POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

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Department of Civil Engineering

Regulation 2021

II Year – III Semester

CE3301 Fluid Mechanics

UNIT-I Problems on Properties of fluid

1. A 15CM diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.1CM. Both cylinders are 25CM high. The space between cylinders is filled with a liquid. If a torque 11.77 N.M is required to rotate the inner cylinder at 100 rpm, determine the viscosity of liquid.

Given data:

Diameter of inside cylinder, $d = 15CM = 0.15M$

Diameter of outer cylinder, $D = 15.1CM = 0.151M$

Height of both cylinder, $L = 25CM = 0.25M$

Torque of inner cylinder, $T = 11.77$ N-M

Speed of cylinder, $N = 100$ rpm.

Solution:

Tangential velocity, $du = u = \frac{\pi d N}{60}$

$= \frac{\pi \times 0.15 \times 100}{60}$

$= 0.785 m/s$

Area of contact of fluid with inner cylinder,

$A = \pi \times d \times l$

$= \pi \times 0.15 \times 0.25$

$= 0.118 m^2$

Clearance between cylinders

$dy = \frac{0.151 - 0.15}{2}$

$= 0.0005M$

2) Find the capillary rise in a glass tube of 4mm diameter when immersed in (i) water, (ii) Mercury. Assume $\sigma_{water} = 0.075$ N/m and $\sigma_{mercury} = 0.45$ N/m

Given Data:

Diameter of glass tube, $d = 4mm = 0.004m$

$\sigma_{water} = 0.075$ N/m

$\sigma_{mercury} = 0.45$ N/m

Solution:

1. Capillary rise for water:

$$h = \frac{4\sigma}{wd}$$

$$= \frac{4 \times 0.075}{9810 \times 0.004}$$ ($\because$ w for water $= 9810 N/m^3$)

$$h = 0.0077m$$

2. Capillary rise for Mercury

$$h = \frac{4\sigma \cos\beta}{wd}$$ ($\because$ For mercury, $\beta = 140^\circ$, $\cos\beta = -0.766$)

$$= \frac{4 \times 0.45 \times (-0.766)}{133416 \times 0.004}$$

$$w = 13.6 \times 9810$$

$$= 133416 \text{ N/m}^3$$

$$w = -0.00258m$$

8) A hydraulic lift shaft of 225mm diameter moves in a cylinder of 227 mm diameter with the length of engagement of 1.2m. The interface is filled with oil of a kinematic viscosity of $3.4 \times 10^{-4}\ m^2/s$ and density $950\ kg/m^3$. Determine the uniform velocity of movement of the shaft if the drag resistance is 480N.

Given data:-

Shaft diameter, $D = 225mm$

$= 0.225M$

(Shear stress, $\tau = \frac{F}{A}$)

Cylinder diameter, $D_c = 227mm = 0.227M$

Length of engagement, $l = 1.2M$

Kinematic viscosity of oil, $\nu = 3.4 \times 10^{-4}\ m^2/s$

Density of oil, $\rho = 950 kg/m^3$

Drag resistance, $F = 480N$

Solution:

Area of contact of oil with shaft.

$A = \pi \times D_s \times l$

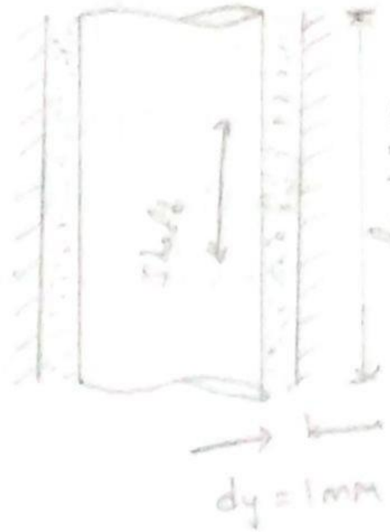
$= \pi \times 0.225 \times 1.2$

$= 0.848\ m^2$

Clearance between cylinder and shaft.

$$dy = \frac{227 - 225}{2}$$

$$= 1\ mm = 0.001 m$$



Shear stress, $\tau = \frac{F}{A}$

$$= \frac{480}{0.848}$$

$$= 566.04\ N/m^2$$

Kinematic viscosity,

$$\nu = \frac{\mu}{\rho}$$

$$3.4 \times 10^{-4} = \frac{\mu}{950}$$

$$\mu = 0.323\ Ns/m^2$$

We know that shear stress

$$\tau = \mu \times \frac{du}{dy}$$

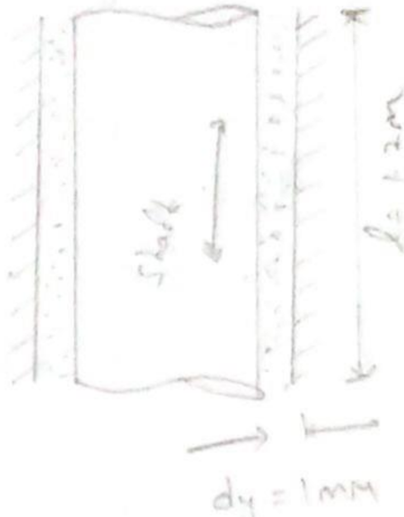
$$566.04 = 0.323 \times \frac{du}{0.001}$$

$$du = 1.75\ m/s$$

Clearance between cylinder and Shaft.

$$dy = \frac{227 - 225}{2}$$

$$= 1\,mm = 0.001\,m$$



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We know that shear stress

$$\tau = \mu \times \frac{du}{dy}$$

$$566.04 = 0.323 \times \frac{du}{0.001}$$

$$u = du = 1.75\ m/s$$

Problem on manometer

1. With a neat sketch of 'U' tube connected to a pipe under pressure, explain the procedure of writing the manometric equation. (Assume the U tube manometer contain mercury on manometric liquid and open to atmosphere). Gauge 'A' attached at the bottom of a tank shown in figure 1.66 reads 350 kPa (abs.) what is the height 'h' of water? What is the reading of gauge 'B'?

Given data:

Pressure of air, $P_{air} = 200\,kPa = 200000\,N/m^2$

Head of water at gauge, $A = h_A$

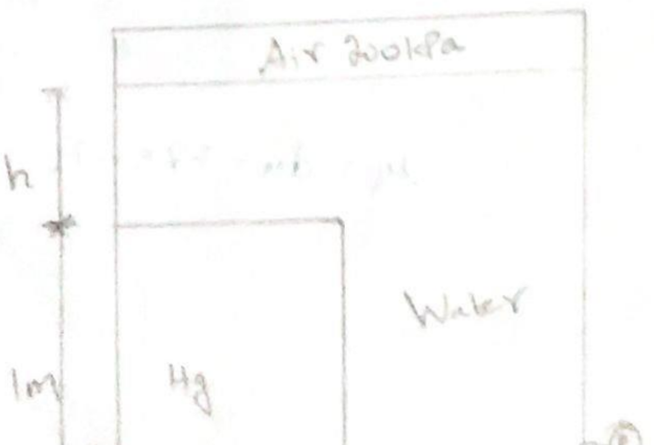
Head of water at gauge, $B = h_B$

$= h_A + 1 = 1 + h_A$

Head of mercury at gauge, $A = 1m$

Pressure at gauge A, $P_A = 350\,kPa$

$= 35000\,N/m^2$



Solution:

As Per the hydrostatic Principle.

Pressure at gauge A, $p_A = p_{Air} + p_{water} + p_{mercury}$

$= p_{Air} + \rho_{water} \times g \times h_A + \rho_{mercury} \times g \times h_{mercury}$

$= 200000 + 1000 \times 9.81 \times h_A + 13600 \times 9.81 \times 1$

$$\left[\because S_{mercury} = \frac{\rho_{Mercury}}{\rho_{water}} \right]$$

$\rho_{mercury} = 13.6$ & $\rho_{water} = 1000 kg/m^3$

$= 13600$

$350000 = 200000 + 9810 h_A + 133416$

$h = 1.69 m$

Pressure at gauge B,

$p_B = p_{Air} + p_{water}$

$= p_{Air} + \rho_{water} * g \times h_B$

$= 20000 + 1000 \times 9.81 \times (1 + 1.69)$

$= 20000 + 1000 \times 9.81 \times (2.69)$

$p_B = 226.39 kPa$

Problem on Forces on Planes

1) A trapezoidal channel 2.5m wide at the bottom and 1.5m deep has side slopes 1:1. Determine the (a) total pressure and (b) center of pressure on the vertical gate closing the channel when it is full of water.

Given data:

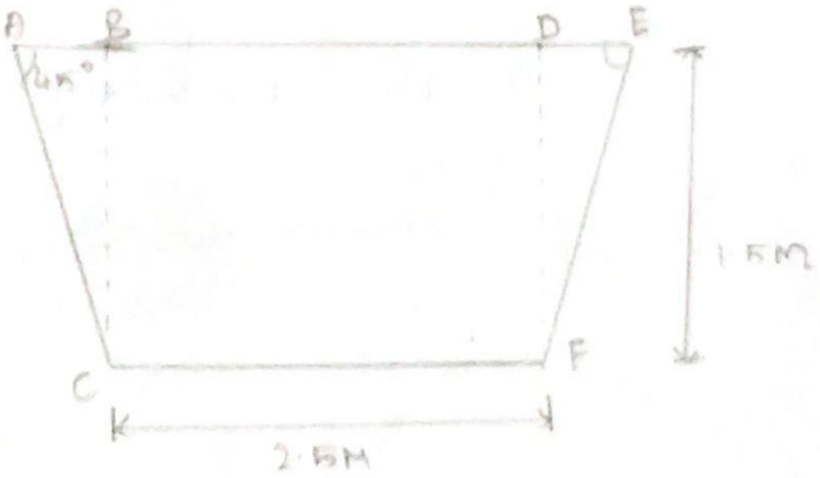
Base, $b = 2.5M$

Depth, $h = 1.5M$

Slope $= 1:1$

Solution:-

First, the given trapezoidal is divided into one rectangle and two right angle triangle as shown in fig. So, these two triangles are considered as a single triangle.



From $\triangle ABC$,

$$AB = \frac{BC}{\tan 45°} = \frac{1.5}{\tan 45°}$$

$$= 1.5m$$

$a = 2.5 + 2AB$

$= 2.5 + 2 \times 1.5$

$= 5.5m$

Now, area of rectangle BCDF,

$A_1 = b \times h$

$= 2.5 \times 1.5$

$= 3.75m^2$

$\bar{x} = \frac{1.5}{2} = 0.75m$

Pressure, $P_1 = wA_1\bar{x}_1$

$= 9.81 \times 3.75 \times 0.75$

$= 27.59 kN$

For rectangle BCDF,

$I_g = \frac{bh^3}{12}$

$= \frac{2.5 \times 1.5^3}{12}$

$= 0.703 m^4$

Depth, $\bar{h}_1 = \bar{x}_1 + \frac{I_{g1}}{A_1\bar{x}_1}$

$= 0.75 + \frac{0.703}{3.75 \times 0.75}$

$\bar{h}_1 = 1m$

Then Area of triangle, $A_2 = \frac{1}{2} \times$ Base $\times$ Height

$= \frac{1}{2} \times 3 \times 1.5$ ($\because$ Base of triangle = 2AB)

$= 2.25 m^2$

$\bar{x} = \frac{h}{3} = \frac{1.5}{3} = 0.5 m$

Pressure, $P_2 = w A_2 \bar{x}_2$

$= 9.81 \times 2.25 \times 0.5$

$= 11.04 kN$

$\therefore$ Total Pressure,

$P = P_1 + P_2$

$= 27.59 + 11.04$

$P = 38.63 kN$

Depth, $\bar{h}_2 = \bar{x}_2 + \frac{I_{g2}}{A_2 \bar{x}_2}$ $\left[\because I_{g2} = \frac{\text{base} \times \text{height}^3}{36}\right)$

$= 0.5 + \frac{3 \times 1.5^3}{36 \times 2.25 \times 0.5}$

$= 0.75 m$

Now, taking moment about the top.

$P\bar{h} = P_1 \bar{h}_1 + P_2 \bar{h}_2$

$38.63 \times \bar{h} = 27.59 \times 1 + 11.04 \times 0.75$

$\bar{h} = 0.93 m$ from the free surface

Problems on buoyance and Flotation

1) A metallic body floats at the interface of mercury of specific gravity 13.6 and water such that 30% of its volume is submerged in mercury and 70% in water. Find the density of the metallic body.

Given data:

specific gravity of mercury = 13.6

Solution:

Let V = Volume of metallic body in m^3

Volume of body submerged in mercury

$$= \frac{30}{100} V$$

$$= 0.3V$$

Volume of body submerged in water,

$$= \frac{70}{100} \times V$$

$$= 0.7V$$

The body will remain in equilibrium, when the

Total buoyant force = weight of the body

$$\text{Total buoyant force} = \begin{pmatrix}\text{Force of buoyancy} \\ \text{due to water}\end{pmatrix} + \begin{pmatrix}\text{Force of buoyancy} \\ \text{due to mercury}\end{pmatrix}$$

$$\left.\begin{matrix}\text{Force of buoyancy} \\ \text{due to water}\end{matrix}\right\} = \begin{matrix}\text{weight of water} \\ \text{displaced by the body}\end{matrix}$$

$$= \rho_{water} \times g \times \text{Volume of water displaced}$$

$$= 1000 \times 9.81 \times 0.7V$$

$$= 6867\ V$$

Similarly, force of buoyancy due to mercury

$$= 1000 \times 13.6 \times 9.81 \times 0.3V$$

$$= 40024.8V$$

Total buoyant force

$$= 6867V + 40024.8V$$

$$= 46891.8V$$

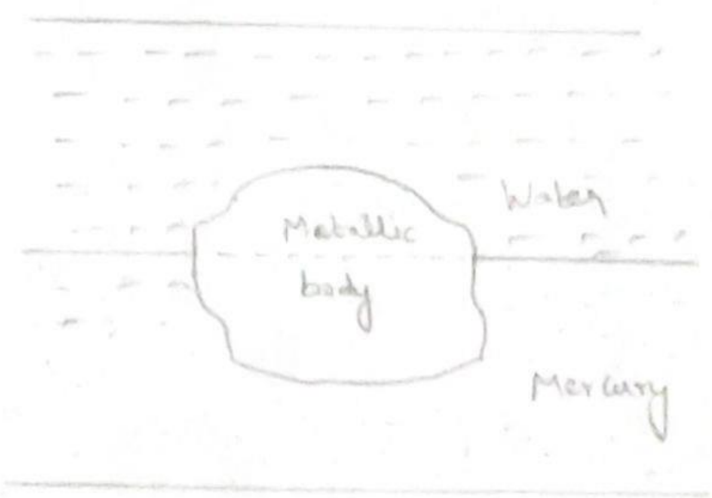
Weight of the metallic body

$$= \rho g V$$

$$= 9.81 \times \rho \times V$$

$$\therefore 46891.8V = 9.81 \times \rho \times V$$

$$\rho = 4780 \, kg/m^3$$



UNIT-II

Problems on fluid kinematics

1. In a two dimensional Potential flow, the velocity Potential is given by $\phi = 4x(3y-4)$, determine the velocity at the Point (2,3). Determine also the value of stream function ψ at the Point (2,3).

Solution:

$$\phi = 4x(3y-4)$$

Differentiating the above equation,

$$u = \frac{\partial \phi}{\partial x} \text{ and } v = -\frac{\partial \phi}{\partial y}$$

$$u = \frac{\partial (4x(3y-4))}{\partial x}$$

$$= -4(3y-4)$$

$$u = 16-12y$$

$$v = \frac{-\partial (4x(3y-4))}{\partial y}$$

$$= -(4x(3-0))$$

$$= -12x$$

At (2,3) $u = 16-12\times 3$

$= -20$ Units

$V = -12\times 2$

$= -24$ Units.

Resultant velocity, $V = \sqrt{u^2+v^2}$

$= \sqrt{(-20)^2+(-24)^2}$

$= 31.24$ m/s

By definition,

$$u = \frac{\partial \psi}{\partial y} \quad \& \quad v = \frac{\partial \varphi}{\partial x}$$

$$16 - 12y = -\frac{\partial \varphi}{\partial y} \quad \& \quad -12x = \frac{\partial \varphi}{\partial x}$$

$$\frac{\partial \psi}{\partial y} = 12y - 16 \quad \& \quad \frac{\partial \varphi}{\partial x} = -12x$$

Integrating the above equation with respect to x

$$\varphi = -6x^2 + C$$

Differentiating the above equation with respect to y,

$$\frac{\partial \varphi}{\partial y} = \frac{\partial C}{\partial y}$$

Equating the above equation with u values.

$$\frac{\partial C}{\partial y} = 12y - 16$$

Integrating the above equation with respect to y,

$$C = 6y^2 - 16y$$

Substituting the C values in equation

$$\varphi = -6x^2 + 6y^2 - 16y$$

Substituting $x = 2$ and $y = 3$,

$$\varphi = -6 \times 2^2 + 6 \times 3^2 - (16 \times 3)$$

$$\psi = -18$$

2) Water flows through a Pipe AB 1.2m diameter at 3m/s and then Passes through a Pipe BC 1.5m diameter. At C, the Pipe branches. branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, velocity in BC, velocity in CD and diameter of CE.

Given data:

$D_{AB} = 1.2m$

$V_{AB} = 3m/s$

$D_{BC} = 1.5m$

$D_{CD} = 0.8m$

$Q_{CD} = \frac{1}{3} Q_{AB}$

$V_{CE} = 2.5 m/s$

Solution:-

Pipe AB:

Cross sectional area of Pipe AB,

$$A_{AB} = \frac{\pi}{4} (1.2)^2$$

$$= 1.13 m^2$$

Pipe BC:

$$A_{BC} = \frac{\pi}{4} D^2_{BC}$$

$$= \frac{\pi}{4} (1.5)^2$$

$$= 1.77 m^2$$

Mass flow rate in pipe AB,

$\therefore A_{AB} \times V_{AB} = 1.13 \times 3$

$Q_{AB} = 3.39 m^3/s$

$\therefore Q_{BC} = Q_{AB} = 3.39 m^3/s$

Velocity in pipe BC,

$A_{BC} \times V_{BC} = 3.39$

$\therefore V_{BC} = \frac{3.39}{1.77} = 1.92 m/s$

Mass flow rate in BC = Mass flow rate in CD + Mass flow rate in CE

$Q_{BC} = Q_{CD} + Q_{CE}$

$Q_{BC} = \frac{1}{3} Q_{AB} + Q_{CE}$

$Q_{BC} = \frac{1}{3} Q_{BC} + Q_{CE}$

$3.39 = \frac{1}{2} \times 3.39 + Q_{CE}$

$\therefore Q_{CE} = 2.26 m^3/s$

Mass flow rate in BC = Mass flow rate in CD + Mass flow rate in CE

$$3.39 = Q_{CD} + 2.26$$

$$\therefore \quad Q_{CD} = 1.13 m^3/s$$

Pipe CD:

$$A_{CD} = \frac{\pi}{4} D^2_{CD} = \frac{\pi}{4}(0.8)^2$$

$$= 0.503 m^2$$

Velocity in pipe CD

$$Q_{CD} = A_{CD} \times V_{CD}$$

$$V_{CD} = \frac{Q_{CD}}{A_{CD}}$$

$$= \frac{1.13}{0.503} = 2.25 m/s$$

Diameter of pipe CE,

$$Q_{CE} = 2.26 m^3/s$$

$$Q_{CE} = A_{CE} \times V_{CE}$$

$$A_{CE} = \frac{Q_{CE}}{V_{CE}}$$

$$= \frac{2.26}{2.5} = 0.904 m^2$$

$$\therefore \quad A_{CE} = \frac{\pi}{4} D^2_{CE} = 0.904$$

$$D_{CE} = \sqrt{\frac{0.904 \times 4}{\pi}}$$

$$= 1.073 m$$

Problems on Bernoulli's Equation

1. A pipe 200m long slopes down at 1 in 100 and tapers from 600 mm diameter at the higher end to 300 mm diameter at the lower end and carries 100 litres/sec of oil having specific gravity 0.8. If the pressure gauge at the higher end reads 60 kN/m², determine the velocities at the two ends and also the pressure at the lower end.

Given data;

Length, L = 200mm

Slope = 1 in 100

$D_1 = 600\text{ mm} = 0.6\text{ m}$

$D_2 = 300\text{ mm} = 0.3\text{ m}$

Discharge, $Q = 100\text{ lit/s}$

$= 100 \times 10^{-3}$

$= 0.1\text{ m}^3/\text{s}$

Specific gravity, $S = 0.8$

$p_1 = 60\text{ kN/m}^2$

Solution;

Area of 0.6m diameter pipe,

$$A_1 = \frac{\pi}{4} D_1^2$$

$$= \pi/4 \times 0.6^2$$

$$= 0.283\text{ m}^2$$

Area of 0.3m diameter pipe,

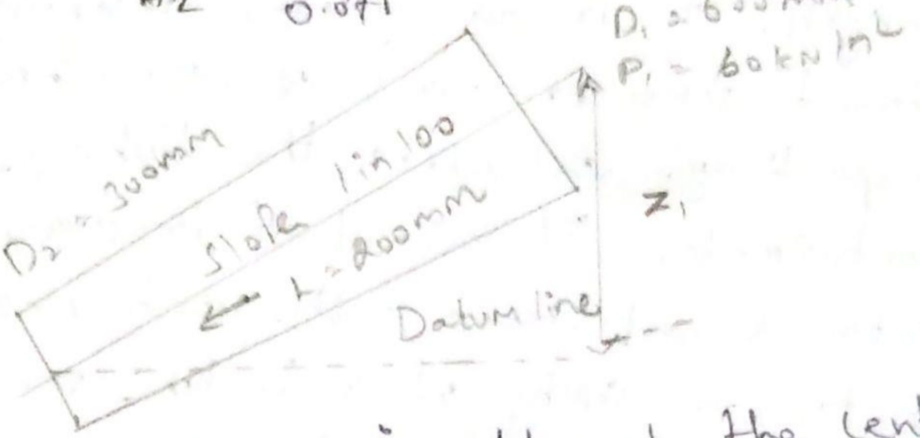
$$A_2 = \pi/4\, D_2^2$$

$$= \pi/4 \times 0.3^2 = 0.071\text{ m}^2$$

$Q = A_1 V_1$

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{0.283} = 0.353\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{0.071} = 1.414\ m/s$$



Let the datum line passing through the centre of the lower end. So,

$$Z_2 = 0$$

But $$Z_1 = \frac{1}{100} \times 200 = 2m$$

Applying Bernoulli's equation.

$$\frac{V_1^2}{2g} + \frac{P_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{w} + Z_2$$

$$\frac{0.353^2}{2 \times 9.81} + \frac{60 \times 10^3}{9810 \times 0.8} + 2 = \frac{1.414^2}{2 \times 9.81} + \frac{P_2 \times 10^3}{9810 \times 0.8} + 0$$

$$P_2 = 74.8\ kN/m^2$$

2) An oil of specific gravity 0.85 is flowing through an inclined venturimeter fitted to a 250mm diameter pipe at the rate of 110 lit/s. The venturimeter is inclined at 60° to the vertical and its 120mm diameter throat is 1m from the entrance along its length. The pressure gauges inserted at entrance and throat show pressures of 0.125 N/mm^2 and 0.08 N/mm^2 respectively. calculate the discharge coefficient of venturimeter. If instead of pressure gauges the entrance and throat of the venturimeter are connected to the two limbs of a U-tube mercury manometer, determine its reading in m of the mercury column.

Given data:

Specific gravity of oil, $s = 0.85$

Diameter of pipe, $d_1 = 250\,mm = 0.25\,m$

Rate of flow, $Q = 110$ lit/s $= 0.11\,m^3/s$

Inclination to vertical, $\theta = 60°$

Throat diameter, $d_2 = 120\,mm = 0.12\,m$

Distance of throat from entrance $= 1\,m$

Pressure at the entrance, $P_1 = 0.125\,N/mm^2$

$= 0.125 \times 10^6\,N/m^2$

Pressure at the throat, $P_2 = 0.08\,N/mm^2$

$= 0.08 \times 10^6\,N/m^2$

Solution:-

Area of entrance, $a_1 = \frac{\pi}{4} d_1^2$

$= \frac{\pi}{4} \times (0.25)^2$

$= 0.049\,m^2$

Area at throat section,

$$a_2 = \frac{\pi}{4} d_2^2$$

$$= \frac{\pi}{4} \times (0.12)^2 = 0.0113 m^2$$

Pressure head at entrance,

$$\frac{P_1}{w} = \frac{0.125 \times 10^6}{9810 \times 0.85} = 14.99 \text{ m of oil}$$

Pressure head at throat,

$$\frac{P_2}{w} = \frac{0.08 \times 10^6}{9810 \times 0.85}$$

$$= 9.59 \text{m of oil}$$

$$Z_1 = 0$$

$$Z_2 = 1 \times \cos 60^\circ = 0.5 m$$

we know that,

$$h = \left(\frac{P_1}{w} + Z_1\right) - \left(\frac{P_2}{w} + Z_2\right)$$

$$= (14.99 + 0) - (9.59 + 0.5)$$

$$= 4.9 m$$

Discharge through the venturimeter is given by,

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$0.11 = C_d \times \frac{0.049 \times 0.0113}{\sqrt{(0.049)^2 - (0.0113)^2}} \times \sqrt{2 \times 9.81 \times 4.9}$$

$$0.11 = C_d \times 0.1139$$

$$C_d = 0.966$$

If a U-tube manometer is connected, then

$$h = x\left(\frac{S_m}{S} - 1\right)$$

$$4.9 = x\left(\frac{13.6}{0.85} - 1\right)$$

$$x = 0.327\,m$$

Problems on Momentum Equation

1) The diameter of a pipe gradually reduces from 1m to 0.7m. The pressure intensity at the centre line of 1m section is 7.848 kN/m^2 and rate of flow of water through the pipe is 600 litres/s. Find the intensity of pressure at the centre line of 0.7m section. Also determine the force exerted by flowing water on the transition of the pipe.

Given data:

$D_1 = 1m$

$D_2 = 0.7m$

$P_1 = 7.848\,kN/m^2$

$Q = 600\ lit/s = 600 \times 10^{-3}\ m^3/s$

Solution:

Velocity of fluid at section 1, $V_1 = \frac{Q}{Area}$

$$= \frac{600 \times 10^{-3}}{\frac{\pi}{4} \times 1^2}$$

$$= 0.76\,m/s$$

Velocity of fluid at section 2, $V_2 = \frac{Q}{Area}$

$$= \frac{600 \times 10^{-3}}{\frac{\pi}{4} \times 0.7^2} = 1.56 m/s$$

Let F_x be the force exerted by Pipe transition on the following water in the directions of flow. So, Net force = Rate of change of momentum

$$P_1 A_1 - P_2 A_2 + F_x = \rho Q (V_2 - V_1)$$

$$\left[7.848 \times 1000 \times \frac{\pi}{4} \times 1^2 - 6.942 \times 1000 \times \frac{\pi}{4} \times 0.7^2 + F_x \right]$$

$$= \left[1000 \times \frac{600}{1000} \times (1.56 - 0.76) \right]$$

$$F_x = -3012.21 N$$

2) A Pipe having a diameter of 300mm carries water under a head of 22m with a velocity of 4m/s. If the axis of the Pipe turns through 135°, find the magnitude and direction of the resultant forces on the bend.

Given data:

Diameter of Pipe. $D_1 = D_2 = 300mm = 0.3m$

Head, $h = 22m$

Velocity, $V_1 = V_2 = 4m/s$

Angle of bend, $\theta = 135°$

Solution:

Area of Pipe, $A_1 = A_2 = \frac{\pi}{4} D_1^2$

$= \frac{\pi}{4} \times (0.3)^2$

$= 0.071 m^2$

Discharge, $Q = A_1 V_1 = 0.071 \times 4$

$= 0.284 m^3/s$

Pressure intensity is same at two sections since the area is same.

$P_1 = P_2 = wh$

$= 9810 \times 22$

$= 215820 N/m^2$

Force along x-direction (from equation 2.27)

$F_x = \rho Q (V_1 - V_2 \cos\theta) - P_2 A_2 \cos\theta + P_1 A_1$

$= (1000 \times 3.08 \times 26.44 \times \sin 45^\circ) + (83.34 \times 10^{2} \times 0.126 \times \sin 45^\circ)$

$= 60652.82\ N$

Resultant force,

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(98737.18)^2 + (60652.82)^2}$$

$$= 115878.36 N$$

Direction of resultant force with x axis is given by,

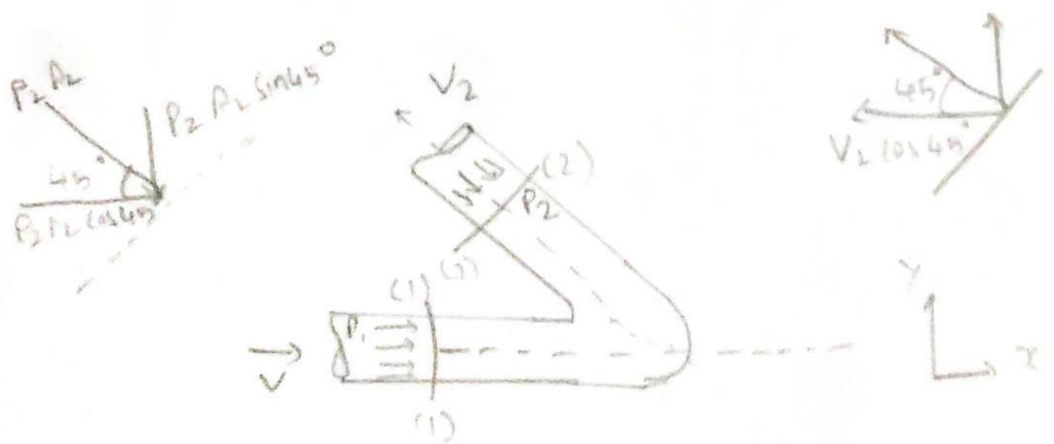
$$\tan\theta = \frac{F_y}{F_x}$$

$$= \frac{60652.82}{98737.18}$$

$$= 0.614$$

$$\therefore \theta = \tan^{-1}(0.614)$$

$$= 31.55^\circ$$



Problems on dimensional homogeneity

1) Determine the dimension of the following Quantities

(i) Discharge (iii) Force and

(ii) kinematic Viscosity (iv) Specific weight

Solution:

Discharge = Area × Velocity

$= L^2 \times \frac{L}{T} = \frac{L^3}{T}$

$= L^3 T^{-1}$

Kinematic viscosity, $\nu = \frac{\mu}{\rho}$

Where μ is given by

$\tau = \mu \frac{du}{dy}$

$\mu = \frac{\tau}{du/dy}$

$= \frac{\text{Shear Stress}}{\frac{L}{T} \times \frac{1}{L}}$

$= \frac{\text{Force} / \text{Area}}{1/T}$

$= \frac{\text{Mass} \times \text{Acceleration}}{\text{Area} \times 1/T}$

$= \frac{M \times L/T^2}{L^2 \times 1/T}$

$$= \frac{MLT}{L^2 \times T^2} = \frac{M}{LT}$$

$$= ML^{-1}T^{-1}$$

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{M}{L^3} = ML^{-3}$$

$\therefore$ kinematic Viscosity,

$$\nu = \frac{\mu}{\rho} = \frac{ML^{-1}T^{-1}}{ML^{-3}}$$

$$= L^2 T^{-1}$$

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$= M \times \frac{\text{Length}}{\text{Time}^2}$$

$$= \frac{ML}{T^2} = MLT^{-2}$$

$$\text{Specific weight} = \frac{\text{Weight}}{\text{Volume}} = \frac{\text{Force}}{\text{Volume}}$$

$$= \frac{MLT^{-2}}{L^3}$$

$$= ML^{-2}T^{-2}$$

Problems on dimensional Analysis.

1) Efficiency η of a fan depends on the density ρ, dynamic viscosity of the fluid μ, angular velocity ω, diameter D of the rotor and discharge Q. Express η in terms of dimensional Parameters.

Solution:

The efficiency η of a fan is a function of

(i) Density, ρ

(ii) Viscosity, μ

(iii) Angular velocity, ω

(iv) Diameter

(v) Discharge, Q

Mathematically,

$$\eta = F(\rho, \mu, \omega, D, Q)$$

$$\eta = C(\rho^a, \mu^b, \omega^c, D^d, Q^e)$$

Where C is a non-dimensional constant.

Using M-L-T system, the corresponding equation for dimension is given by

$$M^0 L^0 T^0 = C\left[(ML^{-3})^a (ML^{-1}T^{-1})^b (T^{-1})^c (L)^d (L^3T^{-1})^e\right]$$

For dimensional homogeneity,

For M, $0 = a+b$

For L, $0 = -3a - b + d + 3e$

For T, $0 = -b - c - e$

There are five variables but only these equations are available.

$$a = -b$$

$$c = -(b+e)$$

$$d = 3a + b - 3e$$

$$= 3(-b) + b - 3e$$

$$= -(2b + 3e)$$

Substituting these values of exponents in equation

$$h = C\left(\rho^{-b} M^{b} \omega^{-(b+e)} D^{-(2b+3e)} Q^{e}\right)$$

$$= C\left(\rho^{-b} M^{b} \omega^{-b} \omega^{-e} D^{-2b} D^{-3e} Q^{e}\right)$$

$$= C\left[\left(\frac{M}{\rho \omega D^{2}}\right)^{b} \left(\frac{Q}{\omega D^{3}}\right)^{e}\right]$$

For exponent $b = c = 1$, the above equation can be modified into

$$= Q\left[\left(\frac{M}{\rho \omega D^{2}}\right), \left(\frac{Q}{\omega D^{3}}\right)\right]$$

2) The efficiency η of a fan depends on the density ρ, dynamic viscosity μ, angular velocity ω, diameter D of the motor and discharge Q. Express the efficiency η in terms of dimensional parameters.

Solutions

The parameters involved in the given analysis are η, ρ, μ, ω, D and Q.

Dimensions of each parameter are,

Efficiency, η = No dimensions

Density, $\rho = ML^{-3}$

Dynamic viscosity, $\mu = ML^{-1}T^{-1}$

Angular velocity, $\omega = T^{-1}$

Discharge, $Q = L^{-3}T^{-1}$

The function relationship can be written as

$$\eta = f(D, \rho, \omega, \mu, Q)$$

Again it can be written as

$$f_1(\eta, D, \rho, \omega, \mu, Q) = 0$$

Here, the total number of variables, $n = 6$

Fundamental variables, $m = 3$

So, the number of π-term has $m+1$ variables.

Here, D, ρ and ω are selected as repeating variables

Functional equation is given by.

$$f(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi_1 = D^{a_1} \times \rho^{b_1} \times \omega^{c_1} \times \eta$$

$$\pi_2 = D^{a_2} \times \rho^{b_2} \times \omega^{c_2} \times \mu$$

$$\pi_3 = D^{a_3} \times \rho^{b_3} \times \omega^{c_3} \times Q$$

π_1 - term:

$$\pi_1 = D^{a_1} \times \rho^{b_1} \times \omega^{c_1} \times \eta$$

Substituting the dimensions of each variables on both sides,

$$M^0 L^0 T^0 = L^{a_1} \times (ML^{-3})^{b_1} \times (T^{-1})^{c_1} \times M^0 L^0 T^0$$

Comparing the coefficients of exponents on both sides,

For M, $0 = b_1 + 0$ → 1.a

For L, $0 = a_1 - 3b_1 + 0$ → 1.b

For T, $0 = -c_1 + 0$ → 1.c

From 1.a $b_1 = 0$

From 1.c $c_1 = 0$

Substituting 1a & 1c in equation 1b

$$a_1 = 0$$

$$\pi_1 = D^0 \times \rho^0 \times \omega^0 \times \eta = \eta$$

π_2 - term:

$$\pi_2 = D^{a_2} \times \rho^{b_2} \times \omega^{c_2} \times \mu$$

Dimensionless equation is given by

$$M^0 L^0 T^0 = L^{a_2} \times (ML^{-3})^{b_2} \times (T^{-1})^{c_2} \times ML^{-1}T^{-1}$$

By comparison the coefficients of exponents on both sides

For M, $0 = b_2 + 1 \rightarrow 2.a$

For L, $0 = a_2 - 3b_2 - 1 \rightarrow 2.b$

For T, $0 = c_2 - 1 \rightarrow 2.c$

From 2.a, $b_2 = -1$

From 2.c, $c_2 = -1$

Substituting b_2 and c_2 in 2.a

$$0 = a_2 - 3(-1) - 1$$

$$a_2 = -3 + 1 = -2$$

Now, $\pi_2 = D^{-2} \times \rho^{-1} \times \omega^{-1} \times \mu = \dfrac{\mu}{\rho D^2 \omega}$

π_3 - term:

$$\pi_3 = D^{a_3} \times \rho^{b_3} \times \omega^{c_3} \times Q$$

Dimensionless equation is given by

$$M^0 L^0 T^0 = L^{a_3} \times (ML^{-3})^{b_3} \times (T^{-1})^{c_3} \times L^3 T^{-1}$$

By exponents coefficients comparison,

For M, $0 = b_3 \rightarrow 3.a$

For L, $0 = a_3 - 3b_3 + 3 \rightarrow 3.b$

For T, $0 = -c_3 - 1$

From 3a, $b_3 = 0$

From 3c, $c_3 = -1$

Substituting b_3 and c_3 in 3b,

$$0 = a_3 - 3 \times 0 + 3$$

$$\therefore a_3 = -3$$

Now,

$$\pi_3 = D^{-3} \times \rho^0 \times \omega^{-1} \times Q$$

$$= \frac{Q}{D^3 \omega}$$

Substituting π_1, π_2 and π_3 in $f(\pi_1, \pi_2, \pi_3) = 0$

$$f_1\left(\eta, \frac{\mu}{\rho D^2 \omega}, \frac{Q}{D^3 \omega}\right) = 0$$

$\therefore$ Efficiency,

$$\eta = \phi\left(\frac{\mu}{\rho D^2 \omega}, \frac{Q}{D^3 \omega}\right)$$

Problems on model studies

1) An oil of specific gravity 0.91 and viscosity of 0.03 poise is to be transported at the rate of $3 m^3/s$ through a 1.3m diameter pipe. Model tests were conducted on a 130 mm diameter pipe using water having viscosity of 0.01 poise. Find the velocity of flow and discharge in the model.

Given:

$D_m = 130 mm$

$Q_m = 3 m^3/s$

$D_p = 1.3 m$

Model fluid - linseed oil

Prototype fluid - Water

$\mu_p = 0.03$ strokes $= 0.003 Ns/m^2$

$\mu_m = 0.01$ poise $= 0.001 Ns/m^2$

specific gravity for model, $S_m = 1$

Specific gravity for Prototype, $S_p = 0.91$

Solution:

Discharge of model, $Q_m = A_m \times V_m$

$$3 = \frac{\pi}{4} \times (0.13)^2 \times V_m$$

$$V_m = 226.02 m/s$$

By dynamic similarity (Re) model = (Re) prototype

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p} \quad \left(\because Re = \frac{\rho v D}{\mu}\right)$$

$$\therefore V_P = V_m \times \frac{D_M}{D_P} \times \frac{P_m}{P_P} \times \frac{\mu_P}{\mu_m}$$

$$= 226.02 \times \frac{0.13}{1.3} \times \frac{1000}{910} \times \frac{0.003}{0.001}$$

$$= 74.51\ m/s$$

Discharge of Prototype,

$$Q_P = A_P \times V_P$$

$$= \frac{\pi}{4} \times (1.3)^2 \times 74.51$$

$$Q_P = 98.899\ m^3/s$$

2) A river carrying a discharge of $3500\ m^3/s$ has a depth of 2.25m and width of 1500m. From the Point of view of availability of space, the horizontal scale of 1:400 is chosen. Assuming slope scale to be unity, determine the depth and discharge scales for the model.

Given data:

Discharge, $Q_P = 3500\ m^3/s$

Depth, $H_P = 2.25\ m$

Width, $B_P = 1500\ m$

Horizontal scale, $L_r = 400$

Solution:

For model, $H_m = L_m$

For Prototype, $H_p = L_p$

$\therefore \dfrac{H_m}{H_p} = \dfrac{L_m}{L_p} = \dfrac{1}{L_r}$ $\left(\because \dfrac{L_p}{L_m} = L_r\right)$

$= \dfrac{1}{400}$

$H_m = \dfrac{1}{400} \times 2.25$

$\therefore H_m = 0.005625\,m$

According to Froude's Model law,

$(F_r)_{model} = (F_r)_{prototype}$

$\dfrac{V_m}{\sqrt{g_m L_m}} = \dfrac{V_p}{\sqrt{g_p L_p}}$ $\left(\because F_r = \dfrac{V}{\sqrt{gL}}\right)$

$\dfrac{V_m}{\sqrt{L_m}} = \dfrac{V_p}{\sqrt{L_p}}$ $(\because g_m = g_p)$

$\dfrac{V_p}{V_m} = \sqrt{\dfrac{L_p}{L_m}} = \sqrt{L_r}$

$= \sqrt{400} = 20$

Rate of flow or discharge of model.

$$Q_m = A_m v_m$$

For Prototype, $Q_p = A_p v_p$

$$\frac{Q_m}{Q_p} = \frac{A_m}{A_p} \times \frac{v_m}{v_p} = \left(\frac{L_m}{L_p}\right)^2 \times \frac{v_m}{v_p}$$

$$Q_m = Q_p \times \left(\frac{1}{L_r}\right)^2 \times \frac{v_m}{v_p}$$

$$= 3500 \times \left(\frac{1}{400}\right)^2 \times \frac{1}{20}$$

$$Q_m = 0.0011 \, m^3/s$$

So Problems on Laminar Flow

1) Oil of mass density 800 kg/m^3 and dynamic viscosity 0.02 Poise flows through 50mm diameter pipe of length 500m at the rate of 0.19 l/s. Determine the (i) Reynolds number of flow, (ii) centreline velocity, (iii) pressure gradient, (iv) loss of pressure in 500m length (v) wall shear stress and (vi) Power required to maintain the flow.

Given data:

Density $\rho = 800\ kg/m^3$

Viscosity of oil, $\mu = 0.02$ poise $= 0.002\ Ns/m^2$

Diameter, $D = 50mm = 0.05m$

Length, $l = 500m$

Discharge, $Q = 0.19\ l/s = 0.00019\ m^3/s$

Solution:

Average velocity,

$$U = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}$$

$$= \frac{0.00019}{\frac{\pi}{4} \times 0.05^2}$$

$$= 0.097\ m/s$$

Reynolds number,

$$R_e = \frac{\rho U D}{\mu}$$

$$= \frac{800 \times 0.097 \times 0.05}{0.002}$$

$$= \underline{1940}$$

Centerline velocity, $U_{max} = 2U$

$$= 2 \times 0.097$$

$$= \underline{0.194 \, m/s}$$

From Hagen - Poiseuille's equation,

Pressure gradient,

$$\frac{P_1 - P_2}{L} = \frac{128 \mu Q}{\pi D^4}$$

$$= \frac{128 \times 0.02 \times 0.00019}{\pi (0.05)^4}$$

$$= 24.77 \, N/m^2$$

Loss of Pressure,

$$\Delta P = \text{Pressure gradient} \times \text{length}$$

$$= 24.77 \times 500$$

$$= 12385 \, N/m^2$$

Shear stress at the Pipe wall,

$$\tau_{max} = \left(- \frac{dP}{dx} \right) \frac{R}{2}$$

$$= \frac{P_1 - P_2}{L} \times \frac{R}{2}$$

$$= 24.77 \times \frac{0.05}{2 \times 2}$$

$$= 0.31 \, N/m^2$$

Power, $P = Q(P_1 - P_2)$

$= 0.00019 \times 12385$

$= 2.35 W$

Answer

Reynolds number, $Re = 1960$

Center line velocity, $U_{max} = 0.194 m/s$

Pressure gradient $= 24.77 N/m^2$

Loss of pressure, $\Delta p = 12385 N/m^2$

Shear stress, $\tau_{max} = 0.31 N/m^2$

Power, $P = 2.35 W$

2) An oil of viscosity $1.5 Ns/m^2$ flows between two Parallel fixed Plates which are kept at a distance of 60mm apart. The maximum velocity of oil is 2m/s. Calculate the, 1. Discharge Per m length 2. shear stress at the Plates, 3. Pressure difference between two Points of 10m apart along the direction of flow, 4. velocity gradient at the Plates. 5. velocity at 18mm from the Plate.

Given data:

Viscosity of oil, $\mu = 1.5 Ns/m^2$

Distance between Plates, $b = 60mm = 0.06m$

Maximum velocity, $U_{max} = 2m/s$

Solution:

Oil Discharge:

Average velocity, $U_{ave} = \frac{2}{3} U_{max}$

$$= \frac{2}{3} \times 2 = 1.33 \, m/s$$

Discharge, $Q = U_{ave} \times area$

$$= 1.33 \times 0.06 \times 1$$

$$= 0.0798 \, m^3/s$$

(ii) shear stress at the Plates:

we know that maximum velocity,

$$u_{max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right) b^2$$

$$2 = -\frac{1}{8 \times 1.5}\left(\frac{\partial p}{\partial x}\right)(0.06)^2$$

$$\frac{\partial p}{\partial x} = -6666.7 \, N/m^2$$

The shear stress is maximum at the Plates

$$\tau_{max} = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right) b$$

$$= -\frac{1}{2} \times (-6666.7) \times 0.06$$

$$= 200 \, N/m^2$$

(iii) pressure difference between two Points of 10m apart, we know that,

$$\frac{\partial p}{\partial x} = -6666.7$$

$$\partial p = -6666.7 \, \partial x$$

Integrating with respect to x. so it becomes,

$$\int_{p_1}^{p_2} \partial p = \int_{x_1}^{x_2} -6666.7 \, \partial x$$

$$P_1 - P_2 = 6666.7\,(x_2 - x_1)$$

$$= 6666.7 \times 10$$

$$= 66667\ N/m^2 \text{ or } = 66.67\ kN/m^2$$

(iv) Velocity gradient at the Plates:

we know that

$$\tau_{max} = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\tau_{max}}{\mu}$$

$$= \frac{200}{1.5}$$

$$= 133.33\ s^{-1}$$

v) velocity at 18mm from the Plate:

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\,(by - y^2)$$

$$= -\frac{1}{2 \times 1.5} \times (-6666.7) \times \left[(0.06 \times 0.018) - (0.018)^2\right]$$

$$= 1.68\ m/s$$

Answer

1) Discharge, $Q = 0.0798\ m^3/s$

2) Shear stress, $\tau_{max} = 200\ N/m^2$

3) Pressure different $= 66667\ N/m^2$

4) velocity gradient $= 133.33\ s^{-1}$

5) velocity $u = 1.68\ m/s$

Problem on Darcy-Weisbach's equation

1) Derive the equation for the friction loss in a pipe line and determine the friction loss in a pipe of 400m long and 200mm diameter when the discharge is $3m^3/min$ and the resistance coefficient $f = 0.006$.

Solution:

Darcy-weisbach's Equation:

The various viscous friction effects associated with fluid are proportional to:

→ the length of pipe, L

→ the wetted perimeter, P and

→ V^n,

where V is the average velocity of flow and n is the index varying from 1.5 to 2.

Bernoulli's equation between the section 1 & 2,

$$\frac{V_1^2}{2g} + \frac{P_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{w} + Z_2 + h_f$$

Since, $V_1 = V_2 = V$ and $Z_1 = Z_2$

$$\frac{P_1}{w} = \frac{P_2}{w} + h_f$$

Loss of head, $h_f = \frac{P_1}{w} - \frac{P_2}{w} = \frac{P_1 - P_2}{w}$

Frictional resistance, $= f' \times \text{Area} \times V^n = f' \times PL \times V^n$

P force 1 = $P_1 A$

P force 2 = $P_2 A$

$P_1 A = P_2 A +$ Frictional resistance

$(P_1 - P_2)A = f' \times PL \times V^n$

Dividing by w, $\frac{P_1 - P_2}{w} = \frac{f'}{w} \times \frac{PL}{A} \times V_n$

Substituting h_f values?.

$$h_f = \frac{f'}{w} \times \frac{LV^n}{m} \longrightarrow (1)$$

Hydraulic mean depth,

$$m = \left(\frac{A}{P}\right) = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4}$$

Substituting m value in (1)

$$h_f = \frac{f'}{w} \times \frac{LV^n}{D/4}$$

$$h_f = f \times \frac{4LV^2}{2gD} \Rightarrow h_f = \frac{4fLV^2}{2gD}$$

f = Darcy's coefficient of friction

The above equation is called Darcy-weisbach equation,

$$h_f = \frac{f_1 LV^2}{2gD}.$$

f_1 = Friction factor.

Given data:

Diameter of Pipe, $D = 200mm$

$= 0.2m$

Length of Pipe, $L = 400mm$

$= 0.4m$

Discharge, $Q = 3m^3/min$

$= 3/60$

$= 0.05 m^3/s$

Resistance coefficient, $f = 0.004$

Solution:

Friction loss,

$$h_f = \frac{flQ^2}{3D^5}$$

$$= \frac{0.004 \times 0.4 \times 0.05^2}{3 \times 0.2^5}$$

$$= 0.00417m$$

$$h_f = 4.17mm.$$

Problem on losses in Pipe flow

1) Water flows at the rate of 200 l/s upwards through a tapered vertical pipe. The diameters at the bottom is 240 mm and at the top 200 mm. The Length is 5 m. The pressure at the bottom is 8 bar and the Pressure at the top side is 7.3 bar. Determine the head loss through the Pipe. Express it as a function of exit velocity head.

Given data:

$Q = 200\ l/s = 200 \times 10^{-3}\ m^3/s$

$D_1 = 240mm = 0.24m$

$D_2 = 200mm = 0.2m$

$l = 5m$

$D = 0.4m$

$P_1 = 8\ bar = 8 \times 10^5\ N/m^2$

$P_2 = 7.3\ bar = 7.3 \times 10^5\ N/m^2$

Solution:

Assuming the datum pacing through the lower end of the pipe line ($z_1 = 0$), the datum head for the upper end of the pipe line is given by.

$$z_2 = L = 5m$$

Discharge, $Q = A_1 V_1$

$$= \frac{\pi}{4} D_1^2 \times V_1$$

$$200 \times 10^{-3} = \frac{\pi}{4} \times (0.24)^2 \times V_1$$

$$V_1 = 4.42\ m/s$$

Similarly,

Discharge, $Q = A_2 V_2$

$$= \frac{\pi}{4} D_2^2 \times V_2$$

$$200 \times 10^{-3} = \frac{\pi}{4} \times (0.2)^2 \times V_2$$

$$V_2 = 6.37 \text{ m/s}$$

Applying Bernoulli's equation,

$$\frac{V_1^2}{2g} + \frac{P_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{w} + Z_2 + h_f$$

$$\frac{4.42^2}{2 \times 9.81} + \frac{8 \times 10^5}{9810} + 0 = \frac{6.37^2}{2 \times 9.81} + \frac{7.3 \times 10^5}{9810} + 10 + h_f$$

$$h_f = 1.07 \text{ m}$$

We know that,

coefficient of contraction,

$$h_c = \frac{k V_2^2}{2g}$$

In this pipe, the pipe contracts uniformly.

So, $h_f = h_c$

$$1.07 = \frac{k \times 6.37^2}{2 \times 9.81}$$

$$k = 0.516$$

$\therefore$ Head loss, $h_c = 0.516 \frac{V_2^2}{2g}$

1. Three pipes of same length L, diameter D and friction factor f are connected in parallel. Determine the diameter of the pipe of length L and friction factor f that will carry the same discharge for the same head loss. Use $h_f = \frac{fLV^2}{2gD}$.

Given data:

Length of each pipe = L

Diameter of each pipe = D

Friction factor of each pipe = f

Head loss, $h_f = \frac{fLV^2}{2gD}$

Solution:

When the pipes are connected in parallel

$h_f = h_{f_1} + h_{f_2} + h_{f_3}$

$Q = Q_1 + Q_2 + Q_3$

$Q_1 = Q_2 = Q_3$

$\therefore \; Q = 3Q_1 = 3A_1V_1$

$Q = 3 \times \frac{\pi}{4} D^2 \times V \quad \rightarrow (1)$

When a single pipe replaces these three pipes which carry the same discharge.

Let d = Diameter of the single pipe

V = velocity through single pipe

$Q = A \times V = \frac{\pi}{4} d^2 \times V \quad \rightarrow (2)$

Equating (1) and (2)

$$3 \times \frac{\pi}{4} D^2 \times V = \frac{\pi}{4} d^2 \times v$$

$$3 \times \frac{D^2}{d^2} = \frac{v}{V} \longrightarrow (3)$$

Head loss for the single pipe of length L and diameter d,

$$h_f = \frac{fLv^2}{2gd} \longrightarrow (4)$$

Head loss for each pipe,

$$h_f = \frac{fLV^2}{2gD} \longrightarrow (5)$$

Equating (4) and (5)

$$\frac{fLv^2}{2gd} = \frac{fLV^2}{2gD}$$

$$\frac{v^2}{d} = \frac{V^2}{D} \Rightarrow \frac{d}{D} = \left(\frac{v}{V}\right)^2$$

$$\frac{v}{V} = \left(\frac{d}{D}\right)^{1/2}$$

Substituting $\frac{v}{V}$ value in equation (3)

$$\frac{3 \times D^2}{d^2} = \left(\frac{d}{D}\right)^{1/2}$$

$$3 = \left(\frac{d}{D}\right)^{1/2} \times \left(\frac{d}{D}\right)^2 = \left(\frac{d}{D}\right)^{5/2}$$

$$\frac{d}{D} = (3)^{2/5}$$

$$d = 1.55D$$

1) A horizontal pipe line 50m is connected to a water tank at one end and discharges freely to atmosphere through the other end. For the first 30m length from the tank, the diameter of pipe is 15cm and for rest, it is 30cm in diameter. The water level in the tank is 8m above the centre of the pipe. Take f = 0.01. By considering all losses, determine the discharge through the pipe. Also draw the hydraulic gradient line & total energy line.

Given data:

Total length of pipe, $L = 50m$

$L_1 = 30m$

$L_2 = 20m$

$D_1 = 15cm = 0.15m$

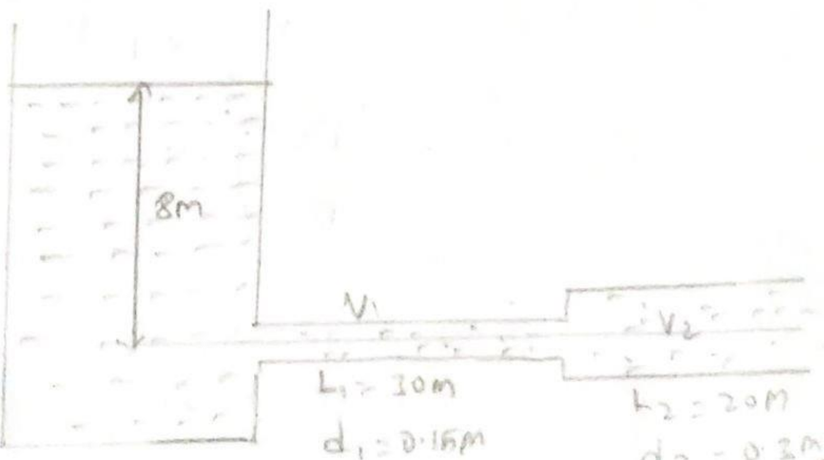
$D_2 = 30cm = 0.3m$

$Z_1 = 8m$

$f = 0.01$

Solution:

Losses in the pipe line:



Head loss at the entrance of the pipe,

$$h_i = \frac{0.5 V_1^2}{2g}$$

Head loss due to friction in the pipe (1),

$$h_{f_1} = \frac{4 f L_1 V_1^2}{2 g D_1}$$

Head loss due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Head loss due to friction in pipe (2),

$$h_{f_2} = \frac{4 f L_2 V_2^2}{2 g D_2}$$

Head loss at the exit from a pipe,

$$h_o = \frac{V_2^2}{2g}$$

From continuity equation,

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_2}{A_1}$$

$$= \frac{\frac{\pi}{4} \times D_2^2 \times V_2}{\frac{\pi}{4} \times D_1^2}$$

$$= \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.12}{0.15}\right)^2 \times V_2$$

$$\therefore V_1 = 4 V_2$$

$$V_2 = 0.25 V_1$$

Substituting the value of V_1 in different head losses

$$h_i = \frac{0.5V_1^2}{2g} = 0.41V_2^2$$

$$h_{f_1} = \frac{4fL_1V_1^2}{2gD_1} = 6.52V_2^2$$

$$h_e = \frac{(V_1-V_2)^2}{2g} = 0.66V_2^2.$$

$$h_{f_2} = \frac{4fL_2V_2^2}{2gD_2} = 2.18V_2^2$$

$$h_o = \frac{V_2^2}{2g} = 0.051V_2^2$$

Applying Bernoulli's theorem.

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 + \text{All losses},$$

$$\longrightarrow \quad V_2 = \sqrt{\frac{8}{9.67}} = 0.91\,m/s$$

Rate of flow,

$$Q = A_2V_2 = \frac{\pi}{4}D_2^2 \times V_2$$

$$= \frac{\pi}{4} \times (0.3)^2 \times 0.91$$

$$Q = 0.0643\,m^3/s.$$

Total energy line and Hydraulic gradient line (H.G.L)

Total energy line $\left(\frac{P}{w} + \frac{V^2}{2g} + Z\right)$

H.G.L $\left(\frac{P}{w} + Z\right)$

$h - h_1 = 7.98m$

$\therefore$ Piezometric head $\left(\frac{P}{w} + z\right)$ at the entrance

$$= 7.98 - \frac{V_1^2}{2g}$$

$$= \underline{7.31m}$$

Total energy before enlargement of the Pipe

$$= T.E - h_{f_1}$$

$$= 7.98 - 6.52\,V_2^2 = \underline{2.58m}$$

Total head,

$$= 7.98 - (6.52V_2^2 + 0.46V_2^2)$$

$$= 2.25m.$$

Piezometric head $\left(\frac{P}{w} + z\right)$ at the enlargement.

$$= 2.25 - \frac{V_1^2}{2g}$$

$$= 0.67m$$

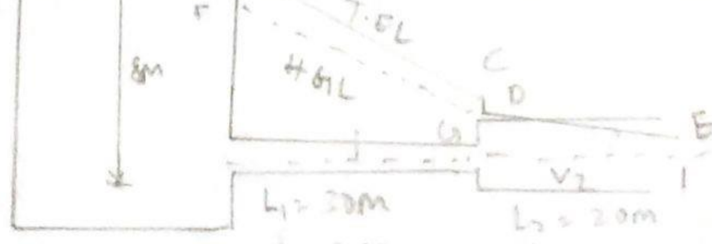
Total energy available at the exit of the Pipe,

$$= T.E \text{ available} - h_{f_2}$$

$$= 2.25 - (2.18V_2^2)$$

$$= 0.45m.$$

Total energy line (T.E.L):



UNIT- 5

Problems on boundary layer concept

1. The velocity distribution in the boundary layer is given by $\frac{u}{U} = \frac{y}{\delta}$ where u = Velocity at a distance y from the flat Plate and u = U at y = δ where δ = Boundary layer thickness

Determine the value of : (i) Displacement thickness (ii) Momentum thickness, (iii) Energy thickness and (iv) $\frac{\delta^*}{\theta}$.

Solution:

(i) Displacement thickness, δ^*

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy \qquad \left(\because \text{Given } \frac{u}{U} = \frac{y}{\delta}\right)$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta$$

$$= \left(\delta - \frac{\delta^2}{2\delta}\right) = \delta - \frac{\delta}{2}$$

$$\delta^* = \frac{\delta}{2}$$

(ii) Momentum thickness, θ:

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^2}{\delta^2} \right) dy$$

$$= \left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2} \right)_0^{\delta}$$

$$= \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3}$$

$$\theta = \frac{\delta}{6}$$

(iii) Energy thickness, δe:

$$\delta e = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u^2}{U^2} \right) dy$$

$$= \int_0^{\delta} \frac{y}{\delta} \left(1 - \frac{y^2}{\delta^2} \right) dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^3}{\delta^3} \right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3} \right]_0^{\delta}$$

$$= \frac{\delta^2}{2\delta} - \frac{\delta^4}{4\delta^3}$$

$$= \frac{\delta}{2} - \frac{\delta}{4}$$

$$\delta e = \frac{\delta}{4}$$

(iv) $$\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6}$$

$$= 3 \,//$$

2) Determine the displacement thickness, momentum thickness and energy thickness in terms of boundary layer thickness δ for the velocity profile in the boundary layer on a flat plate is given by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$ where u is the velocity at a height y above the surface and U is the mainstream velocity.

Solution:

Velocity profile: $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(i) Displacement thickness (δ^*):

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left[1 - \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right)\right] dy$$

$$= \int_0^\delta \left(1 - 2\left(\frac{y}{\delta}\right) + \frac{y^2}{\delta^2}\right) dy$$

$$= \left(y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right)_0^\delta$$

$$= \left[\delta - \frac{2\delta^2}{2\delta} + \frac{\delta^3}{3\delta^2}\right]$$

$$= \delta - \delta + \frac{\delta}{2} = \frac{\delta}{2}$$

(ii) Momentum thickness (θ):

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(2\frac{y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right) dy$$

$$= \int_0^\delta \left(2\frac{y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy$$

$$= \left(\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}\right)$$

$$= \delta - \frac{5}{3}\delta + \delta - \frac{1}{5}\delta$$

$$= \frac{15\delta - 25\delta + 15\delta - 3\delta}{15}$$

$$\theta = \frac{2}{15}\delta$$

(iii) Energy thickness (δe):

$$\delta e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)^2\right] dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{4y^2}{\delta^2} - \frac{y^4}{\delta^4} + \frac{4y^3}{\delta^3}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^3} + \frac{12y^4}{\delta^4} - \frac{6y^5}{\delta^5} + \frac{y^6}{\delta^6}\right) dy$$

$$= \left(\delta - \frac{1}{3}\delta - 2\delta + \frac{12}{5}\delta - \delta + \frac{1}{7}\delta\right)$$

$$= \left(\frac{12}{5}\delta - \frac{1}{3}\delta - 2\delta + \frac{1}{7}\delta\right)$$

$$= \left(\frac{252\delta - 35\delta - 210\delta + 15\delta}{105}\right.$$

$$\delta e = \frac{22}{105}\delta$$

3) A Plate of length 750mm and width 250mm has been Placed longitudinally in a stream of crude oil which flows with a velocity of 5m/s. If the oil has a specific gravity of 0.8 and kinematic viscosity of 1 stroke, calculate the

(i) Boundary layer thickness at the middle of the Plate
(ii) Shear stress at the middle of the Plate and
(iii) friction drag on one side of the Plate.

Given data:

Length of the Plate, $L = 750mm = 0.75m$

Width of the Plate, $b = 250mm = 0.25m$

Velocity, $U = 5 m/s$

Specific gravity, $S = 0.8$

Kinematic viscosity $\nu = 1$ Stoke $= 1 \times 10^{-4} m^2/s$

Solution:

(i) Boundary layer thickness at the middle of the Plate:

At the middle of the Plate

$$x = 0.75/2$$

$$= 0.375m$$

$$Re = \frac{Ux}{\nu}$$

$$= \frac{5 \times 0.375}{0.0001}$$

$$= 18750 < 5 \times 10^5$$

Laminar boundary layer thickness,

$$\delta_{lam} = \frac{5x}{\sqrt{Re}}$$

$$= \frac{5 \times 0.375}{\sqrt{18750}}$$

$$= 0.0137 m$$

$$\Rightarrow \delta_{lam} = 13.7 mm$$

(ii) shear stress at the middle of the Plate end.

By blasius theory, the local coefficient of drag is given by

$$C^*_D = \frac{0.664}{\sqrt{Re_x}}$$

$$= \frac{0.664}{\sqrt{18750}}$$

$$C^*_D = 4.85 \times 10^{-3}$$

we also know that,

Specific gravity of oil,

$$S = \frac{\rho_{oil}}{\rho_{water}}$$

$$0.8 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 800 kg/m^2$$

4) A flat plate 1.5m × 1.5m moves at 50 km/hour in the stationary air of density 1.15 kg/m³. If the coefficient of drag and lift are 0.15 and 0.75 respectively, determine the lift force, drag force, resultant force and power required to keep the plate in motion.

Given data:

Area of the plate, $A = 1.5 \times 1.5$

$= 2.25 m^2$

Velocity, $U = 50$ km/hour

$= \frac{50 \times 1000}{3600}$

$= 13.89$ m/s

$\rho_{air} = 1.15$ kg/m²

Coefficient of drag, $C_D = 0.15$

Coefficient of lift, $C_L = 0.75$

Solution:

(i) Lift force:

$$F_L = C_L A \frac{\rho U^2}{2}$$

$$= \frac{0.75 \times 2.25 \times 1.15 \times 13.89^2}{2}$$

$$F_L = 187.2 N$$

So shear stress,

$$\tau_0 = C^*_D \frac{1}{2} \rho U^2$$

$$= 4.85 \times 10^{-3} \times \frac{1}{2} \times 800 \times 5^2$$

$$\tau_0 = 48.5 \, N/m^2$$

(iii) Friction drag on one side of the Plate:

At the trailing edge of the Plate,

$$L = 0.75 M$$

$$Re_L = \frac{UL}{\nu} = \frac{5 \times 0.75}{0.0001}$$

$$= 37500 < 5 \times 10^5$$

Average drag coefficient,

$$C^*_D = \frac{1.328}{\sqrt{Re_L}}$$

$$= \frac{1.328}{\sqrt{37500}} = 6.858 \times 10^{-3}$$

∴ Friction drag force = stress × Area.

$$F_D = \int_0^L \tau_0 \times b \times dx$$

$$= C^*_D \times \frac{1}{2} \rho U^2 \times b \times L$$

$$= 6.858 \times 10^{-3} \times \frac{1}{2} \times 800 \times 5^2 \times 0.25 \times 0.75$$

$$F_D = 12.86 \, N$$

(ii) Drag force:

$$F_D = C_D A \frac{\rho U^2}{2}$$

$$= 0.15 \times 2.215 \times \frac{1.15 \times 13.89^2}{2}$$

$$F_D = 37.44 N$$

(iii) Resultant force:

$$F_R = \sqrt{(F_L)^2 + (F_D)^2}$$

$$= \sqrt{1872^2 + 37.44^2}$$

$$F_R = 190.91 N$$

(iv) Power developed:

$$P = F_D U$$

$$= 37.44 \times 13.89$$

$$P = 520.04 W$$

5) A free stream of water has a velocity of 4 m/s and a smooth flat plate with a sharp leading edge is placed in it. Find the distance from the leading edge where the boundary layer transition from laminar to turbulent flow occurs. Find also the thickness of the boundary layer at that point. Take ρ for water = 1000 kg/m^3 and μ = 1 centipoise

Given data:

Velocity, $U = 4\,m/s$

Density, $\rho = 1000\,kg\,m^{-3}$

Dynamic viscosity, $\mu = 1$ centipoise

$$= \frac{1}{100\times10}$$

$$= 1\times10^{-3}\,Ns/m^2$$

Solution:

The transition flow takes places where the laminar flow will end. So, the maximum value of R_e for laminar flow is 5×10^5

We know that Reynolds number,

$$R_e = \frac{\rho U x}{\mu}$$

$$5\times10^5 = \frac{1000\times4\times x}{10^{-3}}$$

$$x = \frac{5\times10^5\times10^{-3}}{1000\times4}$$

The distance from leading edge,

$$x = 0.125\,m.$$

The boundary layer thickness for laminar flow

$$\delta = \frac{5x}{\sqrt{R_e}} = \frac{5\times0.125}{\sqrt{5\times10^5}}$$

$$= 8.84\times10^{-4}$$