γ_f

Load combination	Ultimate limit state	Serviceability limit state
1) Dead load & live load	1.5(DL+LL)	DL+LL
2) Dead seismic/wind load	0.9DL+1.5(E2/WL)	DL + EQ/WL
a) Dead load contributes to Stability		
b) Dead load assists overturning	1.5(DL+E2/WL)	DL+EQ/wL
3) Dead, live load and Seismic/wind load	1.2(DL+LL+EQ/WL)	DL+0.7LL+0.8EQ/WL

DL-Dead load, LL- Live load WL- Wind load EQ- Earthquake load

(iii) The code has suggested effective span to effective depth ratios as given below

Basic effective span to effective depth ratio (l/d) basic

The above values are to be modified for (i) the type and amount of tension steel (Fig 4 page 38 of T5456-2000)

Basic effective span to effective depth ratio (l/d) basic

Type of beam one /slab	Span≤10m	Span>10m
1)Cantilever	7	Deflection should be Be calculated
2) Simply supported	20	(20X10)/span
3)continuous beam	26	(26X10)/span

(ii) The amount of compression steel (Fig 5 page 39 of IS456-2000)

The type of beam ie flanged beams etc (Fig 6 page 39 of IS456 - 2000).

For slabs spanning in two directions, the l/d ratio is given below.

For slabs spanning in two directions, the l/d ratio is given below

Type of slab	l/d for grade of steel	
	Fe250	Fe415
1)Simply supported	35	28
2) Continuous	40	32

Characteristic strength and loads

Limit state method is based on statistical concepts. Strength of materials and loads are highly variable in a range of values. The test in laboratory on compressive strength of concrete has indicated coefficient of variation of 10%. Hence in reinforced concrete construction, It is not practicable to specify a precise cube strength. Hence in limit state design uses the concept of 'characteristic strength' f_{ck} indicates characteristics strength of concrete & by characteristic strength of steel. In general f_k indicates the characteristic strength of material.

$$f_k = f_m - 1.646 \sigma \quad (2.6) \text{ here } f_m = \text{mean strength.}$$

Similarly 'characteristic load' is that value of load which has an accepted probability of not being exceeded during the life span of structure. In practice the load specified by IS875 - 1987 is considered as characteristic load. Equation for characteristic load is

$$L_k = L_m + 1.64 \sigma$$

WORKING STRESS METHOD DESIGN

GENERAL PRINCIPLES OF WORKING STRESS DESIGN

(a) General features

During the early part of 20th century, elastic theory of reinforced concrete sections outlined was developed which formed the basis of the working stress or permissible stress method of design of reinforced concrete members. In this method, the working or permissible stress in concrete and steel are obtained applying appropriate partial safety factors to the characteristics strength of the materials. The permissible stresses in concrete and steel are well within the linear elastic range of the materials.

The design based on the working stress method although ensures safety of the structures at working or services loads, it does not provide a realistic estimate of the ultimate or collapse load of the structure in contrast to the limit state method of design. The working stress method of design results in comparatively larger and conservative sections of the structural elements with higher quantities of steel reinforcement which results in conservative and costly design. Structural engineers have used this

method extensively during the 20th century and presently the method is incorporated as an alternative to the limit state method in Annexure -B of the recently revised Indian Standard Code Is : 456 -2000 for specific applications.

The permissible stresses in concrete under service loads for the various stress states of compressive, flexure and bond is compiled in Table 2.1 (Table 21 of IS ; 456 -2000)

Table 12.1 Permissible Shear Stresses in Concrete (τ_c N/mm²) (Table 23 of IS:456 – 2000)

100 A _s / bd	Permissible shear stresses in concrete τ_c N/mm ²					
	M15	M20	M25	M30	M35	M40 &
≤ 0.15	0.18	0.18	0.19	0.20	0.20	0.20
0.25	0.22	0.22	0.23	0.23	0.23	0.23
0.50	0.29	0.30	0.31	0.31	0.31	0.32
0.75	0.34	0.35	0.36	0.37	0.37	0.38
1.00	0.37	0.39	0.40	0.41	0.42	0.42
1.25	0.40	0.42	0.44	0.45	0.45	0.46
1.50	0.42	0.45	0.46	0.48	0.49	0.49
1.75	0.44	0.47	0.49	0.50	0.52	0.52
2.00	0.44	0.49	0.51	0.53	0.54	0.55
2.25	0.44	0.51	0.53	0.55	0.56	0.57
2.50	0.44	0.51	0.55	0.57	0.58	0.60
2.75	0.44	0.51	0.56	0.58	0.60	0.62
3.00 & above	0.44	0.51	0.57	0.60	0.62	0.63

Table 12.2 Maximum Shear Stress ($\tau_{c, \max}$ N/mm²) (Table 24 of IS: 456 – 2000)

Concrete grade ($\tau_{c, \max}$ N/mm ²)	M – 15	M – 25	M – 30	M – 35	M – 40 & above
	1.6	1.8	1.9	2.3	2.5

The permissible stress in different types of steel reinforcement is shown in table 2.2 (Table 22 of IS 456 -2000)

The permissible shear stress for various grades of concrete in beams is shown in Table 12.1 (Table 23 of IS: 456 -2000)

The maximum shear stress permissible in concrete for different grades is shown in Table 12.2 (Table 24 of IS: 456 -2000)

In the case of reinforced concrete slabs, the permissible shear stress in concrete is obtained by multiplying the values given in Table 2.1 by as shown in Table 12.3 (Section 40.2.1.1. of IS; 456 -2000)

Note: A_s is that area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used provided the detailing conforms to 26.2.3.

The maximum shear stress permissible in concrete for different grades is shown in Table 12.2 (Table 24 of IS 456 -2000)

In the case of reinforced concrete slabs, the permissible shear stress in concrete is obtained by multiplying the values in Table 2.1 by a factor shown in Table 12.3 (Section 40.2.1.1. of IS 456 -2000)

(b) General design procedure

In the working stress design, the cross -sectional dimensions are assumed based on the basic span / depth ratios outlined in Chapter 5 (Table 5.1 and 5.2) (Section 23.2.1. of IS: 456 -2000)

The working load moments and shear forces are evaluated at critical sections and the required effective depth is checked by using the relation:

$$d = \sqrt{M / Q \cdot b}$$

Where d = effective depth of section M = working load moment b = width of section

Q = a constant depending upon the working stresses in concrete and steel, neutral axis depth factor (k) and lever arm coefficient (f).

For different grades of concrete provided in Table 2.3. The depth d provided should be equal to or greater than the depth computed by the relation and the area of reinforcement required in the section to resist

The number of steel bars required is selected with due regard to the spacing of bars and cover requirements.

After complying with flexure, the section is generally checked for resistance against shear forces by calculating the nominal shear stress $\tau_v = (V/bd)$ and comparing it with the permissible shear stress τ_{vc} .

Where V = Working shear force at critical section.

The permissible shear stress τ_{vc} depends upon the percentage of reinforcement in the concrete in the cross section and grade of concrete as shown in Table 12.1

If $\tau_v < \tau_{vsuitable}$ shear reinforcements are designed in beams at a spacing s_v given by the relation;

$$S_v = [0.87 f_y A_{sv} d / V_{us}]$$

Where s_v = spacing of stirrups

A_{sv} = cross -sectional area of stirrups legs

f_y = Characteristics strength of stirrup reinforcement

d = effective depth

$$V_s = [V - \tau_c \cdot b \cdot d]$$

If $\tau_v < \tau_c$, nominal shear reinforcements are provided in beams are provided in beams at a spacing given by

$$S_v [0.87 f_y A_{st} / 0.4 b]$$

In case of slabs, the permissible shear stress Also in the case of slabs, should not exceed nominal half the τ_{cmax} shown value in

Table 12.2. In such cases the thickness of the slab is increased and the slab is redesigned.

In the case of compression members, the axial load permissible on a short column reinforced with longitudinal bars and lateral ties is given by

$$P = \tau_{cc} A (\tau_c + \tau_{sc})$$

Where τ_{cc} = permissible stress in concrete in direct compression (Refer Table 2.1)

A_c = cross -sectional area of concrete excluding the area of reinforcements.

τ_{sc} = permissible compressive stress in reinforcement

A_{sc} = cross -sectional area of longitudinal steel bars.

UNIT II DESIGN OF BEAMS

Design Problems:

1. Design a R.C beam to carry a load of 6 kN/m inclusive of its own weight on an effect span of 6m keep the breadth to be $2/3^{\text{rd}}$ of the effective depth .the permissible stressed in the concrete and steel are not to exceed 5N/mm^2 and 140 N/mm^2 .take

$m=18$. Step 1: Design constants.

❖ Modular ratio, $m = 18$.

❖ A Coefficient $n = \frac{m \cdot \sigma_{bc}}{m \cdot \sigma_{bc} + \sigma_{st}} = 0.39$

❖ Lever arm Coefficient, $j = 1 - (n/3) = 0.87$

❖ Moment of resistance Coefficient $Q = \frac{\sigma_{bc}}{2} \cdot n \cdot j = 0.84$

Step 2: Moment on the beam.

$$M = (w \cdot l^2)/8 = (6 \times 6^2)/8 = 27\text{kNm}$$

$$M = Qbd^2$$

$$d^2 = M/Qb = (27 \times 10^6) / (0.84 \times 2/3 \times d)$$

$$d = 245\text{mm}.$$

Step 3: Balanced Moment.

$M_{bal} = Qbd^2 = 0.84 \times 245 \times 365^2 = 27.41\text{kNm} > M$. it can be designed as singly reinforced section.

Step 4: Area of steel.

$$A_{st} = M_{bal} / (\sigma_{st} \cdot j \cdot d) = 616.72\text{mm}^2$$

$$\text{Use } 20\text{mm dia bars } a_{st} = \pi/4 (20^2) = 314.15\text{mm}^2$$

$$\text{No. of bars} = A_{st}/a_{st} = 616.72/314.15 = 1.96 \text{ say } 2\text{nos.}$$

Provide 2#20mm dia bars at the tension side.

2. Design a doubly reinforced beam of section 240X500mm to carry a bending moment of 80kNm. Assume clear cover at top and bottom as 30mm and take $m=18$. adopt working stress method. Assume the permissible stress in the concrete and steel are not to exceed 5N/mm^2 and 140 N/mm^2 .

Step 1: Design constants.

◆ Modular ratio, $m=18$.

◆ A Coefficient $n = \frac{m \cdot \sigma_{bc}}{m \cdot \sigma_{bc} + \sigma_{st}} = 0.39$

◆ Lever arm Coefficient, $j = 1 - (n/3) = 0.87$

◆ Moment of resistance Coefficient $Q = \frac{\sigma_{bc}}{2} \cdot n \cdot j = 0.84$

Step 2: Moment on the beam.

$$M = 80\text{kNm}$$

$$M = Qbd^2$$

$$D = 500\text{mm}, b = 240\text{mm}$$

$$d = 500 - 30\text{mm} = 470\text{mm}$$

.Step 3: Balanced Moment.

$$M_{bal} = Qbd^2 = 0.84 \times 240 \times 470^2 = 44.53\text{kNm} < M. \text{ it can be designed as doubly reinforced section.}$$

Step 4: Area of Tension steel.

$$A_{st} = A_{st1} + A_{st2}$$

$$A_{st1} = \frac{M_{bal}}{(\sigma_{st} \cdot j \cdot d)} = \frac{44.53 \times 10^6}{(140 \times 0.87 \times 470)} = 777.87\text{mm}^2$$

$$\text{Use 20mm dia bars } a_{st} = \frac{\pi}{4} (20^2) = 314.15\text{mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{777.87}{314.15} = 2.47 \text{ say 3nos.}$$

$$A_{st2} = \frac{(M - M_{bal})}{(\sigma_{st} \cdot (d - d^1))} = \frac{(80 \times 10^6 - 44.53 \times 10^6)}{(140 \times (470 - 30))} = 575.8\text{mm}^2$$

$$\text{Use 20mm dia bars } a_{st} = \frac{\pi}{4} (20^2) = 314.15\text{mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{575.8}{314.15} = 1.8 \text{ say 2nos.}$$

Step 5: Area of Compression steel:

$$A_{sc} = \frac{(M - M_{bal})}{(\sigma_{sc} \cdot (d - d^1))} = \frac{(80 \times 10^6 - 44.53 \times 10^6)}{(51.8 \times (470 - 30))} = 1580.65\text{ mm}^2$$

Use 20mm dia bars $a_{st} = \pi/4 (20^2) = 314.15\text{mm}^2$

No. of bars = $A_{st}/a_{st} = 1580.65/314.15 = 5.5$ say 6nos.

Provide 6#20mm dia bars as compression reinforcement.

3. Design a beam subjected to a bending moment of 40kNm by working stress design. Adopt width of beam equal to half the effective depth.

Assume the permissible stressed in the concrete and steel are not to exceed 5N/mm^2 and 140 N/mm^2 . take $m=18$.

Step 1: Design constants.

◆ Modular ratio, $m=18$.

◆ A Coefficient $n = \frac{m \cdot \sigma_{bc}}{m \cdot \sigma_{bc} + \sigma_{st}} = 0.39$

Lever arm Coefficient, $j = 1 - (n/3) = 0.87$

Moment of resistance Coefficient $Q = \frac{\sigma_{bc}}{2} \cdot n \cdot j = 0.84$

Step 2: Moment on the beam.

$M = 40\text{kNm}$

$M = Qbd^2$

$d^2 = M/Qb = (40 \times 10^6) / (0.84 \times 1/2 \times d)$

$d = 456.2$ say 460 mm.

$b = \frac{1}{2} d = 0.5 \times 460 = 230\text{mm}$

Step 3: Balanced Moment.

$M_{bal} = Qbd^2 = 0.84 \times 230 \times 460^2 = 40.88\text{kNm}$. $> M$. it can be designed as singly reinforced section.

Step 4: Area of steel.

$A_{st} = \frac{M_{bal}}{\sigma_{st} \cdot j \cdot d} = \frac{(40.88 \times 10^6)}{(140 \times 0.87 \times 460)} = 729.64\text{mm}^2$

Use 20mm dia bars $a_{st} = \pi/4 (20^2) = 314.15\text{mm}^2$

No. of bars = $A_{st}/a_{st} = 729.64/314.15 = 2.96$ say 3nos.

Provide 3#20mm dia bars at the tension side.

4. Determine the moment of resistance of a singly reinforced beam 160X300mm effective section, if the stress in steel and concrete are not to exceed 140N/mm^2 and 5N/mm^2 . effective span of the beam is 5m and the beam carries 4 nos of 16mm dia bars. Take $m=18$. find also the minimum load the beam can carry. Use WSD method.

Step 1: Actual NA.

$$b x a^2/2 = m.A_{st}.(d - x_a)$$

$$160. x_a^2/2 = 18 \times 804.24(300 - x_a)$$

$$X_a = 159.42\text{mm}$$

Step 2: Critical NA.

$$x_c = \frac{\sigma_{bc}.d}{(\sigma_{st}/m + \sigma_{bc})} = 117.39\text{mm} < X_a = 159.42\text{mm}$$

it is Over reinforced Section.

Step 3: Moment of Resistance

$$M = (b. x_a/2 \cdot \sigma_{bc}) (d - x_a/3) = (160 \times 159.42/2 \times 5)(300 - 159.42/3) = 15.74\text{kNm}$$

Step 4: Safe load.

$$M = (w.L^2)/8$$

$$W = (8 \times 15.74)/5^2 = 5.03 \text{ kN/m}$$

5. Design an interior panel of RC slab 3mX6m size, supported by wall of 300mm thick. Live load on the slab is 2.5kN/m^2 . the slab carries 100mm thick lime concrete (density 19kN/m^3). Use M15 concrete and Fe 415 steel. (NOV-DEC 2009)

Step 1: Type of Slab.

$$l_y/l_x = 6/3 = 2 = 2. \text{ it has to be designed as two way slab.}$$

Step 2: Effective depth calculation.

For Economic consideration adopt shorter span to design the slab.

$$d = \text{span}/(\text{basic value} \times \text{modification factor}) = 3000/(20 \times 0.95) = 270\text{mm} \quad D = 270 + 20 + 10/2 = 295\text{mm}$$

Step 3: Effective Span. For shorter span:

$$L_e = \text{clear span} + \text{effective depth} = 3000 + 270 = 3.27\text{m (or)} \quad L_e = \text{c/c distance b/w supports} = 3000 + 2(230/2) = 3.23\text{m}$$

Adopt effective span = 3.23m least value. For longer span:

$$L_e = \text{clear span} + \text{effective depth} = 6000 + 270 = 6.27\text{m (or)} \quad L_e = \text{c/c distance b/w supports} = 6000 + 2(230/2) = 6.23\text{m} \quad \text{Adopt effective span} = 6.23\text{m least value.}$$

Step 4: load calculation Live load = 2.5kN/m^2

$$\text{Dead load} = 1 \times 1 \times 0.27 \times 25 = 6.75\text{kN/m}^2$$

$$\text{Dead load} = 1 \times 1 \times 0.1 \times 19 = 1.9\text{kN/m}^2$$

$$\text{Floor Finish} = 1\text{kN/m}^2$$

$$\text{Total load} = 12.15\text{kN/m}^2$$

$$\text{Factored load} = 12.15 \times 1.5 = 18.225\text{kN/m}^2$$

Step 5: Moment calculation.

$$M_x = \gamma_x \cdot w \cdot l_x \cdot 0.103 \times 18.225 \times 3.23 = 9.49\text{kNm}$$

$$M_y = \gamma_y \cdot w \cdot l_y \cdot 0.048 \times 18.225 \times 3.23 = 4.425\text{kNm}$$

Step 6: Check for effective depth.

$$M = Qbd^2$$

$$d^2 = M/Qb = 9.49/2.76 \times 1 = 149.39\text{mm say } 150\text{mm. For design consideration adopt } d = 150\text{mm.}$$

Step 7: Area of Steel. For longer span:

$$M_u = 0.87 f_y A_{st} d (1 - (f_y A_{st})/(f_{ck} b d))$$

$$4.425 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 (1 - (415 A_{st})/(20 \times 1000 \times 150)) \quad A_{st} = 180\text{mm}^2$$

Use 10mm dia bars

Spacing ,S = $a_{st}/A_{st} \times 1000 = (78.53/300)1000 = 261\text{mm}$ say 260mm/c Provide 10mm dia @260mm c/c.

For shorter span:

$$\mu = 0.87 f_y A_{st} d (1 - (f_y a_{st})/(f_{ck} b d))$$

$$9.49 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 (1 - (415 A_{st})/(20 \times 1000 \times 150)) \quad A_{st} = 200\text{mm}^2$$

Use 10mm dia bars

Spacing ,S = $a_{st}/A_{st} \times 1000 = (78.53/300)1000 = 281\text{mm}$ say 300mm/c Provide 10mm dia @300mm c/c.

6. A reinforced concrete rectangular section 300 mm wide and 600 mm overall depth is reinforced with 4 bars of 25 mm diameter at an effective cover of 50 mm on the tension side. The beam is designed with M 20 grade concrete and Fe 415 grade steel. Determine the allowable bending moment and the stresses developed in steel and concrete under this moment. Use working stress method.

Step 1: Actual NA.

$$b x a^2/2 = m.A_{st}.(d - x_a)$$

$$300. x a^2/2 = 18 \times 1963.50(550 - x_a) \quad X_a = 117.81\text{mm}$$

Step 2: Critical NA.

$$x_c = \frac{f_{st} d}{f_{st} + f_{cbc}} = 194.66\text{mm} > X_a = 117.81\text{mm} \text{ it is Under reinforced Section.}$$

Step 3: Moment of Resistance For steel:

$$M = (A_{st} \cdot f_{st}) (d - x_a/3) = (1963.5 \times 230)(550 - 117.81/3) = 230.64\text{kNm} \text{ For concrete:}$$

$$M = (b \cdot x_a/2 \cdot f_{cbc}) (d - x_a/3) = (300 \times 117.81/2 \times 7)(550 - 117.81/3) = 63.17\text{kNm}$$

Type of Beam

A structural member that support transverse (Perpendicular to the axis of the member) load is called a beam. Beams are subjected to bending moment and shear force. Beams are also known as flexural or bending members. In a beam one of the dimensions is very large compared to the other two dimensions. Beams may be of the following types:

- a. Singly or doubly reinforced rectangular beams
- b. Singly or doubly reinforced T-beams
- c. Singly or doubly reinforced L-beams

a. **Singly or doubly reinforced rectangular beams**

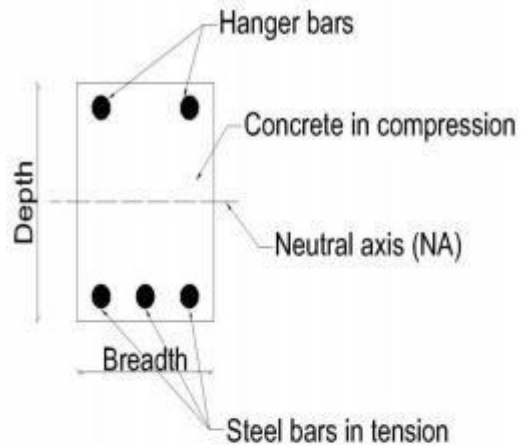


Fig 1: Singly reinforced rectangular beam

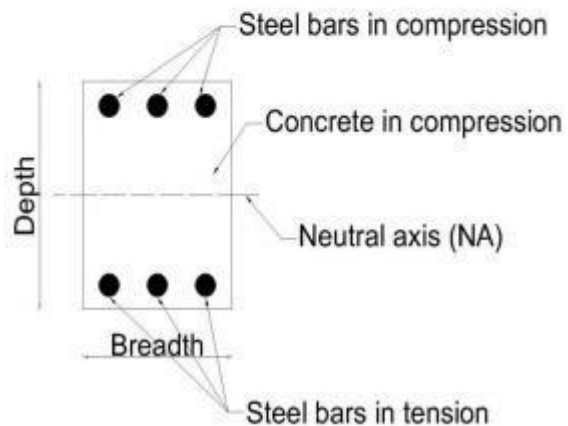


Fig 2: Doubly reinforced rectangular beam

- b. **Singly or doubly reinforced T-beams**

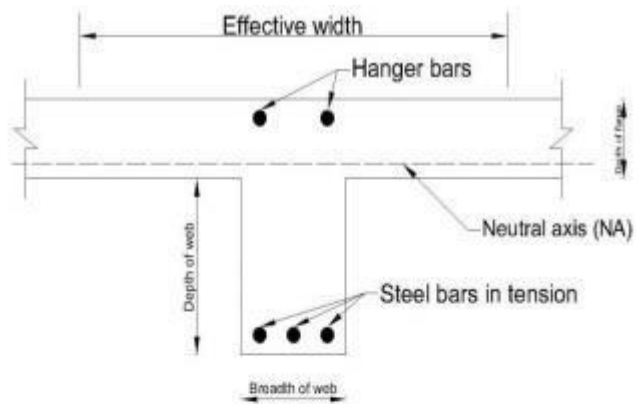


Fig 3: Singly reinforced T beam

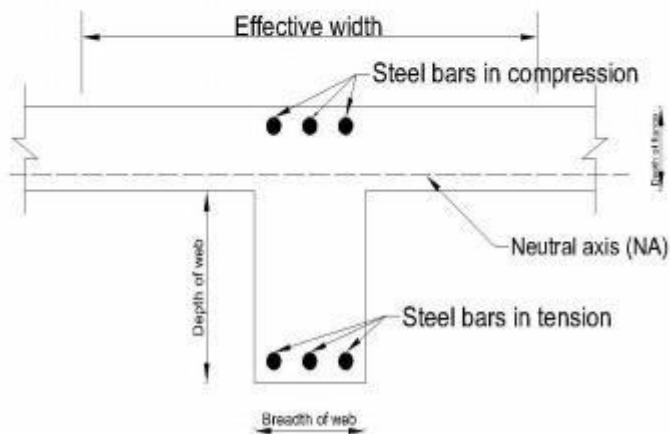


Fig 4: Doubly reinforced T beam

c. Singly or doubly reinforced L-beams

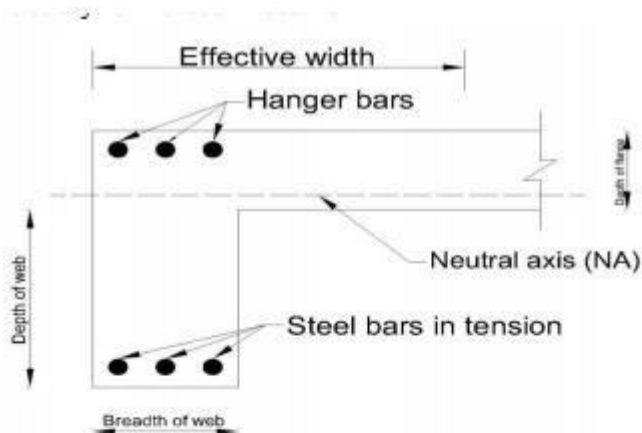


Fig 5: Singly reinforced L beam

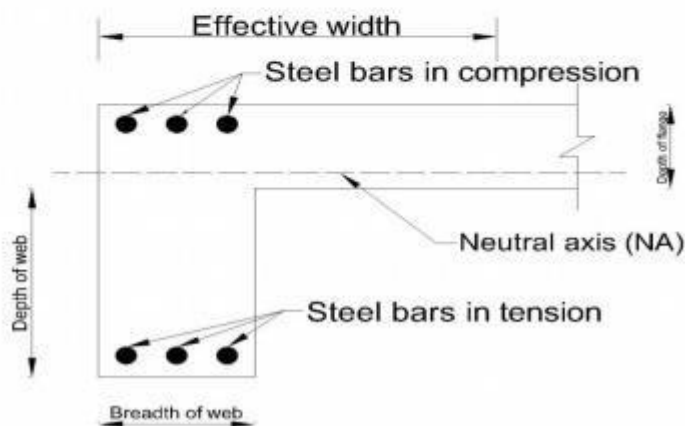


Fig 6: Doubly reinforced L beam

General specification for flexure design of beams

Beams are designed on the basis of limit state of collapse in flexure and checked for other limit states of shear, torsion and serviceability. To ensure safety the resistance to bending, shear, torsion and axial loads at every section should be greater than the appropriate values at that produced by the probable most unfavourable combination of loads on the structure using the appropriate safety factors. The following general specifications and practical requirements are necessary for designing the reinforced cement concrete beams.

- a. Selection of grade of concrete

Apart from strength and deflection, durability shall also be considered to select the grade of concrete to be used. Table 5 of IS 456:2000 shall be referred for the grade of concrete to be used. In this table the grade of concrete to be used is recommended based on the different environmental exposure conditions.

b. Selection of grade of steel

Normally Fe 250, Fe 415 and Fe 500 are used. In earthquake zones and other places where there are possibilities of vibration, impact, blast etc, Fe 250 (mild steel) is preferred as it is more ductile.

c. Size of the beam

The size of the beam shall be fixed based on the architectural requirements, placing of reinforcement, economy of the formwork, deflection, design moments and shear. In addition, the depth of the beam depends on the clear height below the beam and the width depends on the thickness of the wall to be constructed below the beam. The width of the beam is usually equal to the width of the wall so that there is no projection or offset at the common surface of contact between the beam and the wall.

The commonly used widths of the beam are 115 mm, 150 mm, 200 mm, 230 mm, 250 mm, 300 mm.

d. Cover to the reinforcement

Cover is the certain thickness of concrete provided all round the steel bars to give adequate protection to steel against fire, corrosion and other harmful elements present in the atmosphere. It is measured as distance from the outer concrete surface to the nearest surface of steel. The amount of cover to be provided depends on the condition of exposure and shall be as given in the Table 16 of IS 456:2000. The cover shall not be less than the diameter of the bar.

e. Spacing of the bars

The details of spacing of bars to be provided in beams are given in clause 26.3.2 of IS 456. As per this clause the following shall be considered for spacing of bars.

The horizontal distance between two parallel main bars shall usually be not less than the greatest of the following

- i. Diameter of the bar if the diameters are equal
- ii. The diameter of the larger bar if the diameters are unequal
- iii. 5mm more than the nominal maximum size of coarse aggregate

Greater horizontal spacing than the minimum specified above should be provided wherever possible. However when needle vibrators are used, the horizontal distance between bars of a group may be reduced to two thirds the nominal maximum size of the coarse aggregate, provided that sufficient space is left between groups of bars to enable the vibrator to be immersed.

Where there are 2 or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be of the greatest of the following

- i. 15 mm
- ii. Maximum size of aggregate
- iii. Maximum size of bars

Maximum distance between bars in tension in beams:

The maximum distance between parallel reinforcement bars shall not be greater than the values given in table 15 of IS 456:2000.

General Aspects of Serviceability:

The members are designed to withstand safely all loads liable to act on it throughout its life using the limit state of collapse. These members designed should also satisfy the serviceability limit states. To satisfy the serviceability requirements the deflections and cracking in the member should not be excessive and shall be less than the permissible values. Apart from this the other limit states are that of the durability and vibrations. Excessive values beyond this limit state spoil the appearance of the structure and affect the partition walls, flooring etc. This will cause the user discomfort and the structure is said to be unfit for use.

The different load combinations and the corresponding partial safety factors to be used for the limit state of serviceability are given in Table 18 of IS 456:2000.

Limit state of serviceability for flexural members

The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

Limit state of serviceability for flexural members:

Deflection

The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

1 Empirical Method

In this method, the deflection criteria of the member is said to be satisfied when the actual value of span to depth ratio of the member is less than the permissible values. The IS code procedure for calculating the permissible values are as given below

- a. Choosing the basic values of span to effective depth ratios (l/d) from the following, depending on the type of beam

Cantilever = 8

Simply supported = 20

Continuous = 26

- b. Modify the value of basic span to depth ratio to get the allowable span to depth ratio.

$$\text{Allowable } l/d = \text{Basic } l/d \times M_t \times M_c \times M_f$$

Where, M_t = Modification factor obtained from fig 4 IS 456:2000. It depends on the area of tension reinforcement provided and the type of steel.

M_c = Modification factor obtained from fig 5 IS 456:2000. This depends on the area of compression steel used.

M_f = Reduction factor got from fig 6 of IS 456:2000

Note: The basic values of l/d mentioned above is valid upto spans of 10m. The basic values are multiplied by 10 / span in meters except for cantilever. For cantilevers whose span exceeds 10 m the theoretical method shall be used.

2 Theoretical method of checking deflection

The actual deflections of the members are calculated as per procedure given in annexure 'C' of IS 456:2000. This deflection value shall be limited to the following

- i. The final deflection due to all loads including the effects of temperature, creep and shrinkage shall not exceed span / 250.
- ii. The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes shall not exceed span/350 or 20 mm whichever is less.

Cracking in structural members

Cracking of concrete occurs whenever the tensile stress developed is greater than the tensile strength of concrete. This happens due to large values of the following:

1. Flexural tensile stress because of excessive bending under the applied load
2. Diagonal tension due to shear and torsion
3. Direct tensile stress under applied loads (for example hoop tension in a circular tank)

4. Lateral tensile strains accompanying high axis compressive strains due to Poisson's effect (as in a compression test)
5. Settlement of supports

In addition to the above reasons, cracking also occurs because of

1. Restraint against volume changes due to shrinkage, temperature creep and chemical effects.
2. Bond and anchorage failures

Cracking spoils the aesthetics of the structure and also adversely affect the durability of the structure. Presence of wide cracks exposes the reinforcement to the atmosphere due to which the reinforcements get corroded causing the deterioration of concrete. In some cases, such as liquid retaining structures and pressure vessels cracks affects the basic functional requirement itself (such as water tightness in water tank).

Permissible crack width

The permissible crack width in structural concrete members depends on the type of structure and the exposure conditions. The permissible values are prescribed in clause 35.3.2 IS 456:2000 and are shown in table below

Table: Permissible values of crack width as per IS 456:2000

Table: Permissible values of crack width as per IS 456:2000

No.	Types of Exposure	Permissible widths of crack at surface (mm)
1	Protected and not exposed to aggressive environmental conditions	0.3
2	Moderate environmental conditions	0.2

Control of cracking

The check for cracking in beams are done through the following 2 methods specified in IS 456:2000 clause 43.1

1. By empirical method:

In this method, the cracking is said to be in control if proper detailing (i.e. spacing) of reinforcements as specified in clause 26.3.2 of IS 456:2000 is followed. These specifications regarding the spacing have been already discussed under heading general specifications. In addition, the following specifications shall also be considered

- i. In the beams where the depth of the web exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less. (Refer clause 25.5.1.3 IS456:2000)

- ii. The minimum tension reinforcement in beams to prevent failure in the tension

zone by cracking of concrete is given by the following

$$A_s = 0.85 f_y / 0.87 f_y \quad (\text{Refer clause 26.5.1.1 IS 456:2000})$$

- iii. Provide large number of smaller diameter bars rather than large diameter bars of the same area. This will make the bars well distributed in the tension zone and will reduce the width of the cracks.

2. By crack width computations

In the case of special structures and in aggressive environmental conditions, it is preferred to compute the width of cracks and compare them with the permissible crack width to ensure the safety of the structure at the limit state of serviceability. The IS 456-2000 has specified an analytical method for the estimation of surface crack width in Annexure-F which is based on the British Code (BS : 8110) specifications where the surface crack width is less than the permissible width, the crack control is said to be satisfied.

UNIT-4

Design of Columns

Pre-requisite Discussion:

A column is defined as a compression member, the effective length of which exceeds three times the least lateral dimension. Compression members, whose lengths do not exceed three times the least lateral dimension, may be made of plain concrete. A column forms a very important component of a structure. Columns support beams which in turn support walls and slabs. It should be realized that the failure of a column results in the collapse of the structure. The design of a column should therefore receive importance.

Introduction:

A column is a vertical structural member supporting axial compressive loads, with or without moments. The cross-sectional dimensions of a column are generally considerably less than its height. Columns support vertical loads from the floors and roof and transmit these loads to the foundations.

The more general terms compression members and members subjected to combined axial load and bending are sometimes used to refer to columns, walls, and members in concrete trusses or frames. These may be vertical, inclined, or horizontal. A column is a special case of a compression member that is vertical. Stability effects must be considered in the design of compression members.

Classification of columns

A column may be classified based on different criteria such as:

1. Based on shape

- Rectangle
- Square
- Circular
- Polygon
- L type
- T type
- + type



2. Based on slenderness ratio or height

Short column and Long column or Short and Slender Compression Members

A compression member may be considered as short when both the slenderness ratios namely l_{ex}/D and l_{ey}/b are less than 12: Where

l_{ex} = effective length in respect of the major axis, D = depth in respect of the major axis,

l_{ey} = effective length in respect of the minor axis, and b = width of the member.

It shall otherwise be considered as a slender or long compression member.

The great majority of concrete columns are sufficiently stocky (short) that slenderness can be ignored. Such columns are referred to as short columns. Short column generally fails by crushing of concrete due to axial force. If the moments induced by slenderness effects weaken a column appreciably, it is referred to as a slender column or a long column. Long columns generally fail by bending effect than due to axial effect. Long column carry less load compared to long column.

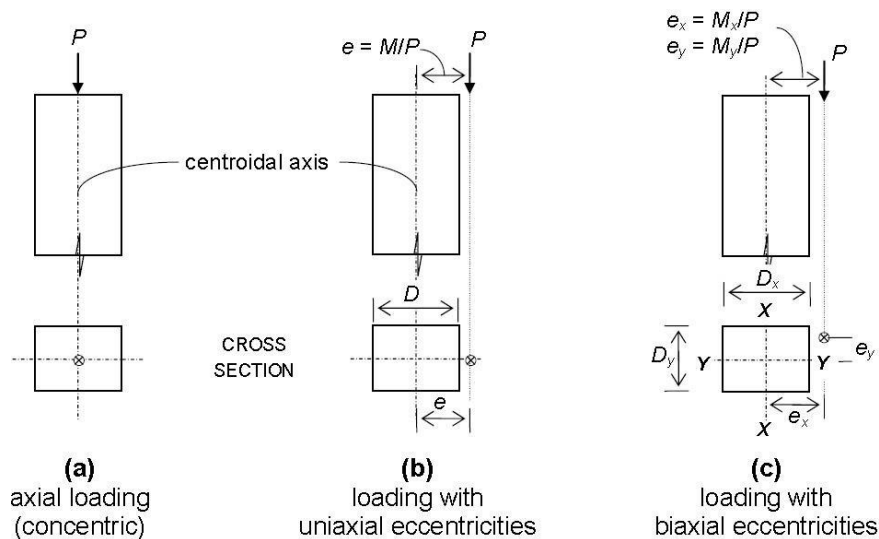
3. Based on pattern of lateral reinforcement

- Tied columns with ties as laterals
- columns with Spiral steel as laterals or spiral columns

Majority of columns in any buildings are tied columns. In a tied column the longitudinal bars are tied together with smaller bars at intervals up the column. Tied columns may be square, rectangular, L-shaped, circular, or any other required shape. Occasionally, when high strength and/or high ductility are required, the bars are placed in a circle, and the ties are replaced by a bar bent into a helix or spiral. Such a column, called a spiral column. Spiral columns are generally circular, although square or polygonal shapes are sometimes used. The spiral acts to restrain the lateral expansion of the column core under high axial loads and, in doing so, delays the failure of the core, making the column more ductile. Spiral columns are used more extensively in seismic regions. If properly designed, spiral column carry 5% extra load at failure compared to similar tied column.

4. Based on type of loading

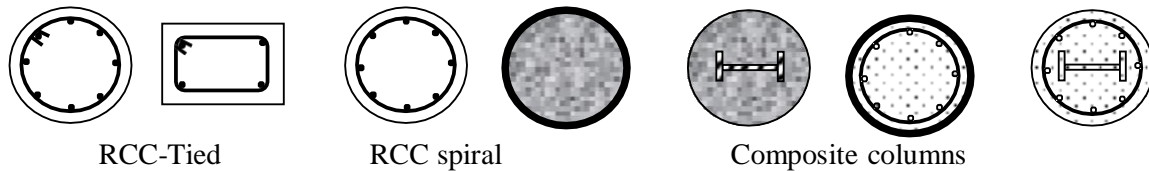
- Axially loaded column or centrally or concentrically loaded column (P_u)
- A column subjected to axial load and uniaxial bending ($P_u + M_{ux}$) or ($P + M_{uy}$)
- A column subjected to axial load and biaxial bending ($P_u + M_{ux} + M_{uy}$)



Different loading situations in columns

5. Based on materials

Timber, stone, masonry, RCC, PSC, Steel, aluminium , composite column



Behavior of Tied and Spiral Columns

Figure shows a portion of the core of a spiral column. Under a compressive load, the concrete in this column shortens longitudinally under the stress and so, to satisfy Poisson's ratio, it expands laterally. In a spiral column, the lateral expansion of the concrete inside the spiral (referred to as the core) is restrained by the spiral. This stresses the spiral in tension. For equilibrium, the concrete is subjected to lateral compressive stresses. In a tied column in a non seismic region, the ties are spaced roughly the width of the column apart and, as a result, provide relatively little lateral restraint to the core. Outward pressure on the sides of the ties due to lateral expansion of the core merely bends them outward, developing an insignificant hoop-stress effect. Hence, normal ties have little effect on the strength of the core in a tied column. They do, however, act to reduce the unsupported length of the longitudinal bars, thus reducing the danger of buckling of those bars as the bar stress approaches yield. load-deflection diagrams for a tied column and a spiral column subjected to axial loads is shown in figure. The initial parts of these diagrams are similar. As the maximum load is reached, vertical cracks and crushing develop in the concrete shell outside the ties or spiral, and this concrete spalls off. When this occurs in a tied column, the capacity of the core that remains is less than the load on the column. The concrete core is crushed, and the reinforcement buckles outward between ties. This occurs suddenly, without warning, in a brittle manner. When the shell spalls off a spiral column, the column does not fail immediately because the strength of the core has been enhanced by the triaxial stresses resulting from the effect of the spiral reinforcement. As a result, the column can undergo large deformations, eventually reaching a second maximum load, when the spirals yield and the column finally collapses. Such a failure is much more ductile than that of a tied column and gives warning of the impending failure, along with possible load redistribution to other members. Due to this, spiral column carry little more load than the tied column to an extent of about 5%. Spiral columns are used when ductility is important or where high loads make it economical to utilize the extra strength. Both columns are in the same building and have undergone the same deformations. The tied column has failed completely, while the spiral column, although badly damaged, is still supporting a load. The very minimal ties were inadequate to confine the core concrete. Had the column ties been detailed according to ACI Code, the column will perform better as shown.

Specifications for covers and reinforcement in column

For a longitudinal reinforcing bar in a column nominal cover shall in any case not be less than 40 mm, or less than the diameter of such bar. In the case of columns of minimum dimension of 200 mm or under, whose reinforcing bars do not exceed 12 mm, a nominal cover of 25 mm may be used. For footings minimum cover shall be 50 mm.

Nominal Cover in mm to meet durability requirements based on exposure

Mild 20, Moderate 30, Severe 45, Very severe 50, Extreme 75

Nominal cover to meet specified period of fire resistance for all fire rating 0.5 to 4 hours is 40 mm for columns only



Effective length of compression member






Column or strut is a compression member, the effective length of which exceeds three times the least lateral dimension. For normal usage assuming idealized conditions, the effective length of in a given plane may be assessed on the basis of Table 28 of IS: 456-2000. Following terms are required.

Following are the end restraints:

- Effectively held in position and restrained against rotation in both ends
- Effectively held in position at both ends, restrained against rotation at one end
- Effectively held in position at both ends, but not restrained against rotation
- Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position
- Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position
- Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position
- Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end

Table.Effective length of compression member

Sl. No.	Degree of End Restraint of Compression Members	Figure	Theo. Value of Effective Length	Reco. Value of Effective Length
1	Effectively held in position and restrained against rotation in both ends		0.50 l	0.65l
2	Effectively held in position at both ends, restrained against rotation at one end		0.70 l	0.80l

3	Effectively held in position at both ends, but not restrained against rotation		1.0 l	1.0l
4	Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position		1.0 l	1.20l
5	Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		-	1.5l
6	Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position		2.0 l	2.0l
7	Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end		2.0 l	2.0l

Unsupported Length

The unsupported length, l , of a compression member shall be taken as the clear distance between end restraints (visible height of column). Exception to this is for flat slab construction, beam and slab construction, and columns restrained laterally by struts (Ref. IS:456-2000),

Slenderness Limits for Columns

The unsupported length between end restraints shall not exceed 60 times the least lateral dimension of a column.

If in any given plane, one end of a column is unrestrained, its unsupported length, l , shall not exceed $100b^2/D$, where b = width of that cross-section, and D = depth of the cross-section measured in the plane under consideration.

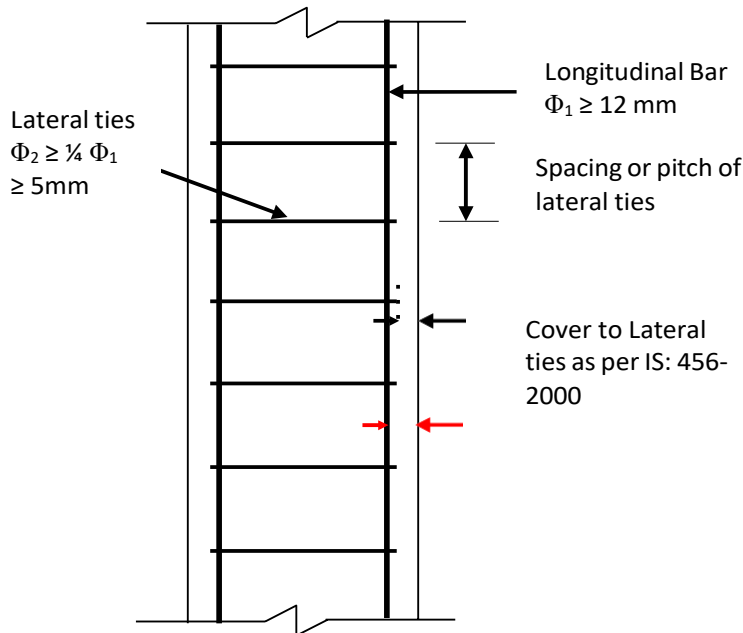
Specifications as per IS: 456-2000

Longitudinal reinforcement

1. The cross-sectional area of longitudinal reinforcement, shall be not less than 0.8 percent nor more than 6 percent of the gross cross sectional area of the column.
2. NOTE - The use of 6 percent reinforcement may involve practical difficulties in placing and compacting of concrete; hence lower percentage is recommended. Where

bars from the columns below have to be lapped with those in the column under consideration, the percentage of steel shall usually not exceed 4 percent.

3. In any column that has a larger cross-sectional area than that required to support the load, the minimum percentage of steel shall be based upon the area of concrete required to resist the direct stress and not upon the actual area.
4. The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and six in circular columns.
5. The bars shall not be less than 12 mm in diameter
6. A reinforced concrete column having helical reinforcement shall have at least six bars of longitudinal reinforcement within the helical reinforcement.
7. In a helically reinforced column, the longitudinal bars shall be in contact with the helical reinforcement and equidistant around its inner circumference.
8. Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.
9. In case of pedestals in which the longitudinal reinforcement is not taken in account in strength calculations, nominal longitudinal reinforcement not less than 0.15 percent of the cross-sectional area shall be provided.



Transverse reinforcement

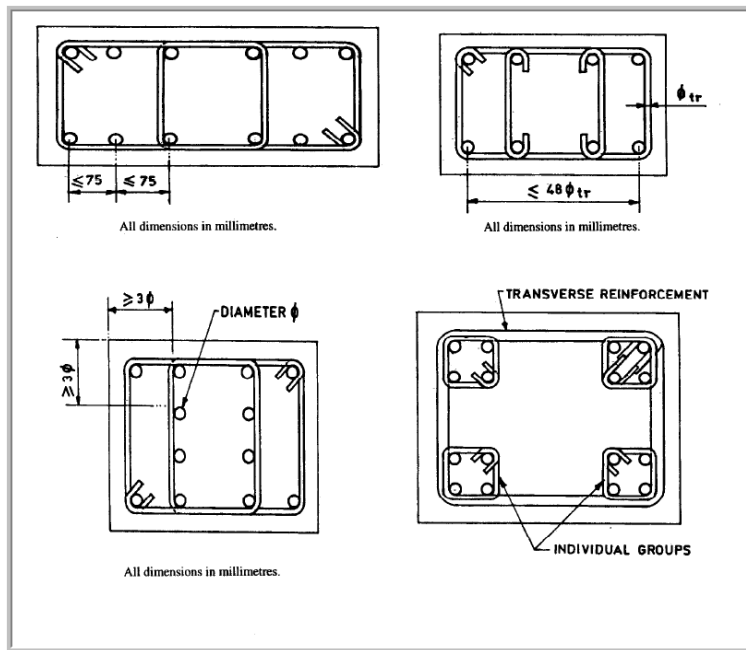
A reinforced concrete compression member shall have transverse or helical reinforcement so disposed that every longitudinal bar nearest to the compression face has effective lateral support against buckling.

The effective lateral support is given by transverse reinforcement either in the form of circular rings capable of taking up circumferential tension or by polygonal links (lateral ties) with internal angles not exceeding 135° . The ends of the transverse reinforcement shall be properly anchored.

Arrangement of transverse reinforcement

If the longitudinal bars are not spaced more than 75 mm on either side, transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports (Ref. IS:456).

If the longitudinal bars spaced at a distance of not exceeding 48 times the diameter of the tie are effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction by open ties (Ref. IS:456).



Pitch and diameter of lateral ties

- 1) Pitch-The pitch of transverse reinforcement shall be not more than the least of the following distances:
 - i) The least lateral dimension of the compression members;
 - ii) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
 - iii) 300 mm.
- 2) Diameter-The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

Helical reinforcement

- 1) Pitch-Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and a half extra turns of the

spiral bar. Where an increased load on the column on the strength of the helical reinforcement is allowed for, the pitch of helical turns shall be not more than 7.5 mm, nor more than one-sixth of the core diameter of the column, nor less than 25 mm, nor less than three times the diameter of the steel bar forming the helix.

LIMIT STATE OF COLLAPSE: COMPRESSION

Assumptions

1. The maximum compressive strain in concrete in axial compression is taken as 0.002.
2. The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

In addition the following assumptions of flexure are also required

3. Plane sections normal to the axis remain plane after bending.
4. The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
5. The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test.
6. An acceptable stress strain curve is given in IS:456-200. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor γ of 1.5 shall be applied in addition to this.
7. The tensile strength of the concrete is ignored.
8. The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in IS:456-2000. For design purposes the partial safety factor equal to 1.15 shall be applied.

Minimum eccentricity

As per IS:456-2000, all columns shall be designed for minimum eccentricity, equal to the unsupported length of column/ 500 plus lateral dimensions/30, subject to a minimum of 20 mm. Where bi-axial bending is considered, it is sufficient to ensure that eccentricity exceeds the minimum about one axis at a time.

Short Axially Loaded Members in Compression

The member shall be designed by considering the assumptions given in 39.1 and the minimum eccentricity. When the minimum eccentricity as per 25.4 does not exceed 0.05 times the lateral dimension, the members may be designed by the following equation:

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

P_u = axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c = area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_s = area of longitudinal reinforcement for columns.

Compression Members with Helical Reinforcement

The strength of compression members with helical reinforcement satisfying the requirement of IS: 456 shall be taken as 1.05 times the strength of similar member with lateral ties.

The ratio of the volume of helical reinforcement to the volume of the core shall not be less than

$$V_{hs} / V_c > 0.36 (A_g/A_c - 1) f_{ck}/f_y$$

A_g = gross area of the section,

A_c = area of the core of the helically reinforced column measured to the outside diameter of the helix,

f_{ck} = characteristic compressive strength of the concrete, and

f_y = characteristic strength of the helical reinforcement but not exceeding 415 N/mm.

Members Subjected to Combined Axial Load and Uni-axial Bending

Use of Non-dimensional Interaction Diagrams as Design Aids

Design Charts (for Uniaxial Eccentric Compression) in SP-16

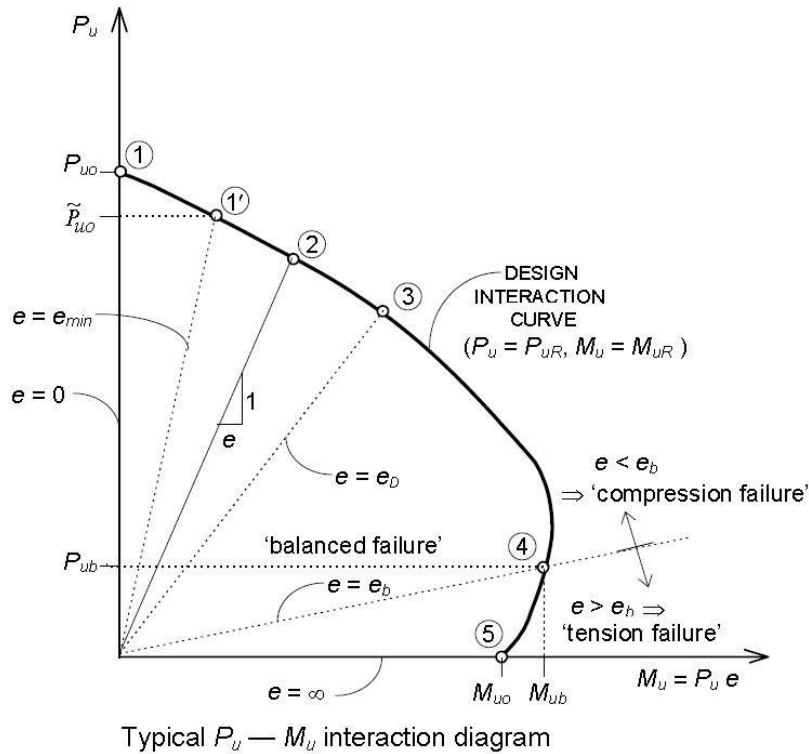
The design Charts (non-dimensional interaction curves) given in the Design Handbook, SP : 16 cover the following three cases of symmetrically arranged reinforcement :

- (a) Rectangular sections with reinforcement distributed equally on two sides (Charts 27 – 38): the ‘two sides’ refer to the sides parallel to the axis of bending; there are no inner rows of bars, and each outer row has an area of $0.5A_s$ this includes the simple 4-bar configuration.
- (b) Rectangular sections with reinforcement distributed equally on four sides (Charts 39 – 50): two outer rows (with area $0.3A_s$ each) and four inner rows (with area $0.1A_s$ each) have been considered in the calculations ; however, the use of these Charts can be extended, without significant error, to cases of not less than two inner rows (with a minimum area $0.3A_s$ in each outer row).
- (c) Circular column sections (Charts 51 – 62): the Charts are applicable for circular sections with at least six bars (of equal diameter) uniformly spaced circumferentially.

Corresponding to each of the above three cases, there are as many as 12 Charts available covering the 3 grades of steel (Fe 250, Fe 415, Fe 500), with 4 values of d'/D ratio for each grade (namely 0.05, .0.10, 0.15, 0.20). For intermediate values of d'/D , linear interpolation may be done. Each of the 12 Charts of SP-16 covers a family of non-dimensional design interaction curves with p/f_{ck} values ranging from 0.0 to 0.26.

From this, percentage of steel (p) can be found. Find the area of steel and provide the required number of bars with proper arrangement of steel as shown in the chart.

Typical interaction curve



Salient Points on the Interaction Curve

The salient points, marked 1 to 5 on the interaction curve correspond to the failure strain profiles, marked 1 to 5 in the above figure.

- The point 1 in figure corresponds to the condition of axial loading with $e = 0$. For this case of 'pure' axial compression.
- The point 1' in figure corresponds to the condition of axial loading with the mandatory minimum eccentricity e_{min} prescribed by the Code.
- The point 3 in figure corresponds to the condition $x_u = D$, i.e., $e = e_D$. For $e < e_D$, the entire section is under compression and the neutral axis is located outside the section ($x > D$), with $0.002 < \epsilon_{cu} < 0.0035$. For $e > e_D$, the NA is located within the section ($x < D$) and $\epsilon_{cu} = 0.0035$ at the 'highly compressed edge'.
- The point 4 in figure corresponds to the balanced failure condition, with $e = e_b$ and $x_u = x_{u,b}$. The design strength values for this 'balanced failure' condition are denoted as P_{ub} and M_{ub} .
- The point 5 in figure corresponds to a 'pure' bending condition ($e = \infty$, $P_{uR} = 0$); the resulting ultimate moment of resistance is denoted M_{uo} and the corresponding NA depth takes on a minimum value $x_{u,min}$.

Procedure for using of Non-dimensional Interaction Diagrams as Design Aids to find steel

Given:

Size of column, Grade of concrete, Grade of steel (otherwise assume suitably)

Factored load and Factored moment

Assume arrangement of reinforcement: On two sides or on four sides

Assume moment due to minimum eccentricity to be less than the actual moment

Assume suitable axis of bending based on the given moment (xx or yy)

Assuming suitable diameter of longitudinal bars and suitable nominal cover

1. Find d^1/D from effective cover d^1
2. Find non dimensional parameters $P_u/f_{ck}bD$ and $M_u/f_{ck}bD^2$
3. Referring to appropriate chart from S-16, find p/f_{ck} and hence the percentage of reinforcement, p
4. Find steel from, $A_s = p bD/100$
5. Provide proper number and arrangement for steel
6. Design suitable transverse steel
7. Provide neat sketch

Members Subjected to Combined Axial Load and Biaxial Bending

The resistance of a member subjected to axial force and biaxial bending shall be obtained on the basis of assumptions given in IS:456 with neutral axis so chosen as to satisfy the equilibrium of load and moments about two axes. Alternatively such members may be designed by the following equation:

$$[M_{ux}/M_{uxl}]^{\alpha_n} + [M_{uy}/M_{uy1}]^{\alpha_n} \leq 1, \text{ where}$$

M_{ux} and M_y = moments about x and y axes due to design loads,

M_{uxl} and M_{y1} = maximum uni-axial moment capacity for an axial load of P_u bending about x and y axes respectively, and α_n is related to P_u/P_{uz} , where $P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y A_{sc}$

For values of $P_u/P_{uz} = 0.2$ to 0.8 , the values of α_n vary linearly from 1.0 to 2.0 . For values less than 0.2 and greater than 0.8 , it is taken as 1 and 2 respectively

NOTE -The design of member subject to combined axial load and uniaxial bending will involve lengthy calculation by trial and error. In order to overcome these difficulties interaction diagrams may be used. These have been prepared and published by BIS in SP:16 titled Design aids for reinforced concrete to IS 456-2000.

IS:456-2000 Code Procedure

1. Given P_u , M_{ux} , M_{uy} , grade of concrete and steel
2. Verify that the eccentricities $e_x = M_{ux}/P_u$ and $e_y = M_{uy}/P_u$ are not less than the corresponding minimum eccentricities as per IS:456-2000
3. Assume a trial section for the column (square, rectangle or circular).

4. Determine M_{ux1} and M_{uy1} , corresponding to the given P_u (using appropriate curve from SP-16 design aids)
5. Ensure that M_{ux1} and M_{uy1} are significantly greater than M_{ux} and M_{uy} respectively; otherwise, suitably redesign the section.
6. Determine P_{uz} and hence α_n
7. Check the adequacy of the section using interaction equation. If necessary, redesign the section and check again.

Slender Compression Members: The design of slender compression members shall be based on the forces and the moments determined from an analysis of the structure, including the effect of deflections on moments and forces. When the effects of deflections are not taken into account in the analysis, additional moment given in 39.7.1 shall be taken into account in the appropriate direction.

Design Problems

1. **Determine the load carrying capacity of a column of size 300 x 400 mm reinforced with six rods of 20 mm diameter i.e, 6-#20. The grade of concrete and steel are M20 and Fe 415 respectively. Assume that the column is short.**

$$f_{ck} = 20 \text{ MPa}, f_y = 415 \text{ MPa}$$

$$\text{Area of steel } A_{sc} = 6 \times \pi \times 20^2/4 = 6 \times 314 = 1884 \text{ mm}^2$$

$$\text{Percentage of steel} = 100A_{sc}/bD = 100 \times 1884/300 \times 400 = 1.57 \%$$

$$\text{Area of concrete } A_c = A_g - A_{sc} = 300 \times 400 - 1884 = 118116 \text{ mm}^2$$

Ultimate load carried by the column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$0.4 \times 20 \times 118116 + 0.67 \times 415 \times 1884$$

$$944928 + 523846 = 1468774 \text{ N} = 1468.8 \text{ kN}$$

$$\text{Therefore the safe load on the column} = 1468.8 / 1.5 = 979.2 \text{ kN}$$

2. **Determine the steel required to carry a load of 980kN on a rectangular column of size 300 x 400 mm. The grade of concrete and steel are M20 and Fe 415 respectively. Assume that the column is short.**

$$f_{ck} = 20 \text{ MPa}, f_y = 415 \text{ MPa}, P = 980 \text{ kN}$$

$$\text{Area of steel } A_{sc} = ?$$

$$\text{Area of concrete } A_c = A_g - A_{sc} = (300 \times 400 - A_{sc})$$

Ultimate load carried by the column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$980 \times 1.5 \times 1000 = 0.4 \times 20 \times (300 \times 400 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$= 960000 - 8 A_{sc} + 278.06 A_{sc}$$

$$A_{sc} = 1888.5 \text{ mm}^2,$$

Percentage of steel = $100A_{sc}/bD = 100 \times 1888.5 / 300 \times 400 = 1.57 \%$ which is more than 0.8% and less than 6% and therefore ok.

Use 20 mm dia. bar, No. of bars = $1888.5/314 = 6.01$ say 6

3. Design a square or circular column to carry a working load of 980kN. The grade of concrete and steel are M20 and Fe 415 respectively. Assume that the column is short.

Let us assume 1.0% steel (1 to 2%)

Say $A_{sc} = 1.0\%$ $A_g = 1/100$ $A_g = 0.01A_g$

$f_{ck} = 20$ MPa, $f_y = 415$ MPa, $P = 980$ kN

Area of concrete $A_c = A_g - A_{sc} = A_g - 0.01A_g = 0.99 A_g$

Ultimate load carried by the column

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$980 \times 1.5 \times 1000 = 0.4 \times 20 \times 0.99 A_g + 0.67 \times 415 \times 0.01 A_g$

$= 7.92 A_g + 2.78 A_g = 10.7 A_g$

$A_g = 137383 \text{ mm}^2$

Let us design a square column:

$B = D = \sqrt{A_g} = 370.6 \text{ mm}$ say $375 \times 375 \text{ mm}$

This is ok. However this size cannot take the minimum eccentricity of 20 mm as $e_{min}/D = 20/375 = 0.053 > 0.05$. To restrict the eccentricity to 20 mm, the required size is $400 \times 400 \text{ mm}$.

Area of steel required is $A_g = 1373.8 \text{ mm}^2$. Provide 4 bar of 22 mm diameter. Steel provided is $380 \times 4 = 1520 \text{ mm}^2$

Actual percentage of steel $= 100A_{sc}/bD = 100 \times 1520 / 400 \times 400 = 0.95 \%$ which is more than 0.8% and less than 6% and therefore ok.

Design of Transverse steel:

Diameter of tie $= 1/4$ diameter of main steel $= 22/4 = 5.5 \text{ mm}$ or 6 mm, whichever is greater. Provide 6 mm.

Spacing: $< 300 \text{ mm}$, $< 16 \times 22 = 352 \text{ mm}$, $< \text{LLD} = 400 \text{ mm}$. Say 300 mm c/c

Design of circular column:

Here $A_g = 137383 \text{ mm}^2$

$\pi \times D^2/4 = A_g$, $D = 418.2 \text{ mm}$ say 420 mm . This satisfy the minimum eccentricity of 20m

Also provide 7 bars of 16 mm, $7 \times 201 = 1407 \text{ mm}^2$

Design of Transverse steel:

Dia of tie $= 1/4$ dia of main steel $= 16/4 = 4 \text{ mm}$ or 6 mm, whichever is greater. Provide 6 mm.

Spacing: $< 300 \text{ mm}$, $< 16 \times 16 = 256 \text{ mm}$, $< \text{LLD} = 420 \text{ mm}$. Say 250 mm c/c

4. Design a rectangular column to carry an ultimate load of 2500kN. The unsupported length of the column is 3m. The ends of the column are effectively held in position

and also restrained against rotation. The grade of concrete and steel are M20 and Fe 415 respectively.

Given:

$$f_{ck} = 20 \text{ MPa}, f_y = 415 \text{ MPa}, P_u = 2500 \text{ kN}$$

Let us assume 1.0% steel (1 to 2%)

$$\text{Say } A_{sc} = 1.0\% A_g = 1/100 A_g = 0.01 A_g$$

$$\text{Area of concrete } A_c = A_g - A_{sc} = A_g - 0.01 A_g = 0.99 A_g$$

Ultimate load carried by the column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$2500 \times 1000 = 0.4 \times 20 \times 0.99 A_g + 0.67 \times 415 \times 0.01 A_g$$

$$= 7.92 A_g + 2.78 A_g = 10.7 A_g$$

$$A_g = 233645 \text{ mm}^2$$

If it is a square column:

$B = D = \sqrt{A_g} = 483 \text{ mm}$. However provide rectangular column of size 425 x 550 mm. The area provided = 333750 mm²

Area of steel = 2336 mm², Also provide 8 bars of 20 mm, $8 \times 314 = 2512 \text{ mm}^2$

Check for shortness: Ends are fixed. $l_{ex} = l_{ey} = 0.65 l = 0.65 \times 3000 = 1950 \text{ mm}$

$l_{ex}/D = 1950/550 < 12$, and $l_{ey}/b = 1950/425 < 12$, Column is short

Check for minimum eccentricity:

In the direction of longer direction

$$e_{min, x} = l_{ux}/500 + D/30 = 3000/500 + 550/30 = 24.22 \text{ mm or } 20 \text{ mm whichever is greater.}$$

$$e_{min, x} = 24.22 \text{ mm} < 0.05D = 0.05 \times 550 = 27.5 \text{ mm. O.K}$$

In the direction of shorter direction

$$e_{min, y} = l_{uy}/500 + b/30 = 3000/500 + 425/30 = 20.17 \text{ mm or } 20 \text{ mm whichever is greater.}$$

$$e_{min, x} = 20.17 \text{ mm} < 0.05b = 0.05 \times 425 = 21.25 \text{ mm. O.K}$$

Design of Transverse steel:

Dia of tie = $1/4$ dia of main steel = $20/4 = 5 \text{ mm}$ or 6 mm, whichever is greater. Provide 6 mm or 8 mm.

Spacing: $< 300 \text{ mm}$, $< 16 \times 20 = 320 \text{ mm}$, $< \text{LLD} = 425 \text{ mm}$. Say 300 mm c/c

5. Design a circular column with ties to carry an ultimate load of 2500kN. The unsupported length of the column is 3m. The ends of the column are effectively held in position but not against rotation. The grade of concrete and steel are M20 and Fe 415 respectively.

Given:

$$f_{ck} = 20 \text{ MPa}, f_y = 415 \text{ MPa}, P_u = 2500 \text{ kN}$$

Let us assume 1.0% steel (1 to 2%)

$$\text{Say } A_{sc} = 1.0\% A_g = 1/100 A_g = 0.01 A_g$$

$$\text{Area of concrete } A_c = A_g - A_{sc} = A_g - 0.01 A_g = 0.99 A_g$$

Ultimate load carried by the column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$2500 \times 1000 = 0.4 \times 20 \times 0.99 A_g + 0.67 \times 415 \times 0.01 A_g$$

$$= 7.92 A_g + 2.78 A_g = 10.7 A_g$$

$$A_g = 233645 \text{ mm}^2$$

$$\pi \times D^2/4 = A_g, D = 545.4 \text{ mm say } 550 \text{ mm.}$$

$$\text{Area of steel} = 2336 \text{ mm}^2, \text{ Also provide 8 bars of } 20 \text{ mm, } 8 \times 314 = 2512 \text{ mm}^2$$

Check for shortness: Ends are hinged $l_{ex} = l_{ey} = l = 3000 \text{ mm}$

$$l_{ex}/D = 3000/550 < 12, \text{ and } l_{ey}/b = 3000/425 < 12, \text{ Column is short}$$

Check for minimum eccentricity:

Here, $e_{min, x} = e_{min, y} = l_{ux}/500 + D/30 = 3000/500 + 550/30 = 24.22 \text{ mm}$ or 20mm whichever is greater.

$$e_{min} = 24.22 \text{ mm} < 0.05D = 0.05 \times 550 = 27.5 \text{ mm. O.K}$$

Design of Transverse steel:

Diameter of tie = $1/4$ dia of main steel = $20/4 = 5 \text{ mm}$ or 6 mm, whichever is greater. Provide 6 mm or 8 mm.

Spacing: $< 300 \text{ mm}, < 16 \times 20 = 320 \text{ mm}, < \text{LLD} = 550 \text{ mm}$. Say 300 mm c/c

Similarly square column can be designed.

If the size of the column provided is less than that provided above, then the minimum eccentricity criteria are not satisfied. Then e_{min} is more and the column is to be designed as

uni axial bending case or bi axial bending case as the case may be. This situation arises when more steel is provided (say 2% in this case).

Try to solve these problems by using SP 16 charts, though not mentioned in the syllabus.

- 6. Design the reinforcement in a column of size 450 mm × 600 mm, subject to an axial load of 2000 kN under service dead and live loads. The column has an unsupported length of 3.0m and its ends are held in position but not in direction. Use M 20 concrete and Fe 415 steel.**

Solution:

Given: $l_u = 3000$ mm, $b = 450$ mm, $D = 600$ mm, $P = 2000$ kN, M20, Fe415

Check for shortness: Ends are fixed. $l_{ex} = l_{ey} = l = 3000$ mm

$l_{ex}/D = 3000/600 < 12$, and $l_{ey}/b = 3000/450 < 12$, Column is short

Check for minimum eccentricity:

In the direction of longer direction

$e_{min, x} = l_{ux}/500 + D/30 = 3000/500 + 600/30 = 26$ mm or 20mm whichever is greater.

$e_{min, x} = 26$ mm $< 0.05D = 0.05 \times 600 = 30$ mm. O.K

In the direction of shorter direction

$e_{min, y} = l_{uy}/500 + b/30 = 3000/500 + 450/30 = 21$ mm or 20mm whichever is greater.

$e_{min, x} = 21$ mm $< 0.05b = 0.05 \times 450 = 22.5$ mm. O.K

Minimum eccentricities are within the limits and hence code formula for axially loaded short columns can be used.

Factored Load

$P_u = \text{service load} \times \text{partial load factor}$
 $= 2000 \times 1.5 = 3000$ kN

Design of Longitudinal Reinforcement

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$ or
 $P_u = 0.4 f_{ck} A_c + (0.67 f_y - 0.4 f_{ck}) A_{sc}$

$$3000 \times 10^3 = 0.4 \times 20 \times (450 \times 600) + (0.67 \times 415 - 0.4 \times 20) A_{sc}$$

$$= 2160 \times 10^3 + 270.05 A_{sc}$$

$$\Rightarrow A_{sc} = (3000 - 2160) \times 10^3 / 270.05 = 3111 \text{ mm}^2$$

In view of the column dimensions (450 mm, 600 mm), it is necessary to place intermediate bars, in addition to the 4 corner bars:

$$\text{Provide } 4-25\phi \text{ at corners ie, } 4 \times 491 = 1964 \text{ mm}^2$$

$$\text{and } 4-20\phi \text{ additional ie, } 4 \times 314 = 1256 \text{ mm}^2$$

$$\Rightarrow A_{sc} = 3220 \text{ mm}^2 > 3111 \text{ mm}^2$$

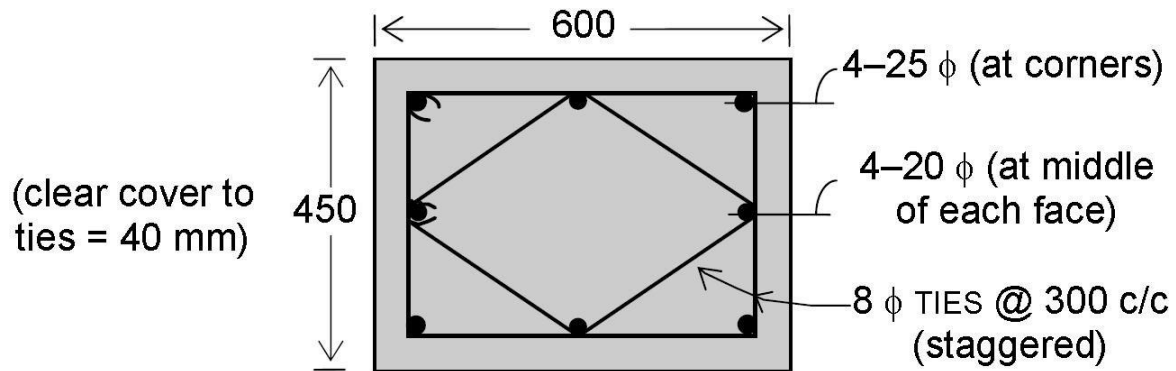
$$\Rightarrow p = (100 \times 3220) / (450 \times 600) = 1.192 > 0.8 \text{ (minimum steel), OK.}$$

Design of transverse steel

Diameter of tie = $\frac{1}{4}$ diameter of main steel = $25/4 = 6.25$ mm or 6 mm, whichever is greater.
Provide 6 mm.

Spacing: < 300 mm, $< 16 \times 20 = 320$ mm, $< \text{LLD} = 450$ mm. Say 300 mm c/c
Thus provide ties 8mm @ 300 mm c/c

Sketch:



Example: Square Column with Uniaxial Bending

7. Determine the reinforcement to be provided in a square column subjected to uniaxial bending with the following data:

Size of column 450 x 450 mm

Concrete mix M 25

Characteristic strength of steel 415 N/mm²

Factored load 2500 kN

Factored moment 200 kN.m

Arrangement of reinforcement:

(a) On two sides

(b) On four sides

Assume moment due to minimum eccentricity to be less than the actual moment

Assuming 25 mm bars with 40 mm cover,

$$d = 40 + 12.5 = 52.5 \text{ mm}$$

$$d^1/D = 52.5/450 = 0.12$$

Charts for $d^1/D = 0.15$ will be used

$$P_u/f_{ck}bD = (2500 \times 1000)/(25 \times 450 \times 450) = 0.494$$

$$M_u/f_{ck}bD^2 = 200 \times 10^6/(25 \times 450 \times 450^2) = 0.088$$

a) Reinforcement on two sides,

Referring to Chart 33,

$$p/f_{ck} = 0.09$$

Percentage of reinforcement,

$$p = 0.09 \times 25 = 2.25 \%$$

$$A_s = p bD/100 = 2.25 \times 450 \times 450/100 \\ = 4556 \text{ mm}^2$$

b) Reinforcement on four sides

from Chart 45,

$$p/f_{ck} = 0.10$$

$$p = 0.10 \times 25 = 2.5 \%$$

$$A_s = 2.5 \times 450 \times 450/100 = 5063 \text{ mm}^2$$

8. Example: Circular Column with Uniaxial Bending

Determine the reinforcement to be provided in a circular column with the following data:

Diameter of column 500 mm

Grade of concrete M20

Characteristic strength 250 N/mm²

Factored load 1600 kN

Factored moment 125 kN.m

Lateral reinforcement :

(a) Hoop reinforcement

(b) Helical reinforcement

(Assume moment due to minimum eccentricity to be less than the actual moment).

Assuming 25 mm bars with 40 mm cover,

$$d^1 = 40 + 12.5 = 52.5 \text{ mm}$$

$$d^1/D = 52.5/500 = 0.105$$

Charts for $d^1/D = 0.10$ will be used.

(a) Column with hoop reinforcement

$$P_u/f_{ck} D D = (1600 \times 1000)/(20 \times 500 \times 500) = 0.32$$

$$M_u/f_{ck} D \times D^2 = 125 \times 10^6/(20 \times 500 \times 500^2) = 0.05$$

Referring to *Chart 52*, for $f_y = 250 \text{ N/mm}^2$
 $p/f_{ck} = 0.87$

Percentage of reinforcement,

$$p = 0.87 \times 20 = 1.74 \%$$

$$A_s = 1.74 \times (\pi \times 500^2/4)/100 = 3416 \text{ mm}^2$$

(b) Column with Helical Reinforcement

According to 38.4 of the Code, the strength of a compression member with helical reinforcement is 1.05 times the strength of a similar member with lateral ties. Therefore, the, given load and moment should be divided by 1.05 before referring to the chart.

$$P_u/f_{ck} D D = (1600/1.05 \times 1000)/(20 \times 500 \times 500) = 0.31$$

$$M_u/f_{ck} D \times D^2 = 125/1.05 \times 10^6/(20 \times 500 \times 500^2) = 0.048$$

Hence, From Chart 52, for $f_y = 250 \text{ N/mm}^2$,

$$p/f_{ck} = 0.078$$

$$p = 0.078 \times 20 = 1.56 \%$$

$$A_s = 1.56 \times (\pi \times 500 \times 500/4)/100 = 3063 \text{ cm}^2$$

According to 38.4.1 of the Code the ratio of the volume of helical reinforcement to the volume of the core shall not be less than

$$0.36 (A_g/A_c - 1) \times f_{ck}/f_y$$

where A_g is the gross area of the section and A_c is the area of the core measured to the outside diameter of the helix. Assuming 8 mm dia bars for the helix,

$$\text{Core diameter} = 500 - 2(40 - 8) = 436 \text{ mm}$$

$$A_g/A_c = 500/436 = 1.315$$

$$0.36 (A_g/A_c - 1) \times f_{ck}/f_y = 0.36(0.315) 20/250 = 0.0091$$

Volume of helical reinforcement / Volume of core

$$= A_{sh} \pi \times 428 / (\pi/4 \times 436^2) s_h$$

$$0.09 A_{sh} / s_h$$

where, A_{sh} is the area of the bar forming the helix and s_h is the pitch of the helix.
 In order to satisfy the code requirement,

$$0.09 A_{sh} / s_h \geq 0.0091$$

For 8 mm dia bar,

$s_h \leq 0.09 \times 50 / 0.0091 = 49.7$ mm. Thus provide 48 mm pitch

Example: Rectangular column with Biaxial Bending

9. Determine the reinforcement to be provided in a short column subjected to biaxial bending, with the following data:

size of column = 400 x 600 mm

Concrete mix = M15

Characteristic strength of reinforcement = 415 N/mm²

Factored load, P_u = 1600 kN

Factored moment acting parallel to the larger dimension, M_{ux} = 120 kNm

Factored moment acting parallel to the shorter dimension, M_{uy} = 90 kNm

Moments due to minimum eccentricity are less than the values given above.

Reinforcement is distributed equally on four sides.

As a first trial assume the reinforcement percentage, $p = 1.2\%$

$$p/f_{ck} = 1.2/15 = 0.08$$

Uniaxial moment capacity of the section about xx-axis :

$$d^1/D = 52.5 / 600 = 0.088$$

Chart for $d^1/D = 0.1$ will be used.

$$P_u/f_{ck} b D = (1600 \times 1000) / (15 \times 400 \times 600) = 0.444$$

Referring to chart 44

$$M_u/f_{ck} b \times D^2 = 0.09$$

$$M_{ux1} = 0.09 \times 15 \times 400 \times 600^2 = 194.4 \text{ kN.m}$$

Uni-axial moment capacity of the section about yy axis :

$$d^1/D = 52.5 / 400 = 0.131$$

Chart for $d^1/D = 0.15$ will be used.

Referring to Chart 45,

$$M_u/f_{ck} b \times D^2 = 0.083$$

$$M_{ux1} = 0.083 \times 15 \times 600 \times 400^2 = 119.52 \text{ kN.m}$$

Calculation of P_{uz} :

Referring to Chart 63 corresponding to

$p = 1.2$, $f_y = 415$ and $f_{ck} = 15$,

$$P_{uz}/A_g = 10.3$$

$$P_{uz} = 10.3 \times 400 \times 600 = 2472 \text{ kN}$$

$$M_{ux}/M_{ux1} = 120/194.4 = 0.62$$

$$M_{uy}/M_{uy1} = 90/119.52 = 0.75$$

$$P_u/P_{uz} = 1600/2472 = 0.65$$

Referring to Churn 64, the permissible value of M_{ux}/M_{ux1} corresponding to M_{uy}/M_{uy1} and P_u/P_{uz} is equal to 0.58

The actual value of 0.62 is only slightly higher than the value read from the Chart.

This can be made up by slight increase in reinforcement.

Using Boris load contour equation as per IS:456-2000

$$P_u/P_{uz} = 0.65 \text{ thus, } \alpha_n = 1 + [(2-1) / (0.8 - 0.2)] (0.65-0.2) = 1.75$$

$[0.62]^{1.75} + [0.75]^{1.75} = 1.04$ slightly greater than 1 and slightly unsafe. This can be made up by slight increase in reinforcement say 1.3%

$$\text{Thus provide } A_s = 1.3 \times 400 \times 600 / 100 = 3120 \text{ mm}^2$$

Provide 1.3 % of steel

$$p/f_{ck} = 1.3/15 = 0.086$$

$$d^1/D = 52.5 / 600 = 0.088 = 0.1$$

From chart 44

$$M_u/f_{ck} b \times D^2 = 0.095$$

$$M_{ux1} = 0.095 \times 15 \times 400 \times 600^2 = 205.2 \text{ kN.m}$$

Referring to Chart 45,

$$M_u/f_{ck} b \times D^2 = 0.085$$

$$M_{ux1} = 0.085 \times 15 \times 600 \times 400^2 = 122.4 \text{ kN.m}$$

Chart 63 : $P_{uz}/A_g = 10.4$

$$P_{uz} = 10.4 \times 400 \times 600 = 2496 \text{ kN}$$

$$M_{ux}/M_{ux1} = 120/205.2 = 0.585$$

$$M_{uy}/M_{uy1} = 90/122.4 = 0.735$$

$$P_u/P_{uz} = 1600/2496 = 0.641$$

Referring to Chart 64, the permissible value of M_{ux}/M_{ux1} corresponding to M_{uy}/M_{uy1} and P_u/P_{uz} is equal to 0.60

Hence the section is O.K.

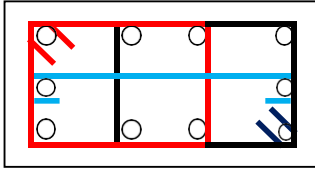
Using Boris load contour equation as per IS:456-2000

$$P_u/P_{uz} = 0.641 \text{ thus, } \alpha_n = 1 + [(2-1) / (0.8 - 0.2)] (0.641-0.2) = 1.735$$

$$[120/205.2]^{1.735} + [90/122.4]^{1.735} = 0.981 \leq 1 \text{ Thus OK}$$

$$A_s = 3120 \text{ mm}^2. \text{ Provide 10 bars of 20 mm dia. Steel provided is } 314 \times 10 = 3140 \text{ mm}^2$$

Design of transverse steel: Provide 8 mm dia stirrups at 300 mm c/c as shown satisfying the requirements of IS: 456-2000



10. Verify the adequacy of the short column section 500 mm x 300 mm under the following load conditions:

$P_u = 1400$ kN, $M_{ux} = 125$ kNm, $M_{uy} = 75$ kNm. The design interaction curves of SP 16 should be used. Assume that the column is a 'short column' and the eccentricity due to moments is greater than the minimum eccentricity.

Solution:

Given: $D = 500$ mm, $b = 300$ mm, $A_s = 2946$ mm², $M_{ux} = 125$ kNm, $M_{uy} = 75$ kNm, $f_{ck} = 25$ MPa, $f_y = 415$ MPa

Applied eccentricities

$$e_x = M_{ux} / P_u = 125 \times 10^3 / 1400 = 89.3 \text{ mm} \Rightarrow e_x / D = 0.179$$

$$e_y = M_{uy} / P_u = 75 \times 10^3 / 1400 = 53.6 \text{ mm} \Rightarrow e_y / D = 0.179$$

These eccentricities for the short column are clearly not less than the minimum eccentricities specified by the Code.

Uniaxial moment capacities: M_{ux1} , M_{uy1}

As determined in the earlier example, corresponding to $P_u = 1400$ kN,

$$M_{ux1} = 187 \text{ kNm}$$

$$M_{uy1} = 110 \text{ kNm}$$

Values of P_n and α_n

$$P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

$$= (0.45 \times 25 \times 300 \times 500) + (0.75 \times 415 - 0.45 \times 25) \times 2946$$

$$= (1687500 + 883800) \text{ N} = 2571 \text{ kN}$$

$$\Rightarrow P_u / P_n = 1400 / 2571 = 0.545 \text{ (which lies between 0.2 and 0.8)}$$

$$\Rightarrow \alpha_n = 1.575$$

Check safety under biaxial bending

$$[125/187]^{1.575} + [75/110]^1$$

$$= 0.530 + 0.547$$

$$= 1.077 > 1.0$$

Hence, almost ok.

UNIT V DESIGN OF FOOTING

. Pre-requisites

Most of the structures built by us are made of reinforced concrete. Here, the part of the structure above ground level is called as the superstructure, where the part of the structure below the ground level is called as the substructure. Footings are located below the ground level and are also referred as foundations. Foundation is that part of the structure which is in direct contact with soil. The R.C. structures consist of various structural components which act together to resist the applied loads and transfer them safely to soil. In general the loads applied on slabs in buildings are transferred to soil through beams, columns and footings. Footings are that part of the structure which are generally located below ground Level. They are also referred as foundations. Footings transfer the vertical loads, Horizontal loads, Moments, and other forces to the soil.

The important purpose of foundation are as follows;

1. To transfer forces from superstructure to firm soil below.
2. To distribute stresses evenly on foundation soil such that foundation soil neither fails nor experiences excessive settlement.
3. To develop an anchor for stability against overturning.
4. To provide an even surface for smooth construction of superstructure.

Due to the loads and soil pressure, footings develop Bending moments and Shear forces. Calculations are made as per the guidelines suggested in IS 456 2000 to resist the internal forces.

Types of Foundations

Based on the position with respect to ground level, Footings are classified into two types;

1. Shallow Foundations
2. Deep Foundations

Shallow Foundations are provided when adequate SBC is available at relatively short depth below ground level. Here, the ratio of $D_f / B < 1$, where D_f is the depth of footing and B is the width of footing. Deep Foundations are provided when adequate SBC is available at large depth below ground level. Here the ratio of $D_f / B \geq 1$.

Types of Shallow Foundations

The different types of shallow foundations are as follows:

- Isolated Footing
- Combined footing
- Strap Footing
- Strip Footing
- Mat/Raft Foundation
- Wall footing

Some of the popular types of shallow foundations are briefly discussed below.

a) Isolated Column Footing

These are independent footings which are provided for each column. This type of footing is chosen when

- SBC is generally high
- Columns are far apart
- Loads on footings are less

The isolated footings can have different shapes in plan. Generally it depends on the shape of column cross section. Some of the popular shapes of footings are;

- Square
- Rectangular
- Circular

The isolated footings essentially consist of bottom slab. These bottom slabs can be either flat, stepped or sloping in nature. The bottom of the slab is reinforced with steel mesh to resist the **two** internal forces namely bending moment and shear force.

The sketch of a typical isolated footing is shown in Fig. 1.

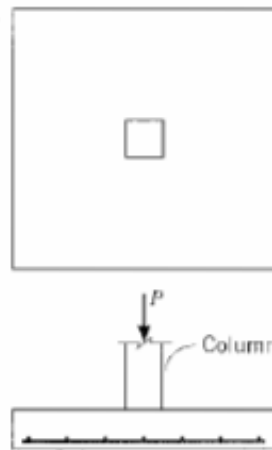


Fig. 1 Plan and section of typical isolated footing

b) Combined Column Footing

These are common footings which support the loads from 2 or more columns. Combined footings are provided when

- SBC is generally less
- Columns are closely spaced
- Footings are heavily loaded

In the above situations, the area required to provide isolated footings for the columns generally overlap. Hence, it is advantageous to provide single combined footing. In some cases the columns are located on or close to property line. In such cases footings cannot **be** extended on **one side**. Here, the footings of exterior and interior columns are connected by the **combined** footing.

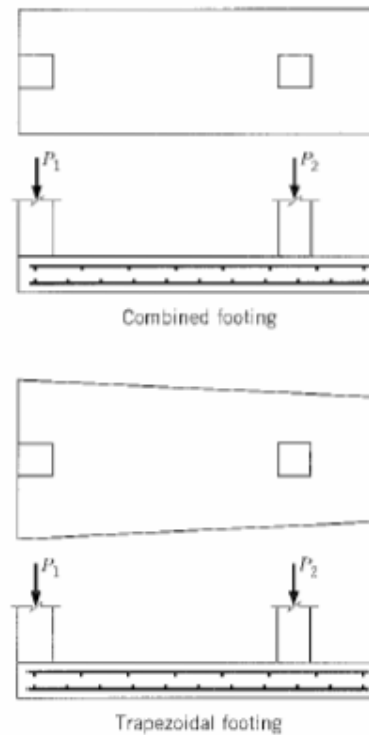


Fig. 2 Plan and section of typical

combined footing

Combined footings essentially consist of a common slab for the columns it is supporting. These slabs are generally rectangular in plan. Sometimes they can also be trapezoidal in plan (refer Fig. 2). Combined footings can also have a connecting beam and a slab arrangement, which is similar to an inverted T – beam slab.

c) Strap Footing

An alternate way of providing combined footing located close to property line is the strap footing. In strap footing, independent slabs below columns are provided which are then connected by a strap beam. The strap beam does not remain in contact with the soil and does not transfer any pressure to the soil. Generally it is used to combine the footing of the outer column to the adjacent one so that the footing does not extend in the adjoining property. A typical strap footing is shown in Fig. 3.

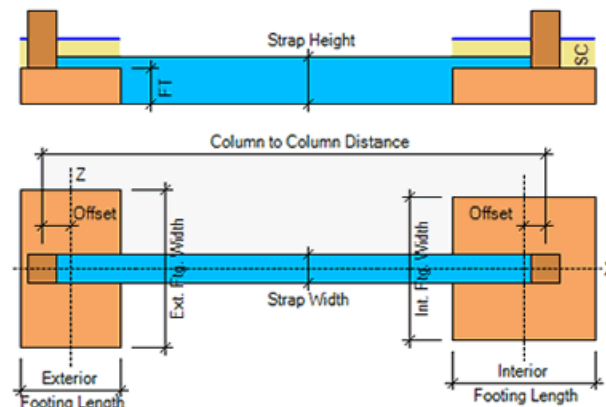


Fig. 3 Plan and section of typical strap footing